Parameter Bias in an Estimated DSGE Model: Does Nonlinearity Matter?

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Outline

Background

- Many economists have estimated DSGE models, following the development of Bayesian estimation techniques.
  - i.a., New Keynesian models
- Most of the estimated DSGE models are linearized around a steady state.
  - Linear state-space representation + normality of shocks
    $\Rightarrow$ efficiently evaluate likelihood using the Kalman filter

Background

- Parameter estimates based on a linearized model can be different from those based on its nonlinear counterpart.
  - e.g., Fernández-Villaverde and Rubio-Ramírez (2005); Fernández-Villaverde, Rubio-Ramírez, and Santos (2006)
- Recently, the importance of considering nonlinearity has been emphasized in models with the ZLB.
  - e.g., Basu and Bundick (2012); Braun, Körber, and Waki (2012); Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015); Gavin, Keen, Richter, and Throckmorton (2015); Gust, López-Salido, and Smith (2012); Nakata (2013a, 2013b); and Ngo (2014)

Background

- Attempts to estimate DSGE models in a fully nonlinear and stochastic setting are still limited.
  - Due to high computational costs:
    - Nonlinear solution method
    - Particle filter
  - Exceptions: Fernández-Villaverde and Rubio-Ramírez (2005); Gust, López-Salido, and Smith (2012); Maliar and Maliar (2015)

Question

Do we need to consider nonlinearity in estimating DSGE models?

Objective

- Examine how and to what extent parameter estimates can be biased in an estimated DSGE model when nonlinearity is omitted in estimation.
  - Suppose that nonlinearity exists in the economy and that an econometrician fits a model without taking account of the nonlinearity.
  - Then, parameter estimates in the model can be biased to some extent.
- Significant biases $\Rightarrow$ Urge researchers to be equipped with nonlinear estimation techniques.
  - Negligible biases $\Rightarrow$ Common practice of estimating linearized models could lead to reliable estimates.

Strategy

- Construct artificial time series simulated from a fully nonlinear New Keynesian model that incorporates the ZLB constraint.
  - The parameters calibrated in the DGP are regarded as true values.
- Monte Carlo experiment: Using the simulated data, a linearized version of the model is estimated without imposing the ZLB.
  - Bayesian estimation
  - Set the prior means equal to the true parameter values.
  - Assess the parameter biases by comparing the posterior means and credible intervals with the true values.

Contribution

- An extension of Hirose and Inoue (2015).
  - DGP: *Quasi-linear* New Keynesian model where nonlinearity is considered only in the ZLB but the remaining equilibrium conditions are linearized.
  - Solution: Erceg and Lindé (2014); Bodenstein, Guerrieri, and Gust (2013)
  - Point to parameter bias only resulting from omitting the ZLB
- The present paper investigates the bias arising from missing nonlinearities regarding both the ZLB and the other equilibrium conditions.
  - DGP: Fully nonlinear model
  - Solution: Projection method
  - Identifies which nonlinearity matters.

Contribution

- The DGP used in our experiments has the richest dynamic structure of all the New Keynesian models with the ZLB solved in a fully nonlinear and stochastic setting.
  1. Habit persistence in consumption preferences
  2. Price indexation of intermediate-good firms
  3. Monetary policy smoothing
- Apply the efficient algorithm for the Smolyak-based projection methods developed by Judd, Maliar, Maliar, and Valero (2014).
  - Alleviate the computational burden associated with the increased number of state variables.

Outline

Household

- Each household $h \in [0,1]$ maximize the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{k=1}^{t} d_k \right)^{-1} \left[\frac{(C_{h,t} - \gamma C_{t-1})^{1-\sigma}}{1-\sigma} - L_{h,t} \right],$$

subject to the budget constraint

$$P_t C_{h,t} + B_{h,t} = P_t W_t L_{h,t} + R_{t-1} B_{h,t-1} + T_{h,t}.$$

- Discount factor shock:

$$\log d_t = \rho_d \log d_{t-1} + \varepsilon_{d,t},$$

where $\varepsilon_{d,t} \sim N(0, \sigma_d)$.

Final-Good Firm

- The representative final-good firm produces output Y_t under perfect competition by choosing a combination of intermediate inputs $\{Y_{f,t}\}$ so as to maximize profit

$$P_t Y_t - \int_0^1 P_{f,t} Y_{f,t} df,$$

subject to a CES production technology

$$Y_t = \left(\int_0^1 Y_{f,t}^{\frac{\theta-1}{\theta}} df \right)^{\frac{\theta}{\theta-1}}.$$

Intermediate-Good Firms

- Each intermediate-good firm f produces one kind of differentiated good $Y_{f,t}$ by choosing a cost-minimizing labor input L_t subject to the production function

$$Y_{f,t} = A_t L_{f,t}.$$

- Productivity shock:

$$\log A_t = \rho_a \log A_{t-1} + \varepsilon_{a,t},$$

where $\varepsilon_{a,t} \sim N(0, \sigma_a)$.

Intermediate-Good Firms

- Intermediate-good firms set prices of their products on a staggered basis as in Calvo (1983).
  - In each period, a fraction $1-\xi \in (0,1)$ of intermediate-good firms reoptimize their prices while the remaining fraction ξ indexes prices to a weighted average of Π_{t-1} and $\bar{\Pi}$.
- The firms that reoptimize their prices in the current period then maximize expected profit

$$\mathbb{E}_t \sum_{j=0}^{\infty} \xi^j \beta^j \left(\prod_{k=1}^{j} d_k \right)^{-1} \frac{\Lambda_{t+j}}{\Lambda_t} \left[\frac{P_{f,t}}{P_{t+j}} \prod_{k=1}^{j} \left(\Pi_{t+k-1}^{\iota} \bar{\Pi}^{1-\iota} \right) - MC_{t+j} \right] Y_{f,t+j}$$

subject to the final-good firm's demand

$$Y_{f,t+j} = \left[\frac{P_{f,t}}{P_{t+j}} \prod_{k=1}^{j} \left(\Pi_{t+k-1}^{\iota} \bar{\Pi}^{1-\iota} \right) \right]^{-\theta} Y_{t+j}.$$

Central Bank

- Monetary policy rule:

$$R_t = \max\left[R_t^*, 1\right],$$

$$R_t^* = (R_{t-1}^*)^{\phi_r}\left[\bar{R}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}\right]^{1-\phi_r}\exp(\varepsilon_{r,t}),$$

where $\varepsilon_{r,t} \sim N(0, \sigma_r)$.

  - The max operator enforces the ZLB.

Parameter Setting

σ	Inv. of intertemporal elasticity of substitution	1.500
γ	Habit persistence	0.500
ξ	Price stickiness	0.750
ι	Price indexation	0.500
ϕ_π	Policy response to inflation	2.000
ϕ_y	Policy response to output	0.500
ϕ_r	Interest rate smoothing	0.500
$\bar{\pi}$	Steady-state inflation rate	0.500
$\bar{r}r$	Steady-state real interest rate	0.250
ρ_d	Persistence of discount factor shock	0.700
ρ_p	Persistence of productivity shock	0.700
$100\sigma_d$	S.D. of discount shock	0.300
$100\sigma_a$	S.D. of productivity shock	0.300
$100\sigma_r$	S.D. of monetary policy shock	0.100

Solution Method

- The policy functions:

$$\mathbb{S} = h(\mathbb{S}_{-1}, \tau),$$

where $\mathbb{S}_{-1} = (Y_{-1}, \Pi_{-1}, R^*_{-1}, \Delta_{-1})$ and $\tau = (d, A, \varepsilon_r)$.

- Computed using the time-iteration method with a Smolyak algorithm in the context of a projection method.
  - Apply an efficient algorithm developed by Judd, Maliar, Maliar, and Valero (2014).
  - Level of approximation: $\mu = 2$; same as Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015)
  - Our solution is very accurate, albeit with the reduced number of grid points.

Outline

Monte Carlo Experiments

1. Generate an artificial time series of output, inflation, and the nominal interest rate from the DGP.
   - Reflect the nonlinearity of the economy including the ZLB.
   - 200 observations; i.e., quarterly observations of 50 years.
   - The economy is at the ZLB for 9.7 percent of quarters, and the average duration of ZLB spells is 2.8 quarters.
     - cf. Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015): Prob. of the ZLB = 5.5%; Average duration of ZLB spells = 2.1 quarters
     - cf. Gust, López-Salido, and Smith (2012): Prob. of the ZLB = 3.1%; Average duration of ZLB spells = about 3 quarters

Monte Carlo Experiments

2. Estimate a linearized version of the model using the simulated data.
    - Bayesian estimation
    - Set the prior means equal to the true parameter values.
3. These steps are replicated 200 times.
    - Posterior means and credible intervals are averaged over the replications.
4. Examine how the resulting posterior means and credible intervals differ from the true parameter values.

Linearized Model

$$\tilde{Y}_t = \frac{1}{1+\gamma}\mathbb{E}_t\tilde{Y}_{t+1} + \frac{\gamma}{1+\gamma}\tilde{Y}_{t-1} - \frac{1-\gamma}{\sigma\left(1+\gamma\right)}\left(\tilde{R}_t - \mathbb{E}_t\tilde{\Pi}_{t+1} - \tilde{d}_t\right),$$

$$\begin{aligned}\tilde{\Pi}_t &= \frac{\beta}{1+\beta\iota}\mathbb{E}_t\tilde{\Pi}_{t+1} + \frac{\iota}{1+\beta\iota}\tilde{\Pi}_{t-1} \\ &\quad + \frac{\left(1-\xi\right)\left(1-\xi\beta\right)}{\xi\left(1+\beta\iota\right)}\left[\frac{\sigma}{1-\gamma}\tilde{Y}_t - \frac{\sigma\gamma}{1-\gamma}\tilde{Y}_{t-1} - \tilde{A}_t\right],\end{aligned}$$

$$\tilde{R}_t = \phi_r\tilde{R}_{t-1} + \left(1-\phi_r\right)\left(\phi_\pi\tilde{\Pi}_t + \phi_y\tilde{Y}_t\right) + \varepsilon_{r,t}.$$

Priors

Parameter	Distribution	Mean	S.D.
σ	Gamma	1.500	0.200
γ	Beta	0.500	0.200
ξ	Beta	0.750	0.200
ι	Beta	0.500	0.200
ϕ_π	Gamma	2.000	0.200
ϕ_y	Gamma	0.500	0.200
ϕ_r	Beta	0.500	0.200
$\bar{\pi}$	Gamma	0.500	0.200
$\bar{r}r$	Gamma	0.250	0.200
ρ_d	Beta	0.700	0.200
ρ_a	Beta	0.700	0.200
$100\sigma_d$	Inv. Gamma	0.300	2.000
$100\sigma_a$	Inv. Gamma	0.300	2.000
$100\sigma_r$	Inv. Gamma	0.100	2.000

Results

Parameter	True value	Mean	90% interval
σ	1.500	1.508	[1.192, 1.820]
γ	0.500	0.510	[0.412, 0.609]
ξ	0.750	0.734	[0.693, 0.774]
ι	0.500	0.548	[0.363, 0.733]
ϕ_π	2.000	1.731	[1.574, 1.887]
ϕ_y	0.500	0.416	[0.307, 0.525]
ϕ_r	0.500	0.498	[0.453, 0.544]
$\bar{\pi}$	0.500	0.384	[0.333, 0.434]
$\bar{r}r$	0.250	0.175	[0.130, 0.220]
ρ_d	0.700	0.723	[0.645, 0.802]
ρ_a	0.700	0.683	[0.557, 0.809]
$100\sigma_d$	0.300	0.337	[0.247, 0.424]
$100\sigma_a$	0.300	0.334	[0.213, 0.451]
$100\sigma_r$	0.100	0.108	[0.099, 0.117]

Explanation for the Biases in ϕ_π and ϕ_y

- In the DGP, the monetary policy reaction function has a kink where the ZLB constraint becomes binding.
  - Unconstrained nominal interest rate $\geq 0 \Rightarrow$ Positive slopes
  - Unconstrained nominal interest rate $< 0 \Rightarrow$ Flat slopes
- If such a kink is omitted in the estimation, the estimated slopes are approximated to lie between the positive and flat slopes.

  $\Rightarrow$ Monetary policy coefficients can be underestimated.

Explanation for the Biases in $\bar{\pi}$ and $\bar{rr}$

- The DGP is characterized by the fully nonlinear model.
- The stochastic steady state can be substantially different from the deterministic steady state.
  - e.g., Nakata (2013a): Once the ZLB constraint is taken into account, the presence of uncertainty can reduce inflation and output by a substantial amount.
  - Mean of π_t simulated from the DGP is 0.39 while $\bar{\pi}$ is calibrated at 0.5.
  - Mean of r_t simulated from the DGP is 0.57 while $\bar{r}$ is calibrated at 0.75.

$\Rightarrow$ $\bar{\pi}$ and $\bar{rr}$ must be underestimated to fill these gaps.

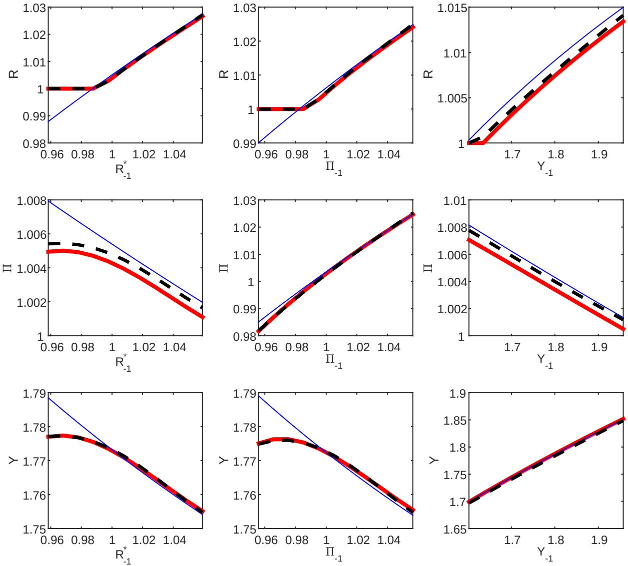
Which nonlinearity matters?

- Which missing nonlinearity causes the biases, the ZLB constraint or the nonlinear equilibrium conditions?
- Two additional experiments: In the DGP,
  1. The ZLB constraint is not imposed but the other nonlinearities remain unchanged.
  2. The ZLB constraint is imposed but all the equilibrium conditions are linearized.

Results: Which Nonlinearity Matters?

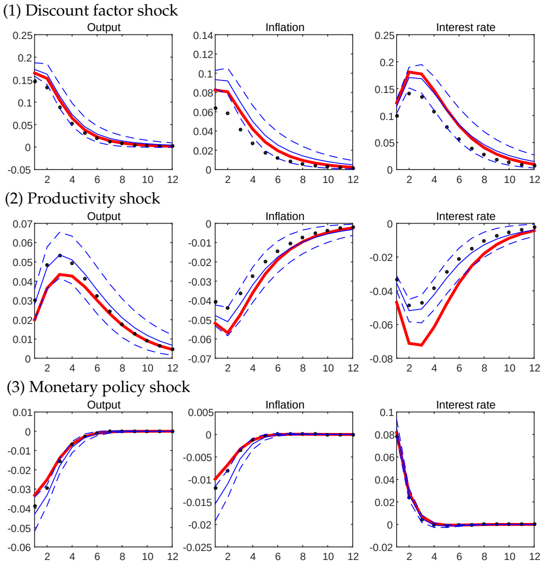
		No ZLB in DGP		Quasi-linear DGP	
	True	Mean	90% interval	Mean	90% interval
σ	1.500	1.471	[1.160, 1.776]	1.492	[1.178, 1.801]
γ	0.500	0.487	[0.385, 0.590]	0.500	[0.401, 0.600]
ξ	0.750	0.743	[0.703, 0.783]	0.738	[0.697, 0.778]
ι	0.500	0.498	[0.311, 0.683]	0.524	[0.340, 0.707]
ϕ_π	2.000	2.001	[1.815, 2.187]	1.838	[1.680, 1.995]
ϕ_y	0.500	0.495	[0.384, 0.605]	0.442	[0.334, 0.548]
ϕ_r	0.500	0.495	[0.447, 0.543]	0.491	[0.447, 0.535]
$\bar{\pi}$	0.500	0.498	[0.459, 0.537]	0.432	[0.383, 0.481]
$\bar{r}r$	0.250	0.248	[0.203, 0.293]	0.203	[0.156, 0.251]
ρ_d	0.700	0.698	[0.614, 0.782]	0.732	[0.655, 0.810]
ρ_a	0.700	0.707	[0.586, 0.829]	0.693	[0.570, 0.817]
$100\sigma_d$	0.300	0.296	[0.219, 0.371]	0.316	[0.235, 0.396]
$100\sigma_a$	0.300	0.293	[0.187, 0.397]	0.323	[0.206, 0.435]
$100\sigma_r$	0.100	0.100	[0.091, 0.108]	0.103	[0.094, 0.112]

Policy Functions



Red: fully nonlinear; Blue: nonlinear without ZLB; Black: quasi-linear with ZLB

Impulse Responses



Red: True nonlinear; Blue: estimated linear; Black: linear with true parameters

Variance-covariance matrices

	Output	Inflation	Interest rate
(1) Nonlinear model with true parameters (DGP)			
Output	0.090	0.038	0.069
Inflation		0.036	0.059
Interest rate			0.129
(2) Linear model with true parameters			
Output	0.066	0.015	0.041
Inflation		0.017	0.032
Interest rate			0.088
(3) Linear model with biased parameters			
Output	0.096	0.037	0.071
Inflation		0.037	0.063
Interest rate			0.139

Summary of Results in Baseline Experiment

1. The estimates of ϕ_π, $\bar{\pi}$, and $\bar{rr}$ are significantly biased.
   - Deep parameters are not biased.
2. These biases arise mainly from neglecting the ZLB rather than linearizing the equilibrium conditions.
3. Estimated IRFs can be substantially different from the true ones.
   - For some of the IRFs, the biased parameters contribute to making the differences small.

Outline

Alternative Parameter Settings

- Baseline DGP: The economy is at the ZLB for 9.7% of quarters, and the average duration of ZLB spells is 2.8 quarters.

- As the probability of hitting the ZLB increases, parameter biases due to excluding the nonlinearity in the estimation might become large.

- Two alternative parameter settings:

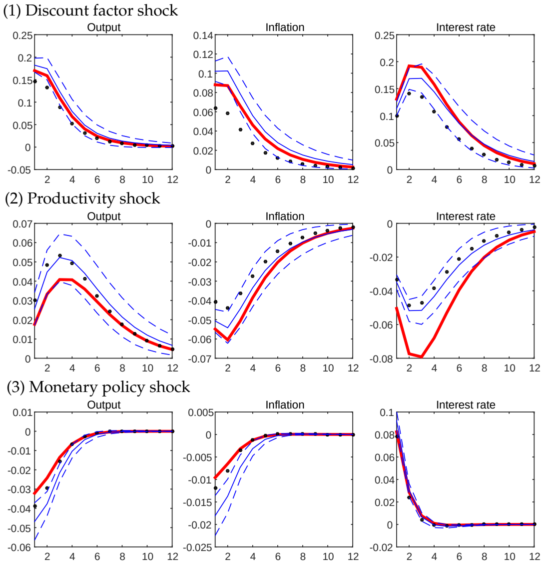
  1. $\bar{r}r$: $0.25 \rightarrow 0.23$
     $\Rightarrow$ The prob. of ZLB = 14.8%
  2. $100\sigma_d$: $0.3 \rightarrow 0.33$
     $\Rightarrow$ The prob. of ZLB = 14.2%

  - In both cases, the average duration of ZLB spells is 3.1 quarters.

Results: Alternative Experiments

		Case of low $\bar{rr}$		Case of large σ_d	
	True	Mean	90% interval	Mean	90% interval
σ	1.500	1.530	[1.212, 1.844]	1.528	[1.209, 1.842]
γ	0.500	0.521	[0.424, 0.619]	0.521	[0.425, 0.619]
ξ	0.750	0.733	[0.693, 0.773]	0.733	[0.694, 0.773]
ι	0.500	0.570	[0.388, 0.754]	0.573	[0.390, 0.755]
ϕ_π	2.000	1.614	[1.456, 1.771]	1.619	[1.459, 1.778]
ϕ_y	0.500	0.376	[0.263, 0.487]	0.376	[0.265, 0.486]
ϕ_r	0.500	0.514	[0.466, 0.562]	0.515	[0.468, 0.561]
$\bar{\pi}$	0.500	0.347	[0.294, 0.400]	0.352	[0.299, 0.405]
$\bar{rr}$	0.22/0.25	0.138	[0.094, 0.182]	0.175	[0.130, 0.219]
ρ_d	0.700	0.721	[0.644, 0.800]	0.716	[0.638, 0.794]
ρ_a	0.700	0.672	[0.544, 0.800]	0.670	[0.543, 0.799]
$100\sigma_d$	0.30/0.33	0.347	[0.251, 0.440]	0.382	[0.277, 0.484]
$100\sigma_a$	0.300	0.355	[0.225, 0.481]	0.356	[0.226, 0.481]
$100\sigma_r$	0.100	0.115	[0.105, 0.124]	0.118	[0.108, 0.128]

Impulse responses in the case of low $\bar{rr}$

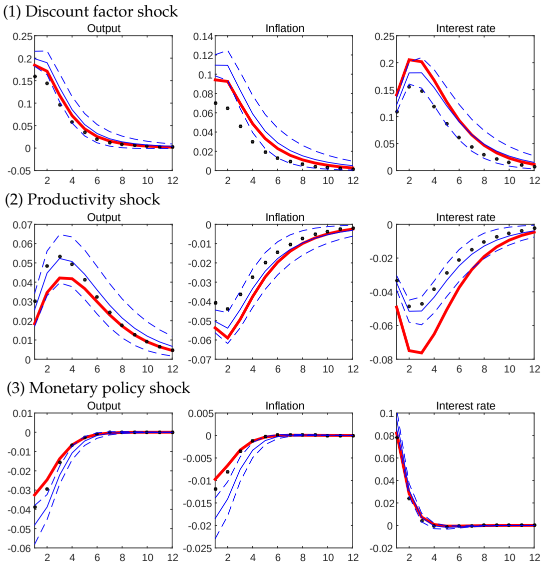


Red: True nonlinear; Blue: estimated linear; Black: linear with true parameters

Variance-covariance matrices in the case of low $\bar{rr}$

	Output	Inflation	Interest rate
(1) Nonlinear model with true parameters (DGP)			
Output	0.101	0.047	0.075
Inflation		0.044	0.065
Interest rate			0.130
(2) Linear model with true parameters			
Output	0.066	0.015	0.041
Inflation		0.017	0.032
Interest rate			0.088
(3) Linear model with biased parameters			
Output	0.108	0.045	0.076
Inflation		0.045	0.069
Interest rate			0.143

Impulse responses in the case of large σ_d



Red: True nonlinear; Blue: estimated linear; Black: linear with true parameters

Variance-covariance matrices in the case of large σ_d

	Output	Inflation	Interest rate
(1) Nonlinear model with true parameters (DGP)			
Output	0.118	0.055	0.089
Inflation		0.048	0.072
Interest rate			0.146
(2) Linear model with true parameters			
Output	0.076	0.020	0.053
Inflation		0.019	0.038
Interest rate			0.103
(3) Linear model with biased parameters			
Output	0.124	0.054	0.091
Inflation		0.049	0.077
Interest rate			0.160

Outline

Concluding Remarks

- Investigated parameter bias in an estimated DSGE model that neglects nonlinearity.
- Significant biases have been detected in the estimates of the monetary policy coefficients and the steady-state inflation and real interest rates.
- These biases are caused by ignoring the ZLB rather than linearizing the equilibrium conditions.
- Demonstrated that the estimated IRFs can be substantially different from the true ones, although the biased parameters partially contribute to making the differences small.
- These findings are a caution to researchers against the common practice of estimating linearized DSGE models in the presence of the ZLB.

Concluding Remarks

- Our finding regarding the source of parameter bias indicates that omitting nonlinearity in estimation would not affect parameter estimates if the ZLB was not an issue.
- However, it might not be the case if the DGP is characterized by a more highly nonlinear model than considered in this paper.
  - Recursive preferences
  - State-dependent pricing
  - Increased uncertainty

Thank you very much for your attention.

Appendix: Accuracy of the solutions

	Equilibrium paths		Entire state space	
	$\log_{10} L_1$	$\log_{10} L_\infty$	$\log_{10} L_1$	$\log_{10} L_\infty$
Baseline	-3.508	-2.527	-2.951	-1.780
No ZLB in DGP	-5.045	-4.570	-4.831	-4.008
Quasi-linear DGP	-4.160	-3.195	-3.522	-2.456
Case of low $\bar{rr}$	-3.477	-2.531	-2.961	-1.770
Case of large σ_d	-3.453	-2.532	-2.948	-1.784

Note: L_1 and L_∞ are the average and maximum of the absolute residuals across all the equilibrium conditions based on 40,000 points of $(\mathbb{S}_{-1}, \tau)$ on the equilibrium paths and 40,000 random points from uniform distributions over the entire state space, respectively.