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Abstract

This study analyzes fiscal policies in a business cycle model with an endogenous borrowing constraint when firms are heavily in debt. The tightness of the borrowing constraint for working capital loans depends on the level of corporate debt. When the level of corporate debt is modest, an increase in corporate debt amplifies corporate tax cut multipliers. Because the difference in debt levels due to the temporary tax cut remains for a long time, the cumulative effect on welfare becomes large. If the debt level exceeds a certain threshold, it remains at this level and depresses an economy permanently. In this situation, a permanent spending expansion changes the firm's capital structure and can eliminate this inefficiency in the long run.

JEL Classification Numbers: E30, E62, G32, H32

Keywords: Fiscal policy, Corporate tax, Business cycle, State-dependent fiscal policy, Corporate debt

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1 Introduction

There is growing concern about the problem of increasing corporate debt. In the United States, the ratio of outstanding corporate debt to GDP has continued to increase beyond its peak in 2008. It is well known that corporate leverage amplifies the effects of adverse shocks to an economy and prolongs recessions.[1] During a recession, the debt problem becomes a serious policy issue, and fiscal policies such as a corporate tax cut and an expansionary spending policy are implemented to solve the problem.

This study examines the effectiveness of fiscal policy, especially in situations where private firms are heavily indebted, which in turn harms the economy. Besides, we also focus on whether the effectiveness of fiscal policy depends on the level of corporate debt and whether the debt problem is temporary or permanent. We consider how expansionary government spending and a corporate tax cut can effectively accelerate debt repayment and their effect on macroeconomic variables such as GDP, consumption, and social welfare, depending on the severity of the corporate debt problem.

To examine the relationship between the corporate debt problem and the effects of fiscal policy, we specifically focus on the effects of fiscal policy on firms' capital structure. The relationship between corporate taxes and firm value is well known as the trade-off theory in corporate finance. Interest on debt is treated as a cost and tax-deductible, whereas dividends on equity are not deductible. Thus, corporate taxes encourage debt financing by firms because it is advantageous for them. A higher tax rate on corporate income leads to higher leverage by firms. When implementing corporate tax cuts as a fiscal policy, the corporate tax cut stimulates production by reducing the tax burden on firms, but it also affects the capital structure.

Our model is a modified version of Kobayashi and Shirai (2018), which shows that once the level of debt exceeds a certain threshold, borrowing constraints remain permanently severe, and inefficiencies also remain forever. Kobayashi and Shirai (2018) show that a debt redistribution shock can worsen the total factor productivity and economic growth in the long run by worsening R&D and production if a certain number of firms incur huge debts. The purpose of this study is to examine the effects of fiscal policies under a corporate debt problem. We develop a business cycle model and explicitly introduce government spending and corporate tax. Government spending increases the utility of a representative household, and a lump-sum tax finances the increase in government spending. Thus, the increase in government spending has both a direct welfare improving effect and an inefficiency effect through the lump-sum tax increase.

Our model deals with two occasionally binding constraints faced by firms: the borrowing constraint and the limited liability constraint (i.e., non-negative dividend constraint).

[1] See, for example, Kiyotaki and Moore (1997) and Kiyotaki (1998).

These lead to the problem of multiple kinks in policy functions and make the model difficult to solve. In recent years, research has been ongoing in solving such non-linear models, including dynamic stochastic general equilibrium (DSGE) models with the zero lower bound (ZLB) of nominal interest rate. Following Hirose and Sunakawa (2019b), the solution method for our model is a fixed-point iteration using a more efficient version of Smolyak (1963)'s method as proposed by Judd, Maliar, Maliar and Valero (2014) and applies an index function approach to deal with occasionally binding constraints. We extend the simulation code in Hirose and Sunakawa (2019b) to deal with two occasionally binding constraints. This method can solve a model nonlinearly even when policy functions have kinks. The nonlinear solution method allows us to analyze the effects of state-dependent fiscal policy, whereas a log-linear approximation cannot.

Based on this method, we find that a corporate tax cut is effective when the debt problem is temporary. In this case, the level of debt is modest and can eventually be an optimal debt level over time without policy intervention. The corporate tax cut helps firms repay their debt by reducing their tax burden. The larger the debt, the greater the welfare gains from the corporate tax cut, as the corporate tax cut helps firms pay off the debt. Thus, corporate tax cut multipliers differ in size depending on the amount of corporate debt. In contrast, government spending is not effective in this situation and does not depend on corporate debt. Government spending crowds out private investment and debt finance and worsens welfare.

Meanwhile, a permanent expansionary fiscal spending policy may solve a permanent debt problem by changing the optimal capital structure of firms. If the debt exceeds a certain level and remains at this level permanently, the inefficiency can continue permanently. Firms facing the permanent debt problem are financed 100 percent with debt. In this situation, the permanent expansionary fiscal spending policy changes the optimal capital structure of these firms, and they repay their debt up to the constrained-efficient level. Thus, when the policy is effective, it is necessary that the ratio of debt-ridden firms is not high. If this condition is not satisfied, the fiscal policy cannot solve the debt problem because it cannot change the optimal capital structure.

Under the permanent debt problem, a permanent corporate tax cut only plays a complementary role by providing more room for increased government spending that helps debt repayment. Firms facing the permanent debt problem earn zero profit and do not pay any corporate tax. Hence, the corporate tax cut cannot change the optimal capital structure for these firms and solve the debt problem. However, it helps the spending policy through the reduction of the tax advantage. The permanent expanding spending policy changes the optimal capital structure, and firms choose to remain in permanent debt to benefit from the tax advantage or continue to repay until the level of debt is optimal. If the corporate tax cut is implemented, the reduction in the tax advantage causes firms to choose to own the optimal debt. In this way, the tax cut assists the effectiveness of the spending expansion policy.

Related literature Our paper relates to recent literature analyzing bailouts for the corporate sector through fiscal policy for the excessive debt problem. Bianchi (2016) examines the effects of debt purchases through an increase in the labor income tax to bail out the corporate sector when financial shocks make corporate borrowing difficult. In recent years, the scope for additional monetary policy accommodation has been limited by zero-interest rate constraints; therefore, the importance of analyzing fiscal policy bailouts, as in Bianchi (2016), has increased. The aim of this study is to examine the effects of policies such as corporate taxes and fiscal spending, which are not considered by Bianchi (2016), via changes in the capital structure of firms.

We also contribute to the literature that examines state-dependent fiscal policy within the context of DSGE models. There is vast literature on this issue. Leeper (1991) is an early work with fiscal policy and monetary policy interaction, while Canzoneri, Cumby and Diba (2010) survey the literature. Much of the recent literature examines fiscal policy multipliers at the ZLB of nominal interest rate; see, for example, Christiano, Eichenbaum and Rebelo (2011), Eggertsson (2011), and Nakata (2017a). Furthermore, recent developments in higher-order approximations and nonlinear solution methods have made it possible to investigate the relationship between continuous state changes and fiscal policy multipliers. For more details on recent developments in solution methods, see Fernández-Villaverde, Rubio-Ramírez and Schorfheide (2016) and Hirose and Sunakawa (2019b). Bi, Shen and Yang (2016) find that a high public debt state reduces government spending multipliers. Sims and Wolff (2018b) study government consumption and investment multipliers which vary across the states of the business cycle. Sims and Wolff (2018a) also study the state-dependent effects of consumption, labor, and capital tax cut multipliers that vary over the business cycle. To the best of our knowledge, our model is the first to address the corporate debt dependent fiscal policy.

Our study focuses on corporate debt problems and is closely related to Caballero, Hoshi and Kashyap (2008). They find that the survival of zombie firms causes credit misallocation and deteriorates productivity growth by limiting the entry and exit of firms. A zombie firm is defined as a firm that is low in productivity, practically insolvent, and unlikely to be rebuilt, surviving on bank support even though it should be bankrupt. They argue that zombie firms are over-indebted because these firms are low-productivity firms. Our study considers that even an intrinsically productive firm can become unproductive due to debt, and this argument has reverse causality. Our argument reveals another possibility: over-indebted firms become unproductive, which is complementary to the argument made by Caballero et al. (2008). This point results in a notably different policy implication.

On the one hand, based on the zombie firm hypothesis, the physical liquidation of zombie firms is desirable. Monetary easing and fiscal expansion policies help zombie firms continue operation and borrowing, and do not contribute to recovering productivity. On the other hand, our theory implies that zombie firms can regain high productivity levels if relieved of their excessive debt by policy intervention. This argument is consistent with the findings of Fukuda and Nakamura (2011) who argued that zombie firms in Japan, which were identified by the method used by Caballero et al. (2008), recovered to become non-zombie firms during the early 2000s. Recently, Banerjee and Hofmann (2018) estimate that the number of zombie firms in advanced economies has increased significantly since the 1980s.

While the relationship between corporate taxes and corporate debt has been studied and is widely known as the trade-off theory in corporate finance since the work of Modigliani and Miller (1963), there is little research, to our knowledge, that considers the impact of a corporate tax cut as a stimulus via the capital structure of firms.

The remainder of this paper is organized as follows. In the next section, we construct the DSGE model and analyze the debt dynamics. Section 3 discusses the calibration procedure and the solution method. Section 4 evaluates comparative statics for a permanent change in fiscal policy. Section 5 evaluates the transitional dynamics associated with responding to temporary policy shocks and permanent policy shocks when the corporate debt problem is realized. In Section 6, we investigate the robustness of our results. Section 7 presents our concluding remarks.

2 The model

Our model is based on Kobayashi and Shirai (2018), and make certain modifications to investigate fiscal policy. Our model introduces two fiscal policies: corporate tax and government expenditure. For simplicity, we do not consider R&D and economic growth.

Time is discrete and continues from zero to infinity: $t = 0, 1, 2, \cdots, \infty$. There are four agents in this model: intermediate goods firms, final goods firms, a representative household, and a government. The intermediate goods firms play an important role in our model. These firms can borrow an inter-period debt b_t and an intra-period debt q_t from the representative household. In this model, the inter-period debt b_t can be interpreted as unsecured debt. The initial debt b_{-1} is given for the intermediate goods firm at $t = 0$, where $r_{-1}b_{-1}$ is the inter-period debt at the end of the previous period, and r_t is the interest rate. The more inter-period debt b_t, the more tightly the borrowing constraints. Hence, the firm's production becomes inefficient. When the initial debt b_{t-1} exceeds a certain threshold, the debt remains at this level permanently and inefficient production continues perpetually.

2.1 The borrowing constraint

In this model, an intermediate goods firm receives revenue $f(q_t)$ from producing and selling output and faces the borrowing constraint that limits its financing for working capital loans q_t,

$$q_t \leq \phi f(q_t) + \max\{\xi S_t - b_t,\ 0\}, \tag{1}$$

where $0 \leq \phi < 1,\ 0 \leq \xi < 1,\ S_t$ is the liquidation value, which is defined by (5). This borrowing constraint is proposed by Kobayashi and Shirai (2018). The derivation of the borrowing constraint is shown in Appendix A. This borrowing constraint means that the firm is constrained to finance working capital loans only in the range of a part of revenue and the liquidation value minus the long-term debt.

The threshold of the borrowing constraint gives rise to two steady states: the *constrained-efficient* (CE) steady-state and the *debt-ridden* (DR) steady-state. When the debt b_t is smaller than ξS, the borrowing constraint is qualitatively similar to existing studies, such as those by Kiyotaki and Moore (1997), Kiyotaki (1998), Bernanke, Gertler and Gilchrist (1999) and Jermann and Quadrini (2012). After a shock, the firm's production eventually converges at the CE steady state for some periods. When the debt b_t is greater than ξS, the borrowing constraint becomes

$$q_t \leq \phi f(q_t) \tag{2}$$

and binds more tightly. The working capital loans are not sufficiently financed $q_{z,t} < q^{ce}$ and production becomes inefficient, where $q_{z,t}$ is defined as the solution to $q_t = \phi f(q_t)$, q^{ce} is the CE production and defined in Definition 1, and superscript *ce* denotes variables associated with the CE steady state. In this situation, the firm's production converges at the CE steady state over a long period. When the debt size is sufficiently large, we can show that inefficiency can continue permanently, and stay at the DR steady state forever. We show this result in Proposition 5.

Throughout this analysis, we assume that

$$\phi < \eta,$$

where $1/(1-\eta)$ is the elasticity of substitution across intermediate goods firms. This assumption means that production becomes inefficient when the borrowing constraint is $q_t \leq \phi f(q_t)$.

2.2 Intermediate goods firms

The intermediate goods firm produces its respective variety of intermediate goods from the capital and labor inputs under monopolistic competition. The intermediate goods firm is required to finance working capital loans. The amount of working capital loans q_t are limited by the borrowing constraint and are used for wage payments $w_t l_t$ and rental capital $r_t^K k_t$,

$$q_t = w_t l_t + r_t^K k_t,$$

where w_t is the wage rate, l_t is the labor input, r_t^K is the rental rate of capital, and k_t is the capital input.[2] The firm pays corporate tax on earnings. Interest paid $r_t b_{t-1}$ is deducted from taxable income as a cost. Thus, the corporate tax rate has a tax advantage for debt financing.

The optimization problem is

$$V_t^N = \max \pi_t + E_t \left[\frac{V_{t+1}^N}{1 + r_{t+1}} \right], \tag{3}$$

$$\text{s.t.} \begin{cases} \pi_t = (1 - \tau_t^{corp}) \left[f(q_t) - q_t - r_t b_{t-1} \right] - b_{t-1} + b_t, & (\lambda_t) \\ q_t \leq \phi f(q_t) + \max \left\{ \xi S_t - b_t,\ 0 \right\}, & (\mu_t) \\ \pi_t \geq 0, & (\lambda_{\pi,t}) \\ b_t \leq b_z, & (\lambda_{b,t}) \end{cases} \tag{4}$$

where V_t^N is the value of a firm's continued operation, π_t is the dividend, E_t is the expectation operator conditioned on time t information, τ_t^{corp} is the corporate tax rate, $\lambda_t, \mu_t, \lambda_{\pi,t}$ and $\lambda_{b,t}$ are the Lagrange multipliers for the budget constraint for the firm, the borrowing constraint, the limited liability constraint, and the debt limit constraint, respectively. The revenue function $f(q_t)$ can be derived as follows: [3]

$$f(q_t) = A_t \left(\frac{\alpha}{r_t^K} \right)^{\alpha\eta} \left(\frac{1-\alpha}{w_t} \right)^{(1-\alpha)\eta} q_t^\eta.$$

S_t and b_{t-1} are given for the firm. The liquidation value S_t is the maximum value that the lender can obtain from operating the seized firm itself. The equilibrium condition is determined S_t as follows

$$S_t = \max_b \quad E_t \left[\frac{V_t^N}{1 + r_{t+1}} \right] + b. \tag{5}$$

[2]The reason that the firm does not use inter-period debt to finance working capital for production is the limited commitment due to agency problems for workers and employees. First, suppose that in period t, the firm pays wage payment for production in the next period, w_{t+1}. In this case, the worker cannot commit to providing the labor input in period $t+1$. Second, suppose the firm saves a part of inter-period borrowing b_t in the form of safe assets to use in period $t+1$ for working capital. In this case, employees in the firm can easily steal and consume the safe asset privately in period t, and the firm cannot use it for working capital in period $t+1$.

[3]$f(q_t)$ is defined as the solution for the following problem:

$$f(q_t) = \max_{k,l} A_t k_t^{\alpha\eta} l_t^{(1-\alpha)\eta},$$

$$\text{subject to} \quad r_t^K k + w_t l \leq q_t,$$

where $A_t k_t^{\alpha\eta} l_t^{(1-\alpha)\eta}$ is the revenue function which is shown in (14).

The firm's owner is protected by limited liability, and the dividend must be non-negative, $\pi_t \geq 0$, as in Albuquerque and Hopenhayn (2004). The discount rate $1/(1+r_t)$ is given by the representative household's stochastic discount factor $\beta E_t U_{C_{t+1}}/U_{C_t}$ and $1/(1+r_t) = \beta E_t U_{C_{t+1}}/U_{C_t}$, where U_{C_t} is the marginal utility of consumption. The b_z is the maximum amount of repayable debt for the firm and $b_t \leq b_z$ implies the debt limit. We assume that parameter values satisfy $\xi S_t < b_z$. This assumption implies that the borrowing constraint becomes $q_t \leq \phi f(q_t)$ and q_z is the solution. Define b_z by

$$b_z \equiv (1-\phi)\frac{f(q_z)}{r}. \tag{6}$$

The envelop theorem implies that $\partial V_t/\partial b_{t-1} = -\lambda_t[1 + r_t(1-\tau_t^{corp})]$. $\tau_t^{corp} r_t$ is the tax advantage for interest paid on debt. Hereafter, we define the effective gross interest rate, $R_t \equiv 1 + r_t(1-\tau_t^{corp})$.

At the beginning of every period, the firm can intentionally choose to borrow the maximum amount of repayable debt, b_z. This is because there are cases when the marginal gain of b_z from the tax advantage is strictly larger than the marginal cost from tightening the borrowing constraint. When the firm decides to borrow b_z, the firm becomes a "debt-ridden firm." We call a firm that owes b_z the debt-ridden (DR) firm. This problem can be described below:

$$V_t^Z = \max \pi_t + E_t\left[\frac{V_{t+1}^Z(b_z)}{1+r_{t+1}}\right], \tag{7}$$

$$\text{s.t.} \begin{cases} \pi_t = (1-\tau_t^{corp})\left[f(q_t) - q_t - r_t b_{t-1}\right] - b_{t-1} + b_z, & (\lambda_t) \\ q_t \leq \phi f(q_t) + \max\left\{\xi S_t - b_z,\ 0\right\}, & (\mu_t) \\ \pi_t = 0, & (\lambda_{\pi t}) \\ b_t = b_z. \end{cases} \tag{8}$$

where V_t^Z is the value of becoming the DR firm. Note that $V_{t+1}^Z = 0$, because once the firm owes b_z, the debt level remains b_z and $\pi_t = 0$ continues for all t. In other words, the DR firm finances with 100 percent debt. Our parameter setting is chosen as $\xi S_t - b_z < 0$. Hence, the borrowing constraint becomes $q_{z,t} \leq \phi f(q_{z,t})$ and production becomes inefficient, $q_{z,t}$, forever, and the firm stays in the DR steady-state permanently, except for policy interventions.

Every period, the firm compares two values:

$$V_t = \max\left\{V_t^N,\ V_t^Z\right\}, \tag{9}$$

and decides whether to borrow b_z or not. If the firm decides to borrow b_z, the firm becomes the DR firm in this period.

Timing of events Figure 1 summarizes the timing of the firm's cash flow and debt finance. At the beginning of the period, the firm borrows working capital to pay wage and

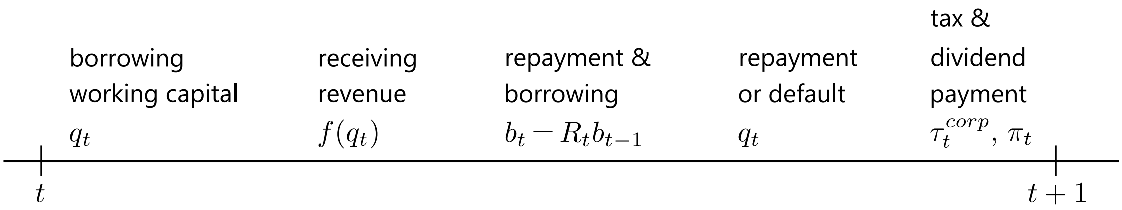

Figure 1: Time of events

rental capital in advance of production for the household, and employs labor and capital by paying q_t. When production is completed, the firm receives revenue $f(q_t)$, repays $R_t b_{t-1}$ and borrows new inter-period debt b_t. After that, the firm chooses to repay the working capital loan q_t to the household or default on working capital q_t. However, as described in Appendix A, this default is an off-the-equilibrium path outcome. Finally, the firm pays corporate tax to the government and pays a dividend to the household.

Default on inter-period debt The firm can choose to default on its inter-period debt b_{t-1} at the beginning of period t, and it will choose to default if and only if the continuation value is negative, i.e., $V_t < 0$. However, the continuation value is never negative because of the limited liability constraint ($\pi_t \geq 0$) even when the firm owes the maximum repayable debt b_z. Thus, a default on the inter-period debt is an off-the-equilibrium path event.

2.3 Debt dynamics

In this subsection, we consider the firm's debt dynamics. For simplicity, we assume the partial equilibrium setting and real prices $\{w_t, r_t^K, r_t\}$, tax τ_t^{corp}, and the debt contract variables $\{S_t, b_z\}$ are exogenously given for a firm. In this setting, the state variable is only b_t.

First, we briefly discuss the debt dynamics using Figure 2. We can derive policy functions and the value function numerically. Figure 2 shows policy functions for working capital q_t and for inter-period debt b_t, and the value function V_t. In the partial equilibrium setting, policy functions and value function are functions of the inter-period debt. Policy functions and the value function have several kinks at thresholds at the level of debt. There are three thresholds: B^{ce}, B_z, and B_c which are defined in this subsection. Capital B is used to represent a threshold. In section 4, we discuss that these thresholds are changed by fiscal policies. When b_{t-1} is larger than the threshold B^{ce}, the limited liability constraint binds, $\lambda_\pi > 0$, and the dividend is equal to zero, $\pi_t = 0$. This means that when the firm owes a certain amount of relatively large debt, the firm does not pay dividends in order to repay b_{t-1}. As shown in Proposition 4, when $b_{t-1} < B^{ce}$, the limited liability constraint no longer binds, and production and debt become the CE (i.e., $q_t = q^{ce}$ and $b_t = b^{ce}$). When $B_z \leq b_{t-1} \leq B_c$, $b_t > \xi S$ and the borrowing constraint becomes

$q_t \leq \phi f(q_t)$, production becomes inefficient, and the slope of the policy function of b_t is close to the slope of a 45-degree line in the standard parameter setting. The speed of the debt repayment is extremely slow in the region where $B_z < b_{t-1} \leq B_c$, because the borrowing constraint becomes $q_t \leq \phi f(q_t)$. The value function also has a kink at B_z due to switching the borrowing constraints. If $b_{t-1} < B_c$, the debt level eventually converges at the CE steady state, b^{ce}. However, if $b_{t-1} \geq B_c$, the firm borrows the maximum amount of debt b_z and becomes a DR firm, because the value of tax benefit obtained by borrowing the maximum amount of repayable debt exceeds the value of continuing repayment of debt (i.e., $V_t^N < V_t^Z$). Once the firm owes b_z, the debt level stays at the DR steady state, b_z, and inefficient production ($q_t = q_z$) continues forever.

Next, we discuss the debt dynamics in more detail to characterize some equilibrium properties.

Proposition 1. *borrowing constraint μ depends on the corporate tax rate τ^{corp},*

$$\mu^{ce} \equiv \mu/\lambda = \tau^{corp}(1-\beta). \tag{10}$$

Proof. The Euler equation for debt stock is $1 - \frac{\mu_t}{\lambda_t} = \frac{\lambda_{t+1}}{\lambda_t}\frac{R_{t+1}}{1+r_t}$. In the CE steady state, $R = 1 + r(1-\tau^{corp})$ and $r = 1/\beta - 1$. Substituting the Euler equation for debt stock for R and r, we get the equation (10). □

Proposition 1 is similar to Jermann and Quadrini (2012)'s Proposition 1. When the corporate tax rate is greater than zero, the borrowing constraint binds even in the CE steady state (i.e., $\mu^{ce} > 0$) because the firm borrows inter-period debt to exploit the tax advantage. Now, we define variables at the CE level.

Definition 1. q^{ce} is defined as the solution to

$$\frac{\partial}{\partial q} f(q) = \frac{1+\mu^{ce}}{1+\phi\mu^{ce}}.$$

Define b^{ce} and π^{ce} as follows:

$$b^{ce} \equiv \phi f(q^{ce}) - q^{ce} + \xi S,$$
$$\pi^{ce} \equiv (1-\tau^{corp})\left[f(q^{ce}) - q^{ce} - rb^{ce}\right].$$

Assumption 1. The parameters satisfy the following condition

$$\pi^{ce} = (1-\tau^{corp})\left[f(q^{ce}) - q^{ce} - rb^{ce}\right] > 0.$$

This assumption implies that the dividend is non-negative in the CE steady state. Once the inter-period debt reaches the CE steady state, it stays there forever.

Next, to characterize the features of the equilibrium path, we define thresholds.

Definition 2. Define B^{ce} and B_z as follows:

$$B^{ce} \equiv \frac{(1-\tau^{corp})\left[f(q^{ce}) - q^{ce}(x)\right] + b^{ce}}{R}, \tag{11}$$

$$B_z \equiv \frac{(1-\tau^{corp})(1-\phi)f(q_z) + \xi S}{R}, \tag{12}$$

and B_c is defined as the solution,

$$V^N(B_c) = V^Z(B_c). \tag{13}$$

We justify later that B^{ce} is the maximum possible amount of debt b_{t-1}, at which the economy stays in the CE steady state; B_z is the minimum amount of debt b_{t-1} that makes $b_t \geq \xi S$ and the borrowing constraint becomes $q_t \leq \phi f(q_t)$. The threshold B_c indicates that $V^N(b_{t-1})$ and $V^Z(b_{t-1})$ are indifferent when $b_{t-1} = B_c$. If $b_t > B_c$, then $V^N(b_{t-1}) < V^Z(b_{t-1})$ and the firm chooses to finance the maximum repayable debt and becomes a DR firm.

Using these definitions, first, we show a feature of debt dynamics in the case of a small debt. The following propositions are almost identical to ones used by Kobayashi and Shirai (2018).

Proposition 2. *If $b^{ce} < b_{t-1} < B^{ce}$, the debt to be repaid in the next period and production are equal to CE level, i.e., $b_t = b^{ce}$ and $q_t = q^{ce}$.*

Proof. Suppose that $b_{t-1} < B^{ce}$. We assume and justify later that $\lambda_{\pi,t} = \lambda_{\pi,t+1} = 0$. The first-order condition for b_t is $\mu_t/\lambda_t = 1 - \frac{\lambda_{t+1}}{\lambda_t}\frac{R_{t+1}}{1+r_t}$, which can be written as $\mu_t/\lambda_t = \mu^{ce}$. μ^{ce} is not dependent on b_{t-1} and decided solely by the corporate tax rate τ_t^{corp} as shown in proposition 1. Therefore, the production also becomes CE, i.e., $q_t = q^{ce}$. This result implies that the inter-period debt in the next period, b_t, should be b^{ce} and π_t is non-negative. Assumption 1 implies that once the economy enters the CE steady state, it stays there forever. Hence, $\pi_{t+1} = \pi^{ce}$, and it justifies the assumption $\lambda_{\pi,t} = \lambda_{\pi,t+1} = 0$. □

Proposition 2 shows that if the debt level is sufficiently small, the economy reaches the CE steady state in the next period immediately. The next proposition shows this feature in the case of a medium-sized debt (i.e., $B^{ce} \leq b_{t-1} < B_z$).

Proposition 3. *Consider the case where b_{t-1} is medium-sized, $B^{ce} \leq b_{t-1} < B_z$, and $q_t < q^{ce}$ in equilibrium. Then, the limited liability constraint binds, $\lambda_{\pi,t} > 0$, and the dividend is equal to zero, $\pi_t = 0$, in equilibrium.*

Proof. We prove the proposition by contradiction. Suppose that $\lambda_{\pi,t} = 0$. The first-order condition for b_t is $\mu_t/\lambda_t = 1 - \frac{\lambda_{t+1}}{\lambda_t}\frac{R_{t+1}}{1+r_t} < \mu^{ce}$. q_t is derived from as the solution to $\frac{\partial}{\partial q_t} f(q_t) = \left(1 + \frac{\mu_t}{1+\lambda_{\pi,t}}\right) / \left(1 + \phi\frac{\mu_t}{1+\lambda_{\pi,t}}\right)$ and it implies that $q_t > q^{ce}$. This is a contradiction. Thus, $\lambda_{\pi,t} = 0$ cannot hold in equilibrium. Hence, when b_{t-1} is medium-sized, the limited liability constraint binds $\lambda_{\pi,t} > 0$ and the dividend is equal to zero, $\pi_t = 0$, in equilibrium. □

Proposition 3 is a modified version of Kobayashi and Shirai (2018)'s Lemma 4. Given that debt b_{t-1} is medium-sized, the firms pass a dividend ($\pi_t = 0$) to repay as much debt as possible and eventually return to the CE steady state within a finite period. These features are quantitatively the same as Albuquerque and Hopenhayn (2004). Note that when $b_{t-1} \geq B^{ce}$, the limited liability constraint remains binding. The next proposition shows this feature in the case of a large debt (i.e., $B_z \leq b_{t-1} < B_c$).

Proposition 4. *Consider the case where b_{t-1} is large, $B_z < b_{t-1} < B_c$, and $q_t = q_z$ in equilibrium. Then, the limited liability constraint binds, $\lambda_{\pi,t} > 0$, the dividend is equal to zero, $\pi_t = 0$, and the borrowing constraint is $q_t \leq \phi f(q_t)$ in equilibrium.*

Proof. The proposition follows immediately from the definition of threshold B_z. □

The proposition implies that the policy function of inter-period debt is $b_t = b_{t-1} - (1-\phi)f(q_z)$, where q_z is constant and has a small value due to the binding borrowing constraint. Hence, the speed of the debt repayment is extremely slow in the region where $B_z < b_{t-1} < B_c$.

By the given definition, given $b_{t-1} \geq B_c$, the firm chooses to finance the maximum amount of repayable debt b_z and becomes a DR firm. Then, the following proposition holds.

Proposition 5. *Once $b_{t-1} = b_z$ in equilibrium, $b_{t+j} = b_z$ and $q_{t+j} = q_z$ for all $j \geq 0$*

Proof. Suppose $b_{t-1} = b_z$. We assume that $b_z > \xi S$. The borrowing constraint is $q \leq \phi f(q_z)$, the limited liability constraint binds ($\pi_t = 0$), and cash flow is $0 = (1 - \tau^{corp})\left[f(q_z) - q_z - rb_z\right] - b_z + b_t$. Then, the debt to be repaid in the next period must be equal to b_z (i.e., $b_t = b_z$). Therefore, once the debt to be repaid in the current period is equal to b_z, the debt level remains b_z and inefficient production continues forever. □

We call such a permanent inefficiency the DR steady state. Even in this situation, the lender (representative household) has no incentive to reduce the debt, because b_z is the maximum amount of repayable debt for the firm.

2.4 Debt-ridden firms

In the benchmark case, we assume that firms $i \in [0, \zeta]$ are DR firms and firms $i \in (\zeta, 1]$ are normal in the equilibrium, where ζ is the ratio of the DR firms. DR firms owe the maximum repayable debt $b_{t-1} = b_z$, and the initial debt is given as an exogenous shock in this model.

In Section 5, we consider some variation in the initial debt for ζ ratio of firms to examine corporate debt dependent fiscal policy.

2.5 Final goods firms

The final goods sector produces a homogeneous good Y to aggregate intermediate goods y_i under perfect competition. We normalize to 1 the number of intermediate goods firms.

$$\min_{y_{i,t}} Y_t - \int_0^1 p_{j,t} y_{j,t} dj,$$

$$s.t.\ Y_t = \left(\int_0^1 y_{j,t}^{\eta} dj \right)^{\frac{1}{\eta}},$$

where $0 < \eta < 1$, $p_{j,t}$ is the real price of the intermediate good j. Intermediate good j employs labor $l_{j,t}$ and capital stock $k_{j,t}$, and produces intermediate goods $y_{j,t}$ by the following Cobb–Douglas type production function:

$$y_{j,t} = a_t k_{j,t}^{\alpha} l_{j,t}^{1-\alpha},$$

where a_t is the productivity shock. Perfect competition in the final goods market implies that

$$p_{j,t} = A_t y_{j,t}^{\eta-1},$$

where $A_t \equiv Y_t^{1-\eta} a_t^{\eta}$. The revenue function of intermediate goods firm j is

$$f(q_{j,t}) \equiv p_{j,t} y_{j,t} = A_t k_{j,t}^{\alpha\eta} l_{j,t}^{(1-\alpha)\eta}. \tag{14}$$

2.6 Household and welfare

The representative household chooses consumption C_t, labor supply L_t, investment K_t and savings $\mathbb{B}_t$ to maximize the utility function

$$\max_{C_t, L_t, \mathbb{B}_t, K_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \gamma_L \frac{L_t^{1+\nu}}{1+\nu} + \gamma_G \ln G_t \right] \right\}, \tag{15}$$

subject to the budget constraint

$$C_t + K_t + \mathbb{B}_t + \tau_t^{lump-sum} \leq w_t L_t + (r_t^K + 1 - \delta) K_{t-1} + (1 + r_t)\mathbb{B}_{t-1} + \int_0^1 \pi_{i,t} di,$$

where β is the subjective discount factor, $\nu > 0$ is the elasticity of labor supply, γ_L is the coefficient of labor disutility relative to contemporaneous consumption utility, γ_G is the coefficient of the public service utility relative to contemporaneous consumption utility, C_t is consumption, G_t is government expenditure, K_t is capital stock, $\mathbb{B}_t$ is inter-period lending to the firms, $\tau_t^{lump-sum}$ is the lump-sum tax, L_t is total labor supply, and δ is the depreciation rate of capital. The first-order condition implies a stochastic discount factor, $\beta E_t C_t / C_{t+1}$.

We define welfare, $\mathbb{W}_t$, as the present discounted value of flow utility, i.e., the value function for the representative household:

$$\mathbb{W}_t = \ln C_t - \gamma_L \frac{L_t^{1+\nu}}{1+\nu} + \gamma_G \ln G_t + E_t \frac{\mathbb{W}_{t+1}}{1 + r_{t+1}}. \tag{16}$$

2.7 Government

The flow budget constraint for the government is given by:

$$G_t = \int_0^1 \tau_t^{corp} \left[f(q_{i,t}) - q_{i,t} - r_t b_{i,t-1} \right] di + \tau_t^{lump-sum}.$$

We assume a balanced budget and that the public debt, increase in government expenditures, and corporate tax cut are financed by a lump-sum tax. We also assume that government expenditures and the corporate tax rate follow the independent stationary AR(1) process:

$$\ln G_t = \rho_G \ln G_{t-1} + (1-\rho_G) \ln \overline{G} + \epsilon_{G,t}, \qquad \epsilon_{G,t} \sim N(0, \sigma_G^2)$$
$$\ln \tau_t^{corp} = \rho_\tau \ln \tau_{t-1}^{corp} + (1-\rho_\tau) \ln \overline{\tau}^{corp} + \epsilon_{\tau,t}, \qquad \epsilon_{\tau,t} \sim N(0, \sigma_\tau^2)$$

where $\overline{G}$ is the non-stochastic steady state value of government expenditures, ρ_G and ρ_τ are the parameters for persistence of the shocks, and $\epsilon_{G,t}$ and $\epsilon_{\tau,t}$ are the independent shocks drawn from the standard normal distribution with zero mean and variances σ_G^2 and σ_τ^2, respectively, and are serially uncorrelated and independent from each other.

2.8 Exogenous processes

The productivity also follows the AR(1) process:

$$\ln a_t = \rho_a \ln a_{t-1} + (1-\rho_a)\overline{a} + \epsilon_{a,t}, \qquad \epsilon_{a,t} \sim N(0, \sigma_a^2)$$

where ρ_a ar the parameter for persistence of the shocks. The disturbance term $\epsilon_{a,t}$ is normally distributed with zero mean and variances σ_a^2, respectively, and are serially uncorrelated and independent from other shocks.

2.9 Equilibrium

The definition of an equilibrium is standard. All budget constrains hold with equality, the representative household holds all corporate debt, and markets for capital services and labor are cleared. The aggregate resource constraint is:

$$C_t + K_t - (1-\delta)K_{t-1} + G_t = Y_t.$$

The market clearing conditions are $L_t = \int_0^1 l_{i,t} di, \;\; K_{t-1} = \int_0^1 k_{i,t} di, \;\; \mathbb{B}_{t-1} = \int_0^1 b_{i,t} di.$

The following conditions must be satisfied on the equilibrium:

$$\xi S > b^{ce}, \tag{17}$$
$$\xi S < b_z < S, \tag{18}$$
$$b^{ce} > 0. \tag{19}$$

The first condition requires that the max operator in the borrowing constraint (1) must be positive in the equilibrium. The second condition is required by the definition of the maximum repayable debt b_z. The last condition requires that firms do not lend funds to other firms and households in the equilibrium.

3 Calibration and solution

Parameter	Value	Description
α	0.3	Cobb–Douglas production function
β	0.99	Subjective discount factor
η	0.7	Intermediate goods elasticity, $1/(1-\eta)$
δ	0.025	Depreciation rate
ν	1	Labor supply elasticity
γ_L	6.55	Labor disutility weight
γ_G	0.3095	Public service weight
ϕ	0.3413	Collateral ratio of revenue
ξ	0.1	Collateral ratio of foreclosure value
ζ	0.13	Debt-ridden firms ratio
$\overline{a}$	1	Steady state productivity
$\overline{L}$	1/3	Steady state labor supply
$\overline{G}/\overline{Y}$	0.1944	Steady state government expenditures-to-GDP ratio
$\overline{\tau}^{corp}$	0.35	Corporate tax rate
ρ_a	0.79	Productivity AR(1)
ρ_G	0.87	Government expenditures AR(1)
ρ_τ	0.8747	Corporate tax rate AR(1)
σ_a	0.007	SD productivity shock
σ_G	0.0014	SD government expenditures shock
σ_τ	0.01229	SD corporate tax rate shock

Table 1: Calibrated parameters

We set the Cobb–Douglas parameter in the production function at $\alpha = 0.3$, the subjective discount factor at $\beta = 0.99$, the depreciation rate at $\delta = 0.025$, the parameter for the elasticity of substitution at $\eta = 0.7$, and the elasticity of labor supply at $\nu = 1$. The steady-state labor supply is set to 1/3. These are the standard settings used in prior studies. The coefficient of labor disutility relative to contemporaneous consumption utility $\gamma_L = 6.55$ is chosen to have a steady-state labor supply. The coefficient of the government expenditure utility relative to contemporaneous consumption utility $\gamma_G = 0.3095$ is chosen to have a steady-state government expenditure-to-GDP ratio. [4]

The collateral ratio of foreclosure value ξ and of revenue ϕ are determined simultaneously. ϕ and ξ are chosen to satisfy the equilibrium conditions, i.e., (17)–(19), and fit the data. The value ξ that can satisfy these conditions is limited to a narrow range of [0.0944, 0.1094] and is set to 0.1. ϕ is chosen to have a steady-state ratio of debt over value-added equal to 1.648. This is the average ratio over the period 1984:I-2017:IV for liability of the nonfinancial corporate business from the board of governors of the federal reserve system (US), *Financial Accounts of the United States* and the bureau of economic analysis, *NIPA Tables*. The required value is $\phi = 0.3413$.

The mean value of corporate tax rate is set to $\tau^{corp} = 0.35$. The borrowing constraint is always binding (i.e., $\mu > 0$) because the firm borrows inter-period debt to exploit the tax advantage.

The ratio of DR firms is set to $\zeta = 0.13$, which is estimated as the average zombie firm ratio of 14 advanced countries in 2016 by Banerjee and Hofmann (2018). They find that this ratio and the probability of remaining a zombie have increased significantly since the 1980s. The DR firm' ratio and the zombie firm ratio are complementary relationships. In Japan, Fukuda and Nakamura (2011) show that a significantly substantial proportion of zombie firms recovered to become non-zombie firms in the early 2000s. According to the result, one can see that the zombie firm ratio includes a substantial number of DR firms.

The parameters of the productivity and government spending processes are chosen from Bi et al. (2016) (i.e., $\rho_a = 0.79$, $\sigma_a = 0.007$, $\rho_G = 0.87$, $\sigma_G = 0.0143$). The values of persistence of the corporate tax ρ_τ and σ_τ are estimated in the AR(1) model using Mertens and Ravn (2013)'s dataset and set to $\rho_\tau = 0.8747$ and $\sigma_\tau = 0.01229$.

The model is solved using a global, nonlinear method that accounts for two occasionally binding constraints. Our solution is a Smolyak-based projection method proposed by Judd et al. (2014). To avoid the heavy computational burden, we solve the model using a fixed-point iteration and approximate expectation functions on the right-hand side of the Euler equations and the value function. Our simulation code is modified and extended Hirose and Sunakawa (2019b) to apply two occasionally binding constraints. The full set of equilibrium conditions is available in Appendix B, and the details of the method are described in Appendix C.

[4] Following Song, Storesletten and Zilibotti (2012), γ_G is calibrated to solve the social welfare maximization problem.

4 Fiscal policies and debt thresholds

4.1 Government expenditures and debt thresholds

This section examines how features of debt dynamics in steady states change when the corporate tax rate and government expenditure change permanently in the general equilibrium. In order to examine this, we analyze changes in debt levels and thresholds for each steady-state value of the corporate tax rate and the government expenditure. The level of the thresholds affects decisions about the corporate capital structure.

Figure 3 (a) shows debt thresholds and debt levels in steady states for each level of government expenditure. This figure implies that government expenditure affects the capital structure of firms. The increase in G is financed by increasing the lump-sum tax for the balanced government budget. The household responds to it by increasing its labor supply. Thus, increasing G increases production, firm's value (V^N), and GDP as a result of increasing labor supply. [5] Financing for the inter-period debt, b_t, increases as production increases. B_c also increases, resulting in an increase in V^N. An increase in thresholds and the debt level shift the policy function for b_t outward. This result can be seen in Figure 3 (b). The superscript $*$ denotes variables associated with the new steady-state after changing G^*. As B_c^* increases to $B_c^* > b_z$ due to the expansionary fiscal policy, the firm decides to repay its debt from $b_{t-1} = b_z$ to the CE steady-state b^{ce*} because the value of becoming a normal firm is greater than the value of staying a DR firm (i.e., $V^N > V^Z$). In subsection 5.2, we show transitional dynamics in response to a permanent change G.

4.2 Corporate tax and debt thresholds

Figure 4 (a) shows debt thresholds and debt levels in steady states for each corporate tax rate, τ^{corp}. τ^{corp} has an upper limit and if τ^{corp} exceeds it, condition (18), $S > b_z$, is not satisfied. The range for τ^{corp} is chosen to satisfy the condition $S > b_z$. The vertical axis corresponds to the debt level in each steady-state and the level of each threshold. Increasing τ^{corp} raises μ/λ, which means that the borrowing constraint becomes more tight, and the optimal debt of b^{ce} decreases as the revenue $f(q)$ decreases, whereas b_z increases slightly due to the decreasing wage rate and increasing revenue. Increasing τ^{corp} increases the tax advantage, and the value of becoming a DR firm (V^Z) increases, and as a result, the threshold B_c decreases. Figure 4 (b) shows a change in the policy function for b due to a corporate tax increase. This figure indicates that high corporate tax rates increase the incentive for firms to owe the maximum repayable debt because of the tax advantage of holding debt. Meanwhile, the permanent corporate tax cut does not encourage DR

[5] In this model, because we assume that Ricardian equivalence holds, and a standard utility function is assumed, changes in macro variables due to fiscal policy are almost identical to those in the standard RBC model.

firms to repay their huge debt because firms with high debt burdens, such as the DR firm, pay little corporate tax. If $b_z < B_c^*$, the lender and the firm agree with debt repayment from b_z to b^{ce*}. However, $b_z < B_c^*$ never happens through changing τ^{corp}. Hence, the permanent corporate tax cut has no marginal effect for DR firms.

To summarize results obtained in this section, if fiscal policies are effective, increasing fiscal expansion changes thresholds, $b_z < B_c^* < b_z^*$, and the amount of debt owed by the DR firm below the threshold B_c^* and the upper limit b_z^*. After that, the lender and the firm agree to proceed with the debt repayment, and DR firms can go back to being normal firms by repaying their debt to b^{ce*} thanks to fiscal policies.

5 Transitional dynamics: debt dependent fiscal policies

Section 4 examines the relationship between debt dynamics and fiscal policies in steady states and finds that a permanent expansionary fiscal spending policy has the effect of promoting debt repayment through changes in debt dynamics for firms. This result raises natural questions. Can a temporary fiscal policy help in the repayment of corporate debt? Does the fiscal policy multiplier depend on the level of corporate debt? How much is government spending needed to help firms repay their debt? To answer these questions, we examine transitional dynamics when the government decides to implement a corporate tax cut or an expansionary fiscal spending in a situation where firms with the ratio of ζ owe a relatively large debt in period 1. We analyze the transition dynamics associated with a response to the following shocks:

1. Temporary corporate tax shock
2. Temporary government expenditures shock
3. Permanent increase in government expenditure
4. Permanent corporate tax cut

As in the previous section, we consider an equilibrium in which there are two types of firms. The difference from the previous section is that we assume different initial amounts of debt for the ζ ratio of firms. In period 0, the economy is initially on the CE steady-state, and all firms are normal. At the end of period 0, initial debt $b_{t-1} \geq b^{ce}$ is realized for firms with the ratio ζ, and a fiscal policy shock is realized. The level of $b_{d,t-1}$ is set to either the optimal debt, a medium-sized debt, a large debt, or the maximum repayable debt. The subscript d denotes variables associated with firms with the ratio of DR firms. Firms with a ratio of $(1-\zeta)$ are normal firms and their initial amount of debt is the optimal debt, $b_{n,t-1} = b^{ce}$, where subscript n denotes variables that associated with the normal firm.

The corporate tax cut policy in our model affects the economy through two channels. First, the corporate tax cut improves welfare due to a decline in the corporate debt level. The corporate tax cut relieves the tax burden for firms and helps in the repayment of debt. The decrease in the debt amount improves the welfare gain directly (i.e., the value function of the aggregate welfare is a function of the corporate debt). Second, the government increases the lump-sum tax to compensate for the corporate tax cut. Increases in the lump-sum tax reduce consumption and investment, which decrease the capital stock and have a negative effect on production.

The expansionary expenditure policy in our model is financed by increasing the lump-sum tax, and affects the economy through four channels. (i) The household derives utility from government services in our model, so the expansionary expenditure has the effect of directly improving welfare. (ii) Stimulating the GDP increases corporate income and promotes debt repayment, which indirectly improves welfare. (iii) Increasing the lump-sum tax has a negative effect on consumption and investment. (iv) The crowding-out effect causes the interest rate to rise, which reduces consumption and investment, and the increase in the interest payment burden has the effect of preventing debt repayment. If the effects of (iv) outweigh the effects of (ii), then policy intervention delays debt repayment.

The impulse responses represent one standard deviation from the case in which no policy intervention is implemented; we divide by the impact response of fiscal policy to interpret the multiplier. In this study, following Sims and Wolff (2018a), we introduce the welfare multiplier and define a corporate tax cut multiplier as a response in a particular state to the response of a tax revenue evaluated in the non-stochastic steady state. For example, the output multiplier is defined as $\left.\frac{dY_t}{dTR^*}\right|_{\epsilon_{\tau,t}=-\sigma_\tau,\ b_{t-1}}$, where dY_t is the difference in GDP between the case where the corporate tax cut is implemented and the case where no policy intervention is implemented, and dTR^* is the permanent change in tax revenue relative to the non-stochastic steady state.[6] This is a modified version of a traditional and commonly used definition. The most common definition of a tax multiplier is derived with respect to total tax revenue, dY_t/dTR_t. The reason why we use dTR^* instead of dTR_t is because dTR_t is also state-dependent and it is difficult to capture state-dependent multipliers.

5.1 Temporary changes in corporate tax / increasing government expenditure

Next, we study state dependence in fiscal policies. Our numerical simulation method provides a full nonlinear solution that can address state dependence. We consider the

[6]Note that the tax revenue coincides with expenditure (i.e., $TR_t = G_t$), because we do not model public debt.

following four situations in which differences in the amount of corporate debt create quantitative differences in the effects of fiscal policies. The four different debt levels are as follows:

1. Optimal level of debt: $b_{t-1} = b_t = b^{ce}$, $\xi S_t - b_t > 0$,
2. Medium-sized debt: $b_{t-1} = (B^{ce} + B_z)/2$, $\xi S_t - b_t > 0$, $B^{ce} < b_{t-1} < B_z$,
3. Large debt: $b_{t-1} = (B_z + B_c)/2$, $\xi S_t - b_t < 0$, $B_z < b_{t-1} < B_c$,
4. Maximum repayable debt: $b_{t-1} = b_t = b_z$, $\xi S_t - b_t < 0$.

First, we consider the situation that the firm owes the optimal level of debt, $b_{t-1} = b^{ce}$. In this situation, the borrowing constraint does not depend on the debt level. As described in detail in subsection 2.3, the small debt case is identical to the optimal level of debt case. Hence, we do not deal with the small debt in this subsection.

Second, we consider the situation that the firm owes a medium-sized debt $b^{ce} < b_{t-1} < B_z$. In this situation, the borrowing constraint is (1) and binds tightly $\mu_t/\lambda_t > \mu^{ce}/\lambda^{ce}$. However, the firm repays as much debt as possible by setting the dividend to zero (i.e., $\pi_t = 0$), to relax the borrowing constraint and reduce its debt to b^{ce} within several periods. Thus, inefficiency is temporary.

Third, we consider the situation when the firm owes a large debt, $B_z < b_{t-1} < B_c$. In this situation, the borrowing constraint becomes (2), which binds more tightly $\mu_t/\lambda_t = \mu_{z,t}/\lambda_{z,t}$ and makes production inefficient, $q_t = q_{z,t}$. Inefficiency remains persistently.

Forth, we consider the situation when the firm owes the maximum amount of repayable debt, b_z. In this situation, the borrowing constraint is (2) and never relaxes except when there is a permanent shock or debt forgiveness. The debt level remains b_z forever, and inefficiency continues permanently.

Figure 5 and Figure 6 show one standard deviation from corporate tax cut shock and positive government spending shock, respectively, when all firms owe the optimum level of debt. The vertical axes indicate the level of the corresponding variable. A response to a corporate tax cut shock slightly increases consumption, output, labor supply, and welfare by decreasing μ_t/λ_t and relaxing the borrowing constraint. Figure 6 shows that a response to a government expenditure shock is similar to a standard real business cycle model. Fiscal policy stimulates labor supply and output and reduces consumption and investment.

Figure 7 shows a response for corporate tax cut conditional on debt level. The upper panel shows impulse responses in the short-run (50 quarters), and the lower panel shows the same in the long-run (200 quarters). These are scaled by the response of tax revenue that is evaluated in the non-stochastic steady state. This figure shows that the effect of the corporate tax cut multipliers varies depending on the level of corporate debt, and the larger the amount of corporate debt, the larger the corporate tax multipliers. Note that the welfare multiplier cannot simply be compared to the size of the other multipliers. Since the welfare is the discounted present value of future utility, and the discount factor β is set to 0.99 in our model, the welfare multipliers would be about one-hundredth of the values presented in the figure. Looking first at the case of medium-sized debt, this is shown in panel (a) on the right scale, where the multipliers for consumption, investment, and output are large. In this case, firms can repay their debt to the optimal level in a few periods as Bernanke et al. (1999). When the debt shock is realized and the corporate tax cut is implemented simultaneously, the multipliers for the macro variables become large because firms can repay much of their debt thanks to policy intervention. We can see it in Figure 8 that plots the impulse responses of debt to a tax cut (left column) or spending shock (right column). The following case with large debt shows that the multipliers for consumption, investment, and output are not large, but the improvement in welfare is long-lasting.[7] In the short run, the macro variables respond positively but then respond negatively, and after the amount of debt becomes small enough and the borrowing constraint switches from (2) to (1), they respond positively again. Initially, positive responses are due to the increase in output of normal firms due to the tax cut and consumption increases as compared to the situation without policy intervention, as the policy promotes debt repayment. In the meantime, investment is suppressed, and capital is reduced as compared to the situation without policy intervention. Consumption responds negatively to this. When the borrowing constraint switches from (2) to (1), because the policy promotes debt repayment and the debt-ridden firms can borrow more working capital, the positive response of the macroeconomic variables continues due to the increase in the capital stock as output improves significantly. Despite the temporary and short-term corporate tax cut, the effect is long-lasting, and the cumulative welfare improvement becomes large because the corporate tax cut can increase debt repayments and the difference in debt level remains in the long-run. This finding implies that a large corporate debt state where firms are repaying their debt enhances the corporate tax multipliers. In the b_z case, the impact is not large because the temporary tax cut does not change debt thresholds, and the debt level remains b_z.

Figure 9 shows a response for the expanding government expenditures shock conditional on the corporate debt level and is scaled by the impact response of the government expenditures in the first period, giving the units a multiplier interpretation. The difference of multipliers by the amount of debt is not significant except for the welfare multiplier. This result is similar to that of Sims and Wolff (2018a), who find that consumption, labor, and capital tax cut multipliers vary substantially across business cycles, whereas spending multipliers do not. As the initial debt increases, welfare losses from the policy interven-

[7]Note that, in the large debt case, the jump occurs in the period 96, because the borrowing constraint switches from (2) to (1).

tions get worse. The increase in government expenditure raise the interest rate through the crowding-out effect, making debt repayment more difficult. The effects on debt repayment are that the crowding-out effect outweighs the GDP-stimulating effect (i.e., the effects of (iv) outweigh the effects of (ii), as mentioned earlier in this section), and debt repayment is delayed more when policy intervention takes place. Hence, reducing fiscal spending rather than expanding it improves welfare by promoting debt repayment through the lower interest rate.

5.2 Can a permanent change in fiscal policy stimulate debt repayment?

The temporary fiscal policy change does not affect the steady-state, thus, the debt of the DR firms remains at the steady-state level because lenders have no incentive to repay their loans. Next, we study whether a permanent fiscal policy change can enhance debt repayment even for DR firms when there is the ζ ratio of DR firms. First, we consider the effect of permanent changes in government expenditures. If fiscal policy is effective, even DR firms can proceed with debt repayment and their debt level converges to the optimal debt, $b_z \to b^{ce*}$, where an asterisk $(*)$ denotes variables in the new steady-state due to the permanent fiscal policy. Ultimately, the debt level of all firms becomes optimal (i.e., $\zeta = 0$). The new steady-state is the result of two effects: the permanent change in fiscal policy and debt repayment to the CE steady state.

First, we address the following question: can the permanent expanding fiscal expenditures help debt repayment? The answer is a qualified yes. There are three conditions for the effectiveness of fiscal policies:

(i) $b_z^* > b_z$,

(ii) $V_t^N > V_t^Z$ for all t,

(iii) $\mathbb{W}^* > \mathbb{W}$.

First, condition (i) requires that the maximum repayable debt in the new steady state b_z^* is greater than the initial steady-state b_z. We have already shown in Figure 3 that the permanent fiscal expansion increases the maximum repayable debt b_z^*, and then DR firms do not owe the maximum repayable debt and decide whether to borrow the maximum repayable debt or to repay their debt to the optimal level b^{ce*}. This decision depends on condition (ii). After the permanent fiscal policy is implemented, DR firms decide to repay their debt if the repayment value exceeds the value of gaining a tax advantage, $V_t^N > V_t^Z$ for all t. This condition is a feature of transitional dynamics. Lastly, condition (iii) implies that fiscal policies are required to improve welfare in the new steady state, $\mathbb{W}^* \geq \mathbb{W}$. In our model, fiscal expansion has direct and indirect effects on welfare. The direct effect is that the household derives utility from government expenditure. The indirect effects are negative effects of fiscal policy distorting resource allocation and positive effects of improving efficiency by solving the debt problem. The size relation of these effects determines whether welfare ultimately improves in the new steady state. The yellow region of Figure 10 shows the domain of the parameters $(G^*,\ \zeta)$ that satisfies condition (i) $\sim$ (iii). The vertical axis indicates the level of G^* in the new steady-state and, the horizontal axis indicates the ratio of DR firms ζ. When fiscal policies satisfy these three conditions, the current amount of debt of DR firms falls below the new threshold, $b_{t-1} = b_z < B_c^* < b_z^*$, DR firms start debt repayment, and the debt level eventually converges to the optimum level b^{ce*}. Note that $\mathbb{W}^*$ is calculated on the assumption that DR firms repay their debt to the CE level. This figure implies that when the DR firms' ratio ζ is relatively low, the fiscal expansion promotes debt repayment even for DR firms. However, if the DR firms' ratio ζ is over 0.64, conditions (i) and (iii) are not satisfied, and the fiscal expansion cannot help in debt repayment.

Figure 11 shows a numerical example of transitional dynamics to the new steady state. We set the new steady-state government expenditure-GDP ratio to $G^*/Y^* = 0.29$ and other parameters are calibrated in section 3. In period 50, the government permanently increases government expenditure, and the DR firms start debt repayment. The welfare continues to improve until it reaches the new steady state. Repaying large debt takes a long time, about 80 years, in our quarterly model. In period 316, switching the borrowing constraint from (2) to (1) generates a kink in macroeconomic variables: Welfare, C_t, L_t, K_t, Y_t and w_t.

Thus, fiscal policies are restricted by constraints, $b_z^* > b_z$ and $\mathbb{W}^* > \mathbb{W}$. This result depends on the specification of the utility function. If we assume that $\gamma_G = 0$ (i.e., the utility is not derived from government expenditure), fiscal policies cannot improve welfare (i.e., $\mathbb{W}^* > \mathbb{W}$ is never satisfied).

The other question is whether a permanent corporate tax cut promotes debt repayment. The effect of permanent corporate tax cuts is limited and only supplementary. Figure 12 shows the results of permanent changes in the corporate tax rate as well as increases in government spending. For comparison, we show that Figure 12 (b) is the same as Figure 10, which is the benchmark case. Figure 12 (a) is a case in which permanent corporate tax is reduced to 1% along with increasing G^* and indicates that the range of fiscal expansion satisfies condition (ii) and is expanding. Figure 12 (c) shows the combined results of a permanent change in the corporate tax rate to 50% and permanently increasing G^* and the region satisfying condition (ii) is narrowed. Hence, an increase in corporate tax narrows the range of fiscal expansion.

6 Robustness analysis

This section investigates the robustness of the results to an alternative specification for the household's utility function. It is particularly controversial matter concerning whether the multiplier for consumption is positive or negative. There are several settings for the consumption multiplier to show a positive response: a rule-of-thumb consumer, a non-separable utility function, and complementarity between consumption and government spending. A non-separable functional form such as a CES-type utility function cannot be solved analytically without a linear approximation and cannot be used in our solution method. Leeper, Traum and Walker (2017) compare model performances between rule-of-thumb specification and government-spending-in-utility with Edgeworth complementarity/substitution between private consumption and government expenditures and conclude that models introducing Edgeworth complementarity can provide a capable wide range of multipliers? the positive/negative consumption multiplier and the output multiplier, which is greater than one. Hence, in this section, we examine the Edgeworth complementarity specification and whether our results are robust to the introduction of Edgeworth complementarity.

Following Fève and Sahuc (2017)'s specification, we introduce Edgeworth complementarity/substitutability between private consumption and government expenditures:

$$\max_{C_t, L_t, \mathbb{B}_t, K_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln\left(C_t + \alpha_G G_t\right) - \gamma_L \frac{L_t^{1+\nu}}{1+\nu} + \gamma_G \ln G_t \right] \right\},$$

and the social welfare function becomes

$$\mathbb{W}_t = \ln\left(C_t + \alpha_G G_t\right) - \gamma_L \frac{L_t^{1+\nu}}{1+\nu} + \gamma_G \ln G_t + E_t \frac{\mathbb{W}_{t+1}}{1+r_{t+1}}, \tag{20}$$

where α_G is the degree of complementarity / substitutability between private consumption and public expenditure. The specification of (15) is the special case $\alpha_G = 0$. If the parameter $\alpha_G < 0$, government expenditures complement private consumption. Recent empirical studies support $\alpha_G < 0$ (e.g., Karras, 1994; Fève, Matheron and Sahuc, 2013; Fève and Sahuc, 2017; Leeper et al., 2017) and we set the value $\alpha_G = -0.6340$ which is estimated by Fève et al. (2013) using U.S. data.

First, we consider the temporary debt problem. Figure 13 and Figure 14 show the numerical simulation results of the corporate tax cut multipliers and government expenditure multipliers. On the one hand, the corporate tax cut multipliers are almost indistinguishable from the results in Figure 7, but on the other hand, the government expenditure multipliers are different from those in Figure 9. Consumption responds positively, and output responds more significantly for a while due to Edgeworth complementarity and then turns negative. The welfare multipliers worsen because $\alpha_G < 0$. Thus, the levels of multipliers change, but the conclusion remains the same: the effect of the corporate tax cut depends on the level of corporate debt and is more effective for the temporary debt problem.

Next, we consider the permanent debt problem. The qualitative results remain the same, even if Edgeworth complementarity is introduced.[8] As in Figure 10, Figure 15 shows the parameter region where permanent fiscal policy is effective. Comparing Figure 10 and Figure 15, the yellow region where permanent fiscal policy is effective has expanded. This is because the output multiplier becomes larger after introducing Edgeworth complementarity, as a result of which the gain in tax benefits from borrowing the maximum amount of repayable debt becomes relatively small.

7 Conclusion

We have examined the effects of fiscal policies, such as expansion of government spending and corporate tax cuts on the macroeconomy and welfare under the corporate debt problem using a nonlinear solution method. It is well known that a corporate tax cut has the tax advantage that affects the capital structure for firms, and this study showed that the government expansion policy also affects the capital structure. During a recession, implementing the corporate tax cut or expanding government spending as an economic stimulus measure affects firms through changes in their capital structure. We show that the corporate tax cut multipliers vary with the level of corporate debt and become large when the debt level is large, whereas the government expenditure multipliers do not.

We show that borrowers who owe the maximum repayable debt fall into a DR state in which they can repay only the interest and cannot reduce the principal repayable amount of the debt, which means that they continue inefficient production forever. In this situation, permanent fiscal expansion policies are effective. Firms and lenders agree to proceed with debt repayment because the debt thresholds are changed by permanent policy intervention. Firms choose to operate with debt repayment, and eventually, the corporate debt level is reduced to the CE level, and the economy reaches the CE steady state. However, if the ratio of DR firms is large, the required fiscal expansion becomes large. When the ratio of DR firms exceeds a certain level, the welfare losses caused by distortionary fiscal policy outweighs the welfare improvement due to the resolution of the excess debt problem, and then the policy intervention is not justified.

Several extensions of our framework would be helpful to explore in future research.

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[8] In this model, because private consumption and government expenditures are non-separable, there is a condition for $\overline{G}/\overline{Y}$ to guarantee finite marginal consumption in the steady state:

$$\frac{1-\delta\frac{\overline{K}}{\overline{Y}}}{1-\alpha_G} > \frac{\overline{G}}{\overline{Y}}. \tag{21}$$

First, it would be desirable to consider changes in the distribution of corporate debt. To simplify the analysis, we considered two types of firms: those with the optimal debt and those with a large debt. By introducing heterogeneity, it would be interesting to consider how the level of debt differs across firms and how its distribution produces quantitative differences in the effects of fiscal policy. The next step in this study is to focus on government spending and the corporate tax cut, but also to consider other fiscal policy instruments such as subsidies and bond purchases, as in Bianchi (2016).

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A Derivation of the borrowing constraint

In this appendix, we derive the borrowing constraint (1) as the no-default condition. The lender lends the firm working capital loans q_t that satisfy the no-default condition. The firm never chooses to default on working capital loans. Hence, defaulting is an off-the equilibrium path event. We describe the events that follow a counterfactual default on working capital loans. This derivation follows Kobayashi and Shirai (2018) and their argument is similar to that of Jermann and Quadrini (2012).

Working capital loans q_t are an intra-period debt, financed at the beginning of the current period and repaid after the realization of revenues. This timing is described in section 2.2. At the end of period t, the firm has a choice between repayment or defaulting on q_t.

If the firm chooses to default q_t, on the one hand, the lender seizes a part of the firm's revenue unilaterally, $\phi f(q_t)$, where $0 \leq \phi \leq 1$. The lender has the option to liquidate the firm. If the lender decides to liquidate the firm, the lender succeeds in getting control with probability ξ and fails to get control with probability $1-\xi$. Thus, the expected liquidation value that the lender can obtain is ξS_t. By contrast, if the lender decides to allow the firm to continue to operate, the lender can collect the inter-period debt b_t in the next period. Therefore, the lender's liquidation net value when the firm chooses to default q_t is $\xi S_t - b_t$. On the other hand, the firm begins to negotiate with the lender for operation continuation. For simplicity, we assume that the firm has all the bargaining power in the renegotiation as with Jermann and Quadrini (2012). The firm offers a continuation fee to the lender. If the continuation fee is greater than or equal to $\xi S - b_t$, the lender allows the firm to continue to operate. Hence, this renegotiation agreement depends on whether ξS_t is greater than or less than b_t.

Case where $\xi S_t > b_t$: In this case, the firm makes an offer to pay a continuation fee equal to $\xi S_t - b_t$. This payment is indifferent between liquidation and continuation for the lender, and the lender accepts this offer. The firm makes payment $\xi S_t - b_t$ and promises to pay $(1+r_t)b_t$ at the beginning of the next period. Therefore, the ex-post counterfactual default value for the firm is

$$(1-\tau_t^{corp})\left[f(q_t) - r_t b_{t-1}\right] - b_{t-1} + b_t - (1-\tau_t^{corp})\left[\phi f(q_t) + \{\xi S - b_t\}\right] + \beta E_t\left[\frac{V_{t+1}}{1+r_{t+1}}\right].$$

Case where $\xi S_t \le b_t$: In this case, the lender's liquidation net value is negative, $\xi S_t - b_t < 0$, and the continuation fee from the firm equals zero. Hence, the lender never chooses liquidation, and the optimal choice for the lender is to wait until the next period and receive repayment of intra-period debt $(1+r_t)b_t$. Thus, the ex-post counterfactual default value is

$$(1-\tau_t^{corp})\left[f(q_t) - r_t b_{t-1}\right] - b_{t-1} + b_t - (1-\tau_t^{corp})\phi f(q_t) + \beta E_t\left[\frac{V_{t+1}}{1+r_{t+1}}\right].$$

Therefore, the default value is expressed as

$$(1-\tau_t^{corp})\left[(1-\phi)f(q_t) - r_t b_{t-1}\right] - b_{t-1} + b_t - \max\left\{(1-\tau_t^{corp})\left(\xi S - b_t\right),\ 0\right\} + \beta E_t\left[\frac{V_{t+1}}{1+r_{t+1}}\right].$$

Enforcement requires that the value of not defaulting is no smaller than the value of defaulting, that is,

$$\begin{aligned}&(1-\tau_t^{corp})\left[f(q_t) - q_t - r b_{t-1}\right] - b_{t-1} + b_t \ge \\ &\quad (1-\tau_t^{corp})\left[(1-\phi)f(q_t) - r_t b_{t-1}\right] - b_{t-1} + b_t - \max\left\{(1-\tau_t^{corp})\left(\xi S - b_t\right),\ 0\right\}\end{aligned}$$

which can be rearranged as (1).

B Equilibrium Conditions

This Appendix lists the complete set of equilibrium conditions for the model. The main model which is described in Section 2 is the special case $\alpha_G = 0$.

B.1 Household optimality conditions

The optimality conditions for the household problem described in subsection 2.6 are

$$\begin{aligned} w_t &= \gamma_L (C_t + \alpha_G G_t) L_t^{\nu}, \\ \frac{1}{C_t + \alpha_G G_t} &= E_t \left[\frac{1}{C_{t+1} + \alpha_G G_{t+1}} \beta (1 - \delta + r_{t+1}^K) \right], \\ \frac{1}{C_t + \alpha_G G_t} &= E_t \left[\frac{1}{C_{t+1} + \alpha_G G_{t+1}} \beta (1 + r_{t+1}) \right], \\ \mathbb{W}_t &= \ln(C_t + \alpha_G G_t) - \gamma_L \frac{L_t^{1+\nu}}{1+\nu} + \gamma_G \ln G_t + E_t \frac{\mathbb{W}_{t+1}}{1 + r_{t+1}}. \end{aligned} \tag{22}$$

B.2 Intermediate goods firms optimality conditions

The optimality conditions for the intermediate goods firm problem described in subsection 2.2 are

$$V_{i,t} = \max \left\{ V_{i,t}^N, V_{i,t}^Z \right\},$$

$$V_{i,t}^N = \max \ \pi_{i,t} + E_t \left[\frac{V_{i,t+1}}{1 + r_{t+1}} \right], \tag{23}$$

$$V_{i,t}^Z = \max \ (1 - \tau_t^{corp}) \left[A_t k_{i,t}^{\alpha\eta} l_{i,t}^{(1-\alpha)\eta} - w_t l_{i,t} - r_t^K k_{i,t} - r_t b_{i,t-1} \right] - b_{i,t-1} + b_z,$$

$$\pi_{i,t} = (1 - \tau_t^{corp}) \left[A_t k_{i,t}^{\alpha\eta} l_{i,t}^{(1-\alpha)\eta} - w_t l_{i,t} - r_t^K k_{i,t} - r_t b_{i,t-1} \right] - b_{i,t-1} + b_{i,t}, \tag{$\lambda_{i,t}$}$$

$$w_t l_{i,t} + r_t^K k_{i,t} \leq \phi A_t k_{i,t}^{\alpha\eta} l_{i,t}^{(1-\alpha)\eta} + \max \left\{ \xi S_t - b_{i,t}, \ 0 \right\}, \tag{$\mu_{i,t}$}$$

$$\pi_{i,t} \geq 0, \tag{$\lambda_{\pi_{i,t}}$}$$

$$\begin{cases} \text{if } \xi S_t - b_{i,t} > 0, & 1 - \frac{\mu_{i,t}}{\lambda_{i,t}} = E_t \left[\frac{\lambda_{i,t+1}}{\lambda_{i,t}} \frac{R_{t+1}}{1 + r_{t+1}} \right], \\ \text{if } \xi S_t - b_{i,t} \leq 0, & 1 = E_t \left[\frac{\lambda_{i,t+1}}{\lambda_{i,t}} \frac{R_{t+1}}{1 + r_{t+1}} \right], \end{cases} \tag{24}$$

$$1 + \lambda_{\pi_{i,t}} - \lambda_{i,t} = 0$$

$$r_t^K = \alpha\eta \frac{1 - \tau_t^{corp} + \phi \frac{\mu_{i,t}}{\lambda_{i,t}}}{1 - \tau_t^{corp} + \frac{\mu_{i,t}}{\lambda_{i,t}}} \frac{A_t k_{i,t}^{\alpha\eta} l_{i,t}^{(1-\alpha)\eta}}{k_{i,t}},$$

$$w_t = (1 - \alpha)\eta \frac{1 - \tau_t^{corp} + \phi \frac{\mu_{i,t}}{\lambda_{i,t}}}{1 - \tau_t^{corp} + \frac{\mu_{i,t}}{\lambda_{i,t}}} \frac{A_t k_{i,t}^{\alpha\eta} l_{i,t}^{(1-\alpha)\eta}}{l_{i,t}},$$

$$y_{i,t} = a_t k_{i,t}^{\alpha} l_{i,t}^{1-\alpha},$$

where $i \in \{n, d\}$.

B.3 Final goods firm optimality conditions

The optimality conditions for the final goods firm problem described in subsection 2.5 are

$$
\begin{aligned}
Y_t &= \left[\zeta y_{d,t}^{\eta} + (1-\zeta) y_{n,t}^{\eta}\right]^{\frac{1}{\eta}}, \\
A_t &= a_t^{\eta} Y_t^{1-\eta},
\end{aligned}
$$

where $(1-\zeta)$ is the ratio of normal firms that owe the CE amount of debt in the equilibrium. If $b_{d,t-1} = b_z$, firms with a ζ ratio are DR firms. When $b_{d,t-1}$ is smaller than B_c, all firms eventually become normal firms in the steady state (i.e., $b_d = b_n = b^{ce}$).

B.4 Government

The government's budget constraint and fiscal policy rules as described in subsection 2.7 are

$$
\begin{aligned}
G_t =& \tau_t^{corp} \zeta \left[A_t k_{d,t}^{\alpha\eta} l_{d,t}^{(1-\alpha)\eta} - w_t l_{d,t} - r_t^K k_{d,t} - r_t b_{d,t-1}\right] \\
&+ \tau_t^{corp}(1-\zeta)\left[A_t k_{n,t}^{\alpha\eta} l_{n,t}^{(1-\alpha)\eta} - w_t l_{n,t} - r_t^K k_{n,t} - r_t b_{n,t-1}\right] + \tau_t^{lump-sum},
\end{aligned}
$$

B.5 Exogenous Processes

The exogenous processes in the model are given by:

$$
\begin{aligned}
\ln a_{t+1} &= \rho_a \ln a_t + (1-\rho_a)\ln \overline{a} + \epsilon_{a,t+1}, \\
\ln \tau_{t+1}^{corp} &= \rho_\tau \ln \tau_t^{corp} + (1-\rho_\tau)\ln \overline{\tau}^{corp} + \epsilon_{\tau,t+1}, \\
\ln G_{t+1} &= \rho_G \ln G_t + (1-\rho_G)\ln \overline{G} + \epsilon_{G,t+1}.
\end{aligned}
$$

B.6 Market clearing conditions

$$
\begin{aligned}
& C_t + K_t - (1-\delta)K_{t-1} + G_t = Y_t, \\
& \zeta l_{d,t} + (1-\zeta) l_{n,t} = L_t, \\
& \zeta k_{d,t} + (1-\zeta) k_{n,t} = K_{t-1}, \\
& \zeta b_{d,t-1} + (1-\zeta) b_{n,t-1} = \mathbb{B}_{t-1}, \\
& R_t = 1 + (1-\tau_t^{corp}) r_t, \\
& S_t = E_t\left[\frac{V_{n,t+1}}{1+r_{t+1}}\right] + b_{n,t}.
\end{aligned}
$$

C Solving the dynamic general equilibrium model

In this Appendix, we explain solving the dynamic general equilibrium model to obtain policy functions. We apply a fixed-point approach using a modified Smolyak's method which is proposed by Judd et al. (2014). Our numerical simulation is heavily based on Hirose and Sunakawa (2019b). They provide an excellent review for nonlinear solver and estimation for the ZLB on the nominal interest rate. They also provide Matlab codes that are available at `https://github.com/tkksnk/NKZLB`. We modify the code to deal with two occasionally binding constraints, whereas Hirose and Sunakawa (2019b) deal with one occasionally binding constraint. Our model has two occasionally binding constraints: the borrowing constraint and the limited liability constraint.

The standard projection method interpolates using the Chebyshev polynomials function that is often used in this literature. However, kinked functions are difficult to approximate by the Chebyshev polynomials because the Chebyshev polynomials are a linear combination of differentiable functions. It is well-known that occasionally binding constraints often generate policy function with kinks. Solving the policy functions with kinks is not an easy task. Following Aruoba, Cuba-Borda and Schorfheide (2018), Gust, Herbst, López-Salido and Smith (2017), Hirose and Sunakawa (2016, 2019a,b) and Nakata (2017b), we adapt an index function approach to deal with kinks. We decompose policy functions into three parts using index functions:

$$\begin{aligned}\psi_x(\cdot) =& \mathbb{1}_{\{\xi S_t - b_t > 0,\ \pi_t > 0\}}\psi_{x,nn}(\cdot) + \mathbb{1}_{\{\xi S_t - b_t > 0,\ \pi_t \leq 0\}}\psi_{x,nb}(\cdot) \\ &+ \left(1 - \mathbb{1}_{\{\xi S_t - b_t > 0,\ \pi_t > 0\}} - \mathbb{1}_{\{\xi S_t - b_t > 0,\ \pi_t \leq 0\}}\right)\psi_{x,bb}(\cdot).\end{aligned}$$

where ψ_x is policy functions and x represents each endogenous variable, and $\mathbb{1}$ is an index function and defined by:

$$\begin{aligned}\mathbb{1}_{\{\xi S_t - b_t > 0,\ \pi_t > 0\}} &= 1 \quad \text{if } \xi S_t - b_t > 0 \text{ and } \pi_t > 0, \\ &= 0 \quad \text{otherswise}, \\ \mathbb{1}_{\{\xi S_t - b_t > 0,\ \pi_t \leq 0\}} &= 1 \quad \text{if } \xi S_t - b_t > 0 \text{ and } \pi_t \leq 0, \\ &= 0 \quad \text{otherswise}, \\ 1 - \mathbb{1}_{\{\xi S_t - b_t > 0,\ \pi_t > 0\}} - \mathbb{1}_{\{\xi S_t - b_t > 0,\ \pi_t \leq 0\}} &= 1 \quad \text{if } \xi S_t - b_t \leq 0 \text{ and } \pi_t \leq 0, \\ &= 0 \quad \text{otherswise}.\end{aligned}$$

The first part, which we called nn regime, is assumed that the borrowing constraint is (1), and the limited liability constraint does not bind. The second part, which we called nb regime, is assumed that the borrowing constraint is (1) and the limited liability constraint is binding. The last part, which we called bb regime, is assumed that the borrowing constraint is (2) and the limited liability constraint is binding. These regimes are summarized by Figure 16.

Note that Proposition 4 proves that when the borrowing constraint is (2), the limited liability shall always bind. Hence, we need not consider the situation where the borrowing constraint is (2) and the limited liability constraint does not bind.

We obtain three policy functions corresponding to three regimes for each variable by the Smolyak-based projection method, which is described in detail in Appendix C.1. For example, to obtain the policy function in the bb regime, we assume that the borrowing constraint is (2) and the limited liability constraint is always binding even when $b_t < B_z$. This assumption implies that constraints are not occasionally binding and policy functions of each three regimes are smooth functions. Figure 17 shows policy functions for the corporate debt as an example. The true policy function has kinks and a jump and is difficult to approximate using the standard projection method. This figure shows that the true policy function is approximated by the combination of three smooth policy functions.

C.1 Smolyak's method

Smolyak (1963) proposes a solution method which is one of the methods to avoid the curse of the dimensionality problem associated with the use of a large-scale model, and its application is increasing in economics in recent years. Judd et al. (2014) propose a more efficient implementation of the Smolyak method for interpolation to replace the conventional unidimensional nested-set generators with equivalent unidimensional disjoint-set generators. The conventional Smolyak method involves the same kind of repetitions, and it has inefficient and expensive.

Following Judd et al. (2014), we construct the Smolyak polynomials using extrema of second-order Chebyshev polynomials and unidimensional second-order Chebyshev polynomials. In the algorithm, the level of approximation is set at 2, following Fernández-Villaverde, Gordon, Guerrón-Quintana and Rubio-Ramírez (2015) and Hirose and Sunakawa (2019a). We first show the unidimensional grid points using extrema of second-order Chebyshev polynomials: [9]

$$\mathcal{S}_1 = \{x_j\}_{j=0}^{1-1} = \left\{\cos\left(\frac{j\pi}{N-1}\right)\right\}_{j=0}^{0} = \{0\}\,,$$
$$\mathcal{S}_2 = \{x_j\}_{j=0}^{3-1} = \left\{\cos\left(\frac{j\pi}{N-1}\right)\right\}_{j=0}^{2} = \{0,-1,1\}\,,$$

and the disjoint sets of the unidimensional grid points:

$$\mathcal{A}_1 = \mathcal{S}_1 = \{0\}\,,$$
$$\mathcal{A}_2 = \mathcal{S}_2 \backslash \mathcal{S}_1 = \{-1,1\}\,.$$

[9] These are sets of Chebyshev extrema at degrees 1 and 3, and required to satisfy two conditions:

- *Condition 1*: A set $\mathcal{S}_i,\ i = 1,2,\cdots$, has $m(i) = 2^{i-1}+1$ points for $i \geq 2$ and $m(1) \equiv 1$.
- *Condition 2*: Each subsequent set contains all points of the previous set, $\mathcal{S}_i \subset \mathcal{S}_{i+1}$. Such sets are called *nested*.

See Judd et al. (2014) section 2.2.1 for more detail.

Smolyak (1963)'s tensor products selection rule is

$$\# \text{ of state variables} \leq i_{b_n} + i_{b_d} + i_K + i_a + i_{\tau^{corp}} + i_G \leq \# \text{ of state variables} + \text{approximation level},$$

where approximation level $\in \{0, 1, 2, \cdots\}$, $i_* \in \{1, 2, \cdots\}$ is an index for disjoint sets $\mathcal{A}_{i_*}$ for each state variable, and $* \in \{b_n, b_d, K, a, \tau^{corp}, G\}$. This rule implies that which terms must be selected from tensor products, and the sum of indices i_* must be between this inequality. In our model, the number of state variables is six. We set the approximation level to 1. Hence, $6 \leq i_{b_n} + i_{b_d} + i_K + i_a + i_\tau + 1_G \leq 7$, and the disjoint sets of the unidimensional grid points must be chosen by $(i_{b_n}, i_{b_d}, i_K, i_a, i_\tau, i_G) \in \{(1,1,1,1,1,1),\ (2,1,1,1,1,1),\ (1,2,1,1,1,1),\ (1,1,2,1,1,1),\ (1,1,1,2,1,1), (1,1,1,1,2,1),\ (1,1,1,1,1,2)\ \}$. We have the selected unidimensional disjoint sets:

$$\begin{aligned}
\mathcal{A}_{1,1,1,1,1,1} &= \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 = \{(0,0,0,0,0,0)\}\,, \\
\mathcal{A}_{2,1,1,1,1,1} &= \mathcal{A}_2 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 = \{(-1,0,0,0,0,0),(1,0,0,0,0,0)\}\,, \\
\mathcal{A}_{1,2,1,1,1,1} &= \mathcal{A}_1 \otimes \mathcal{A}_2 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 = \{(0,-1,0,0,0,0),(0,1,0,0,0,0)\}\,, \\
\mathcal{A}_{1,1,2,1,1,1} &= \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_2 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 = \{(0,0,-1,0,0,0),(0,0,1,0,0,0)\}\,, \\
\mathcal{A}_{1,1,1,2,1,1} &= \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_2 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 = \{(0,0,0,-1,0,0),(0,0,0,1,0,0)\}\,, \\
\mathcal{A}_{1,1,1,1,2,1} &= \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_2 \otimes \mathcal{A}_1 = \{(0,0,0,0,-1,0),(0,0,0,0,1,0)\}\,, \\
\mathcal{A}_{1,1,1,1,1,2} &= \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \mathcal{A}_2 = \{(0,0,0,0,0,-1),(0,0,0,0,0,1)\}\,.
\end{aligned}$$

We have construct the Smolyak grid points as:

$$\begin{aligned}
&\mathcal{A}_{1,1,1,1,1,1} \cup \mathcal{A}_{2,1,1,1,1,1} \cup \mathcal{A}_{1,2,1,1,1,1} \cup \mathcal{A}_{1,1,2,1,1,1} \cup \mathcal{A}_{1,1,1,2,1,1} \cup \mathcal{A}_{1,1,1,1,2,1} \cup \mathcal{A}_{1,1,1,1,1,2} \\
&= \{(0,0,0,0,0,0),(-1,0,0,0,0,0),(1,0,0,0,0,0),(0,-1,0,0,0,0),(0,1,0,0,0,0), \\
&\quad (0,0,-1,0,0,0),(0,0,1,0,0,0),(0,0,0,-1,0,0),(0,0,0,1,0,0),(0,0,0,0,-1,0),(0,0,0,0,1,0), \\
&\quad (0,0,0,0,0,-1),(0,0,0,0,0,1)\}.
\end{aligned}$$

We rewrite the matrix form for the Smolyak grid points:

$$\mathcal{H} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Note that the number of columns is the number of state variables.

Similarly, we construct the Smolyak basis functions and disjoint sets of basis functions:

$$\mathcal{F}_1(x) = \{T_0(x)\},$$
$$\mathcal{F}_2(x) = \{T_1(x), T_2(x)\}.$$

where $T_i(x)$ is the i-th order Chebyshev basis function and

$$T_0(x) = 1,$$
$$T_1(x) = x,$$
$$T_2(x) = 2x^2 - 1.$$

We have the selected the tensor products of the unidimensional disjoint sets for basis functions:

$$\mathcal{F}_{1,1,1,1,1,1}(h) = \mathcal{F}_1(x_1) \otimes \mathcal{F}_1(x_2) \otimes \mathcal{F}_1(x_3) \otimes \mathcal{F}_1(x_4) \otimes \mathcal{F}_1(x_5) \otimes \mathcal{F}_1(x_6) = \{1\},$$
$$\mathcal{F}_{2,1,1,1,1,1}(h) = \mathcal{F}_2(x_1) \otimes \mathcal{F}_1(x_2) \otimes \mathcal{F}_1(x_3) \otimes \mathcal{F}_1(x_4) \otimes \mathcal{F}_1(x_5) \otimes \mathcal{F}_1(x_6) = \{T_1(x_1),\ T_2(x_1)\},$$
$$\mathcal{F}_{1,2,1,1,1,1}(h) = \mathcal{F}_1(x_1) \otimes \mathcal{F}_2(x_2) \otimes \mathcal{F}_1(x_3) \otimes \mathcal{F}_1(x_4) \otimes \mathcal{F}_1(x_5) \otimes \mathcal{F}_1(x_6) = \{T_1(x_2),\ T_2(x_2)\},$$
$$\mathcal{F}_{1,1,2,1,1,1}(h) = \mathcal{F}_1(x_1) \otimes \mathcal{F}_1(x_2) \otimes \mathcal{F}_2(x_3) \otimes \mathcal{F}_1(x_4) \otimes \mathcal{F}_1(x_5) \otimes \mathcal{F}_1(x_6) = \{T_1(x_3),\ T_2(x_3)\},$$
$$\mathcal{F}_{1,1,1,2,1,1}(h) = \mathcal{F}_1(x_1) \otimes \mathcal{F}_1(x_2) \otimes \mathcal{F}_1(x_3) \otimes \mathcal{F}_2(x_4) \otimes \mathcal{F}_1(x_5) \otimes \mathcal{F}_1(x_6) = \{T_1(x_4),\ T_2(x_4)\},$$
$$\mathcal{F}_{1,1,1,1,2,1}(h) = \mathcal{F}_1(x_1) \otimes \mathcal{F}_1(x_2) \otimes \mathcal{F}_1(x_3) \otimes \mathcal{F}_1(x_4) \otimes \mathcal{F}_2(x_5) \otimes \mathcal{F}_1(x_6) = \{T_1(x_5),\ T_2(x_5)\},$$
$$\mathcal{F}_{1,1,1,1,1,2}(h) = \mathcal{F}_1(x_1) \otimes \mathcal{F}_1(x_2) \otimes \mathcal{F}_1(x_3) \otimes \mathcal{F}_1(x_4) \otimes \mathcal{F}_1(x_5) \otimes \mathcal{F}_2(x_6) = \{T_1(x_6),\ T_2(x_6)\},$$

We have construct the Smolyak basis function as follows:

$$\mathfrak{T}(x) \equiv \mathcal{F}_{1,1,1,1,1,1}(x) \cup \mathcal{F}_{2,1,1,1,1,1}(x) \cup \mathcal{F}_{1,2,1,1,1,1}(x) \cup \mathcal{F}_{1,1,2,1,1,1}(x) \cup$$

$$\mathcal{F}_{1,1,1,2,1,1}(x) \cup \mathcal{F}_{1,1,1,1,2,1}(x) \cup \mathcal{F}_{1,1,1,1,1,2}(x),$$
$$= \{1,\ T_1(x_1),\ T_2(x_1),\ T_1(x_2),\ T_2(x_2),\ \cdots,\ T_1(x_6),\ T_2(x_6)\},$$

and there are thirteen Smolyak basis functions.

Now we are ready to interpolate a function using the following form:

$$\begin{bmatrix} e(\mathcal{H}_1) \\ e(\mathcal{H}_2) \\ \vdots \\ e(\mathcal{H}_{13}) \end{bmatrix} = \begin{bmatrix} 1 & T_1(\mathcal{H}_{1,1}) & T_2(\mathcal{H}_{1,1}) & T_1(\mathcal{H}_{1,2}) & T_2(\mathcal{H}_{1,2}) & \cdots & T_1(\mathcal{H}_{1,6}) & T_2(\mathcal{H}_{1,6}) \\ 1 & T_1(\mathcal{H}_{2,1}) & T_2(\mathcal{H}_{2,1}) & T_1(\mathcal{H}_{2,2}) & T_2(\mathcal{H}_{2,2}) & \cdots & T_1(\mathcal{H}_{2,6}) & T_2(\mathcal{H}_{2,6}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & T_1(\mathcal{H}_{13,1}) & T_2(\mathcal{H}_{13,1}) & T_1(\mathcal{H}_{13,2}) & T_2(\mathcal{H}_{13,2}) & \cdots & T_1(\mathcal{H}_{13,6}) & T_2(\mathcal{H}_{13,6}) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{13} \end{bmatrix}$$

or

$$e(\mathcal{H}) = \mathfrak{T}(\mathcal{H})\boldsymbol{\theta},$$

where e is an expectation function which is defined in the following subsection and approximated by the Smolyak polynomials, $\mathcal{H}_j$ is j-th row vector of $\mathcal{H}$, $\mathcal{H}_{j,i}$ is an element in row j and column i of $\mathcal{H}$, and $\{\theta_j\}_{j=1}^{13}$ are unknown coefficients for Smolyak basis functions. The solution to this system is given by

$$\boldsymbol{\theta} = \mathfrak{T}(\mathcal{H})^{-1} e(\mathcal{H}).$$

In our setting, we have

$$\mathfrak{T}(\mathcal{H}) = \begin{bmatrix} 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 1 & 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 0 & -1 & 1 & 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 0 & -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 0 & -1 & 0 & -1 & 1 & 1 & 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 0 & -1 & 0 & -1 & 0 & -1 & -1 & 1 & 0 & -1 & 0 & -1 \\ 1 & 0 & -1 & 0 & -1 & 0 & -1 & 1 & 1 & 0 & -1 & 0 & -1 \\ 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & -1 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 1 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & -1 & 1 \\ 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 1 & 1 \end{bmatrix}$$

and

$$
\mathfrak{I}(\mathcal{H})^{-1} = \begin{bmatrix}
-2 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4}
\end{bmatrix}.
$$

C.2 Fixed-point iteration

In this subsection, we solve our DSGE model using the fixed-point iteration with the index function approach. The model used in this subsection is described in section 6. The main model is the special case $\alpha_G = 0$. Fixed-point iteration is one of the solution methods and is commonly used for the DSGE model. We approximate expectation terms in Euler equations and the value function using Gauss-Hermite quadrature and solve by the fixed-point iteration, which is also called the parameterized expectation approach. See, for example, Judd (1998), Marcet and Lorenzoni (1999), Christiano and Fisher (2000), Collard (2002), Heer and Maussner (2009) Ch.5, Judd et al. (2014), Gust et al. (2017). The number of Gauss–Hermite quadrature is set to three.

Following Gust et al. (2017) and Hirose and Sunakawa (2019b), we define the expectation functions for expectation terms of the right-hand side in Euler equations (Equation 22 and 24) and the value function (Equation 23) as follows:

$$
\begin{aligned}
e_{C,jj}(h) &\equiv E\left[\frac{C' + \alpha_G G'}{\beta(1+r')}\right], \quad jj = nn, nb, bb, \\
e_{\mu_n,jj}(h) &\equiv E\left[\frac{\lambda'}{\lambda}\frac{R'}{1+r'}\right], \quad jj = nn, nb, bb, \\
e_{V,jj}(h) &\equiv E\left[\frac{V'}{1+r'}\right], \quad jj = nn, nb, bb, \\
\psi^{(i)}_{e_*,jj}(h;\boldsymbol{\theta}) &\approx e_{*,jj}(h), \quad * = \{C, \mu_n, V\}
\end{aligned}
$$

where $\psi^{(i)}_{*,jj}(h;\boldsymbol{\theta})$ is a policy function, $h = [K, b_n, b_d, a, \tau^{corp}, G]$ and jj is an index for regimes. In this Appendix, to clarify the notation, we shall use letters without time-subscript to denote current period values and a prime to denote the next period's value.

Initialization

1. Set an upper bound and a lower bound for each state variable.

2. Make a grid matrix for state variables using the matrix for the Smolyak grid points:

$$h = \mathbf{1} \cdot h_c \odot \mathcal{H} + \mathbf{1} \cdot \overline{h}$$

where $\mathbf{1}$ is a row vector of ones and every element is equal to one, $h_c \equiv \left[\frac{K^{\max}-K^{\min}}{2},\ \frac{b_n^{\max}-b_n^{\min}}{2},\ \frac{b_d^{\max}-b_d^{\min}}{2},\ \frac{a^{\max}-a^{\min}}{2},\ \frac{\tau^{corp\,\max}-\tau^{corp\,\min}}{2},\ \frac{G^{\max}-G^{\min}}{2}\right]$, $x^{\max}$ and $x^{\min}$ are the upper bound and the lower bound for each state variable, $\odot$ denotes element-by-element multiplication, $\overline{h} \equiv [\overline{K},\ \overline{b},\ \overline{b},\ \overline{a},\ \overline{\tau}^{corp},\ \overline{G}]$, and overbars indicate the steady state value of the corresponding variable.

Step 1 Make an initial guess for the expectation functions:

$$\begin{aligned}
e^{(0)}_{C,j,jj} &= \frac{\overline{C} + \alpha_G \overline{G}}{\beta(1+\overline{r})}, && \text{for } j = 1, 2, \cdots, J, \quad jj = nn, nb, bb,\\
e^{(0)}_{\mu_n,j,jj} &= \beta \overline{R} && \text{for } j = 1, 2, \cdots, J, \quad jj = nn, nb, bb,\\
e^{(0)}_{V,j,jj} &= \frac{\overline{V}}{1+\overline{r}}, && \text{for } j = 1, 2, \cdots, J, \quad jj = nn, nb, bb,
\end{aligned}$$

where j is an index for state variables, J is the total number of grid points and equal to 13 in our setting.

Step 2 Compute the coefficients for Smolyak polynomials $\boldsymbol{\theta}$:

$$\begin{aligned}
\boldsymbol{\theta}_{C,jj} &= \mathfrak{T}(\mathcal{H})^{-1} e^{(i-1)}_{C,jj},\\
\boldsymbol{\theta}_{\mu_n,jj} &= \mathfrak{T}(\mathcal{H})^{-1} e^{(i-1)}_{\mu_n,jj},\\
\boldsymbol{\theta}_{V,jj} &= \mathfrak{T}(\mathcal{H})^{-1} e^{(i-1)}_{V,jj},
\end{aligned}$$

where $\boldsymbol{\theta}_{*,jj} = [\theta_{*,jj,0},\ \theta_{*,jj,1},\ \cdots\ ,\theta_{*,jj,J}]'$, and $e^{(i-1)}_{*,jj} = [e^{(i-1)}_{*,1,jj},\ \cdots\ e^{(i-1)}_{*,J,jj}]'$.

Step 3 Choose a grid: $h_j = [K_j, b_{d,j}, b_{n,j}, a_j, \tau^{corp}_j, G_j]$. Exogenous variables are set using the grid: $K_{t-1} = K_j,\ b_{d,t-1} = b_{d,j},\ b_{n,t-1} = b_{n,j},\ a_t = a_j,\ \tau^{corp}_t = \tau^{corp}_j,\ G_t = G_j$.

Step 4 Taking as given the expectation functions previously obtained, solve the dynamics equations for each regime $jj = nn,\ nb,\ bb$.

$$\begin{aligned}
&C_{j,jj} = e^{(i-1)}_{C,j,jj}(h_j) + \alpha_G G_j,\\
&\frac{\mu_{n,j,jj}}{\lambda_{n,j,jj}} = 1 - e^{(i-1)}_{\mu_n,j,jj}(h_j),
\end{aligned}$$

$$
\begin{cases}
\text{If regime is in } nn, & \dfrac{\mu_{d,j,jj}}{\lambda_{d,j,jj}} = \dfrac{\mu_{n,j,jj}}{\lambda_{n,j,jj}} \\
\text{If regime is in } nb, & \dfrac{\mu_{d,j,jj}}{\lambda_{d,j,jj}} \text{ is given by solving the nonlinear equation (50).} \\
\text{If regime is in } bb, & \begin{cases} \dfrac{1-\tau_j^{corp}+\phi\frac{\mu_{d,j,jj}}{\lambda_{d,j,jj}}}{1-\tau_j^{corp}+\frac{\mu_{d,j,jj}}{\lambda_{d,j,jj}}} = \dfrac{\phi}{\eta}, \\ \dfrac{\mu_{d,jj}}{\lambda_{d,jj}} = \dfrac{(\eta-\phi)(1-\tau_j^{corp})}{\phi(1-\eta)} \end{cases}
\end{cases}
$$

Calculate each equation sequentially at time t.

$$
\Omega_{nz,j,jj} \equiv \frac{f(q_{d,j,jj})}{f(q_{n,j,jj})} = \left(\frac{\frac{1-\tau_j^{corp}+\phi\frac{\mu_{d,j,jj}}{\lambda_{d,j,jj}}}{1-\tau_j^{corp}+\frac{\mu_{d,j,jj}}{\lambda_{d,j,jj}}}}{\frac{1-\tau_j^{corp}+\phi\frac{\mu_{n,j,jj}}{\lambda_{n,j,jj}}}{1-\tau_j^{corp}+\frac{\mu_{n,j,jj}}{\lambda_{n,j,jj}}}} \right)^{\frac{\eta}{1-\eta}} \tag{25}
$$

$$
\Psi_{y,j,jj} \equiv \left(\zeta\Omega_{nz,j,jj}+1-\zeta\right)^{\frac{1}{\eta}}, \tag{26}
$$

$$
\Psi_{k,j,jj} \equiv \zeta\Omega_{nz,j,jj}^{\frac{1}{\eta}}+1-\zeta, \tag{27}
$$

$$
k_{n,j,jj} = \frac{K_j}{\Psi_{k,j,jj}}, \tag{28}
$$

$$
l_{n,j,jj} = \left[\frac{(1-\alpha)\eta \frac{1-\tau_j^{corp}+\phi\frac{\mu_{n,j,jj}}{\lambda_{n,j,jj}}}{1-\tau_j^{corp}+\frac{\mu_{n,j,jj}}{\lambda_{n,j,jj}}} \Psi_{y,j,jj}^{1-\eta} a_j k_{n,j,jj}^{\alpha}}{\Psi_{k,j,jj}^{\nu}\gamma_L(C_{j,jj}+\alpha_G G_j)} \right]^{\frac{1}{\alpha+\nu}}, \tag{29}
$$

$$
L_{j,jj} = \Psi_{k,j,jj} l_{n,j,jj}, \tag{30}
$$

$$
w_{j,jj} = \gamma_L\left(C_{j,jj}+\alpha_G G_j\right) L_{j,jj}^{\nu}, \tag{31}
$$

$$
y_{n,j,jj} = a_j k_{n,j,jj}^{\alpha} l_{n,j,jj}^{1-\alpha}, \tag{32}
$$

$$
Y_{j,jj} = \Psi_{y,j,jj} y_{n,j,jj}, \tag{33}
$$

$$
A_{j,jj} = a_j^{\eta} Y_{j,jj}^{1-\eta}, \tag{34}
$$

$$
r_{j,jj}^{K} = \frac{\alpha\eta\left(1-\tau_j^{corp}+\phi\frac{\mu_{n,j,jj}}{\lambda_{n,j,jj}}\right)}{1-\tau_j^{corp}+\frac{\mu_{n,j,jj}}{\lambda_{n,j,jj}}} \frac{A_{j,jj} k_{n,j,jj}^{\alpha\eta} l_{n,j,jj}^{(1-\alpha)\eta}}{k_{n,j,jj}}, \tag{35}
$$

$$
r_{j,jj} = r_{j,jj}^{K} - \delta, \tag{36}
$$

$$
K'_{j,jj} = Y_{j,jj} - G_{j,jj} - C_{j,jj} + (1-\delta)K_j, \tag{37}
$$

$$
q_{n,j,jj} = w_{j,jj} l_{n,j,jj} + r_{j,jj}^{K} k_{n,j,jj}, \tag{38}
$$

$$
f(q_{n,j,jj}) = A_{j,jj} k_{n,j,jj}^{\alpha\eta} l_{n,j,jj}^{(1-\alpha)\eta}, \tag{39}
$$

$$
R_{j,jj} = 1 + r_{j,jj}(1-\tau_j^{corp}), \tag{40}
$$

$$
k_{d,j,jj} = \left[\frac{1-\tau_j^{corp}+\phi\frac{\mu_{d,j,jj}}{\lambda_{d,j,jj}}}{1-\tau_j^{corp}+\frac{\mu_{d,j,jj}}{\lambda_{d,j,jj}}} \eta A_{j,jj} \left(\frac{r_{j,jj}^{K}}{\alpha}\right)^{(1-\alpha)\eta-1} \left(\frac{1-\alpha}{w_{j,jj}}\right)^{(1-\alpha)\eta} \right]^{\frac{1}{1-\eta}}, \tag{41}
$$

$$
l_{d,j,jj} = \frac{(1-\alpha) r_{j,jj}^{K} k_{d,j,jj}}{\alpha w_{j,jj}}, \tag{42}
$$

$$q_{d,j,jj} = w_{j,jj} l_{d,j,jj} + r^K_{j,jj} k_{d,j,jj}, \tag{43}$$

$$f(q_{d,j,jj}) = A_{j,jj} k^{\alpha\eta}_{d,j,jj} l^{(1-\alpha)\eta}_{d,j,jj}, \tag{44}$$

$$b'_{n,j,jj} = \frac{1}{1-\xi}\left[\phi f(q_{n,j,jj}) - q_{n,j,jj} + \xi e^{(i-1)}_{V,jj}(h_j)\right], \tag{45}$$

$$S_{j,jj} = e^{(i-1)}_{V,jj}(h_j) + b'_{n,j,jj}, \tag{46}$$

$$\pi_{n,j,jj} = (1-\tau^{corp}_j)\left[f(q_{n,j,jj}) - q_{n,j,jj} - r_{j,jj} b_{n,j,jj}\right] - b_{n,j} + b'_{n,jj}, \tag{47}$$

$$V_{n,j,jj} = \pi_{n,j,jj} + e^{(i-1)}_{V,jj}(h_j), \tag{48}$$

$$\begin{cases} \text{If } jj = nn, & b'_{d,j,jj} = \phi f(q_{d,j,jj}) - q_{d,j,jj} + \xi S_{j,jj}, \\ & \pi_{d,j,jj} = (1-\tau^{corp}_j)\left[f(q_{d,j,jj}) - q_{d,j,jj} - r_{j,jj} b_{d,j}\right] - b_{d,j} + b'_{d,j,jj}, \\ \text{If } jj = nb, & b'_{d,j,jj} = \phi f(q_{d,j,jj}) - q_{d,j,jj} + \xi S_{j,jj}, \\ & \pi_{d,j,jj} = 0, \\ \text{If } jj = bb, & b'_{d,j,jj} = b_{d,j} - (1-\tau^{corp}_j)\left[f(q_{d,j,jj}) - q_{d,j,jj} - r_{j,jj} b_{d,j}\right], \\ & \pi_{d,j,jj} = 0. \end{cases} \tag{49}$$

If regime is in nb, solve for $\mu_{d,j,jj}/\lambda_{d,j,jj}$ with the equation below numerically: [10]

$$0 = (1-\tau^{corp}_j)\left[f(q_{d,j,jj}) - q_{d,j,jj} - r_{j,jj} b_{d,j}\right] - b_{d,j} + b'_{d,j,jj}. \tag{50}$$

Compute the next period productivity, government expenditures, and the corporate tax rate for $m = 1, 2, \cdots, M$:

$$\begin{aligned} \ln a'_{j,m} &= \rho_a \ln a_j + (1-\rho_a)\ln \overline{a} + \epsilon'_{a,m}, \\ \ln G'_{j,m} &= \rho_G \ln G_j + (1-\rho_G)\ln \overline{G} + \epsilon'_{G,m}, \\ \ln \tau^{corp'}_{j,m} &= \rho_\tau \ln \tau^{corp}_j + (1-\rho_\tau)\ln \overline{\tau}^{corp} + \epsilon'_{\tau,m}. \end{aligned}$$

where $\epsilon'_{*,m}$ are structural shocks and approximated by the Gauss–Hermite quadrature, m is an index for grid points of the shock, and M is the total number of grid points for the shock.

Interpolate between grids using Smolyak polynomials:

$$\begin{aligned} \hat{e}_C(h'_{j,jj,m}; \boldsymbol{\theta}_{C,jj}) &= \mathfrak{T}(\varphi(h'_{j,jj,m}))\boldsymbol{\theta}_{C,jj}, \\ \hat{e}_{\mu_n}(h'_{j,jj,m}; \boldsymbol{\theta}_{\mu_n,jj}) &= \mathfrak{T}(\varphi(h'_{j,jj,m}))\boldsymbol{\theta}_{\mu,jj}, \\ \hat{e}_V(h'_{j,jj,m}; \boldsymbol{\theta}_{V,jj}) &= \mathfrak{T}(\varphi(h'_{j,jj,m}))\boldsymbol{\theta}_{V,jj}, \end{aligned}$$

where $h'_{j,jj,m} = [K'_{j,jj},\ b'_{n,j,jj},\ b'_{d,j,jj},\ a'_{j,m},\ \tau^{corp'}_{j,m}, G'_{j,m}]$. The domain of Chebyshev polynomials is the interval $[-1,1]$, and in order to approximate a function by the Chebyshev polynomials, it is necessary to transform the interval $h_j \in [h^{\min},\ h^{\max}]$ into the interval of $x_j \in [-1,1]$, $h^{\min}$ and $h^{\max}$ are maximum and minimum values

[10] For example, fsolve is a numerical solver in Matlab.

of each state variable chosen to encompass a wide interval. For each state variable in h, we use $\varphi : [h^{\min},\ h^{\max}] \rightarrow [-1,\ 1]$ for $\{K, b_n, b_d, a, \tau^{corp}, G\}$,

$$x_j = \varphi(h_j) = \frac{2(h_j - h^{\min}) - (h^{\max} - h^{\min})}{h^{\max} - h^{\min}}.$$

In calculating $t+1$, we first assume that $\xi S'_{j,jj,m} - b''_{d,j,jj,m} < 0$ and the regime is in bb.

$$\begin{aligned}
C'_{j,jj,m} &= \hat{e}_C(h'_{j,jj,m}; \boldsymbol{\theta}_{C,bb}) + \alpha_G G'_{j,m}, \\
\frac{\mu'_{n,j,jj,m}}{\lambda'_{n,j,jj,m}} &= 1 - \hat{e}_{\mu_n}(h'_{j,jj,m}; \boldsymbol{\theta}_{\mu,bb}), \\
\frac{\mu'_{d,j,jj,m}}{\lambda'_{d,j,jj,m}} &= \frac{(\eta - \phi)(1 - \tau^{corp\prime}_{j,m})}{\phi(1-\eta)}.
\end{aligned}$$

Calculate (25)–(49) at time $t+1$. Check $\xi S'_{j,jj,m} - b''_{d,j,jj,m} < 0$, and if it is satisfied, go to Step 5. If it is not satisfied, next assume that $\xi S'_{j,jj,m} - b''_{d,j,jj,m} > 0$, $\pi'_{d,j,jj,m} > 0$, the regime is in nn, and calculate as the following:

$$\begin{aligned}
C'_{j,jj,m} &= \hat{e}_C(h'_{j,jj,m}; \boldsymbol{\theta}_{C,nn}) + \alpha_G G'_{j,m}, \\
\frac{\mu'_{n,j,jj,m}}{\lambda'_{n,j,jj,m}} &= 1 - \hat{e}_{\mu_n}(h'_{j,jj,m}; \boldsymbol{\theta}_{\mu,nn}), \\
\frac{\mu'_{d,j,jj,m}}{\lambda'_{d,j,jj,m}} &= \frac{(\eta - \phi)(1 - \tau^{corp\prime}_{j,m})}{\phi(1-\eta)}.
\end{aligned}$$

Calculate (25)–(49) at time $t+1$. Check $\pi'_{d,j,jj,m} > 0$, and if it is satisfied, go to Step 5. If it is not satisfied, next we assume that $\xi S'_{j,jj,m} - b''_{d,j,jj,m} > 0$, $\pi'_{d,j,jj,m} < 0$, the regime is in nb, and calculate as the following:

$$\begin{aligned}
C'_{j,jj,m} &= \hat{e}_C(h'_{j,jj,m}; \boldsymbol{\theta}_{C,nb}) + \alpha_G G'_{j,m}, \\
\frac{\mu'_{n,j,jj,m}}{\lambda'_{n,j,jj,m}} &= 1 - \hat{e}_{\mu_n}(h'_{j,jj,m}; \boldsymbol{\theta}_{\mu,nb}), \\
\frac{\mu'_{d,j,jj,m}}{\lambda'_{d,j,jj,m}} & \text{ is given by solving the nonlinear equation (50).}
\end{aligned}$$

Calculate (25)–(49) at time $t+1$.

Step 5 Calculate $\lambda'_{d,j,jj,m}/\lambda_{d,j,jj,m}$:

If regime is in nb, $$\frac{\lambda'_{d,j,nb,m}}{\lambda_{d,j,nb,m}} = \left(1 - \frac{\mu_{d,j,nb,m}}{\lambda_{d,j,nb,m}}\right) \frac{1 + r'_{j,nb,m}}{R'_{j,nb,m}},$$

otherwise, $$\frac{\lambda'_{d,j,jj,m}}{\lambda_{d,j,jj,m}} = 1, \quad jj = nn, bb.$$

Step 6 Calculate numerical integrals by the Gauss-Hermite quadrature:

$$E\left[\frac{C'_{j,jj}+\alpha_G G'_j}{\beta(1+r'_{j,jj})}\right]=\sum_{m=1}^{M}\omega_m\frac{C'_{j,jj,m}+\alpha_G G'_{j,m}}{\beta(1+r'_{j,jj,m})},$$
$$E\left[\frac{R'_{j,jj}}{1+r'_{j,jj}}\right]=\sum_{m=1}^{M}\omega_m\frac{R'_{j,jj,m}}{1+r'_{j,jj,m}},$$
$$E\left[\frac{V'_{n,j,jj}}{1+r'_{j,jj}}\right]=\sum_{m=1}^{M}\omega_m\frac{V'_{j,jj,m}}{1+r'_{j,jj,m}},$$

where ω_m are Gauss–Hermite quadrature weights.

Step 7 Next, substitute in the policy functions:

$$\psi^{(i)}_{*,j,jj}=*_{j,jj},$$
$$e^{(i)}_{C,j,jj}=E\left[\frac{C'_{j,jj}+\alpha_G G'_j}{\beta(1+r'_{j,jj})}\right],$$
$$e^{(i)}_{\mu_n,j,jj}=E\left[\frac{R'_{j,jj}}{1+r'_{j,jj}}\right],$$
$$e^{(i)}_{V,j,jj}=E\left[\frac{V'_{j,jj}}{1+r'_{j,jj}}\right],$$

where $*_{j,jj}=\Big\{C_{j,jj},V_{n,j,jj},K'_{j,jj},r_{j,jj},w_{j,jj},\pi_{n,j,jj},\pi_{d,j,jj},\ b'_{n,j,jj},b'_{d,j,jj},y_{n,j,jj},y_{d,j,jj},$ $\mu_{n,j,jj},\mu_{d,j,jj}\Big\}$.

Step 8 If $||\psi^{(i)}-\psi^{(i-1)}||>1e-6$, update the policy functions and expectation functions by $\psi(i)=\delta_\psi\psi^{(i-1)}+(1-\delta_\psi)\psi^{(i)}$ and $e^{(i)}=\delta_\psi e^{(i-1)}+(1-\delta_\psi)e^{(i)}$, respectively, where δ_ψ is set to 0.8, and go back to Step 3. If $||\psi^{(i)}-\psi^{(i-1)}||\leq 1e-6$, end.

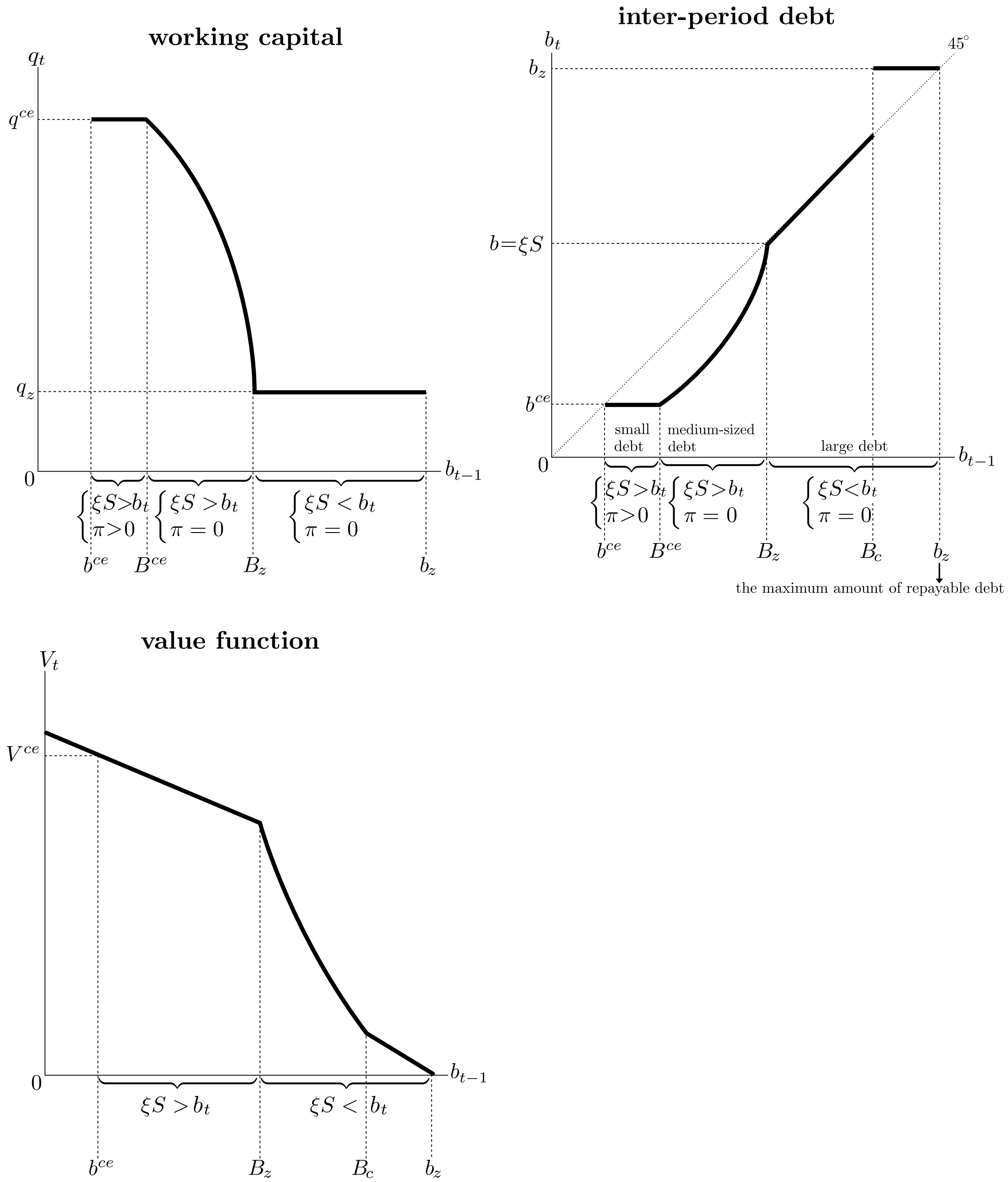
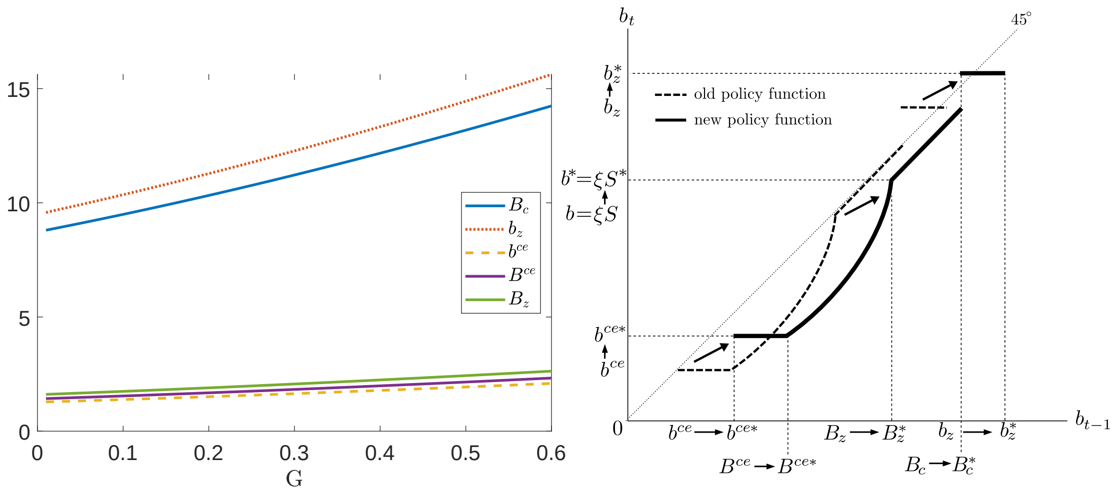


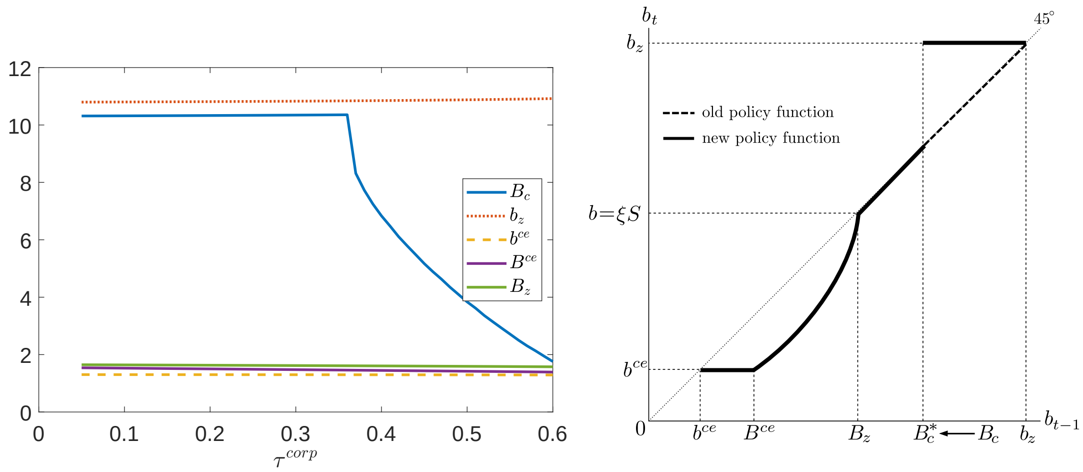
Figure 2: Policy functions and value function



(a) Thresholds and debt level

(b) Change in policy function for b due to expanding government expenditures

Figure 3: Changing the government expenditures



(a) Thresholds and debt level

(b) Change in policy function for b due to corporate tax increases

Figure 4: Increase in Corporate tax

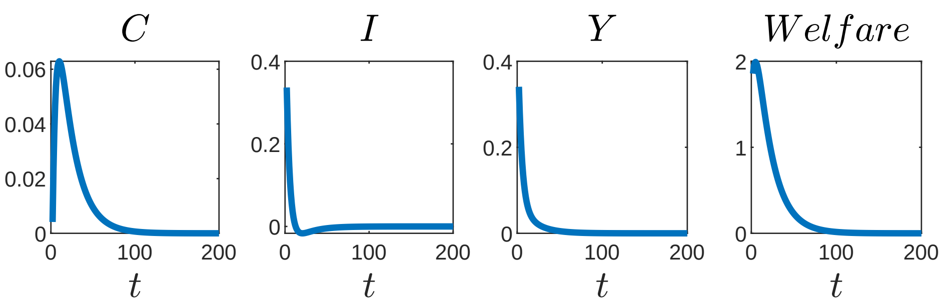


Figure 5: Benchmark case: temporary corporate tax cut without a debt shock

Note: This figure plots impulse responses that are scaled by the impact response of tax revenue evaluated in the non-stochastic steady state, giving the units a multiplier interpretation.

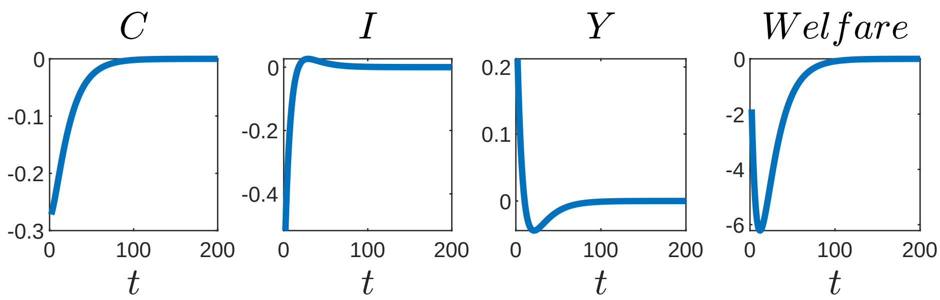


Figure 6: Benchmark case: temporary government spending without a debt shock

Note: This figure plots impulse responses that are scaled by the inverse of the response of the government spending shock on impact, giving the units a multiplier interpretation.

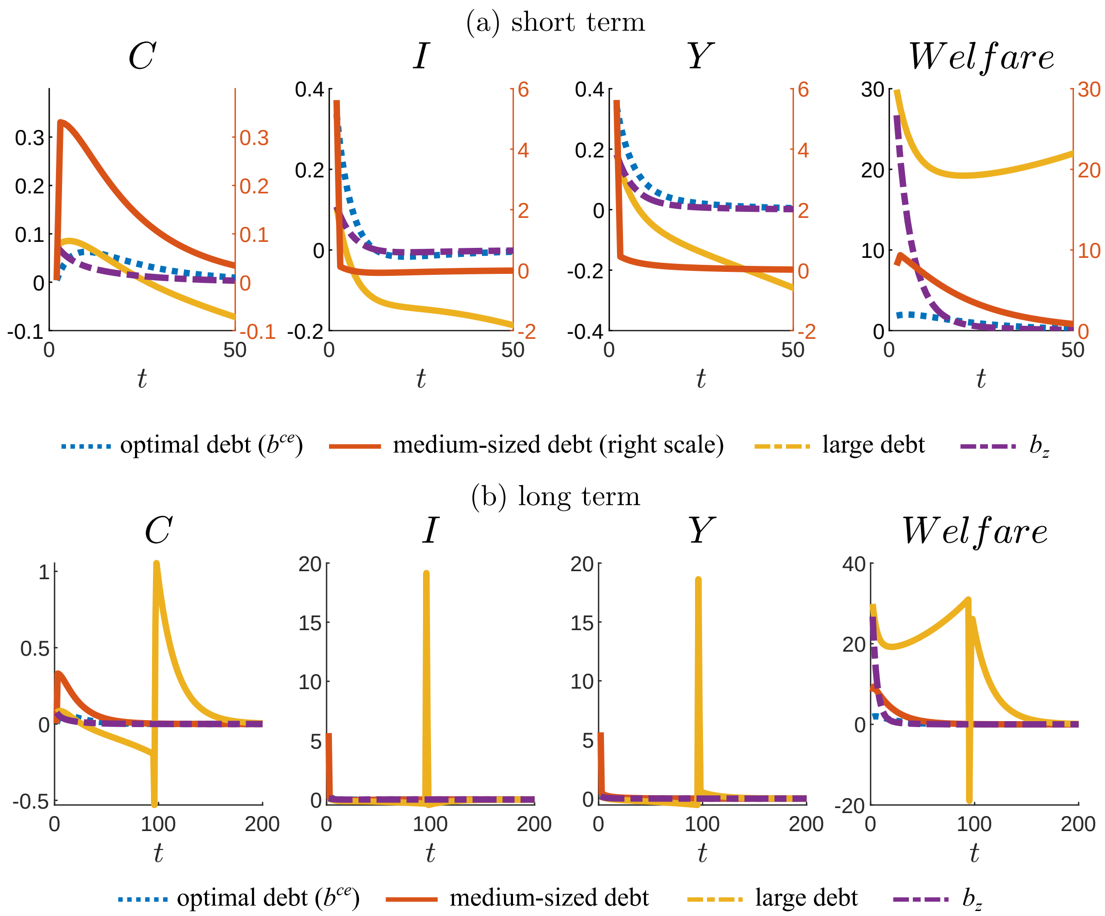


Figure 7: Impulse responses: Corporate tax cut shock

Note: This figure plots impulse responses that are measured gaps between the "with" and "without" shock cases relative to the non-policy intervention and scaled by the impact response of tax revenue evaluated in the non-stochastic steady state, giving the units a multiplier interpretation.

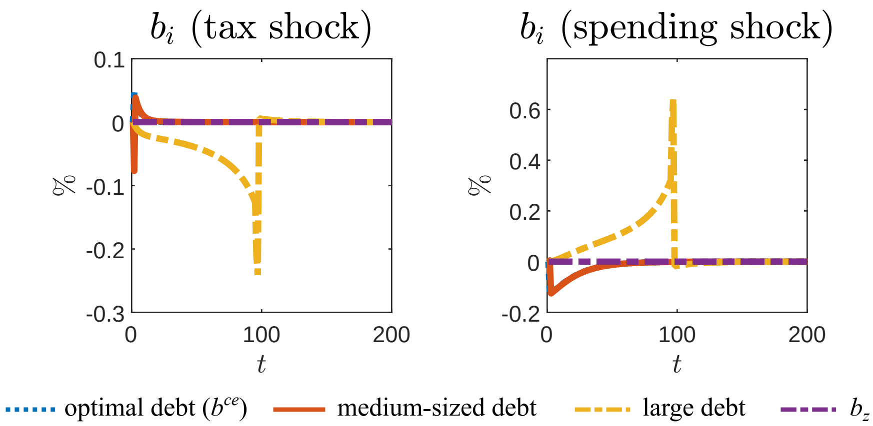


Figure 8: Impulse responses: corporate debt

Note: This figure plots impulse responses for the corporate tax cut shock and the government spending shock, respectively, and measures gaps between the ‘‘with” and ‘‘without” shock cases relative to no policy intervention.

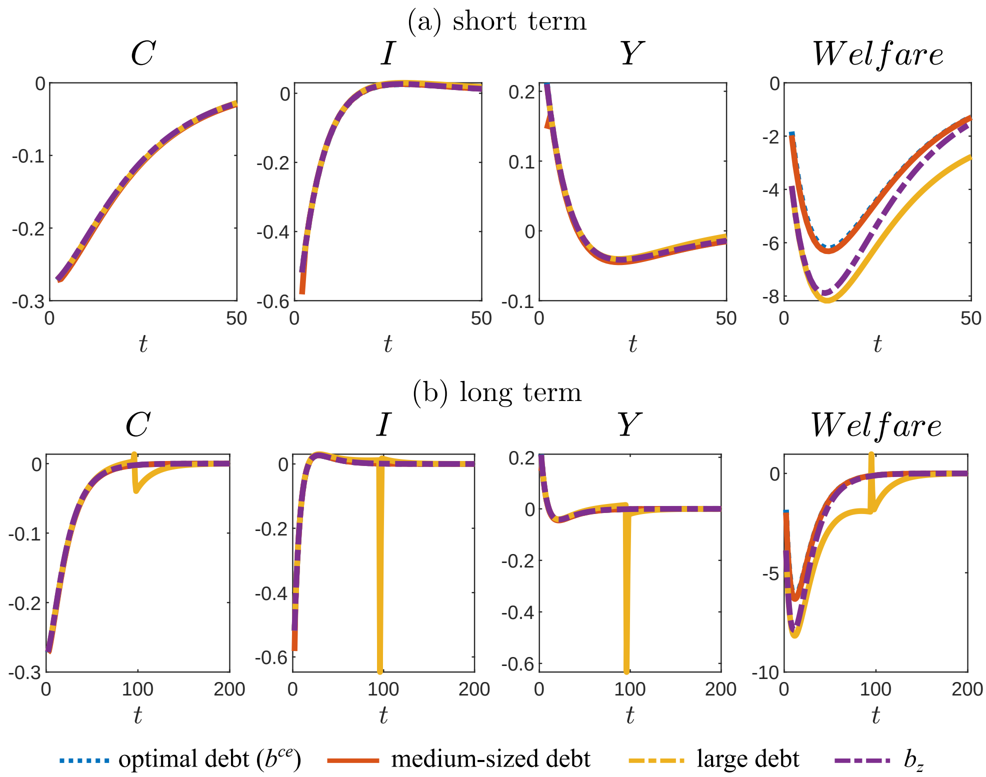


Figure 9: Impulse responses: Government spending shock

Note: This figure plots impulse responses that are scaled by the inverse of the response of the government spending shock on impact, giving the units a multiplier interpretation.

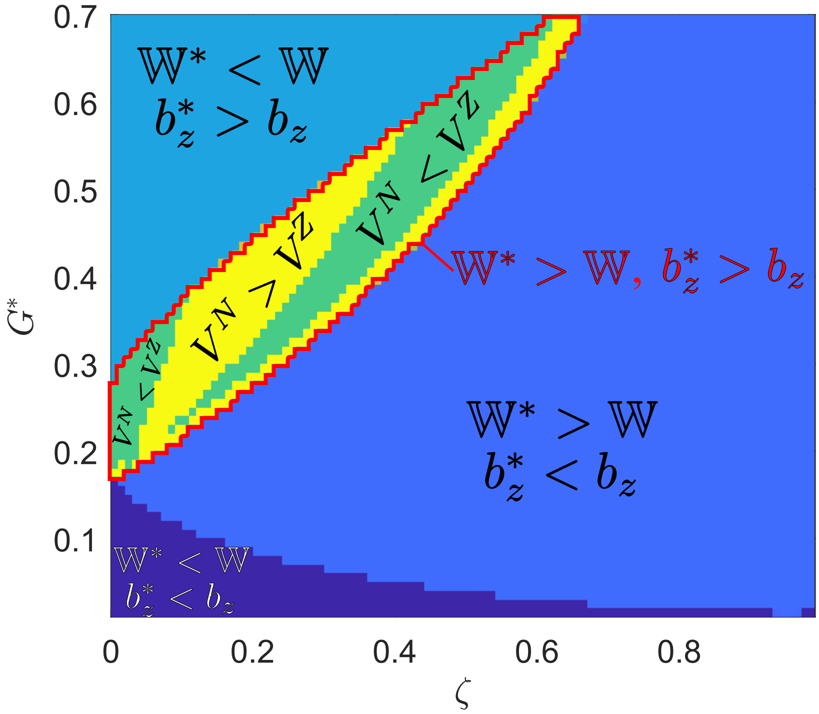


Figure 10: The parameter region for efficient fiscal expansion

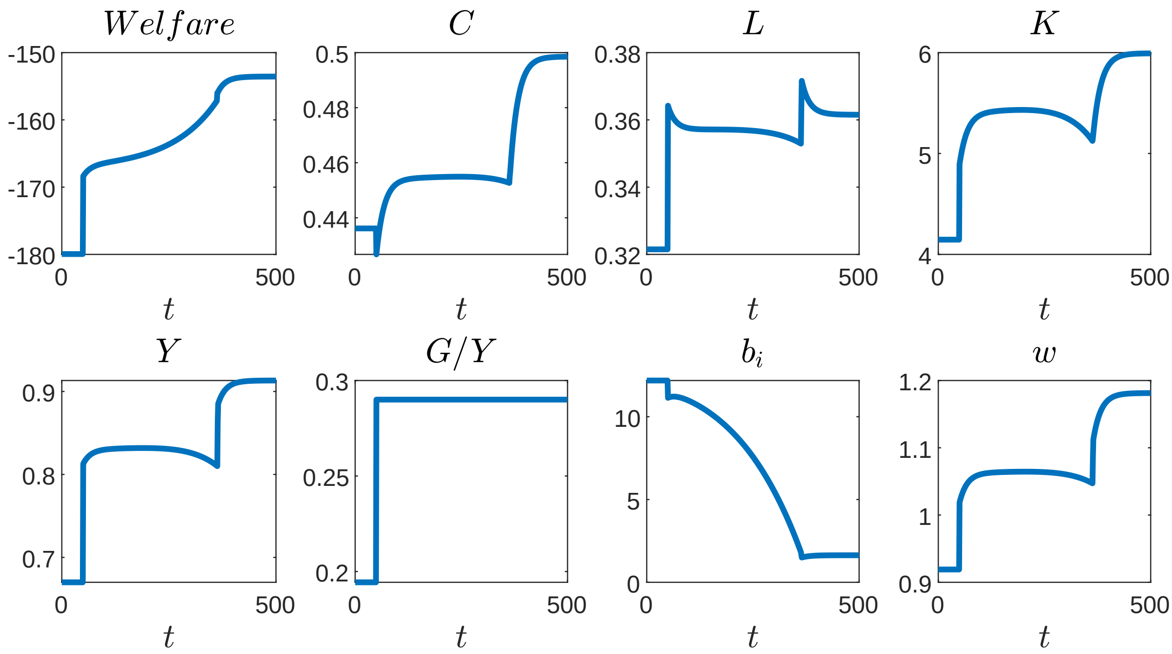


Figure 11: Permanent fiscal policy shock

Notes: This figure plots a response to a permanent increase in government expenditure that we set to $G^*/Y^* = 0.29$. The y-axis measures the level of the corresponding variable.

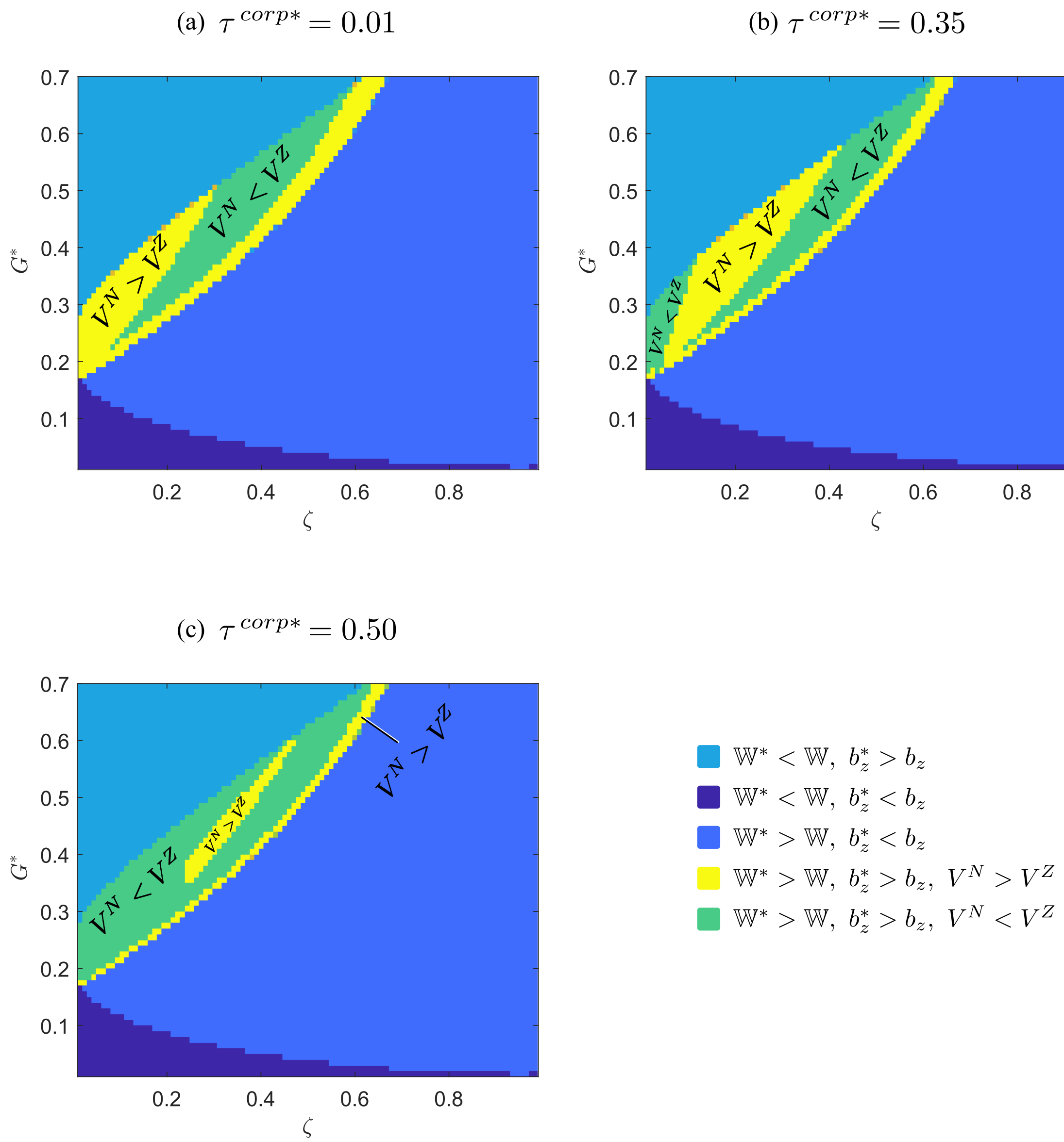


Figure 12: Corporate tax rate and fiscal policy efficiency

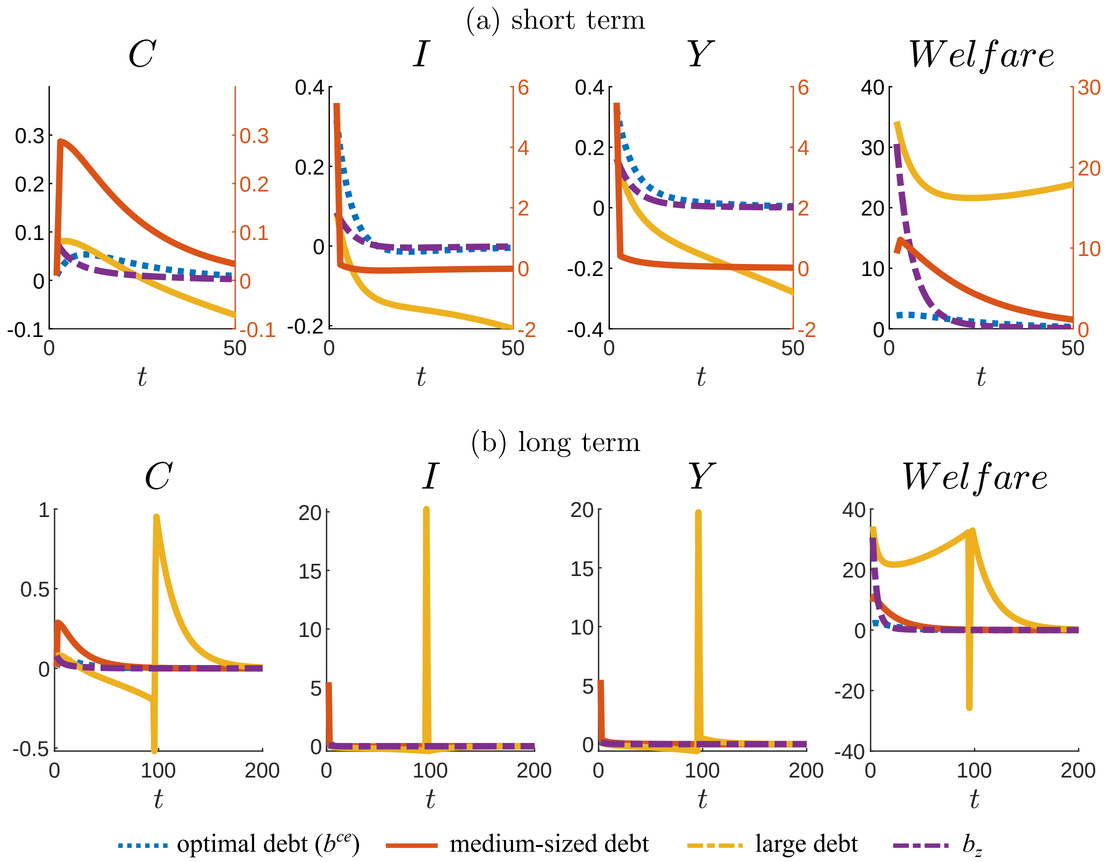


Figure 13: Impulse responses: Corporate tax cut shock (Edgeworth Complementarity)

Note: This figure plots impulse responses that are scaled by the impact response of tax revenue evaluated in the non-stochastic steady state, giving the units a multiplier interpretation.

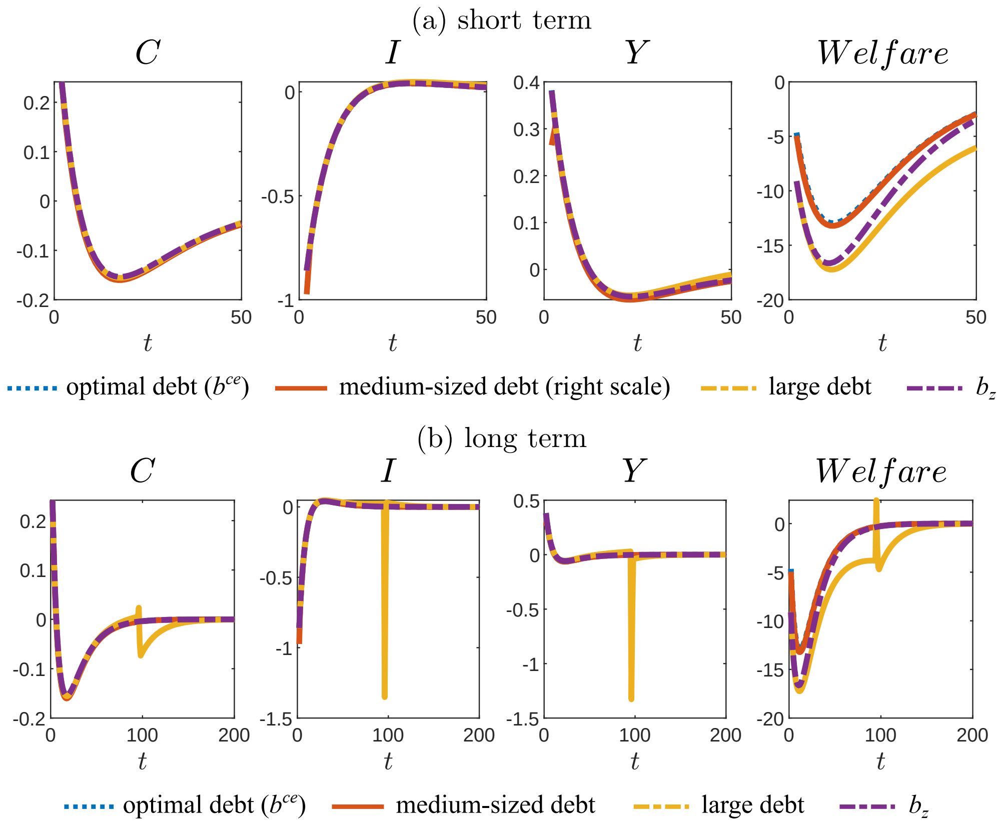


Figure 14: Impulse responses: Government spending shock (Edgeworth Complementarity)

Note: This figure plots impulse responses that are scaled by the inverse of the response of the government spending shock on impact, giving the units a multiplier interpretation.

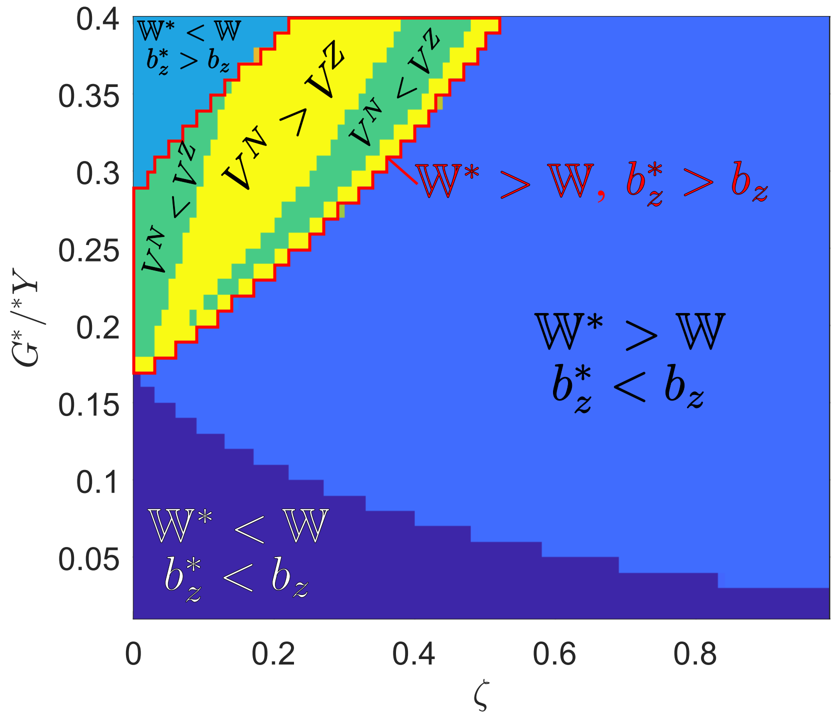


Figure 15: The parameter region for efficient fiscal expansion: considering Edgeworth complementarity

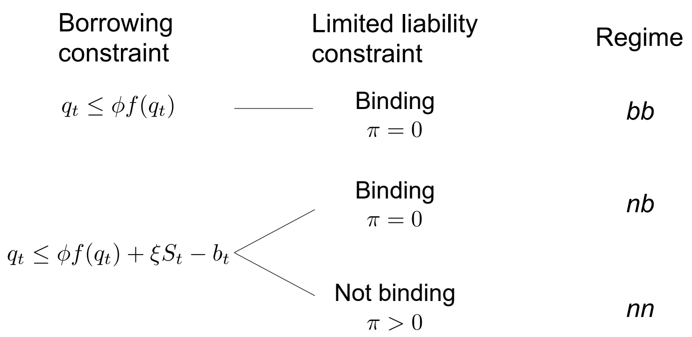


Figure 16: Regime for index functions

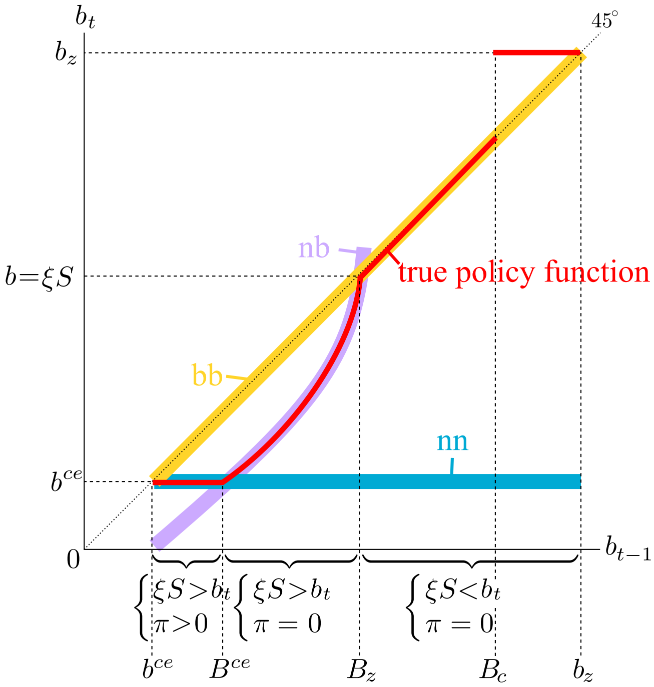


Figure 17: Index function for PEA