Some Pleasant Development Economics Arithmetic

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Recent literature emphasis on inter-firm distortions in the allocation of inputs

Firm-specific wedges

Understand mapping between distortions and aggregate productivity?

On the inference from data

The role of firm dynamics
Understanding firm level distortions: The undistorted economy

- **Simplified Lucas style model**

- **Production function:** \( y_i = e_i n_i^\alpha \)

- **Total labor endowment:** \( N \)

- **Optimal allocation:**
  
  \[
  \ln n_i = a + \left( \frac{1}{1-\alpha} \right) e_i
  \]

  - \( y_i / n_i \) should be equated across firms (TFPR in HK jargon)
Aggregation

- Aggregate production function
- Homogeneous of degree one in firms (given distribution) and labor

\[ y = AM^{1-\alpha} N^\alpha \]

\[ A = \left( Ee_i^{\frac{1}{1-\alpha}} \right)^{1-\alpha} \]
The Distorted Economy

- $y_i/n_i$ not equated across firms

- Two types of distortions:
  - $n_i$ not equal for all firms with same $e_i$ (uncorrelated distortion)
  - average $\ln n_i(e) \neq a + \frac{1}{1-\alpha} \ln e$ (correlated distortion)
Let a firm’s profits be \((1 - \tau_i) y_i - w l_i - r k_i\), where \(\tau_i\) denotes a sales tax variance and covariance.

<table>
<thead>
<tr>
<th>% Estab. taxed</th>
<th>Uncorrelated</th>
<th>Correlated</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(\tau_t)</td>
<td>(\tau_t)</td>
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<tr>
<td></td>
<td>0.2 0.4</td>
<td>0.2 0.4</td>
</tr>
<tr>
<td>90%</td>
<td>0.84 0.74</td>
<td>0.66 0.51</td>
</tr>
<tr>
<td>50%</td>
<td>0.96 0.92</td>
<td>0.80 0.69</td>
</tr>
<tr>
<td>10%</td>
<td>0.99 0.99</td>
<td>0.92 0.86</td>
</tr>
</tbody>
</table>

potentially large effects

More when correlated
Distortions result in deviations of output from optimal:

\[ n(\tau, e) = (1 - \tau)^{\frac{1}{1-\eta}} n(e) \]

Using \( \theta = (1 - \tau)^{\frac{1}{1-\eta}} \) distorted employment is \( \theta n(e) \).

**Definition 1.** A feasible distortion is a conditional probability distribution \( P(\theta|e) \) such that \( N = \int n(e) \theta dP(\theta|e) dG(e) \).

Using a change of variable on order of integration, can define \( Q(n|\theta) \) and \( F(d\theta) \) define measure \( N(d\theta) \) as follows:

\[ dN(\theta) = dF(\theta) \int ndQ(n|\theta) \]
A measure of distortions

- $N(\theta)$ is the measure of total original employment that was distorted with some $\theta' \leq \theta$.

- It is silent about the productivity of the firms underlying these distortions.
  - Example: $n_1 = 10$, $n_2 = 100$. Suppose equal number (e.g. 20) of each.
  - Distorting by $\theta$ half of the firms of type 1 and by $1/\theta$ the remaining half gives the same measure as distorting $1/20$th of the type 2 firms with $\theta$ and $1/20$th with $1/\theta$.
  - $N(\theta) : \{(\theta, 100), (1/\theta, 100), (1, 2000)\}$

- It integrates to total employment

\[
N = \int dN(\theta).
\]
First define total output: \( y = \int e (n (e))^{\eta} dP (\theta|e) dG (e) \)

Using \( n (e) = ae^{\frac{1}{1-\eta}} \) for some constant \( a \)

\[
y = \int e \left( \theta ae^{\frac{1}{1-\eta}} \right)^{\eta} dP (\theta|e) dG (e) \\
= a^{\eta} \int e^{\frac{1}{1-\eta} \theta^{\eta}} dP (\theta|e) dG (e) \\
= a^{\eta-1} \int n (e) \theta^{\eta} dP (\theta|e) dG (e).
\]

Since it is linear in \( n (e) \) we can use our measure:

\[
y = a^{\eta-1} \int \theta^{\eta} dN (\theta).
\]
We obtained our formula:

\[ y = a^{\eta-1} \int \theta^{\eta} dN(\theta). \]

Undistorted economy has \( N(\theta) \) mass point at one.

\[ y_{eff} = a^{\eta-1} N \]

It follows that:

\[ \frac{TFP}{TFP_{eff}} = \frac{y}{y_{eff}} = \frac{1}{N} \int \theta^{\eta} dN(\theta) \]

The effect of distortions depends on \( \eta \) and the distribution of distortions.
TFP and the concentration of distortions

\[
\frac{TFP}{TFP_{eff}} = \frac{y}{y_{eff}} = \frac{1}{N} \int \theta^\eta dN(\theta)
\]

- \(dN(\theta)/N\) is a probability measure
- Mean preserving spreads in this measure reduce \(TFP/TFP_{eff}\)
- And mean preserving spreads give rise to the same aggregate employment!

\[
N = \int \theta dN(\theta)
\]

- Mean preserving spreads \(\iff\) more concentrated distortions.
- Lower \(\eta\) implies more risk aversion so larger effect of a mean preserving spread.
Examples of mean preserving spreads

Uncorrelated taxes to larger firms are worse for productivity than for smaller firms (holding the number of firms affected constant)

- Increasing the variance of the $\theta'$s for large firms will put more employment at the tails than if done for small firms
- But might not be so when taking into account that there are more small firms.
- It all depends on the share of employment of small vs. large firms.
Examples of mean preserving spreads

Increasing the share of firms taxed (while subsidizing others to keep employment constant)

- Let a share $s$ of firms have $\theta_t < 1$
- Then share $(1 - s)$ must have $\theta_s = \frac{s}{1-s} \theta_t$
- This corresponds to a mean preserving spread of the employment that was subsidized initially at a lower rate
Inference from the size distribution of firms

- Size distribution of firms vary a lot across economies
- "missing middle" of underdeveloped economies
What inferences can be made by comparing size distributions?

Nothing in general: distortions might not be revealed in size distribution

- Can generate the efficient distribution by taxing all efficient firms out of the market and a distribution of taxes across the most inefficient ones that replicates the efficient size distribution.

Special case:

- Both economies with same underlying $G(de)$
- One of the economies with no distortions
- Can easily calculate lower bound on distortions for other economy.
A lower bound on distortions

- Take efficient distribution of firm sizes with cdf $F(n)$
- Distorted economy with cdf $D(n)$
- Class of candidate distortions $P(\theta, n)$ such that:

$$D(n) = \int_{\theta n' \leq n} P(\theta, n') dF(n)$$

- This is a large class
Lower bound on distortions

- Objective: minimize spread

- Two key principles:
  - $P(\theta|n)$ should be concentrated at one point for each $n$
  - Preserve ordering: $n_2 \geq n_1$ if $\theta_2 n_2 \geq \theta n_1$

- Identifies unique solution function $\theta(n)$ defined implicitly by:

$$F(n) = D(\theta n)$$

$$\frac{\text{TFP}}{\text{TFP}_{eff}} = \frac{1}{\bar{n}} \int \theta(n)^\eta n dF(n)$$

- TFP India/TFP US = 0.4 (Hsieh-Klenow report 0.38)
Wedges, curvature and productivity

- HK use $\eta = 0.5$ others $\eta = 0.85$. How does curvature affect the impact of distortions?

- Relationship subtle

- Curvature affects the underlying efficient employment at different levels of $TFP_0$:

$$TFP_0 = \frac{\sum e_i^{\frac{1}{1-\eta}}(1 - \tau_i)^{\frac{\eta}{1-\eta}}}{\left[\sum e_i^{\frac{1}{1-\eta}}(1 - \tau_i)^{\frac{1}{1-\eta}}\right]^\eta}$$

- If $\eta = 0$, interfirm elasticity of substitution is zero, no effect.

- If $\eta = 1$, interfirm elasticity is infinite: no effect for uncorrelated taxes, large for correlated.

- Non-monotonic relationship.
Measuring distortions (H-K)

- Measure $y_i, n_i, k_i$
- Compute $e_i$ and wedges.
- Counterfactual experiments.
- TFP gains of 30-50% in China and 40-60% in India
## Dispersion in ln MP

<table>
<thead>
<tr>
<th></th>
<th>US (97)</th>
<th>China (98)</th>
<th>India (94)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>ln AP</td>
<td>$\frac{1-\tau_{25}}{1-\tau_{75}}$</td>
<td>ln AP</td>
</tr>
<tr>
<td>SD</td>
<td>0.49</td>
<td></td>
<td>0.74</td>
</tr>
<tr>
<td>75-25</td>
<td>0.53</td>
<td>1.7</td>
<td>0.97</td>
</tr>
<tr>
<td>90-10</td>
<td>1.19</td>
<td>3.3</td>
<td>1.87</td>
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ratio $(1-\tau_{25}) / (1-\tau_{75})$. E.g. assuming the decile 75 corresponded to no taxes in China, decile 25 would have a subsidy of 160%
- Distribution of productivities depends on $\eta$
- Implicit distortions also vary with $\eta$
- Data: $(n_1, y_1, n_2, y_2, ..., n_M y_M)$
- Production function $y_i = e_i n_i^{\eta}$
- Given parameter $\eta$, solve for $e_i$ and do counterfactuals.
TFP gains

- Aggregate TFP in economy: \( TFP = \frac{y}{n^\eta} \)

- Efficient: \( TFP_e = \sum \left( e_i^{\frac{1}{1-\eta}} \right)^{1-\eta} \)

Substitute measured \( e_i \)

\[
TFP_e = \left( \sum \left( \frac{y_i}{n_i^\eta} \right)^{\frac{1}{1-\eta}} \right)^{1-\eta}
\]

\[
\frac{TFP_e}{TFP} = \left( \sum \left( \frac{y_i}{n_i y} \right)^{\frac{1}{1-\eta}} \right)^{1-\eta} = \left( \sum \frac{n_i}{n} \left( \frac{y_i}{y} \right)^{\frac{1}{1-\eta}} \right)^{1-\eta}
\]
TFP gains

\[
\frac{TFP_e}{TFP} = \left( \frac{n_i}{n} \left( \frac{LPR_i}{y/n} \right)^{1-\eta} \right)^{1-\eta}
\]

where \( LPR_i = \frac{y_i}{n_i} \frac{y}{n} \)

\[
\left( \frac{TFP_e}{TFP} \right)^{1-\eta} = \sum \frac{n_i}{n} \left( LPR_{1i} \right)^{1-\eta}
\]

**Proposition.** \( TFP_e / TFP \) is the certainty equivalent of the lottery \( \left\{ \frac{n_i}{N}, LPR_i \right\} \) with CRRA \( \frac{-\eta}{1-\eta} \). It is thus increasing in \( \eta \).

- Extreme: equal to one when \( \eta = 0 \).
Suppose \( n_1/n = n_2/n = 1/2 \)

<table>
<thead>
<tr>
<th>( \eta )</th>
<th>0.2</th>
<th>0.5</th>
<th>0.8</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>relative ( y_i/n_i )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>1.09</td>
<td>1.28</td>
<td>1.57</td>
<td>1.74</td>
</tr>
<tr>
<td>0.4</td>
<td>1.05</td>
<td>1.17</td>
<td>1.39</td>
<td>1.55</td>
</tr>
<tr>
<td>0.6</td>
<td>1.02</td>
<td>1.08</td>
<td>1.22</td>
<td>1.35</td>
</tr>
<tr>
<td>0.8</td>
<td>1.01</td>
<td>1.02</td>
<td>1.07</td>
<td>1.16</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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The role of entry and firm dynamics

- These models abstract from entry/exit margin
- How do results change when this margin can adjust?
- Constrained social planner and entry
- Potential role of distortions to entry
The dynamic economy

- firms productivities \( F(ds'; s) \). Exogenous death rate \( 1 - \delta \).
- sequence of entries of firms \( \{m_0, ..., m_t\} \)

\[
M_t = \delta^t m_0 + \delta^{t-1} m_1 + \ldots + \delta m_{t-1} + m_t
\]

- firms producing in period \( t \) with probability distribution

\[
\mu_t = M_t^{-1} \left( m_t \tilde{\mu}_0 + \delta m_{t-1}^t \tilde{\mu}_1 + \ldots + \delta^t m_0 \tilde{\mu}_t \right).
\] (1)

- aggregation

\[
y_t = \left( \int e^{\frac{1}{1-\eta} d\mu_t(e)} \right)^{1-\eta} M_t^{1-\eta} N^\eta.
\]


**Competitive equilibrium**

- $v_t(e;w)$ value for a firm at time $t$ for a given sequence of wages
  
  $w = \{w_s\}_{s=0}^\infty. v_t(e;w) = \max_n e n^\eta - w_t n + \beta \delta E v_{t+1}(e';w|e)$.

- $v^e_t = \int v_t(e;w) \, dG(e) - w_t c_e$ expected value for an entrant.

**Definition 2.** A competitive equilibrium is a sequence $\{m_t, n_t(e), v_t\}$ and wages $\{w_t\}$ that satisfy the following conditions:

1. Employment decisions are optimal given wages
2. The value functions are as defined above
3. $v^e_t \leq 0$ and $m_t v^e_t = 0$
4. $m_t c_e + \int n_t(e) \mu_t(de) = N$
The distorted economy

- Firm specific output taxes $\tau_i$

- From firm’s point of view, same as productivity shock $(1 - \tau_i) e_i$

\[ r_i = (1 - \tau_i) y_i = (1 - \tau_i) e_i n_i^\eta \]
\[ = \alpha (e_i (1 - \tau_i)) \frac{1}{1 - \eta} , \]

- Joint distribution of $(e, \tau)$ for each age cohort: $\mu_s (e, \tau)$

\[ \tilde{r}_s = \int e^{\frac{1}{1 - \eta}} (1 - \tau)^\frac{1}{1 - \eta} d\tilde{\mu}_s (e, \tau) \]

- ("distorted" social planner) revenue-value of sequence $[m_0, ..., m_t, m_{t+1}, ...]$

\[ \sum_{t=0}^{\infty} \beta^t (N - c_e m_t)^\eta \left( \sum_{s=0}^{t} \delta^{t-s} m_s \tilde{r}_s \right)^{1-\eta} \]
First order conditions for $m_t$ at steady state $m$:

$$N - c_e m = \frac{\eta c_e}{(1 - \eta)} \frac{\sum_{s=0}^{\infty} \delta^s \tilde{r}_s}{\sum_{s=0}^{\infty} \beta^s \delta^s \tilde{r}_s}$$

$m$ is increasing in:

$$\frac{\sum_{s=0}^{\infty} \beta^s \delta^s \tilde{r}_s}{\sum_{s=0}^{\infty} \delta^s \tilde{r}_s}$$

Equilibrium $m$ depends on the age-structure of distortions (Fattal-Jaef, Hopenhayn)

- $m$ is independent of distortions either if $\delta = 0$ or $\beta = 1$.
- $m$ is independent of distortions if $\tilde{r}_s / r_s$ is independent of $s$
Definition 3. The sequence \( \{\delta^s r_s\} \) dominates (is dominated by) the sequence \( \{\delta^s \tilde{r}_s\} \) if and only if

\[
\frac{\sum_{s=0}^{t} \delta^s r_s}{\sum_{s=0}^{\infty} \delta^s r_s} \leq \left( \geq \right) \frac{\sum_{s=0}^{t} \delta^s \tilde{r}_s}{\sum_{s=0}^{\infty} \delta^s \tilde{r}_s}
\]

for all \( t \).

Proposition 4. Suppose \( \{\delta^s r_s\} \) dominates (is dominated by) \( \{\delta^s \tilde{r}_s\} \) Then \( m \geq (\leq) m_0 \).

- **dominates** if distortions tend to be higher for older firms
  - sufficient condition \( \tilde{r}_s / r_s \) decreasing in \( s \)

- Fattal-Jaef (2010) shows accounting for the response of entry to wedges can lead to substantively different values.
Equilibrium and Optimal entry

- **private vs. social value of a cohort:**

  \[
  \tilde{r}_s = \int e^{\frac{1}{1-\eta}} (1 - \tau)^{\frac{1}{1-\eta}} d\tilde{\mu}_s (e, \tau)
  \]

  \[
  \tilde{y}_s = \int e^{\frac{1}{1-\eta}} (1 - \tau)^{\frac{\eta}{1-\eta}} d\tilde{\mu}_s (e, \tau)
  \]

- **Social planner’s objective:**

  \[
  \max_m \sum_{m_t} \beta^t \left[ \left( \sum_{s=0}^{t} m_{t-s}^s \tilde{r}_s^s \right)^{1-\eta} (N - c_em_t)^\eta \right] 
  \]

  \[
  P_t(m_0, m_1, ...) \quad D_t(m_0, m_1, ...)
  \]

  - \(P_t\) stands for *private return* and \(D_t\) for *distortion*

- **First order conditions:**

  \[
  \sum_{s \geq t} \beta^s \frac{\partial P_s}{\partial m_t} D_s + \sum \beta^s \frac{\partial D_s}{\partial m_t} P_s
  \]

  - **Note:** first term is zero at steady state (\(D_s\) is constant)
The planner and distortions

\[ \sum_{s \geq t} \beta^s \frac{\partial P_s}{\partial m_t} D_s + \sum \beta^s \frac{\partial D_s}{\partial m_t} P_s \]

**Proposition 5.** In a steady state \( \sum \beta^s \frac{\partial D_s}{\partial m_t} P_s \) has sign of: \( \sum_{s=0}^{\infty} \beta^s \left( \frac{\delta^s \tilde{y}_s}{\delta^s \tilde{r}_s} - \frac{\delta^s \tilde{y}_s}{\delta^s \tilde{r}_s} \right) \)

- Negative if \( \delta^s \tilde{r}_s \) is dominated by \( \delta^s \tilde{y}_s \)
- Sufficient condition \( \tilde{r}_s / \tilde{y}_s \) decreasing in \( s \)

\[
\frac{\tilde{r}_s}{\tilde{y}_s} = \left( \frac{\int (1 - \tau) y(e, \tau) d\tilde{\mu}_s(e, \tau)}{y(e, \tau) d\tilde{\mu}_s(e, \tau)} \right)
\]

- Larger in less distorted cohorts
- If distortions are higher (lower) or more (less) correlated with output for younger cohorts then planner wants more (less) entry than equilibrium.
■ Take economy where older firms are more distored than younger firms (e.g. taxes positively correlated with age/size)

■ In that economy, there will be more entry: entrants are "less taxed" than typical incumbent

■ Entry will be excessive: at the equilibrium, the marginal social value of an entrant is less than the marginal social value of an incumbent.

■ Conjecture: If distortions are size/age related and older firms are larger, planner would want less entry than in the undistored equilibrium.

■ Optimal entry at distorted economy < Optimal entry at undistorted economy < equilibrium entry at distorted economy.
Final remarks

- What matters for aggregate TFP is the concentration of distortions
- Correlation with size/efficiency not as important
- Effects of distortions sensitive to curvature
- In HK effects increase with curvature
- Dichotomy between distortions and the productivity of firms
- Distortions and entry/exit