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Secular Stagnation and Low Interest Rates under the Fear of a Government Debt Crisis

Keiichiro Kobayashi* Kozo Ueda†

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Abstract

In this study, we explain the driving forces behind the secular stagnation associated with a persistent decrease in interest rates by employing a model that incorporates a crisis risk triggered by government debt accumulation. The model shows that fear of large-scale capital taxation and capital misallocation in future debt crises accounts for almost half the economic slowdown in Japan over the past two decades. Over the same period, the government bond yield declines, because a decrease in the expected returns on capital makes investing in government bonds more attractive than investing in capital.

JEL Classification Numbers: E32, E62, G18, H12, H63
Keywords: default; government bond; capital levy; lost decades; bank run

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1 INTRODUCTION

The Great Recession of the late 2000s raised concerns about future secular stagnation (Hansen 1939) that would cause the U.S. and European economies to stagnate persistently in the coming decades (Gordon 2012, Krugman 2013, Summers 2013). This effect is evident in Japan in the so-called "lost decades" following the collapse of the asset market bubble around 1990. There are many possible reasons for secular stagnation;[1] however, we focus on the fact that these concerns arose when government bonds outstanding (GBO) started to expand substantially in these countries. Moreover, it is equally important to emphasize that despite this development the interest rate on government bonds did not increase but actually decreased to historically lowest levels.

This study offers a new explanation for the factors that drive secular stagnation and lower government bond yields; the key factor being a *government debt crisis*. A complete loss of market confidence in government debt forces the government to collect extremely large tax revenues. The government reduces its debt by imposing a once-off increase in taxes on, for example, GBO (a partial default of government debt), capital stock (capital levy), and consumption. By contrast, taxes are low in normal periods, and thus the government budget is not balanced. Consequently, government debt continues to rise. A crisis occurs with an exogenous probability that increases with GBO. Using a simple neoclassical closed-economy model, we show that fear of a government debt crisis causes a persistent economic slowdown in normal times.

The model works as follows. In normal periods, people anticipate that returns on capital will decline during a crisis due to a heavy tax on capital stock and/or misallocations of capital caused by bank runs triggered by a partial default on GBO. This fear increases the required return on capital and discourages capital investment. Government debt accumulation intensifies this adverse effect because the probability of a government debt crisis increases. Moreover, the tax distortion in a crisis increases with GBO because the government imposes higher tax rates to meet its debt obligations. Consequently, a growing risk of government debt crisis persistently depresses output.

The model also shows that fear of a capital levy and capital misallocation decreases the government bond yield, which is consistent with the Japanese data. People expect their capital to lose value in a crisis, whereas the government does not tax bond holdings heavily. Consequently, people perceive capital investment as riskier than government bond investment and choose to buy more bonds and less capital. This increase in demand for bonds reduces the bond yield, despite the accumulation of GBO.

In our model, the key to the increase in the required return on capital is the tax on

[1] Examples include a slowdown in innovations (Gordon 2012), demand shortfall (Summers 2013, Eggertsson, Mehrotra, and Robbins 2019), and pessimism (Benigno and Fornaro 2018). Focusing on Japan, Hayashi and Prescott (2002) note the effect of a decrease in the total factor productivity (TFP) growth rate, and Caballero, Hoshi, and Kashyap (2008) emphasize the malfunctioning financial sector.

capital stock. However, it seems problematic to posit that increases in government debt trigger widespread fear of a capital levy but not fear of a government bond default. To quantify the model, we calibrate key parameter values associated with taxes by examining Japan's economic crisis after World War II.[2] We show that the post-war Japanese government imposed a heavy wealth tax to avoid defaulting on their debt obligations. Furthermore, our model incorporates bank runs that cause severe misallocation of capital stock during a crisis, which reinforce the adverse GBO effect on capital investment. Our simulation calibrated to Japan demonstrates that the expectation of heavy capital taxation and capital misallocation in a crisis accounts for approximately half of the output slowdown in Japan during the first two decades of the 21st century.

We also examine the case of a preemptive tax hike, where the government introduces a distortionary consumption tax in normal times to prevent a debt crisis. We find that a preemptive tax hike increases social welfare.

The remainder of this paper proceeds as follows. Section 2 provides several facts on Japan's secular stagnation and reviews related literature. Section 3 introduces the model and specifies the equilibrium. Section 4 presents the results of the numerical simulation. Section 5 presents several extensions of our analyses, and Section 6 concludes.

2 SOME BACKGROUND ON JAPAN AND A LITERATURE REVIEW

In this section, we explain the background of Japan's secular stagnation and review the related literature.

2.1 Background

Economic slowdowns coincide with debt increases in many economies. Figure 1 shows the trend in real GDP per capita and government debt for Japan, the United States, 17 countries in the euro area, and Italy. The graph begins in 1975 for Japan and in 1992 for the other countries, representing the 15-year periods before the financial crises in each region (i.e., 1990 and 2007, respectively, shown as vertical dashed lines). The thick solid line represents the logarithm of real GDP per capita, and the thin solid line represents its linear trend (left axis). The lines with crosses and circles represent the ratio of gross and net government debt, respectively, to nominal GDP (right axis). The figure shows that government debt increased after a financial crisis in all regions. Notably, Japan's gross government debt now exceeds 200% of its nominal GDP and net government debt is around 120% of its nominal GDP. Together with the increase in GBO, real GDP

[2] Kozlowski, Veldkamp, and Venkateswaran (2020) assume that learning from history leads to the formation of expectations on tail events.

decreased persistently compared with its trend. Hereafter, we provide further empirical evidence related to Japan.

Although this government debt accumulation is commonly cited as the result rather than the cause of the stagnation, Figures 2 and 3 show that increased government debt does indeed cause anxiety in Japan. According to a household survey conducted by Japan's Cabinet Office, "Overview of the Public Opinion Survey on the Life of the People," an increasing number of Japanese worry in their everyday lives and are concerned about their prospects, with one-third indicating that the fiscal balance is one of the reasons for this.[3]

The left-hand panel of Figure 3 shows how often specific words appear in the morning and evening editions of the Nihon Keizai Shinbun, Japan's financial newspaper, from 1981 to 2019. The frequency with which terms that suggest a government debt crisis—such as "fiscal failure" (*zaisei hatan*) or "fiscal crisis" (*zaisei kiki*)— appear has been increasing since 1981. Moreover, combinations of "tax increase" (*zozei*) and either "fiscal failure" or "default" are increasingly common, which seems to suggest a growing fear of a tax increase in the event of a government debt crisis. The euro crisis around 2010 caused a sharp increase in these terms; however, we confirm a steady increase even when we exclude the word "Europe."

The right-hand panel of Figure 3 shows the time-series movements of the sovereign credit default swap (CDS) spread for five-year Japanese government bonds from 2003 to 2019. Sovereign CDS contracts protect investors against sovereign default, which enables us to calculate the probability of sovereign default numerically. Similar to the occurrence of specific words in the left-hand panel, the euro crisis around 2010 contributed to the increase in the CDS spread. If we exclude this episode, we find that the CDS spread in the latter half of the 2010s is several percentage points higher than that in the mid-2000s. To investigate the effect of government debt on the CDS spread more quantitatively, we estimate the following equation using the ordinary least squares (OLS) method:

$$CDS_t^{Japan} = c + \alpha \cdot b_t + \beta \cdot CDS_t^{Italy} + \varepsilon_t,$$

where CDS_t^{Japan}, b_t, and CDS_t^{Italy} represent the logarithm of the CDS spread for Japan, logarithm of the debt ratio for Japan, and logarithm of the CDS spread for Italy, respectively, in month t. The observation period is from March 2003 to July 2017.[4] The

[3]Health and natural disasters are the top two reasons, followed by concerns about public services. This worry is likely to be related to the government debt accumulation because it prevents the government from providing sufficient public services in the future, such as pensions, medical services, and investment in infrastructure that is resilient to disasters.

[4]Although the debt data are annual, we use monthly data to secure a sufficient number of observations for estimation. We calculate the debt ratio as follows. First, we collect annual data on the net debt to nominal GDP ratio for Japan (source: OECD, as in Figure 1). Second, we calculate the annual net debt, B_T in year T, by multiplying the debt-to-GDP ratio by the annual nominal GDP for Japan

explanatory variable CDS_t^{Italy} aims to capture the international spillover effect, particularly the euro crisis effect, on Japan's CDS spread. The estimates of α and β are 1.00 and 0.61, respectively, with 95% confidence intervals of [0.37, 1.63] and [0.48, 0.73]. Thus, both coefficients are significantly positive; that is, the CDS spread for Japanese government bonds is positively associated with both Japan's net debt ratio and Italy's CDS spread. We confirm the robustness of the results using the monthly logarithm difference for all three variables in the equation. The estimates of α and β are 1.47 and 0.59, respectively, with 95% confidence intervals of [0.49, 2.44] and [0.45, 0.73]. These results suggest that the elasticity of the CDS spread to debt is approximately one.

The probability of a government debt crisis for Japan is also studied by Morikawa (2016, 2017). By conducting surveys, he finds that Japanese consumers and firm managers believe that a debt crisis will occur by 2030 with a probability of approximately 24% (consumers) and 27% (firm managers), on average. However, the survey is open to interpretation by respondents.

A noteworthy puzzle is that the price (yield) of government bonds is high (low). The left-hand panel of Figure 4 shows the government bond yield in real terms for Japan, defined as the nominal bond yield with five-year maturity minus the annual CPI inflation rate in the following year.[5] The figure shows that bond yields decreased, which hardly suggests a mounting public default risk.

Simultaneously, the return on capital seems to be increasing. The right-hand panel of Figure 4 shows the increase in the credit spread, defined as the bank loan rate with one-year maturity or longer minus the government bond yield with five-year maturity. Data taken from the JIP Database 2015, compiled by the Research Institute of Economy, Trade and Industry in Japan, suggest that the capital cost spread, a proxy for the credit spread, has been increasing. Consequently, the ratio of investment to capital has been decreasing.

(source: the Cabinet Office). Third, we set annual net debt in March (i.e., the end of the fiscal year) in year T to B_T and linearly interpolate the values for the other months to construct monthly data on net debt B_t for month t (e.g., debt in September 2010 equals the average of debt in March 2010 and debt in March 2011). Last, we divide B_t by the monthly CPI (excluding fresh food, source: the Statistical Bureau) and the monthly Indices of Industrial Production (source: the Ministry of Economy, Trade and Industry) to construct the debt ratio. It should be noted that we do not use the net debt to nominal GDP ratio for the regression but only to construct the monthly debt ratio. Furthermore, the CDS data are missing for several months during the observation period.

[5] For 2019, we use the realized inflation rate from 2018 to 2019.

2.2 Literature Review: Sovereign Default and Public Debt Overhang

Empirical studies report that economies tend to stagnate when government debt is high,[6] referred to by Reinhart, Reinhart, and Rogoff (2012) as the *public debt overhang*. They review 26 cases of high government debt in advanced countries, reporting that in 23 cases, economic growth remained stagnant for more than a decade. Based on their finding of a nonlinear correlation between higher debt and lower growth, they argue that an increase in government debt causes lower economic growth. Our study provides a theoretical basis for the public debt overhang phenomenon.

Aguiar, Amador, and Gopinath (2009), Aguiar and Amador (2011), Bocola (2016), Balke (2018), Arellano, Bai, and Mihalache (2018, 2019), and Roldan (2019), among others, propose models in which sovereign default risk dampens real economic activity and causes a recession before a default occurs. The mechanism depends on the model. In Aguiar, Amador, and Gopinath (2009) and Aguiar and Amador (2011), expectations that the government will impose a tax on investment returns during a default dampens capital formation before a default occurs. In Bocola (2016), banks hold government bonds and loans to non-financial firms. The fear of sovereign default tightens banks' funding constraints and increases the stochastic discount factor, both of which make banks require higher returns on the loans, dampening capital formation before a default. Balke (2018) models a contraction in labor demand. The fear of sovereign default decreases the price of government bonds and hurts the balance sheets of banks that hold government bonds as an asset. This decreases bank loans to non-financial firms, which in turn contracts labor demand. In Arellano, Bai, and Mihalache (2018), default risk decreases capital inflows, which prevents investment, particularly in nontraded sectors, whereas Arellano, Bai, and Mihalache (2019) discuss the importance of both monetary and fiscal policy. Roldan (2019) constructs a model of incomplete markets to show that sovereign risk decreases consumption demand. Our model has a similar mechanism to that of Aguiar, Amador, and Gopinath's (2009), Aguiar and Amador's (2011), and Bocola's (2016) models. In our model, as GBO increases, people are prepared for a higher capital levy during a crisis and increase their stochastic discount factors. This causes banks to require a higher return from capital investment and, in turn, decreases capital investment.

However, these standard sovereign default models often predict an increase in the government bond yield as GBO increases. The most important departure from these models is that we seek to explain the decrease in the government bond yield, despite the GBO increase peculiar to episodes of secular stagnation (e.g., the lost decades of

[6] See Reinhart and Rogoff (2010), Reinhart, Reinhart, and Rogoff (2012), Checherita-Westphal and Rother (2012), and Baum, Checherita-Westphal, and Rother (2013) for the negative effect of government debt on output. Barro and Sala-i-Martin (1995) show that government consumption has a negative effect on output. Fischer (1991) shows that a fiscal deficit has a negative effect on output.

Japan). The key feature of the model is a capital levy. When people anticipate relatively heavier capital taxation than government bond taxation during a crisis, investment in government bonds becomes less risky as the probability of a crisis increases, whereas capital investment becomes riskier. Thus, the government bond yield decreases, while the credit spread increases, both of which are consistent with the data. Furthermore, the increase in the credit spread helps explain the stagnation in real economic activity quantitatively better.

It should be noted that the type of government debt crisis we describe in the model does not necessarily refer to a large-scale sovereign default. The government repays most of its debt by taxing private agents heavily (notably, a capital levy), which contributes to a stable government bond price and an increasing credit spread. From a historical perspective, Japan's Ministry of Finance (MOF 1976) and Eichengreen (1989) study capital levies.

Other differences from the early studies are that we incorporate domestic rather than external debt using a closed-economy model. No investor in our model has access to global financial markets, and thus, to a riskless world interest rate. This structure helps to stabilize the price of government bonds, despite the increase in GBO. Another difference is that a crisis occurs with an exogenous probability in our model, whereas in many of the models mentioned above, the government's default decision is endogenous.

2.3 Other Strands of the Literature

Regarding the low, stable government bond yield, Hoshi and Ito (2014) point out that Japanese government bonds are held predominantly by Japanese institutional investors, arguing that they have a strong home bias. However, their simulation results show that government debt will exceed private sector financial assets within 10 years, and thus they warn of a potential fiscal crisis. Sakuragawa and Sakuragawa (2016) use the absence of safe assets to explain the low government bond yield. Caballero and Simsek (2017) point out a secular increase in risk intolerance.

Our study is also related to the literature in the 1990s on the non-Keynesian effect of fiscal policy developed by Giavazzi and Pagano (1990), Alesina and Perotti (1996), Alesina and Ardagna (1998), and Perotti (1999). Perotti (1999) shows theoretically and empirically that an increase in government debt has a contractionary effect on consumption when government debt is large. Alesina and Perotti (1996) and Alesina and Ardagna (1998) show that government expenditure cuts have a longer-lasting effect on improving the economy than do tax increases. Our study has similar implications, in that an increase in government debt is contractionary.

The model we present here has its basis in the neoclassical model of a rare disaster, following the work of Rietz (1988), Barro (2006, 2009), Gabaix (2012), and Gourio (2012, 2013). Specifically, our model is a simplified version of Gourio's (2013). However, the "disaster" properties in our case are different because a debt crisis is an abrupt redistribution of wealth, whereas a disaster destroys resources. Therefore, we refer to a crisis, rather than a disaster, which affects mainly the household (through taxes), whereas a disaster in Gourio (2013) affects firms through changes in their productivity and capital values.[7]

3 MODEL

The model economy consists of a representative household, firm, bank, and government. For simplicity, we assume that, in normal times, the government makes transfers, but collects no tax.[8] Thus, government debt keeps increasing. When a crisis occurs, the government imposes once-off taxes in order to repay its debt. The crisis probability is given exogenously and increases with GBO. We assume a closed economy because around 90% of government bonds are held by domestic investors in Japan. The asset market is incomplete, meaning that the crisis risk is not insured.

3.1 Crisis Risk

We define the crisis indicator x_t as $x_t = 0$ in normal times and as $x_t = 1$ when a government debt crisis occurs. The variable x_t is an exogenous sunspot shock to the economy. A government debt crisis occurs when debt holders lose confidence in the government debt and rush to exchange it for real assets and goods, which is exogenous to the government's actions. More specifically, when $x_t = 0$, the government exchanges maturing GBOs for new GBOs. When $x_t = 1$, (i) a sunspot shock changes people's preferences such that they demand real goods (as much as a portion ω of $B_t^G + G_t$)

[7]Kozlowski, Veldkamp, and Venkateswaran (2020) use Gourio's model to analyze the exogenous disaster risk of a financial crisis. Isore and Szczerbowicz (2017) later extend this work to the New Keynesian model. Kunieda (2015) applies Barro's (2006) model to the risk of a natural disaster.

[8]In reality, there are at least four issues related to government policy in normal times. First, the government may increase taxes to reduce its debt. Bohn (1998) and Lo and Rogoff (2015) report empirical results that governments tend to improve their fiscal balance in response to an increase in their debt (see also Aguiar et al. 2015, Bianchi, Hatchondo, and Martinez 2018). Second, the government may impose several distortionary taxes. Third, government spending is counter-cyclical and tax revenue is pro-cyclical. Finally, G_t may not necessarily be a transfer, but government spending on goods. However, these tax and spending policies in normal times do not affect our main results qualitatively because we focus on how changes in tax policy from normal times to a crisis event influence economic activity. Moreover, with regard to the first issue, the political costs are large, given that very few countries implement policies that are sufficiently conservative to eliminate the possibility of a crisis. The third issue strengthens our results quantitatively because the risk of a government debt crisis causes a larger decrease in output and investment when GBO increases more rapidly. The fourth issue may have implications for the optimality of tax smoothing. However, this does not matter for our main results because our main analysis is not normative.

rather than government bonds. (ii) The government chooses either to impose taxes, as much as $\omega(B_t^G + G_t)$, or not to impose taxes. (iiia) If the government chooses the latter, then maturing GBO holders refuse to purchase new GBOs. This results in the default and zero value for government bonds. The government suffers considerable nonpecuniary disutility. (iiib) If the government chooses to impose taxes of $\omega(B_t^G+G_t)$, then no default occurs. The government does not suffer disutility, and thus, chooses this option. Note that in this environment, maturing GBO holders have no incentive to deviate from the strategy of demanding that the government exchange the maturing bond for real goods. That is, if only one individual stops demanding the exchange and the others continue with their demand, then he/she can still receive real goods, which does not change his/her utility. However, if all holders stop demanding the exchange, then the government will not impose taxes, which makes them unable to receive real goods and decreases their utility. Thus, deviating from the strategy never increases bondholders' utility, and possibly decreases their utility, meaning that the strategy of demanding the exchange is weakly dominant.

The government debt crisis in our model is essentially a rollover crisis, in that the government does not need to repay all of its debt. However, even part of its debt is likely to be sizable because the Japanese government relies on the issuance of new bonds for as much as 30% of its annual expenditure. Thus, when a crisis causes a loss of market confidence in government debt, it is plausible that the government will be forced to collect extremely large tax revenues. Indeed, this was the case in post-war Japan.

A crisis has two main consequences. First, it forces the government to raise a substantial amount in tax revenue. The government imposes a once-off tax only at the time of the crisis, where τ_t^C is the consumption tax rate, τ_t^K is the tax rate on capital stock, and τ_t^G is the tax rate on GBO. Note that the latter two taxes are wealth taxes, not taxes on the net returns from holding these assets. The tax on GBO is essentially equivalent to a partial default (a full default when $\tau^G = 1$).[9] The tax on capital stock, also called a capital levy, plays the most important role in explaining persistent recessions.

Second, when a crisis occurs, banks become insolvent. Because banks and other financial institutions hold government debt as a large part of their assets, it is straightforward that a decrease in the value of government debt will make them insolvent. Then, all depositors rush to withdraw early, and banks are forced to sell both productive capital and government bonds to finance the early withdrawals (bank runs), causing fire sales of productive capital. These fire sales misallocate capital stock within the economy, leading to its inefficient use and decreasing its value. From the household's perspective, this reduction in capital value works as a capital levy.

The probability that a crisis occurs in period $t + 1$ is $p_t = p(B_t^G/z_t) = \Pr(x_{t+1} =$

[9] Although our model is a real model without an explicit role for the nominal variables, we can interpret a partial default in our model as a debt reduction by inflation tax or seigniorage.

$1|B_t^G/z_t)$, where B_t^G is the quantity of government bonds and z_t is productivity (TFP) given by $\log z_{t+1} = \log z_t + \mu + \sigma_e e_{t+1}$, where $e_{t+1} \sim N(0,1)$. This probability depends positively on GBO divided by the TFP. We assume the following logistic form:

$$p_t = p(B_t^G/z_t) = \frac{1}{1 + (1/d_0 - 1)\exp(-d_1 B_t^G/z_t)}, \tag{1}$$

where d_0 represents the crisis probability in the next period when there is no government debt today, and d_1 represents the steepness of the crisis probability curve with respect to GBO divided by the TFP.[10]

The crisis probability, given by equation (1), is exogenous. Abstracting away from modeling the government's decision, we discuss the equilibrium dynamics of macroeconomic variables by focusing on optimizations by private agents.[11]

3.2 Household

There exists a unit mass of identical households. A household has the nonseparable lifetime utility U_t, defined as

$$U_t^{1-\psi} = (1-\beta)(C_t^{\nu}(1-N_t)^{1-\nu})^{1-\psi} + \beta E_t(U_{t+1}^{1-\psi}), \tag{2}$$

[10]Specifically, we assume that p_t becomes one (a crisis occurs for sure) if the tax rate on either capital or GBO during the crisis becomes one or greater.

[11]A crisis is more likely to occur as GBO increases (Reinhart and Rogoff 2010). Arellano (2008) constructs a small open-economy model, showing that a default is more likely to occur in a recession, which is consistent with the data. She points out that this result is due to the incomplete asset market, whereas models based on a complete asset market tend to predict the opposite result. D'Erasmo and Mendoza (2016) show theoretically that a default on domestic government debt is more likely to occur when GBO increases and the tax revenue decreases. See also Aguiar, Amador, and Gopinath (2009) and Arellano, Bai, and Mihalache (2018, 2019). In contrast, the probability of a default is arbitrary in the self-fulfilling crisis model of Cole and Kehoe (2000). They consider an optimal government policy in which government debt is rolled over every period. In this case, there is a risk of a self-fulfilling default caused by a loss of confidence in the government. However, this does not deny the assumption that a self-fulfilling crisis is more likely to occur as GBO increases.

Though the sovereign debt in these models is external debt, similar reasoning would hold for the domestic debt in our model and lead to an endogenous crisis probability that is increasing in GBO. We can consider the following decision problem for the government on whether to impose heavy taxes to repay its debt now or in the future. Here, we assume that the sunspot shock on the preference occurs exogenously with a constant probability. The shock makes private agents demand real goods rather than government bonds, regardless of the government's decision. Given that the tax distortion grows as the debt increases, the government faces tension between taxing today or in the future. Then, the government optimally taxes if its debt exceeds an endogenous threshold. Adding a preference shock on the government due to the stochastic event of losing office (Aguiar, Amador, and Gopinath 2009) would lead to an endogenous crisis probability that the government imposes taxes today, which is increasing in GBO. The crisis probability would be the sum of the exogenous probability that a preference shock on households occurs and the endogenous probability that the government chooses taxation. However, providing such a model with an endogenous crisis probability is beyond the scope of this study.

where β represents a discount factor, ψ represents the intertemporal elasticity of the substitution of consumption, and ν represents the utility weight on consumption. The utility function takes the standard Cobb–Douglas form in consumption C_t and leisure $1 - N_t$, where N_t is the labor supply, which is consistent with balanced growth.

Owing to the inefficiency described below, the household holds certificates of bank deposit D_t rather than capital stock K_t and government bonds B_t^G directly.[12] The rate of return on the bank deposit is $R_t^D(x_t)$. The budget constraint for the household is

$$(1 + x_t\tau_t^C)C_t + x_tT_t + D_{t+1} \leq R_t^D(x_t)D_t + G_t + W_tN_t, \tag{3}$$

where T_t is the lump-sum tax, G_t is the lump-sum transfer from the government, and W_t represents the real wage. We assume that a technical constraint exists for the government's ability to impose a lump-sum tax such that T_t cannot exceed a certain limit, given by a parameter ω^T, which we define shortly. Alternatively, T_t can be interpreted as a decrease in the lump-sum transfer during a crisis, representing that the government makes some transfers at all times and that in the crisis the government could cut this transfer by T_t. The household maximizes the lifetime utility (2) subject to the budget constraint (3). This optimization problem implies that

$$E_t[M_{t+1}R_{t+1}^D(x_{t+1})] = 1, \tag{4}$$

where M_{t+1} is the stochastic discount factor:

$$M_{t+1} = \beta\left(\frac{1 + x_t\tau_t^C}{1 + x_{t+1}\tau_{t+1}^C}\right)\left(\frac{C_{t+1}}{C_t}\right)^{\nu(1-\psi)-1}\left(\frac{1 - N_{t+1}}{1 - N_t}\right)^{(1-\nu)(1-\psi)}. \tag{5}$$

3.3 Firm

There is a unit mass of identical non-financial firms that face perfect competition. Its production function is $Y_t = K_t^\alpha(z_tN_t)^{1-\alpha}$, where Y_t represents output. The static profit maximization is

$$\pi(K_t, z_t; W_t) = \max_{K_t, N_t \geq 0}\{K_t^\alpha(z_tN_t)^{1-\alpha} + (1-\delta)K_t - R_t^KK_t - W_tN_t\}, \tag{6}$$

which yields the return on capital $R_t^K = 1-\delta+\alpha Y_t/K_t$ and $N_t = K_t\left(z_t^{1-\alpha}(1-\alpha)/W_t\right)^{\frac{1}{\alpha}}$, where δ represents the capital depreciation rate.

The household cannot operate K_t efficiently for two reasons. First, the household can use only $\lambda_t^YK_t$ for production, where $\lambda_t^Y < 1$. Second, $(1 - \lambda_t^D)K_t$ is destroyed if the household directly holds and uses K_t, where $\lambda_t^D < 1$. In a crisis, the household

[12] The household can hold government bonds B_t^G, but chooses not to do so because the rate of return is higher for the bank deposit; that is, $E_t[M_{t+1}R_t^D] \geq E_t[M_{t+1}(1 - x_{t+1}\tau_{t+1}^G)/q_t^G]$, in equilibrium. We provide the definitions of the variables below.

uses K_t inefficiently because banks become insolvent, whereas the banks operate capital K_t efficiently in normal times. Consequently, the return on capital $R_t^K(x_t)$ becomes a random variable that depends on x_t:

$$R_t^K(1) = \alpha \frac{Y_t(1)}{K_t} + (1-\delta)\lambda_t^D, \tag{7}$$

$$R_t^K(0) = \alpha \frac{Y_t(0)}{K_t} + (1-\delta), \tag{8}$$

$$Y_t(1) = (\lambda_t^Y K_t)^\alpha (z_t N_t)^{1-\alpha}, \tag{9}$$

$$Y_t(0) = (K_t)^\alpha (z_t N_t)^{1-\alpha}. \tag{10}$$

3.4 Bank

There is a unit mass of one-period-lived banks. A bank is born, collects deposits, and invests them in capital stock and government bonds at the end of period t to maximize the return on the deposit. It then pays out all the return to depositors and ceases to operate in period $t+1$. The deposit contract can be contingent on z_{t+1}, but not on x_{t+1}. The banks can offer only $R_{t+1}^D(0)$, which is the return on a deposit when $x_{t+1} = 0$:

$$R_{t+1}^D(0) = \frac{R_{t+1}^K(0)K_{t+1} + B_{t+1}^G}{K_{t+1} + q_t^G B_{t+1}^G}, \tag{11}$$

where q_t^G is the price of government bonds. The deposit contract is such that depositors can withdraw $R_{t+1}^D(0)D_{t+1}$ whenever they want during period $t+1$, even before output is produced.[13] When $x_{t+1} = 0$, they withdraw after output is produced in equilibrium.[14]

When $x_{t+1} = 1$, the banks adopt the first-come-first-served principle for depositors. Thus, depositors can withdraw the full amount of their deposits $R_{t+1}^D(0)D_{t+1}$ at any time in period $t+1$, as long as the bank has assets remaining. However, they receive nothing if the bank pays all of its assets out to early withdrawers. Thus, when $x_{t+1} = 1$, all

[13]We impose the incompleteness such that the deposit contract cannot specify the timing of withdrawal. Diamond and Dybvig (1983) provide a model that justifies such a contract as constrained optimal. We impose incompleteness as a constraint rather than providing a Diamond–Dybvig type model to simplify our analysis.

[14]If one depositor withdraws early and all the other depositors do not, then the depositor can withdraw the full amount. If all depositors withdraw early, then the bank is forced to sell their asset (physical capital) to households at a discounted price because households are inefficient in operating capital. Then, the depositors cannot withdraw the full amount of their deposits. Therefore, the depositors' dominant strategy is to withdraw after output is produced, and all depositors choose to withdraw late in equilibrium. Rigorously speaking, there exists another equilibrium in which all depositors withdraw early before output is produced when $x_{t+1} = 0$ because withdrawing early is the best response for a depositor given that all other depositors withdraw early. Following Allen and Gale (1998), we exclude this equilibrium since it is Pareto inefficient, whereas the equilibrium where all depositors withdraw late is Pareto efficient.

depositors try to withdraw before production and all capital is paid out to households. As the households produce output inefficiently, the average return on a deposit decreases to

$$R^D_{t+1}(1) = \frac{(1-\tau^K_{t+1})R^K_{t+1}(1)K_{t+1} + (1-\tau^G_{t+1})B^G_{t+1}}{K_{t+1} + q^G_t B^G_{t+1}}. \tag{12}$$

Given deposit D_{t+1}, the bank's problem is

$$E_t[M_{t+1}R^D_{t+1}(x_{t+1})]D_{t+1} = \max_{K_{t+1},B^G_{t+1}} E_t[M_{t+1}\{(1-x_{t+1}\tau^K_{t+1})R^K_{t+1}(x_{t+1})K_{t+1} + (1-x_{t+1}\tau^G_{t+1})B^G_{t+1}\}], \tag{13}$$

subject to the balance sheet constraint $D_{t+1} = q^G_t B^G_{t+1} + K_{t+1}$, where q^G_t is the price of the government bonds. Then, the first-order condition with respect to K_{t+1} implies that

$$E_t[M_{t+1}(1-x_{t+1}\tau^K_{t+1})R^K_{t+1}(x_{t+1})] = \frac{1}{q^G_t}E_t[M_{t+1}(1-x_{t+1}\tau^G_{t+1})]. \tag{14}$$

Furthermore, we assume that the inefficiency λ^Y_t at the crisis is determined endogenously by

$$\lambda^Y_t = F(\xi_t), \tag{15}$$

where $F'(\xi) < 0$ and ξ_t represents the ratio of nonperforming loans to assets:

$$\xi_t \equiv 1 - \frac{(1-\tau^K_t)R^K_t(1)K_t + (1-\tau^G_t)B^G_t}{R^K_t(0)K_t + B^G_t}. \tag{16}$$

This assumption is based on the premise that production activity by the household necessitates bank deposits as a means of payment, while the function of the means of payment is disrupted if the ratio of nonperforming loans is large. In the next section, we provide empirical evidence and calibrate parameter values for this specification.

We define $R^G_t \equiv 1/q^G_t$, and call $R^G_t - 1$ the government bond yield. The price of government bonds, q^G_t, is normally less than one because M_{t+1} is less than one.

3.5 Government

The government spends through a lump-sum transfer $G_t > 0$, which has a constant ratio to the ratio of TFP, z_t, for all x_t. Tax is zero if $x_t = 0$ and non-negative if $x_t = 1$. The government budget constraint is

$$q^G_t B^G_{t+1} + x_t\tau^C_t C_t + x_t\tau^K_t R^K_t K_t + x_t\tau^G_t B^G_t + x_t T_t = B^G_t + G_t. \tag{17}$$

To determine the tax rates, we assume a tax weight of ω^i ($i = C, K, G, T$), which is exogenous and satisfies

$$\begin{aligned}
\tau_t^C C_t &= \omega^C (B_t^G + G_t), &(18)\\
\tau_t^K R_t^K K_t &= \omega^K (B_t^G + G_t), &(19)\\
\tau_t^G B_t^G &= \omega^G (B_t^G + G_t), &(20)\\
T_t &= \omega^T (B_t^G + G_t), &(21)\\
0 < \omega \equiv\ & \omega^C + \omega^K + \omega^G + \omega^T \leq 1. &(22)
\end{aligned}$$

If $\omega^C + \omega^K + \omega^G + \omega^T = 1$, the government issues no new bonds; in other words, it owes nothing after the crisis period.

Although we assume a tax rate of zero in non-crisis periods ($x_t = 0$), this assumption is not critical to our results as long as the government collects insufficient tax to cover its expenditure.

3.6 Market Clearing

The goods market is cleared when

$$Y_t = C_t + I_t, \tag{23}$$

where investment I_t equals $K_{t+1} - (1-\delta)K_t$.

The labor market is cleared when

$$(1-\alpha)\frac{Y_t}{N_t} = W_t = \frac{1-\nu}{\nu}\frac{(1+\tau_t^C)C_t}{1-N_t}. \tag{24}$$

3.7 Equilibrium

In the economy, the state variables are $\{K_t, B_t^G, x_t, z_t\}$; however, as in Gourio (2013), the equilibrium can be expressed using $\{k_t \equiv K_t/z_t,\ b_t^G \equiv B_t^G/z_t,\ x_t\}$. See Appendix A for the calculation of the equilibrium.

In the model, the fear of a government debt crisis can cause persistent stagnation beforehand when people share the expectation that a considerable distortion occurs at the time of a crisis. In particular, two distortions are important. The first is a capital levy, a one-time tax on all wealth holders with the goal of retiring government debt, denoted by ω^K or τ_t^K. The second is capital-use inefficiency resulting from bank runs, denoted by λ_t^Y and λ_t^D. Both distortions increase the required return to capital, $R_{t+1}^K(0)$, in a normal state and, in turn, dampen capital investment before a crisis occurs.

3.8 How the Crisis Risk Affects the Economy

Before describing our simulation, it is useful to discuss how the crisis risk affects the economy in the model. The most important equation, equation (14), is related to the bank's optimization problem with respect to investing in either capital or government bonds. We can rewrite this as

$$
\begin{aligned}
&(1-p_t)E_t\left[M_{t+1}R^K_{t+1}(0)\right] \\
=&\frac{1}{q^G_t}E_t\left[M_{t+1}\left\{(1-p_t)+p_t(1-\tau^G_{t+1})\right\}\right]-p_tE_t\left[M_{t+1}(1-\tau^K_{t+1})R^K_{t+1}(1)\right] \\
=&\frac{1}{q^G_t}E_t\left[M_{t+1}\left\{(1-p_t)+p_t(1-\tau^G_{t+1})\right\}\right] \\
&-p_tE_t\left[M_{t+1}\right]E_t\left[(1-\tau^K_{t+1})R^K_{t+1}(1)\right] \\
&-p_tCov_t\left[M_{t+1},(1-\tau^K_{t+1})R^K_{t+1}(1)\right].
\end{aligned}
\tag{25}
$$

This equation suggests that the return on capital, $R^K_{t+1}(0)$, in normal times ($x_{t+1}=0$) increases as GBO increases through the following three factors. Note that a higher $R^K_{t+1}(0)$ decreases the desired level of capital, and in turn, investment, before a crisis occurs through equation (7).

First and most important is that the expectation of a capital levy increases $R^K_{t+1}(0)$. Unless $\omega^K=0$, τ^K_{t+1} during a crisis is positive and increases as GBO increases. Thus, the second term on the right-hand side of the equation increases, which increases $R^K_{t+1}(0)$ on the left-hand side of the equation. Banks require a higher return on capital investment.

Second, the expectation of a higher crisis risk increases $R^K_{t+1}(0)$. As GBO increases, p_t increases. As long as $R^K_{t+1}(0)>(1-\tau^K_{t+1})R^K_{t+1}(1)$; that is, when the return on capital in normal times is higher than that during a crisis,[15] an increase in p_t leads to an increase in $R^K_{t+1}(0)$ through the second term on the right-hand side of the equation.[16]

Third, banks require a higher risk premium. As GBO increases, the stochastic discount factor M_{t+1} increases, while the post-tax return on capital decreases. This negative comovement increases $R^K_{t+1}(0)$ through the third term on the right-hand side of the equation.

This clarification is similar to that in Bocola (2016). Bocola (2016) decomposes the excess return on bank loans to a liquidity premium and a risk premium. The former

[15]The condition that $R^K_{t+1}(0)>(1-\tau^K_{t+1})R^K_{t+1}(1)$ is not restrictive because it holds when there exists either a capital levy ($\omega^K>0$) or capital-use inefficiency (either $\lambda^Y<1$ or $\lambda^D<1$).

[16]The increase in $R^K_{t+1}(0)$ is partially offset by the first term on the right-hand side of the equation if $\tau^G_{t+1}>0$; that is, $\omega^G>0$. When both τ^K_{t+1} and τ^G_{t+1} are positive, the credit risk decreases returns from investing in capital and government bonds. If the latter decrease is larger, then banks will choose to invest in capital rather than government bonds, which decreases $R^K_{t+1}(0)$. However, this is likely of low possibility, especially when GBO is large because as GBO increases, τ^G_{t+1} decreases (if $G_t=0$, it is constant), while τ^K_{t+1} increases. Thus, the credit risk increasingly affects return on capital, rather than return on government bonds, as GBO increases.

is similar to the first and second factors in our model while the latter corresponds to the third factor in our model. However, the source of the loss of return during a crisis is different. It is banks' funding constraints in Bocola (2016), while it is a capital levy ($\omega^K > 0$) in our model. Moreover, there is no riskless asset in our model, which makes it difficult to define a premium by comparing the return on capital to a riskless rate.

Finally, it should be noted that the above equation influences not only $R^K_{t+1}(0)$ but also q^G_t. Thus, the three factors we discussed also influence the government bond yield differently and the government bond yield may increase or decrease as GBO increases. It likely decreases when the distortion associated with the capital stock increases. In other words, banks prefer to hold government bonds rather than capital. This makes the government bond yield lower due to the no-arbitrage condition between capital and government bonds.

In Appendix B, we provide more rigorous analytical results using a simplified two-period version of the model. For example, we show that the derivatives of τ^K_{t+1} and K_t with respect to GBO are non-negative and non-positive, respectively. The derivative of q^G_t with respect to GBO can be both positive and negative depending on the relative sizes of τ^K and τ^G.

4 SIMULATION

4.1 Simulation Method

Time is measured in years. The equilibrium can be expressed using $\{k_t = K_t/z_t, b^G_t = B^G_t/z_t, x_t\}$. In what follows, we denote the variables divided by z_t using lower-case letters (e.g., $y_t = Y_t/z_t$), with the exception of u_t, which is defined as $U_t^{1-\psi}/z_t^{\nu(1-\psi)}$. Note that the GBO, b^G_t, does not explode, although there is no deterministic steady state. Equation (1) indicates that the crisis probability increases with the increase in b^G_t, provided $d_1 > 0$, which stabilizes b^G_t.[17]

Following Gourio (2013), we solve the model by employing a projection method. The policy functions are approximated using two-dimensional Chebychev polynomials with respect to k_t and b^G_t, with degree five in each dimension.

[17]For the numerical calculation, we set an upper bound on the value of b^G_t; that is, $\bar{b}^G = 0.85$. This setting implies that the government imposes a lump-sum tax when necessary to keep b^G_t within the upper bound; hence, this satisfies the transversality condition. According to the simulation, the mean of y_t is 0.39, and thus, $\bar{b}^G$ implies that the highest debt-to-GDP ratio is approximately 2.2, whereas it is currently around 1.2 (net).

4.2 Calibration

Table 1 shows the benchmark parameter values we use for the simulation. Many are standard and based on Gourio (2013); others are chosen to fit Japan, and are based on Sugo and Ueda (2008) and Hirose and Kurozumi (2012). The TFP trend growth rate, μ, is 0.0182, which is the actual annual growth rate of real GDP per capita from 1990 to 2019 for Japan (see Figure 1).[18] The standard deviation of the productivity shock σ_e is 0.023, calibrated to match the standard deviation of the change in output ($\Delta \log Y_t$) for Japan. Government spending $g = G_t/z_t$ is chosen as 0.014, consistent with the speed of Japan's government debt accumulation.[19]

There are three main parameter categories, each of which needs careful calibration: the tax policy during the crisis (ω^i $(i = C, K, G, T)$), capital-use inefficiency during the crisis (λ_t^Y and λ_t^D), and probability of the crisis (d_0 and d_1). We explain our calibration strategy below.

4.2.1 Tax in a Crisis

As in Kozlowski, Veldkamp, and Venkateswaran (2020), we assume that expectations of the tax policy during a crisis, ω^i $(i = C, K, G, T)$, are formed by learning from history. A crisis is a typical tail event, about which agents in the economy have no knowledge besides historical episodes of similar events. Here, the capital levy ω^K is particularly important, so we review episodes in interwar Europe and post-war Japan.

Eichengreen (1989) notes that prominent British economists and policymakers debated the use of a capital levy in the 1920s when the UK government was suffering from the public debt overhang due to the war debt from World War I. This active debate about the capital levy exemplifies the strength of the *ex post* temptation for policymakers to introduce a one-time capital levy when government debt builds up.[20] Similar debates took place in Italy, Czechoslovakia, Austria, Hungary, Germany, and France. Although Eichengreen concludes that the capital levies in those countries failed,[21] if history repeats

[18]As Gourio (2012, 2013) and Isore and Szczerbowicz (2017) argue, occasional disasters affect the calculation of a trend from data. However, Japan did not experience a government debt crisis between 1975 and 2019, and thus, we do not exclude any sample observations when calculating the trend.

[19]The mean of 0.39 for y_t in our simulation implies that the ratio of government spending to GDP (g/y_t) is approximately 3%. Note that G_t in our model corresponds to government expenditure minus tax in normal times. According to the OECD, the mean of Japan's actual general government net lending as a percentage of GDP is −5% for 1990–2019. Given the considerable size of the volatility for this variable (the standard deviation is 3 percentage points), the size of g is comparable with the actual size of the government deficit.

[20]Eichengreen (1989) states that, "[i]n modern times, capital levies have come under consideration following every period of major military expenditure and rapidly rising debt/income ratios. (...) None of these proposals was adopted. For examples where capital levies were actually implemented, we must turn to the 20th century."

[21]Eichengreen (1989) finds that Italy and Czechoslovakia were the closest to success. A factor in the

itself or people learn from previous events, these episodes suggest that some people anticipate the capital levy when a crisis strikes. Moreover, post-World War II Japan is a noteworthy example of a successful implementation.

In Japan, the government debt inherited from the wartime period amounted to 267% of the national income in 1944, more than 99% of which was internal debt (MOF 1976, Kawamura 2013). Although Eichengreen (1989) emphasizes that the absolute power of the Supreme Commander of the Allied Powers that occupied Japan was crucial in the successful implementation of the capital levy of 1946–47, the MOF (1976) and Kawamura (2013) show memorandums that prove that it was the MOF that decided to impose a capital levy to avoid an outright default of government debt. The Supreme Commander actually recommended declaring a partial default. The capital levy, or wealth tax, taxed all real and financial assets owned by Japanese residents, including land, houses, government bonds, bank deposits, and machinery. Tax rates varied progressively from 25% to 90% depending on the taxpayer's income class. The capital levy worked effectively to reduce wealth inequality among the Japanese.[22] "With important elements of democracy in suspension, the levy could be quickly and effectively implemented" (Eichengreen, 1989) using the deposit blockade and withdrawing the legal tender status of old yen. The package of these policies, which could not have been implemented in normal times, helped the Japanese government seize domestic wealth efficiently.[23]

We use this post-war Japanese experience reported by the MOF (1976) to calibrate the parameters of the tax weights ω^i $(i = K, C, G, T)$. The government debt outstanding (B) was 215,867 million yen at the end of 1945, and the GBO and war indemnity (T^G) were 199,454 million yen (Chapter 11 in the MOF 1976) and 16,413 million yen (Chapter 7), respectively. The government owed war indemnity because it suspended payments of subsidies, loss compensation, wartime insurance, evacuation costs, and so on. This war indemnity was virtually unpaid because 100% of the tax was imposed under the War Indemnity Special Measures Law. The share of the unpaid debt constitutes ω^G; thus, we obtain the fraction of the partial default as $\omega^G = T^G/B = 0.076$. The capital levy (T^K)

failure of the capital levies in these countries was the democratic decision-making processes because political resistance from property owners led to extreme delays and opportunities for capital flight.

[22]According to the MOF (1976), the main aim of the capital levy was to avoid default, whereas the Supreme Commander was attempting to reduce inequality. Kawamura (2013) points out that although the super rich class was taxed most heavily, the middle class paid the largest proportion of the total revenue generated by the capital levy.

[23]Saito (2017) points out that the exchange of old yen for new yen was an effective way for the government to seize private assets concealed on the black market. However, note that the tax revenue from the capital levy was less than the amount needed to restore the sustainability of government debt. By contrast, Hattori and Oguro (2016) estimate that the seigniorage revenue resulting from hyperinflation immediately after World War II was nearly 29% of the GDP. However, this large seigniorage revenue was not intended by the government or MOF officials. Rather, the MOF was concerned about the risk of hyperinflation, as in Germany, which prompted it to introduce the capital levy (MOF, 1976).

was 43,500 million yen (Chapter 7); thus, we obtain $\omega^K = T^K/B = 0.202$. For ω^C, we use the fact that the government increased personal income tax and corporation tax to raise revenues by $T^C = 3,907$ million yen in 1946 (Chapter 7). Considering that consumption tax plays virtually the same role as income tax, we calculate ω^C as $T^C/B = 0.018$. Finally, we calibrate ω^T based on the decrease in government expenditure under the government consolidation plan in 1946 ($T^T = 3,398$ million yen); we find no evidence of an increase in lump-sum tax collection. Then, we obtain $\omega^T = T^T/B = 0.016$.

4.2.2 Capital-Use Inefficiency in a Crisis

We calibrate the parameters associated with the capital-use efficiency of production λ_t^Y and depreciation λ_t^D. In the former case, we use the database on systemic banking crises from 1970 to 2017 constructed by Laeven and Valencia (2018). We estimate the following equation using the OLS method:

$$\frac{\Delta Y_i}{Y_i} = \theta_0 + \theta_1 \log(1 + \xi_i) + \varepsilon_i, \tag{26}$$

where ΔY_i and ξ_i represent the output loss and the ratio of nonperforming loans to total loans, respectively, for crisis episode i, and ε_i is a residual. The output loss is measured as the cumulative sum of the differences between the actual and trend real GDP for a period of four years after a crisis, divided by four (i.e., average annual decrease in output). The coefficient θ_0 indicates the size of the output loss in the event of a banking crisis when there are no nonperforming loans, whereas the coefficient θ_1 indicates the elasticity of the output loss to the ratio of nonperforming loans to total loans. We use 29 of the 151 systemic banking crisis episodes by restricting our estimation to the OECD member countries when a crisis occurred. Figure 5 shows the scatter plot of the results, where each circle indicates a systemic banking crisis episode. The estimates of θ_0 and θ_1 are 0.059 and 0.202, respectively; their standard errors are 0.016 and 0.097, and their p-values are 0.0013 and 0.048, respectively. This result suggests that a significant output loss occurs in the event of a banking crisis. Furthermore, as nonperforming loans increase, the degree of the loss increases significantly at the 5% level.[24] Combining this result with the production function (9) in a crisis, we formulate λ_t^Y as follows:

$$\lambda_t^Y = F(\xi_t) = (1 - \theta_0 - \theta_1 \log(1 + \xi_t))^{1/\alpha}, \tag{27}$$

where we assume that the crisis does not affect labor N_t, which is a quantitatively good approximation in our simulation.

We assume that λ_t^D is constant ($\lambda_t^D = \lambda^D$) because we are unable to find informative empirical evidence. We determine the parameter value using Japan's Flow of Funds

[24]When we use the following equation: $\Delta Y_i/Y_i = \theta_0 + \theta_1 \xi_i + \varepsilon_i$, the estimates of θ_0 and θ_1 become 0.062 and 0.154, respectively; their standard errors are 0.015 and 0.079, and their p-values are 0.0005 and 0.064, respectively.

Accounts compiled by the Bank of Japan. When Japan's systemic banking crisis occurred in 1997 (recorded in Laeven and Valencia 2018), the liability of non-financial private corporations decreased by 6.074%.[25] Considering that this corresponds to a change in the private firms' real firm value, we obtain $\lambda^D = 1 - 0.0607$.

4.2.3 Probability of a Crisis

We need to calibrate two parameters associated with the probability of a crisis, d_0 and d_1, in equation (1). For domestic debt defaults, empirical data are scarce, as shown by Reinhart and Rogoff (2010). However, as we discussed in the Introduction, several sources of data indicate a mounting concern about a government debt crisis in Japan. Of these, we employ the CDS spread as a target variable to calibrate d_0 and d_1, mainly because it can be linked numerically to the probability of a crisis. Based on our model, the CDS spread for the five-year maturity can be priced as follows:[26]

$$CDS \sum_{j=1}^{5} E_t \left[\left(\prod_{k=1}^{j} M_{t+k}(x_{t+k} = 0)(1 - p_{t+k-1}) \right) \right]$$
$$= (1 - R) \sum_{j=1}^{5} E_t \left[\left(\prod_{k=1}^{j-1} M_{t+k}(x_{t+k} = 0)(1 - p_{t+k-1}) \right) \cdot M_{t+j}(x_{t+j} = 1) p_{t+j-1} \right], \quad (28)$$

where p_t and R represent the arrival rate of a crisis in period $t + 1$ and the government bond recovery rate during a crisis, respectively. The left-hand side of the equation represents the present value of the premiums paid by the protection buyer contingent upon a crisis event not occurring. The right-hand side of the equation represents the present value of the payment by the protection seller contingent on a crisis event. Note that the stochastic discount factor $M_{t+1}(x_{t+1})$ changes depending on whether a crisis occurs in $t + 1$ (i.e., $x_{t+1} = 1$) or not (i.e., $x_{t+1} = 0$). Because ω^G represents the degree of partial default, the recovery rate of government bonds R equals $1 - \omega^G$.

Using this formula and the observed CDS values, we calibrate d_0 and d_1. Specifically, we target the following two variables. The first target variable is the level of the CDS spread when the debt ratio is low. Because CDS data are available only from 2003 and the global financial crisis around 2008 led to a surge in the CDS spread, we use the average value for 2003–2006, which is 0.008 (i.e. 0.8 percentage points). The second target is the elasticity of the CDS spread to the debt ratio. As we stated in Section 2.1, the estimate of the elasticity is approximately one.

Based on the model, we calculate the level and elasticity of the CDS spread for the given values of d_0 and d_1. To this end, we first generate the path of the CDS spread as well

[25] At the end of March 1997, private bank borrowings and equity were 400 and 375 trillion yen, respectively. At the end of March 1998, they had decreased to 389 and 339 trillion yen, respectively.

[26] Refer to Pan and Singleton (2008) and Longstaff et al. (2011).

as the debt ratio.[27] Second, we calculate the average CDS spread when the simulated debt ratio is low, which we compare with the first target variable above.[28] Third, we regress the logarithm of the simulated CDS spread by the logarithm of the simulated debt ratio using the OLS method to obtain the elasticity of the CDS spread.

We find that the combination of $d_0 = 0.025$ and $d_1 = 4$ is the best fit to the data. This makes the level and elasticity of the CDS spread 0.008 and 0.96, respectively. The surveys by Morikawa (2016, 2017) show that the average crisis probability from 2016 to 2030 is around 0.02 ($\sim 1 - (1 - 0.25)^{1/15 \text{ years}}$). While this value is comparable with the value of d_0 (i.e., the crisis probability when the debt ratio is low), the former is much lower than the crisis probability simulated for the same period (i.e., when the debt ratio is above 1.2); that is, 0.19. In contrast, Hoshi and Ito (2014) imply that there is a fairly high crisis probability because the amount of government debt will exceed that of private sector financial assets within 10 years. Our model simulation shows that the probability of a crisis in 2024, 10 years after their study, is 0.29. This suggests that our benchmark calibration lies between those of Morikawa (2016, 2017) and Hoshi and Ito (2014).

4.3 Simulation Results Based on Standard Sovereign Default Models

In this subsection, we investigate the extent to which a standard model of sovereign default can or cannot explain Japan's secular stagnation. Our model becomes equivalent to a standard model when the government imposes tax only or heavily on GBO during a crisis. Specifically, we set $\omega^G = 0.2$ and $\omega^C = \omega^K = \omega^T = 0$, which implies that the government declares a partial default for 20% of its debt.

Figure 6 shows the simulation results based on this model (the solid line) and the data for Japan from 1990 to 2019 (the solid line with circles). For the simulation, we assume that normal times, $x_t = 0$, continue from $t = 1$ to 35 years. Log productivity, $\log z_t$, grows with the speed of μ and no shock ($e_t = 0$). We set the initial value of b_t^G to 0.05 to match the actual value in 1990. The initial value of k_t is set as $k_0 = k_{SS} e^{0.11}$, where k_{SS} is the steady-state value of capital stock. This is calculated as the mean of the time-series paths of k_t obtained using the method in Subsection 4.4.1. The value of $e^{0.11}$ comes from the actual log deviation of the capital stock from its linear trend in 1990, based on the JIP 2015 Database.

For comparison, we plot the actual paths for Japan from 1990 to 2019 as the solid

[27]See Section 4.4.1 for the detailed simulation procedure. In the CDS spread simulation, we assume that the future productivity shock, e_{t+1}, is zero for simplicity. In other words, whether a crisis occurs or not is the only uncertain event.

[28]To be precise, we calculate the average when the simulated debt ratio falls within the range of the actual debt ratio for 2003–2006 (i.e., between 0.66 and 0.73). The average value hardly changes when we calculate the average CDS spread when the simulated debt ratio is below the 2006 level (i.e., 0.73).

line with circles. Here, real GDP and investment per capita are shown as deviations from the linear trends of the respective variables from 1975 to 2019 (see Figure 1). Output (y_t) and investment (i_t) based on the model are detrended by z_t, and then demeaned so that their average from 1990 to 2019 equals that in the data for the same period.

The figure shows that as the debt-to-GDP ratio (b^G/y) increases, output (y) decreases, which is consistent with the data. Output decreases because capital-use efficiency in a crisis (λ_t^Y) deteriorates as banks hold nonperforming assets if a crisis occurs. However, the size of the output decrease based on the simulation is smaller, and investment (i_t) is more or less unchanged. The more important simulation results are that the government bond yield ($R^G - 1$) increases while the credit spread ($R^K - R^G$) decreases, both of which are inconsistent with what we observe in the data. The gap in the government bond yield between the model and data widens in the 21st century. As the government bond yield increases, the CDS spread and the probability of a crisis increase, both at an accelerated pace.[29]

These simulation results emerge because investment in government bonds becomes riskier as the probability of a crisis increases, while investment in capital becomes relatively less risky. Thus, the government bond yield increases and the credit spread decreases, while investment does not decrease much. Introducing a capital levy (i.e., $\omega^K > 0$) is a simple and powerful way to improve the model fit, as we see in the next subsection.

4.4 Simulation Results Based on the Benchmark Model

Now we assume the tax weights of $\omega^K = 0.202$, $\omega^C = 0.018$, $\omega^G = 0.076$, and $\omega^T = 0.016$ that are calibrated in Section 4.2.1.

4.4.1 Moments of Variables: Does the Model Have a Good Fit?

In what follows, we discuss two sets of simulation results. The first simulation checks the fit of our model with the data by comparing the key first and second moments of variables, such as the mean of the debt-to-output ratio, the standard deviation of the change in consumption, the correlation coefficient between the change in output and the debt-to-output ratio in the previous period, and the correlation coefficient between the change in investment and the credit spread.

Using the model, we calculate the time-series paths of key economic variables. We generate the time-series path of TFP (z_t) for $t = 1, 2, \cdots, 45$ years. The initial values of the state variables are $b_0^G = 0.05$ for GBO, which we set to be constant with the value

[29]The size of ω^G that we assume here may be modest because in principle we can consider the case of a full default (i.e., $\omega^G = 1$). We find that a higher value of ω^G accelerates changes in b^G/y, $R^G - 1$, the CDS spread, and the probability of a crisis, so that the probability becomes one before 2019.

until 1990, and $k_0 = 0.91$, which is the midpoint of the range of k in our numerical computation. The simulated key economic variables include the debt-to-output ratio (b_t^G/y_t), the change in log output ($\Delta \log Y_t$) , and the interest-rate spreads ($R_t^G - 1$ and $E_t\left[R_{t+1}^K(0)\right] - R_t^G$). In addition, we simulate the deposit spread ($E_t\left[R_{t+1}^D(0)\right] - R_t^G$). The moments are calculated by excluding the crisis periods because Japan has not experienced a government debt crisis for more than half a century. For the same reason, we calculate the interest rates $R_{t+1}^K(x_{t+1})$ and $R_{t+1}^D(x_{t+1})$ in the normal state ($x_{t+1} = 0$). We set $x_t = 0$ for $t = 1, 2, \cdots, 45$; agents in the model are prepared for the risk of a crisis according to equation (1). We repeat this calculation 50 times.

We tabulate the actual moment values for Japan from 1975 to 2019, except for the credit and deposit spreads, which run from 1993 to 2019 and 1988 to 2019, respectively. We express government debt in net rather than gross terms for B_t^G, except for the figures shown in parentheses. The spread data are the same as those reported in the left-hand panel of Figure 4. The deposit rate is the interest rate on the five-year fixed-term deposit.

Table 2 compares the moments in the actual data with those based on the benchmark model. Although we do not target moments other than $\sigma(\Delta \log Y_t)$, the fit of the model is good. The mean and standard deviation are of the same order as those in the data. The main caveat seems to be that in the benchmark model, the mean simulated government bond yield, ($R_t^G - 1$), is 4.4 percentage points, which is 2.5 percentage points higher than the actual mean. In other words, the actual interest rate is lower than that suggested by the actual TFP growth rate for Japan. In reality, government bonds act as a medium of exchange, like money, in that they serve as collateral in short-term transactions in the interbank market. This should add a liquidity premium to the price of government bonds, decreasing the government bond yield. One of the major causes of the discrepancy between the simulated and actual bond yields may be that our model does not consider such a liquidity premium.

Comparing the correlation coefficients, we find that the benchmark model successfully explains the negative coefficients between output growth and both the ratio of GBO (model -0.24 and data -0.30) and the credit spread (model -0.19 and data -0.15). Furthermore, output growth is positively correlated with the government bond yield (model 0.35 and data 0.28). Investment growth has a similar correlation coefficient with the credit spread and the government bond yield, although that between investment growth and the ratio of GBO is almost zero in the data. According to the model, the deposit spread is negatively correlated with output and investment growth; however, this is not the case in the data.

4.4.2 Time-series Path: Does the Model Explain the Persistent Stagnation?

The second simulation is similar to that in Section 4.3. We generate the time-series paths of the economic variables by assuming that the crisis indicator x_t has an exogenous path. Then, we discuss whether our model can explain Japan's persistent stagnation. We assume that normal times ($x_t = 0$) continue from $t = 1$ to 39 years, then a crisis ($x_t = 1$) occurs at $t = 40$, followed by $x_t = 0$ thereafter.[30]

Figure 7 shows that the simulated path of the government debt-to-GDP ratio (b_t^G/y_t; solid line) is similar to the actual path (solid line with circles). This is not surprising because we choose government expenditure g so that the two paths match.

Our model accounts for nearly half the decrease in output (y_t) during the first two decades of the 21st century. Output decreases between 2000 and 2019 as capital investment (i_t) slows down in this period. The model accounts for actual investment during 2000–2019 fairly well. Investment decreases because agents become increasingly cautious about a future crisis, for two reasons. First, the crisis probability increases with GBO. Second, given the crisis probability, agents anticipate a greater capital tax rate during a crisis (τ_t^K) as GBO increases because the government will need greater tax revenue to repay its increased debt. Furthermore, banks' losses during a crisis (ξ_t) increase as GBO increases, which worsens capital-use efficiency (λ_t^Y). The higher capital levy and lower capital-use efficiency both lead to an increase in the credit spread ($E_t\left[R_{t+1}^K(0)\right] - R_t^G$) in normal periods, which further discourages capital investment as GBO accumulates.[31]

Last, but not least, the model shows that the bond yield ($R_t^G - 1$) decreases rather than increases with GBO. Although a tax on government bonds is essentially a default, the parameter value of ω^G is low in our benchmark simulation. Thus, agents invest more of their savings in government bonds and less in capital. In other words, Japanese people do not anticipate a large-scale government bond default, although they do fear tax increases when a crisis strikes. This pattern is consistent with the Japanese data. The decrease in the bond yield in the model is similar to that in the data. However, there are caveats. The level of $R_t^G - 1$ based on the model is several percentage points higher than that in the data, as we discussed in Section 4.4.1. Moreover, in the model,

[30] The agents in our model do not know this predetermined event; instead, they form expectations on the crisis probability based on equation (1). Thus, the simulated paths do not change until year $t = 39$, as long as the crisis occurs at $t = 40$ or later. The longer the delay before the crisis, the larger will be the crisis effect because the government introduces larger tax increases.

[31] In the model, we assume, for simplicity, that a crisis ends in one period, unless x_t happens to be one in two or more consecutive periods. Our results remain quantitatively similar, even if we assume that a crisis continues for more than one period, as long as the distortionary taxes during that crisis have a similar discounted present value. This is a type of the Ricardian equivalence, in that the timing or duration of a tax imposition does not affect the *ex ante* slowdown in investment and output, as long as the total tax distortion remains the same.

Output based on the model increases during the 1990s when GBO is low. This is driven by an increase in consumption. In the 1990s, the probability of a crisis is low and, even if a crisis occurs, the resulting distortion caused by the capital levy and capital-use inefficiency is small. As a result, the household chooses to consume in the current period rather than save. In reality, the decrease in actual output during the 1990s might have been caused by factors outside our model, such as the collapse of the asset-price bubble in the early 1990s and the subsequent decade-long period of financial distress.

the increase in the credit spread ($E_t\left[R^K_{t+1}(0)\right] - R^G_t$) is more sluggish in the first two decades and steeper in the subsequent decade than the data suggest.

4.5 Differing Expectations of Tax Scenarios in the Crisis Period

In the benchmark model, we assumed tax weights of $\omega^K = 0.202$, $\omega^C = 0.018$, $\omega^G = 0.076$, and $\omega^T = 0.016$ as the government tax policy following a government debt crisis. In this subsection, we investigate how the simulation results change when Japanese people have differing expectations of tax scenarios at the point of a crisis. This exercise helps clarify the underlying transmission mechanism caused by the taxes. We illustrate the results in Figures 7 and 8.

We begin by considering a first-best case in which the government imposes a lump-sum tax (T_t) only. The size of the tax collection is the same as that in the benchmark case, namely $\omega^T = 0.312$. Because banks remain solvent, bank runs do not occur, and capital use is efficient (i.e., $\lambda^Y = \lambda^D = 1$). This hypothetical case partly reflects a decrease in government transfers (e.g., a decrease in pension obligations or medical care) during a crisis. "All T tax" in Figure 7 indicate almost no change in real economic activity as GBO increases. Slight changes occur because the capital stock starts from $k_0 = k_{SS}e^{0.11}$. Note that the growth rate of the debt ratio (b^G_t/y_t) slows down because output no longer decreases.

In order to investigate the effects of various taxes independently, we set the tax weight ω^i ($i = K, C, G$) at zero, one by one, and simulate the time-series paths of the economic variables. By the size of the decrease in ω^i, the value of ω^T increases to maintain the total amount of tax collected during a crisis. The results are shown in Figure 8. When we eliminate the capital levy during the crisis ("No K tax" in the figure), the decreases in output and investment are greatly mitigated, and the credit spread decreases, rather than increases. In this scenario, capital use during the crisis remains inefficient ($\lambda^Y, \lambda^D < 1$), which contributes to a decrease in investment. However, the figure suggests that the contribution of capital-use inefficiency is quantitatively much smaller than that of the capital levy. The capital-use efficiency for output (λ^Y_t) deteriorates only slightly because the ratio of banks' nonperforming loans does not increase much without the capital levy.

"No G tax" in Figure 8 shows that eliminating the tax on GBO during a crisis causes little change, particularly before 2010. However, the loss of fear of a government bond default decreases the government bond yield, which slows the accumulation of the debt ratio. This reduces the crisis probability compared with that of the benchmark model, which mitigates the output stagnation after 2010. "No C tax" in the figure shows that eliminating the consumption tax during a crisis does not alter our simulation results considerably. In our calibration, both ω^C and ω^G are small, and thus, they have quantitatively limited effects.

There are many alternatives to the tax policies considered above. One of the most realistic options is an income tax imposed on the return on capital (e.g., $E_t\left[R^K_{t+1}(0)\right]-1$ in our model), return on government bonds (e.g., R^G_t-1), or labor income (e.g., W_tN_t). However, such taxes on incomes (flow variables) are insufficient to repay government debt because their revenue cannot exceed the income. Thus, expectations of income taxes have small quantitative effects on pre-crisis output. Qualitatively, the effect of a capital income tax is similar to that of a capital levy, and the effect of a tax on the interest on government bonds is similar to that on GBO. The effect of a labor income tax is similar to that of a consumption tax, both qualitatively and quantitatively.

5 FURTHER ANALYSES

5.1 A Smaller Loss of Capital-Use Efficiency

In our model, Japan's persistent stagnation is caused by two main expectations: the capital levy and capital-use inefficiency at the time of a crisis. In Figure 9, we investigate the latter effect in more detail by assuming a smaller loss of capital-use efficiency. To do so, we first assume no additional capital depreciation during the crisis (i.e., $\lambda^D=1$), and we keep the production inefficiency (λ^Y_t) at the same value as the benchmark. The dotted line shows that the pace of the government bond yield decrease and credit spread increase becomes slower. Moreover, the size of investment decrease declines, which suggests that the capital-use inefficiency reinforces the adverse effect of GBO on capital investment. We observe an increase, rather than a decrease, in output one period after the crisis occurs because in this case, the crisis does not depreciate capital.

Second, we assume that θ_1 is zero. Although we obtained a positive estimate for the elasticity of output loss to the ratio of nonperforming loans to total loans, it was not significant at the 1% level. To check the robustness of our results, we simulate the model by setting $\theta_1=0$. The simulated paths, shown as solid lines, are almost the same as those based on the benchmark model.

5.2 Low and Constant Crisis Probability

We examine the robustness of our results to changes in the parameter associated with the crisis probability, particularly, d_1. We assume $d_1=0$, meaning that the crisis probability is unchanged (constant at d_0) even if the debt ratio changes. Figure 10 shows the simulation results. Since agents face a smaller, constant crisis risk, the increase in the debt ratio leaves key economic variables such as output, investment, bond yield, credit spread, and CDS spread, almost unaffected.

5.3 Foreign Direct Investment

Although a closed-economy model serves as a good starting point to describe the case of Japan because around 90% of GBO is held by domestic investors, domestic investors have the opportunity to invest abroad. Thus, it is valuable to consider implications for foreign direct investment under the fear of the government debt crisis.

In Appendix B, we construct a simple two-period model, in which private agents can invest in not only government bonds and capital at home, but also in capital abroad. In the model, the benefit of foreign direct investment is that it ensures a riskless return irrespective of a crisis. The government does not impose tax on this return. However, there is a cost, represented by a convex cost function for foreign direct investment. In this environment, we show analytically that foreign direct investment increases as the government debt increases as long as $\omega^K > 0$. Because investing in capital at home becomes riskier as the debt increases, private agents increase investment overseas.

Figure 11 shows the actual changes in foreign direct investment from Japan abroad (i.e., outward) in the solid line and from abroad to Japan (i.e., inward) in the dashed line. It shows a secular increase in outward investment from 1990 to 2019 consistent with the model prediction. By contrast, inward investment is almost unchanged and remains low. Although factors besides taxes and crisis risk influence foreign direct investment, the fear of large-scale taxation on capital and misallocations of capital in future debt crises may be a factor that discourages inward investment.

5.4 Welfare

Using the model, we calculate the loss in welfare caused by the risk of crises. We generate time-series paths for TFP (z_t) and the crisis indicator (x_t) randomly for 1000 years, discard the first one-third of the samples, and take the mean. A welfare loss can arise from not only the expectation of a crisis before it occurs, but also the distortion at the time of the crisis. Thus, we calculate the welfare loss by including crisis periods, unlike the simulation conducted in Section 4.4. Table 3 reports the simulation results based on the benchmark model and the various tax scenario models that we described in Section 4.5. Output (y_t) and welfare (U_t, in terms of consumption equivalence) are indicated as differences from the values in the benchmark model.[32]

If the tax is not distortionary ("All T tax"), both output and welfare improve by 19% and 7%, respectively. These large improvements are the result of a non-distortionary tax and capital-use efficiency. Of the distortionary taxes, the capital levy is the most impor-

[32]Welfare in terms of consumption equivalence is measured as follows. We calculate lifetime utility in the case in question and the benchmark case. We then determine the permanent percentage increase in consumption needed in the benchmark case such that households will be indifferent between the two cases. This permanent increase is the consumption equivalence.

tant, contributing to the decreases in output and welfare by 11% and 1%, respectively. The tax on GBO plays the second most important role, decreasing output and welfare by 5% and 1%, respectively.

5.5 Permanent Distortionary Tax to Prevent a Crisis

Should a government raise tax rates in normal times to avoid a crisis? Because a crisis event causes not only a depression at the time but also stagnation beforehand, it may be better to introduce higher tax rates preemptively. To answer this question, we consider a model in which the government always aims to maintain a bounded range of government debt. Specifically, we assume the following tax policy:

$$\tau_t^C = \max\left(0, \min(\tau_t^{C*}, 0.3)\right),$$
$$\tau_t^{C*} = \left(1 - \frac{\bar{b^G}}{b_t^G + g} q_t^G e^{\mu}\right) \frac{b_t^G + g}{c_t}. \tag{29}$$

The government imposes a consumption tax only to maintain b_{t+1}^G around its target $\bar{b^G}$, where the maximum tax rate for τ_t^C is 30%, and the target $\bar{b^G}$ is 0.3.[33]

Table 3 presents the simulation results. The bottom row shows that the debt ratio (b^G/y) stabilizes at 70%, and that output increases by 11%. Consequently, welfare increases by 6% in the consumption unit. Therefore, preemptive tax increases improve social welfare.[34]

5.6 Final Thoughts on Government Tax Policy

Given the results on welfare, one may think that the Japanese government will (or should) not adopt a capital levy, but should instead use a less distortionary tax on government bonds or a partial default if a debt crisis occurs. Moreover, one may wonder why the Japanese government does not raise taxes now, before a crisis occurs. In the final part of this study, we discuss several issues related to government policy, although our model cannot address these questions formally because it does not endogenize government actions.

With regard to the tax policy during a crisis, three points are worth making. First, as we noted in Section 4.2.1, the post-war Japanese government did not resort to a lump-

[33]Because the future TFP shock e_{t+1} is unknown, the government cannot perfectly stabilize b_{t+1}^G at its target. In our numerical calculation, we set an upper bound b^U for b_t^G, where $b^U \gg \bar{b}^G$, and impose a lump-sum tax only when the consumption tax alone cannot maintain $b_{t+1}^G \leq b^U$.

[34]An alternative approach is to calculate lifetime utility at t, conditional on the state in which the debt-to-GDP ratio, b_{t-1}^G/y_{t-1}, is as high as it is today. We confirm that this hardly changes our results. Because discount factor β in our model is close to one, the household is concerned about the long-run state of the economy, rather than its short-run transition, even if a high debt ratio induces a very high temporary consumption tax rate.

sum tax or a partial default as main means, although it decreased expenditure. Instead, the government imposed a heavy capital levy. Second, a capital levy has political appeal because it reduces wealth inequality, as demonstrated in post-war Japan (Eichengreen 1989). In contrast, a lump-sum tax hurts the poor relatively more than it does the rich. Third, as noted by Eichengreen (1989) as well as Chamley (1986) and Chari, Christiano, and Kehoe (1994), a once-off capital levy has no distortionary effect on economic activity *ex post facto*, although this point applies to a lump-sum tax and a tax on government bonds as well.

With regard to the tax policy before a crisis, a possible reason that the Japanese government does not try to decrease its debt today by raising the tax rate may be its short-time perspective. Parliament elections for the upper and lower houses occur at least every three years. Thus, if the government does not believe a crisis will occur in the near future, it has little incentive to raise taxes because doing so hurts utility in the short run. Indeed, Prime Minister Shinzo Abe postponed a promised increase in the consumption tax rate from 8% to 10% twice, in 2014 and 2016.

6 CONCLUDING REMARKS

We analyzed an economy at risk of a government debt crisis and provided a new perspective to explain the secular stagnation and low interest rates that occur. We demonstrated that most of the persistent slowdown can be accounted for by the increasing fear of a capital levy and capital misallocation in a future debt crisis associated with increasing government debt.

Because our framework is simple, it can be extended and enriched in numerous ways. One possibility is to introduce nominal variables (e.g., Aguiar et al. 2015). The nominal version of our model would be useful for analyzing price dynamics such as hyperinflation, and the implications for monetary policy in the event of secular stagnation with deflation. Importantly, inflation is essentially equivalent to a tax on GBO (i.e., a default) because inflation decreases the real value of nominal bonds directly, whereas it does not affect the real value of capital much. Thus, we believe our analysis of a tax on GBO approximates the effects of high inflation in a nominal model. Second, our model can be extended to incorporate uncertainty during a crisis. A crisis usually entails a large degree of uncertainty in the market and government responses, which may quantitatively alter our results. Finally, it would be worth extending our model to include heterogeneous agents. As such, when a crisis occurs, taxes on capital stock and government bonds can influence the holdings of both assets, which plays an important role in the self-insurance of heterogeneous households.

These extensions may help us explore whether the increasing risk of a crisis causes other economic difficulties in addition to the persistent stagnation and low interest rates demonstrated in this study.

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APPENDIX A MODEL DETAILS

We denote the variables divided by z_t by their lower case letters (e.g., $y_t \equiv Y_t/z_t$), with the exception of $u_t \equiv U_t^{1-\psi}/z_t^{\nu(1-\psi)}$. In summary, we have 16 equations for 16 unknown endogenous variables, $\{c_t,\ k_t,\ N_t,\ y_t,\ M_{t+1},\ u_t,\ x_t,\ \lambda_t^Y,\ q_t^G,\ R_t^K,\ R_t^D,\ b_t^G,\ \tau_t^K,\ \tau_t^C,\ \tau_t^G,\ t_t\}$:

$$(1-\alpha)\frac{y_t}{N_t} = \frac{1-\nu}{\nu}\frac{(1+x_t\tau_t^C)c_t}{1-N_t}, \tag{30}$$

$$1 = E_t\left[M_{t+1}R_{t+1}^D(x_{t+1})\right], \tag{31}$$

$$R_{t+1}^D(x_{t+1}) = \frac{(1-x_{t+1}\tau_{t+1}^K)R_{t+1}^K(x_{t+1})K_{t+1} + (1-x_{t+1}\tau_{t+1}^G)B_{t+1}^G}{K_{t+1}+q_t^G B_{t+1}^G}, \tag{32}$$

$$E_t[M_{t+1}(1-x_{t+1}\tau_{t+1}^K)R_{t+1}^K(x_{t+1})] = \frac{1}{q_t^G}E_t[M_{t+1}(1-x_{t+1}\tau_{t+1}^G)], \tag{33}$$

$$R_t^K = (1-\delta)(x_t\lambda^D + 1 - x_t) + \alpha\frac{y_t}{k_t}, \tag{34}$$

$$y_t = \left(x_t\lambda_t^Y + (1-x_t)\right)^\alpha (k_t)^\alpha N_t^{1-\alpha}, \tag{35}$$

$$M_{t+1} = \beta\left(\frac{1+x_t\tau_t^C}{1+x_{t+1}\tau_{t+1}^C}\right)e^{(\nu(1-\psi)-1)(\mu+\sigma_e e_{t+1})}\left(\frac{c_{t+1}}{c_t}\right)^{\nu(1-\psi)-1}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\nu)(1-\psi)} \tag{36}$$

$$u_t = (1-\beta)c_t^{\nu(1-\psi)}(1-N_t)^{(1-\nu)(1-\psi)} + \beta E_t\left(e^{\nu(1-\psi)(\mu+\sigma_e e_{t+1})}u_{t+1}\right), \tag{37}$$

$$y_t = c_t + k_{t+1}e^{\mu+\sigma_e e_{t+1}} - (1-\delta)(x_t\lambda^D + 1 - x_t)k_t, \tag{38}$$

$$\Pr(x_{t+1}=1|b_t^G) = \frac{1}{1+(1/d_0-1)\exp(-d_1 b_t^G)} \tag{39}$$

$$\begin{aligned} &q_t^G b_{t+1}^G e^{\mu+\sigma_e e_{t+1}} + x_t\tau_t^C c_t + x_t\tau_t^K k_t \\ &+x_t\tau_t^G b_t^G + x_t t_t = b_t^G + g, \end{aligned} \tag{40}$$

$$\tau_t^K R_t^K K_t = \omega^K(b_t^G + g), \tag{41}$$

$$\tau_t^C c_t = \omega^C(b_t^G + g), \tag{42}$$

$$\tau_t^G b_t^G = \omega^G(b_t^G + g), \tag{43}$$

$$t_t = \omega^T(b_t^G + g), \tag{44}$$

where $\lambda_t^Y = F(\xi_t)$ and e_t follows $N(0,1)$.

APPENDIX B ANALYSIS USING A SIMPLIFIED TWO-PERIOD MODEL

B.1 Setup

In this section, we provide an overview of the model by presenting the analytical results of a simplified two-period version of the model, in which the banking sector is abstracted away and the household directly holds government bonds and capital stock. Time runs from period 0 to 1. In period 0, the crisis does not occur. In period 1, the crisis occurs if $x = 1$ and does not occur if $x = 0$. For simplicity of exposition, we assume full depreciation (i.e., $\delta = 1$) and no consumption tax during a crisis (i.e., $\tau^C = 0$). Household utility is expressed simply as log consumption with fixed labor supply (i.e., $N = 1$). The probability that a crisis occurs is $p(B_0)$, which is increasing in B_0 at the beginning of period 0.

The government budget constraint in period 1 is

$$B + G = x\tau^K R_1^K K + x\tau^G B + xT + (1-x)\tilde{T},$$

where we assume that the debt B is repaid only by the lump-sum tax $\tilde{T}$ if the crisis does not occur (i.e., $x = 0$).

In period 0, the representative household is given the initial asset holdings, K_0 and B_0. The household solves the following problem in period 0:

$$U_0 = \max_{C_0, C_1, K, B} (1-\beta)\log C_0 + \beta E_0 \log C_1,$$

subject to the budget constraint:

$$C_0 + qB + K \leq R_0^K K_0 + B_0 + W_0 + G, \tag{45}$$

and

$$C_1 \leq (1 - x\tau^K) R_1^K(x) K + (1 - x\tau^G) B + W(x) - xT - (1-x)\tilde{T}.$$

The firm is modeled as before, producing goods using capital and employment. The goods market is cleared as in equation (23).

If a crisis occurs, the government imposes taxes as

$$\begin{aligned} \tau^K R_1^K(x=1) K &= \omega^K (B+G), & (46)\\ \tau^G B &= \omega^G (B+G), & (47)\\ T &= \omega^T (B+G), & (48) \end{aligned}$$

where $\omega^K + \omega^G + \omega^T = 1$ $(0 \leq \omega^K, \omega^G, \omega^T \leq 1)$. If a crisis does not occur, the lump-sum tax $\tilde{T}$ covers all necessary payments such that $\tilde{T} = B + G$.

The probability of a crisis is

$$p_t = p(B_0) = \frac{1}{1 + (1/d_0 - 1)\exp(-d_1 B_0)}. \tag{49}$$

The CDS spread is priced as

$$CDS \cdot M_1(x=0)(1 - p(B_0)) = (1 - R)M_1(x=1)p(B_0), \tag{50}$$

where $M_1(x)$ is the stochastic discount factor.

B.2 Equilibrium

Because of (45), we can rewrite the period 0 problem as

$$\max_{K,B}(1-\beta)\log[R_0^K K_0 + B_0 + W_0 + G - qB - K]$$
$$+\beta E_0 \log[(1 - x\tau^K)R_1^K(x)K + (1 - x\tau^G)B + W(x) - xT - (1-x)\tilde{T}].$$

The first-order conditions with respect to K and B imply

$$(1-\beta)\frac{1}{C_0} = \beta\left\{p(B_0)\frac{(1-\tau^K)R^K(1)}{C_1(1)} + (1 - p(B_0))\frac{R^K(0)}{C_1(0)}\right\}, \tag{51}$$
$$(1-\beta)\frac{q}{C_0} = \beta\left\{p(B_0)\frac{1-\tau^G}{C_1(1)} + (1 - p(B_0))\frac{1}{C_1(0)}\right\}. \tag{52}$$

Now, we can calculate the equilibrium by combining the solutions to the household problem and the firm's problem. We have the following equilibrium values:

$$\begin{aligned}
C_0 &= z^{1-\alpha}K_0^\alpha, \\
C_1(0) &= C_1(1) = z^{1-\alpha}K^\alpha, \\
W(0) &= W(1) = (1-\alpha)z^{1-\alpha}K^\alpha, \\
R^K(0) &= R^K(1) = \alpha\left(\frac{z}{K}\right)^{1-\alpha} = \alpha z^{1-\alpha}K^{\alpha-1}.
\end{aligned}$$

Then, we can derive the following equilibrium conditions from (51) and (52):

$$\frac{1-\beta}{\beta}K = \alpha z^{1-\alpha}K_0^\alpha\left\{p(B_0)(1-\tau^K) + 1 - p(B_0)\right\}, \tag{53}$$
$$\frac{1-\beta}{\beta}K^\alpha q = K_0^\alpha\left\{p(B_0)(1-\tau^G) + 1 - p(B_0)\right\}. \tag{54}$$

The equilibrium value of B is the government budget in period 0; that is,

$$qB = G + B_0, \tag{55}$$

where the tax rates are

$$\tau^K = \min[1, \omega^K(B+G)/(\alpha z^{1-\alpha}K^\alpha)], \tag{56}$$
$$\tau^G = \min[1, \omega^G(B+G)/B]. \tag{57}$$

Conditions (53), (54), and (55) determine the equilibrium with respect to K, q, and B. Note that from (53) and (54), we can obtain the arbitrage condition between investing in capital and investing in government bonds:

$$\left\{p(B_0)(1-\tau^K)+1-p(B)\right\}R^K = \left\{p(B_0)(1-\tau^G)+1-p(B)\right\}/q. \tag{58}$$

The CDS spread is simply written as

$$CDS = (1-R)\frac{p(B_0)}{1-p(B_0)}. \tag{59}$$

B.3 Implications

Specifically, suppose $(1-\beta)/\beta = \alpha z^{1-\alpha}K_0^{\alpha-1} > 1$, $z = 1$, and $G = 0$. We denote the derivative of y with respect to B_0 by y'.

PROPOSITION 1 *The signs of B' and q' are ambiguous. Specifically, $q' < 0$ if and only if*

$$\frac{p'(B_0)\tau^G}{p(B_0)(1-\tau^G)+1-p(B_0)} > \frac{p'(B_0)\tau^K + p(B_0)\tau^{K'}}{p(B_0)(1-\tau^K)+1-p(B_0)}\alpha. \tag{60}$$

We provide the proofs of this and the following propositions in the next subsection.

This proposition states that an increase in B_0 does not necessarily lead to a decrease in the price of government bonds, q (i.e., an increase in the government bond yield). When the capital levy (τ^K) is less than the size of the government bond default (τ^G), q tends to decrease as B_0 increases. However, when the capital levy (τ^K) is greater than the size of the government bond default (τ^G), q tends to increase rather than decrease as B_0 increases. This occurs because the household chooses to invest in government bonds rather than capital. Moreover, when the elasticity of the crisis probability to debt is zero (i.e., $p'(B_0) = 0$), the above inequality is likely violated, and thus, q tends to increase rather than decrease as B_0 increases.

Furthermore, this proposition shows that somewhat surprisingly, the sign of B' is not necessarily positive. In other words, an increase in B_0 does not necessarily lead to an increase in debt in the next period B. When the capital levy is relatively more important than the size of the government bond default, q' becomes positive, which helps decrease the amount of government bonds issued in the next period.

However, the case of $B' < 0$ appears to be a little extreme. Although the Japanese economy has been experiencing an increase in q as B increases (i.e., $q' > 0$), the amount of debt, B, has not decreased (i.e., $B^{'} > 0$). Thus, in the following analysis, we consider the case in which $B^{'} > 0$.

The following proposition shows that a slowdown of real economic activity occurs if $\omega^K > 0$.

PROPOSITION 2 *Suppose $B^{'} > 0$ and $\alpha p(B_0) < 1 - p(B_0)$. Then, $K^{'} \leq 0$ and $\tau^{K'} \geq 0$. The strict inequality of $K^{'} < 0$ holds if $\omega^K > 0$ and either $\tau^K < 1$ or $p^{'}(B_0) > 0$. The strict inequality of $\tau^{K'} > 0$ holds if $\omega^K > 0$ and $\tau^K < 1$.*

If the government imposes a capital levy during a crisis (i.e., $\omega^K > 0$), an increase in B_0 leads to an increase in the capital levy rate (i.e., $\tau^{K'} > 0$) unless it already reaches the maximum rate (i.e. one). This decreases investment in capital K (i.e., $K^{'} < 0$), which causes real economic activity to slow down. The decrease in investment in capital is also caused by an increase in the crisis probability (i.e., $p^{'}(B_0) > 0$), and thus, $K^{'}$ is negative even if the capital levy rate is already one.

Propositions 1 and 2 suggest that the model can explain both a larger slowdown in real economic activity and a decrease in the government bond yield as government debt increases if people expect a relatively larger ω^K than ω^G during a crisis.

Finally, the next proposition provides a sharp prediction about the elasticity of the CDS spread to B_0.

PROPOSITION 3 *The elasticity of the CDS spread to B_0 equals d_1.*

B.4 Model Extension: Incorporating Foreign Direct Investment

We extend the two-period model by allowing the household to invest in not only government bonds and capital at home, but also capital abroad (denoted by F). The household solves the following problem in period 0:

$$U_0 = \max_{C_0,K,B} (1-\beta)\log C_0 + \beta E_0 \log C_1,$$

subject to the budget constraint:

$$C_0 + qB + K + \Phi(F) \leq R_0^K K_0 + B_0 + W_0 + G,$$

and

$$C_1 \leq (1 - x\tau^K)R_1^K(x)K + (1 - x\tau^G)B + RF + W(x) - xT - (1-x)\tilde{T}.$$

Foreign direct investment ensures a riskless return, given by R, irrespective of a crisis. The government does not impose a tax on this return. However, it is costly, as indicated by the convex function $\Phi(F)$, where $\Phi(F) \geq 0$, $\Phi^{'}(F) > 0$, and $\Phi^{''}(F) > 0$ with $\Phi(0) = 0$.

Then, we can obtain the following proposition illustrating that foreign direct investment F increases as B_0 increases.

PROPOSITION 4 *Suppose $B^{'} > 0$ and $\alpha p(B_0) < 1 - p(B_0)$. Then, $F^{'} \geq 0$. The strict inequality holds if $\omega^K > 0$ and either $\tau^K < 1$ or $p^{'}(B_0) > 0$.*

B.5 Proof of Propositions

B.5.1 Proof of Proposition 1

Differentiating (53) to (57) with respect to B_0 yields

$$\begin{aligned}
\frac{1-\beta}{\beta}K^{'} &= \alpha z^{1-\alpha}K_0^{\alpha}\left\{p^{'}(B_0)(-\tau^K) - p(B_0)\tau^{K'}\right\},\\
\frac{1-\beta}{\beta}(\alpha K^{\alpha-1}K^{'}q + K^{\alpha}q^{'}) &= K_0^{\alpha}\left\{p^{'}(B_0)(-\tau^G) - p(B_0)\tau^{G'}\right\},\\
q^{'}B + qB^{'} &= 1,\\
\tau^{K'} &= \omega^K B^{'}/(\alpha z^{1-\alpha}K^{\alpha}) - \alpha\omega^K(B+G)/(\alpha z^{1-\alpha}K^{\alpha+1})K^{'},\\
\tau^{G'} &= -\omega^G G/B^2 B^{'}.
\end{aligned}$$

Then, under the assumption of $(1-\beta)/\beta = \alpha z^{1-\alpha}K_0^{\alpha-1} > 1$, $z = 1$, and $G = 0$, the equilibrium condition is summarized as

$$\begin{aligned}
&K/K_0 = p(B_0)(1-\tau^K) + 1 - p(B_0),\\
&(1-\beta)/\beta \cdot (K/K_0)^{\alpha}q = p(B_0)(1-\tau^G) + 1 - p(B_0),\\
&qB = B_0,\\
&\tau^K = \min[1, \omega^K B/(\alpha K^{\alpha})],\\
&\tau^G = \omega^G,\\
&K^{'}/K_0 = p^{'}(B_0)(-\tau^K) - p(B_0)\tau^{K'},\\
&(1-\beta)/\beta \cdot (K/K_0)^{\alpha}(\alpha(K^{'}/K)q + q^{'}) = p^{'}(B_0)(-\tau^G),\\
&q^{'}B + qB^{'} = 1,\\
&\tau^{K'} = \omega^K B^{'}/(\alpha K^{\alpha}) - \alpha\omega^K B/(\alpha K^{\alpha+1})K^{'},\\
&\tau^{G'} = 0.
\end{aligned} \tag{61}$$

With respect to $q^{'}$, we can obtain

$$\begin{aligned}
(1-\beta)/\beta \cdot (K/K_0)^{\alpha}q^{'} &= -p^{'}(B_0)\tau^G - (1-\beta)/\beta \cdot (K/K_0)^{\alpha}\alpha(K^{'}/K)q\\
&= -p^{'}(B_0)\tau^G - \{(1-\beta)/\beta \cdot (K/K_0)^{\alpha}q\}\alpha\frac{K^{'}}{K_0}\frac{K_0}{K}\\
&= -p^{'}(B_0)\tau^G + \frac{p(B_0)(1-\tau^G) + 1 - p(B_0)}{p(B_0)(1-\tau^K) + 1 - p(B_0)}\alpha\left\{p^{'}(B_0)\tau^K + p(B_0)\tau^{K'}\right\}.
\end{aligned}$$

This result suggests that $q^{'} < 0$ if and only if

$$\frac{p^{'}(B_0)\tau^G}{p(B_0)(1-\tau^G) + 1 - p(B_0)} > \frac{p^{'}(B_0)\tau^K + p(B_0)\tau^{K'}}{p(B_0)(1-\tau^K) + 1 - p(B_0)}\alpha.$$

With respect to $B^{'}$, it is clear that $B^{'} > 0$ if $q^{'} < 0$ because $q^{'}B + qB^{'} = 1$. However, $B^{'}$ can be negative when $q^{'}$ is positive. To see this, suppose a special case, in which $\tau^G = 0$ and $\tau^K = 1$. Then, we have

$$q^{'} = \alpha p^{'}(B_0)q/(1 - p(B_0)) > 0$$

and

$$qB^{'} = 1 - \alpha B_0 \frac{d_1}{1 + (1/d_0 - 1)\exp(-d_1 B_0)}.$$

which can be negative if either B_0, d_0 or d_1 is sufficiently large.

B.5.2 Proof of Proposition 2

In equation (61), we have

$$\begin{aligned} K^{'}/K_0 &= p^{'}(B_0)(-\tau^K) - p(B_0)\tau^{K'}, \\ \tau^{K'} &= \omega^K B^{'}/(\alpha K^{\alpha}) - \alpha\omega^K B/(\alpha K^{\alpha+1})K^{'}, \end{aligned} \tag{62}$$

which leads to

$$\begin{aligned} \tau^{K'} &= \omega^K B^{'}/(\alpha K^{\alpha}) - \alpha\omega^K B/(\alpha K^{\alpha+1})K_0 \left\{ p^{'}(B_0)(-\tau^K) - p(B_0)\tau^{K'} \right\}, \\ (1 - \alpha\tau^K p(B_0)K_0/K)\tau^{K'} &= \omega^K B^{'}/(\alpha K^{\alpha}) + \alpha\omega^K B/(\alpha K^{\alpha+1})K_0 p^{'}(B_0)\tau^K \geq 0. \end{aligned}$$

Note that $0 < p \leq 1$, $p^{'} \geq 0$, and $0 \leq \tau^K \leq 1$. The coefficient on $\tau^{K'}$ on the left-hand side of the equation is positive because

$$\begin{aligned} 1 - \alpha\tau^K p(B_0)K_0/K = &1 - \frac{\alpha\tau^K p(B_0)}{p(B_0)(1 - \tau^K) + 1 - p(B_0)} \\ \geq &1 - \alpha\frac{p(B_0)}{1 - p(B_0)} \\ > &0. \end{aligned}$$

The first line is derived from the first equation of (61), and in the second line, equality holds when $\tau^K = 1$. The third line comes from the assumption in this proposition, which requires that the probability of a crisis is not too high. Given $\alpha = 1/3$, this assumption holds when $p(B_0) < 3/4$, and thus, this assumption is not restrictive. Therefore, we obtain $\tau^{K'} \geq 0$, where the strict inequality holds if $\omega^K > 0$ and $\tau^K < 1$. Substituting $\tau^{K'}$ into equation (62) yields $K^{'} \leq 0$, where the strict inequality holds if $\omega^K > 0$ and either $\tau^K < 1$ or $p^{'} > 0$.

B.5.3 Proof of Proposition 3

Using

$$
\begin{aligned}
p_t = p(B_0) &= \frac{1}{1 + (1/d_0 - 1)\exp(-d_1 B_0)}, \\
p^{'}(B_0) &= \frac{(1/d_0 - 1)d_1 \exp(-d_1 B_0)}{\{1 + (1/d_0 - 1)\exp(-d_1 B_0)\}^2} > 0,
\end{aligned}
$$

we obtain the derivative of the CDS spread as

$$
\begin{aligned}
\frac{CDS'}{CDS} &= \frac{p^{'}(B_0)}{p(B_0)} + \frac{p^{'}(B_0)}{1 - p(B_0)} \\
&= \frac{(1/d_0 - 1)d_1 \exp(-d_1 B_0)}{1 + (1/d_0 - 1)\exp(-d_1 B_0)} + \frac{d_1}{1 + (1/d_0 - 1)\exp(-d_1 B_0)} \\
&= d_1.
\end{aligned}
$$

B.5.4 Proof of Proposition 4

The first-order conditions with respect to K and B are the same as before. The first-order condition with respect to F is given by

$$
(1 - \beta)\frac{\Phi^{'}(F)}{C_0} = \beta \left\{ p(B_0)\frac{R}{C_1(1)} + (1 - p(B_0))\frac{R}{C_1(0)} \right\} = \beta \frac{R}{C_1}. \tag{63}
$$

This leads to

$$
\begin{aligned}
\Phi^{'}(F) &= \frac{\beta}{1 - \beta}\frac{C_0}{C_1}R \\
&= \frac{\beta}{1 - \beta}\left(\frac{K_0}{K}\right)^{\alpha} R.
\end{aligned}
$$

Because of Proposition 2 and the convexity of $\Phi(F)$, it is clear that $F^{'} \geq 0$.

Table 1: Calibrated Parameters

Parameters	Values	Sources
Capital share α	0.3	Gourio (2013)
Discount factor β	0.995	Sugo and Ueda (2011)
Utility weight on C ν	0.3	Gourio (2013)
IES utility ψ	1.5	Sugo and Ueda (2008); Hirose and Kurozumi (2012)
Depreciation δ	0.08	Gourio (2013)
Trend growth of TFP μ	0.0182	Mean of $\Delta log Y_t$
SD of TFP shock σ_e	0.023	SD of $\Delta log Y_t$
Gov spending g	0.02	Mean of $\Delta(b_t^G/y_t)$
Tax weights		
capital levy ω^K	0.2015	Post-war Japan
consumption tax ω^C	0.0181	Post-war Japan
government bonds (default) ω^G	0.0760	Post-war Japan
lump-sum tax ω^T	0.0157	Post-war Japan
Capital-use inefficiency		
production θ_0	0.059	Laeven and Valencia (2018)
production; dependence on bad loans θ_1	0.202	Laeven and Valencia (2018)
depreciation λ^D	$1-0.061$	Flow of Funds in 1997 and 1998
Crisis probability		
at $b=0$ d_0	0.025	Level of CDS spreads when b is low and ω^G
elasticity to b d_1	4	Elasticity of CDS spreads and ω^G

Table 2: Comparison of Untargeted Moments

	Data	Benchmark model		Data	Benchmark model
b/y	0.5205 (1.1501)	0.756	σ(dY)	0.021	0.021
$R^G - 1$	0.019	0.044	σ(dI)	0.062	0.040
$R^K - R^G$	0.008	0.007	σ(dC)	0.016	0.018
$R^D - R^G$	0.001	0.005	cor(dY,B/Y)	-0.302	-0.239
			cor(dI,B/Y)	0.002	-0.212
			cor(dY,$R^G - 1$)	0.284	0.353
			cor(dY,$R^K - R^G$)	-0.149	-0.188
			cor(dY,$R^D - R^G$)	0.292	-0.198
			cor(dI,$R^G - 1$)	0.058	0.165
			cor(dI,$R^K - R^G$)	-0.012	-0.188
			cor(dI,$R^D - R^G$)	0.023	-0.196

Notes: The data are from 1975 to 2019 for Japan. The model-based moments exclude the time of a crisis. Data b/y represents the ratio of net government debt-to-output, whereas the value in parentheses is the ratio of gross government debt-to-output. σ(dlogY) is the only targeted moment.

Table 3: Welfare Comparison

Model	b/y	y	U
Benchmark	1.16	0.000	0.000
All T tax	0.80	0.192	0.071
No K tax	1.00	0.114	0.014
No C tax	1.15	0.004	0.000
No G tax	1.00	0.046	0.013
Always C tax	0.70	0.108	0.058

Notes: The model includes the crisis periods. Output (y) and welfare (U, in the unit of consumption) are indicated as differences from the values in the benchmark model.

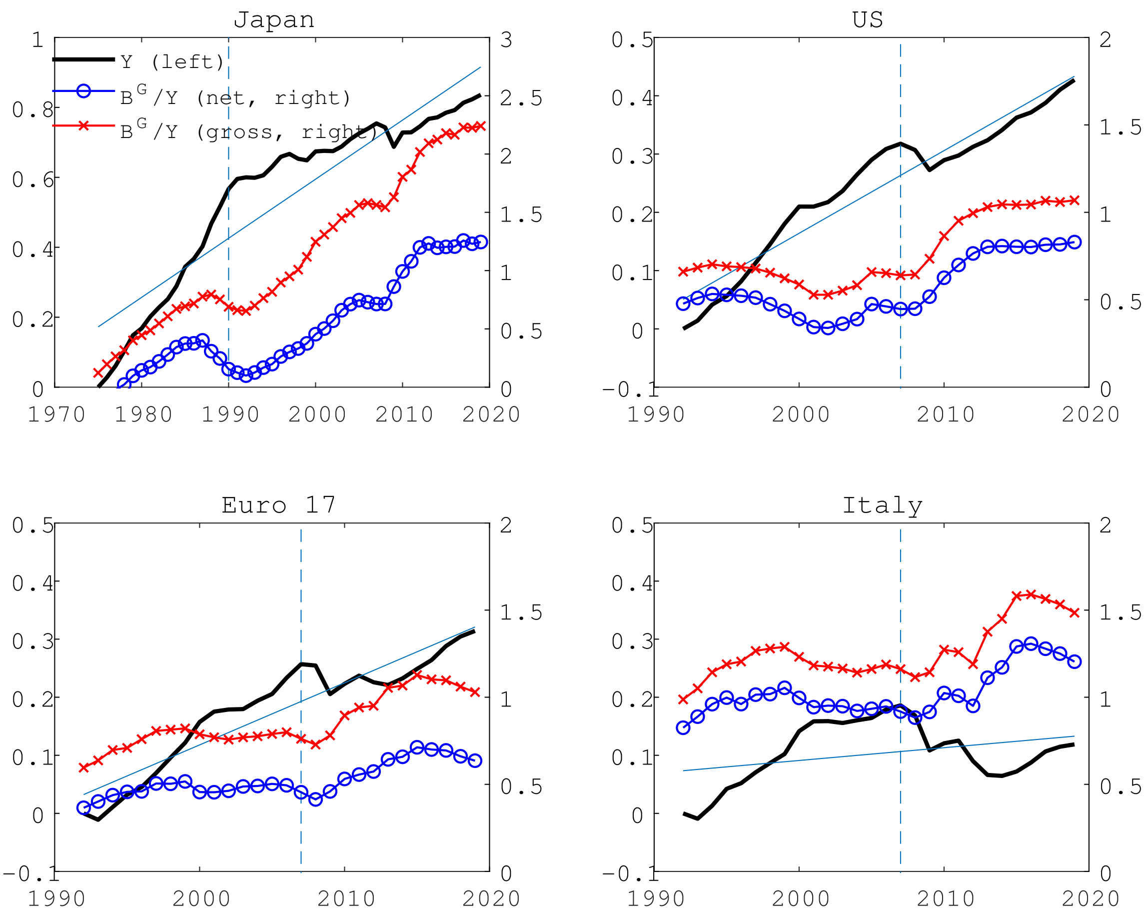

Figure 1: Government Debt and GDP

Note: The beginning year is 1975 for Japan and 1992 for the other countries, representing a 15-year period before the financial crises in each region (i.e., 1990 and 2007, respectively, shown as the vertical dashed line). The thick solid line represents the logarithm of real GDP per capita (set to zero in the first year), shown on the left axis. The thin solid line represents its linear trend. On the right axis, the line with crosses and the line with circles represent the ratio of gross and net government debt, respectively, to nominal GDP. The data are taken from the OECD.

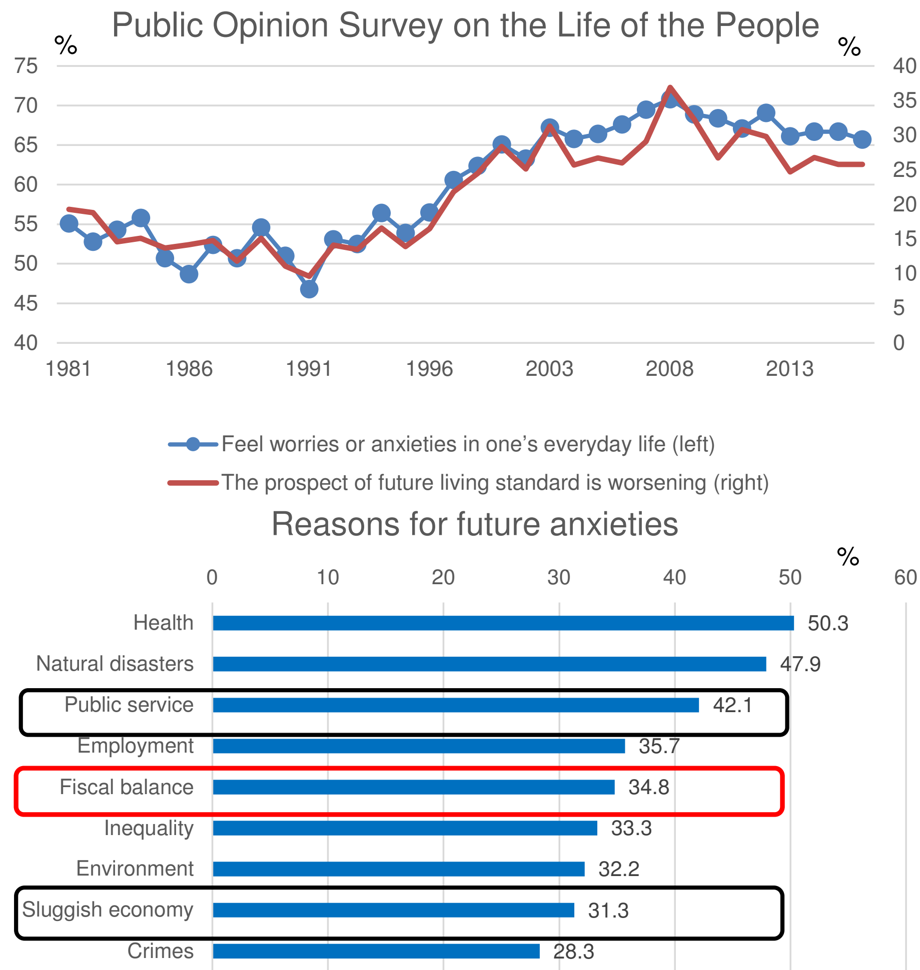

Figure 2: Japanese Sentiment

Source: Cabinet Office "Overview of the Public Opinion Survey on the Life of the People"

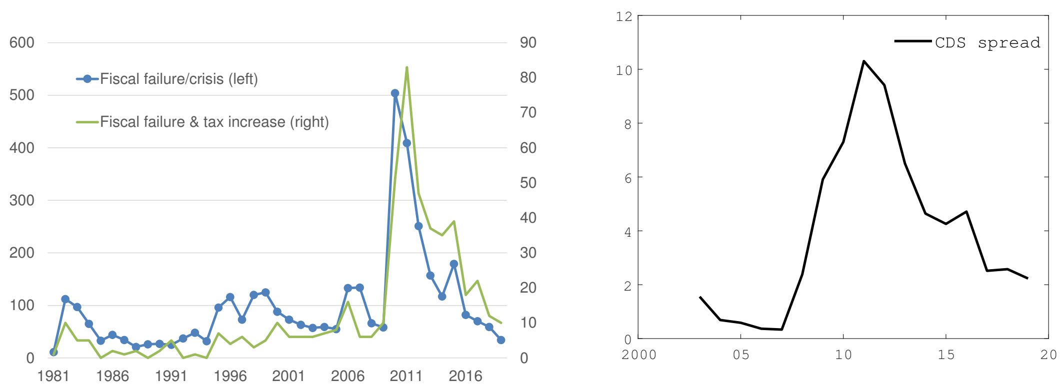

Figure 3: Concerns about a Government Debt Crisis

Note: The left-hand figure shows the occurrences of specific words in the morning and evening editions of the Nihon Keizai Shinbun, Japan's financial newspaper, for each year. The right-hand figure shows the sovereign CDS spread with a five-year maturity (in percentage points), which is taken from Bloomberg.

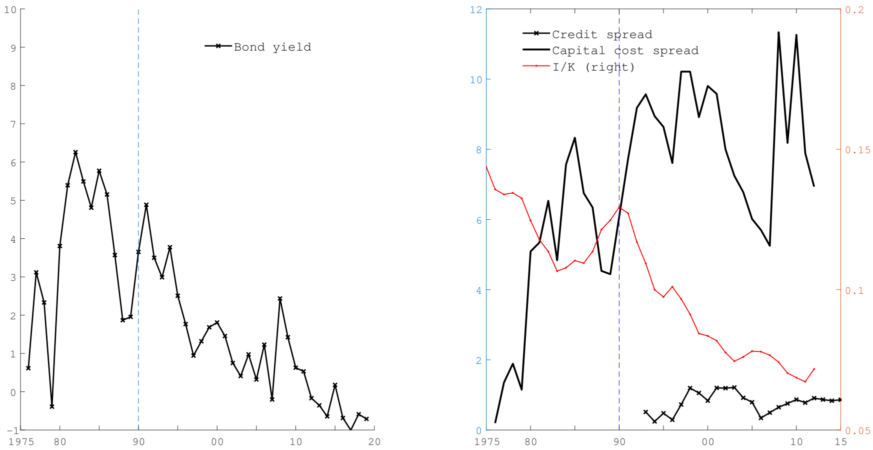

Figure 4: Bond Yield, Credit Spread, and Investment

Note: The government bond yield in real terms is defined as that with a five-year maturity minus the annual CPI inflation rate in the following year. The credit spread is defined as the bank loan rate with one-year maturity or longer minus the government bond yield with five-year maturity. The data on investment, capital, and capital cost are obtained from the JIP database (https://www.rieti.go.jp/en/database/jip.html). The capital cost spread is defined as the rental cost minus the five-year government bond yield.

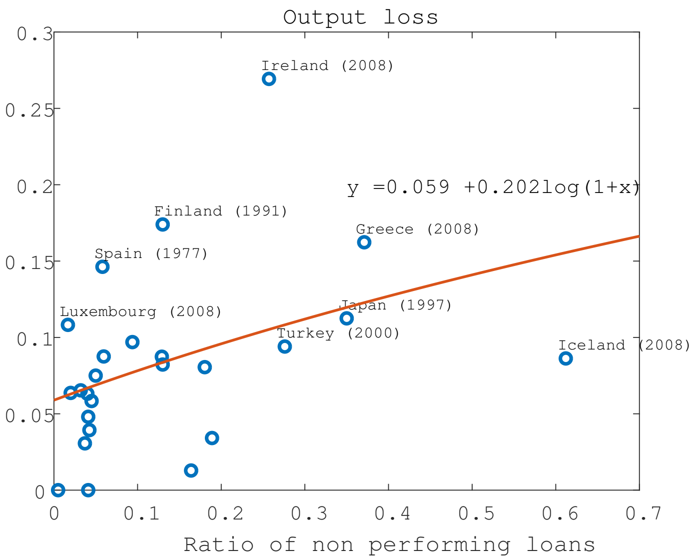

Figure 5: Relation Between Output Loss and Banks' Nonperforming Loans

Source: Laeven and Valencia (2018).

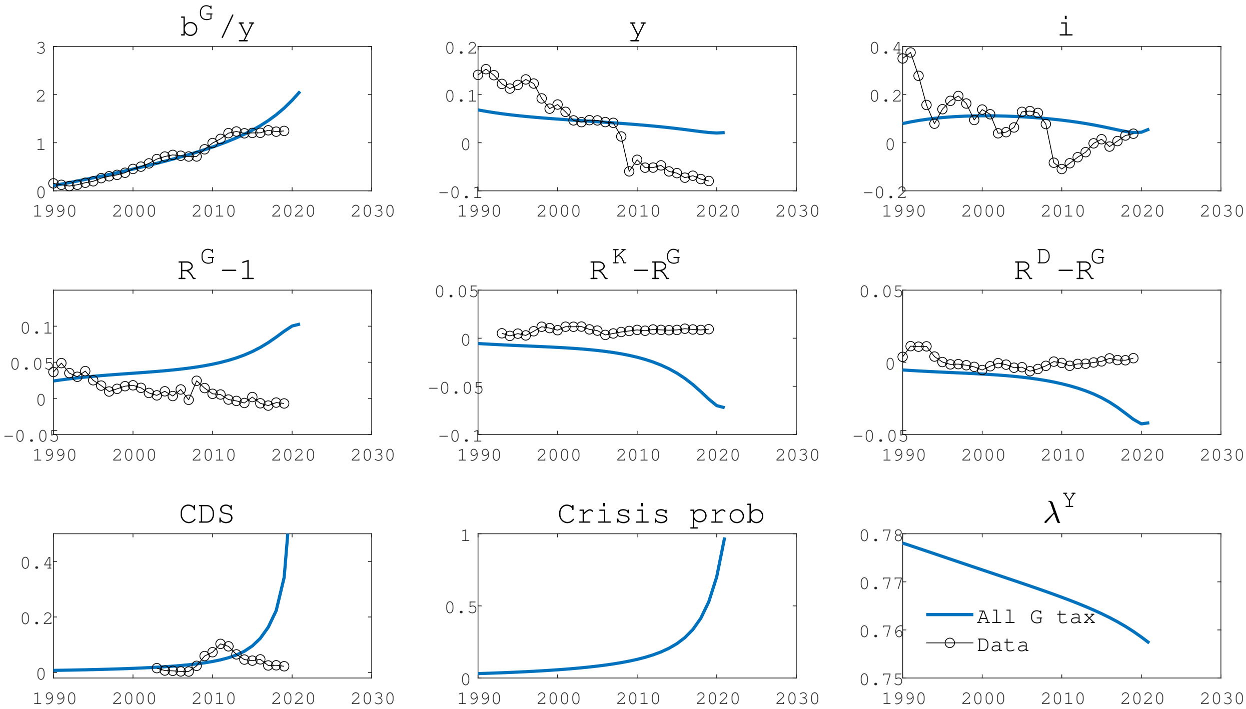

Figure 6: Simulation Results when Tax is Imposed Only on Government Bonds

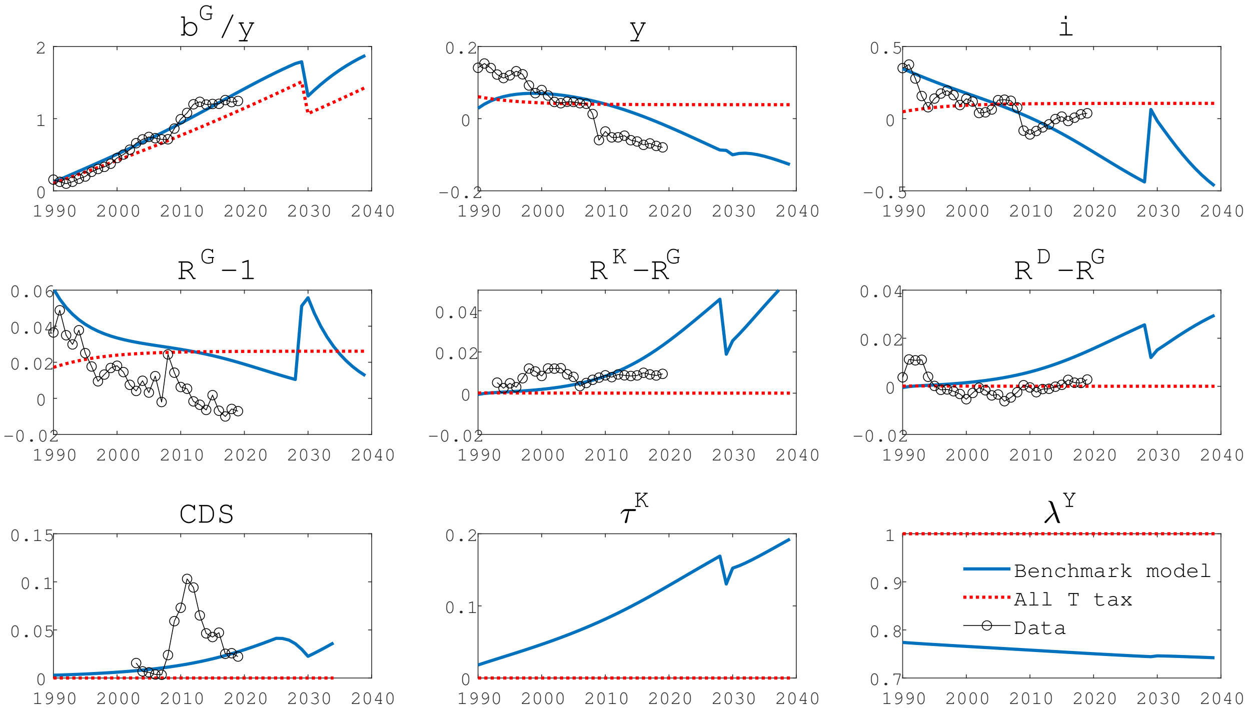

Figure 7: Simulation Results of the Benchmark Model

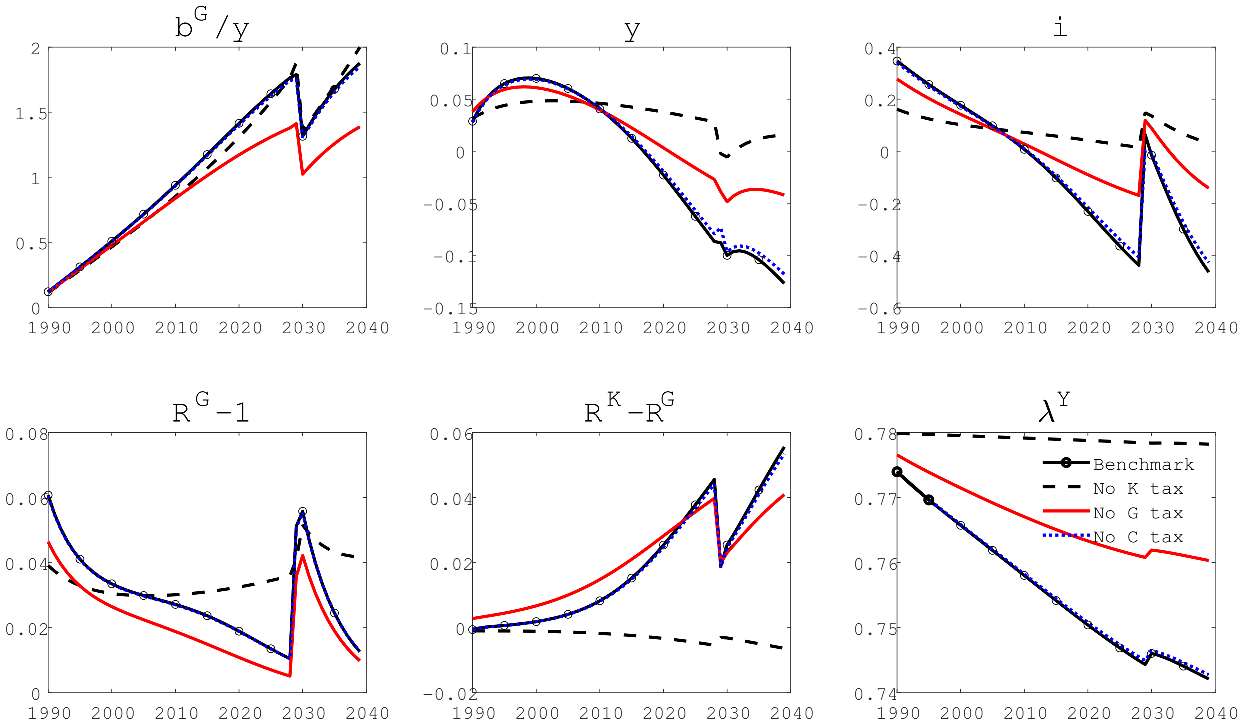

Figure 8: Simulation Results in Different Tax Scenarios at a Crisis

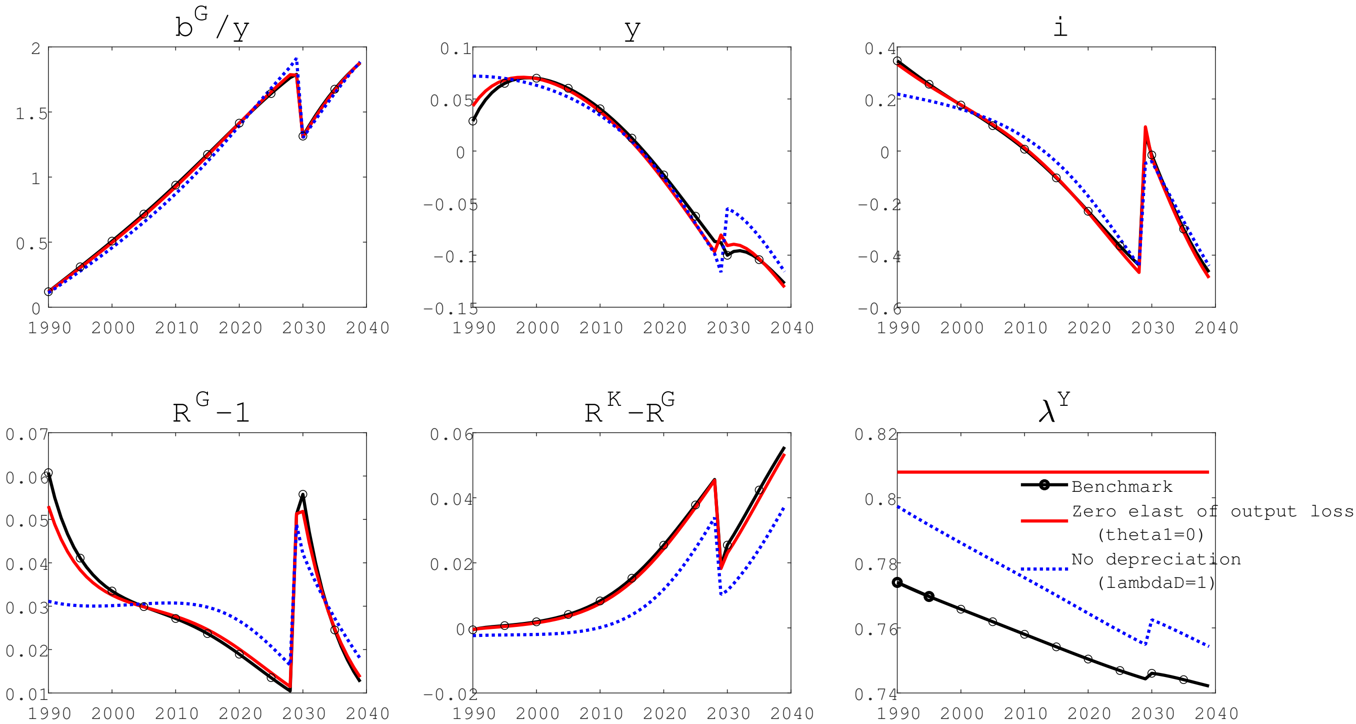

Figure 9: Simulation Results under Low Capital-Use Inefficiency

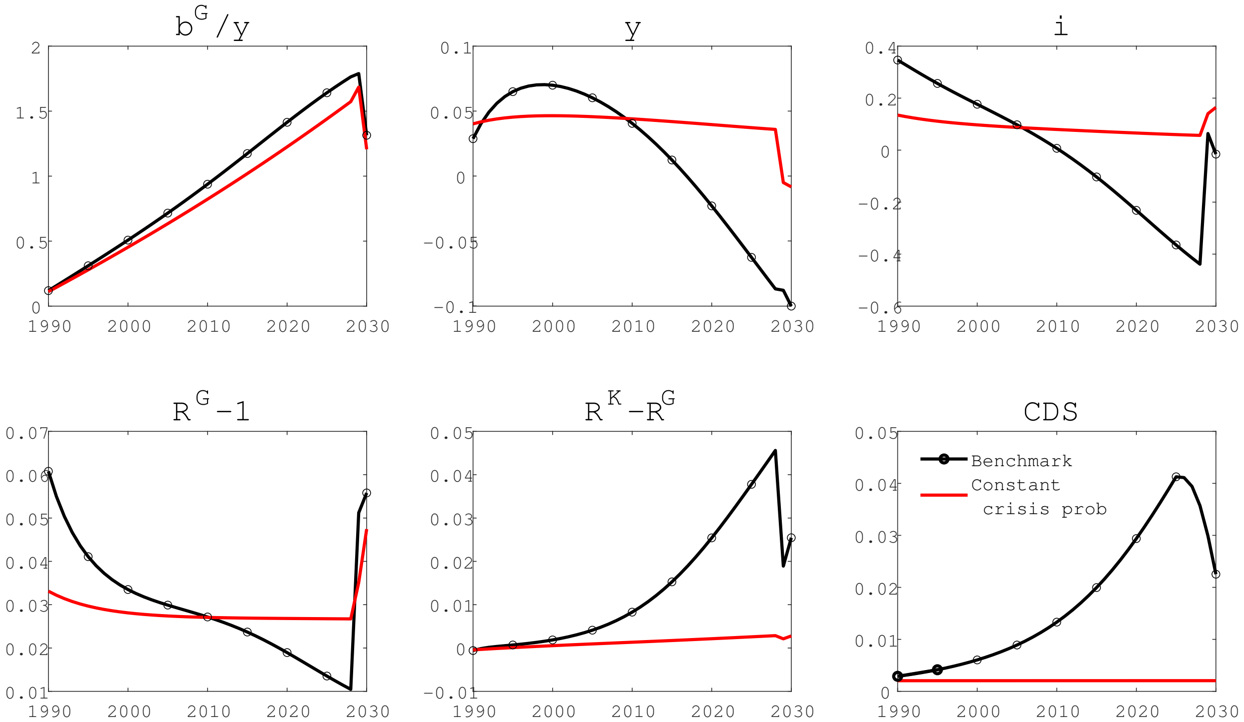

Figure 10: Simulation Results under a Constant Crisis Probability

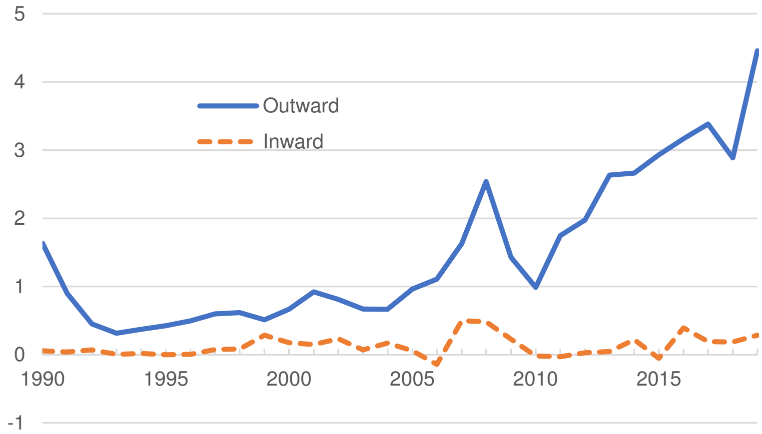

Figure 11: Foreign Direct Investment

Note: The data show the foreign direct investment financial flows (outward and inward in the solid and dashed lines, respectively) as a share of GDP (%). The data are taken from the OECD.