Intergenerational expectations and deflationary equilibrium

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We observe **decade-long deflationary stagnation**

- The last three decades in Japan
- Great Recession in the United States

**Puzzling fact:**

Coexistence of **deflation and increase in gov’t debt** (including money)

**Can we have the coexistence as an equilibrium outcome?**

- Total value of debt grows indefinitely
- Transversality condition (TVC) is not satisfied
  - TVC: the PDV of debt in the future converges to zero
Popular prescription to escape from deflationary recession

- Increasing gov’t debt by monetary and fiscal expansion is effective
  - Krugman (1998); Bernanke (2000); Benhabib, et al. (2002); Eggertsson and Woodford (2003); Auerbach and Obstfeld (2005)

All these theories depend on the premise that TVC must be satisfied in equilibrium. The logic goes as follows.

- Suppose that the gov’t makes debt grows at a sufficiently high rate
- Then, TVC is violated, if deflation continues permanently.
- As TVC must be satisfied in equilibrium, deflation cannot continue.

Now our experience in the last decades seems inconsistent with this logic.
Neo-Fisherian explanation

- Fisher equation implies the low interest rate policy makes deflationary expectations, given that
  - TVC is satisfied because people believe that tax will be increased sufficiently in the future
  - Cochrane (2017), Schmit-Grohe and Uribe (2017)

- The mainstream does not agree with Neo-Fisherians, because
  - the government can easily commit to the irresponsible policy (Krugman 1998), i.e., no tax increase, and violate TVC
Can government-debt expansion with the low nominal interest rate (reflationary policy) realize a higher inflation?
- Answer: Not necessarily.

Can the economy get stuck in a deflationary equilibrium, given that the government commits itself to the reflationary policy and the commitment is fully trusted?
- Answer: Yes, it can, even if people believe there will be no tax increase.

Can the government debt keep growing indefinitely in the deflationary equilibrium?
- Answer: Yes, it can. (↔ Finite upper bound in rational bubble models)
Disinflation in the United States

**Figure:** CPI inflation rate in US economy
Debt growth in the United States

Figure: Government debt to GDP ratio in the United States
1 Introduction

Deflation in Japan

Figure: CPI inflation rate in Japan
Debt growth in Japan

Figure: Debt to GDP ratio in Japan
We show deflation with growing debt can be an equilibrium

- Intergenerational altruism: assets are bequeathed indefinitely
- (Bubbly) expectations: current generation is happy with money as they believe the future generation will be happy with money
- TVC is not necessarily satisfied in equilibrium

Extreme monetary easing may not be effective in fighting deflation

- Same as the Neo-Fisherian argument: $1 + \pi_t = (1 + i_t)\beta$ with $i_t = 0$ implies $\pi_t < 0$.
- Secular stagnation can be a steady state generated by the policy $i_t = 0$. 

Empirical support

1. TVC is violated in Japan (Doi, 2004)
   - The Bohn condition is not satisfied in Japan for 1965–2000
     - Bohn: When the debt increases, the primary balance should be improved
   - The Bohn condition is a sufficient condition for TVC

2. Intergenerational altruism is present in Japan (Horioka, 2008)
   - The survey shows that 70 percent of the inheritees (parents) are subjectively altruistic to their inheritors (children)
   - The same survey shows that the inheritors feel that 20–30 percent of their inheritees are altruistic

3. Bequest motive increased the household savings, since the mid-2000s in Japan (Hamaaki and Hori, 2018)
   - Zero interest rate since the early 2000s.
1 Introduction

2 Baseline model

3 Secular Stagnation in a New Keynesian economy

4 Conclusion
Setting (1/2)

- A closed economy with the representative household and the government
- The economy is deterministic. Time is discrete: $t = 0, 1, 2, \ldots, \infty$,
- Representative household with intergenerational altruism
  - are endowed with output $y_t$ every period,
  - the generation $t$ lives only for period $t$ and die at the end of $t$,
  - period utility for the generation $t$ is $U(c_t)$,
  - the lifetime utility of generation $t$ is $V_t = U(c_t) + \beta W_{t+1}$, where
    - $W_{t+1}$ is the generation $t$'s expectation on the lifetime utility of generation $t + 1$
    - $\beta$ is the degree of altruism (and also the time discount factor)

Note: the model can be generalized such that each generation lives $N$ periods, with

$$V_t = \left\{ \sum_{j=0}^{N-1} \beta^j U(c_{t+j}) \right\} + \beta^N W_{t+N}$$

- We focus on the case where $N = 1$. 

Government issues the nominal bond $B^s_{t+1}$, i.e., the economy is cashless.

- bond evolves by

\[
(1 + \pi_{t+1})b^s_{t+1} = (1 + i_t)b^s_t + \tau_t,
\]

where $b^s_t = \frac{B^s_t}{P_t}$ and $1 + \pi_{t+1} = \frac{P_{t+1}}{P_t}$,

- Gov’t conducts fiscal policy, that decides the real transfers (or lump-sum taxes):

\[
\{\tau_{t+j}\}_{j=0}^{\infty},
\]

assuming that $\tau_t$ satisfies

\[
\underline{\tau} \leq \tau_t \leq \bar{\tau},
\]

where $\underline{\tau} < 0 < \bar{\tau}$.

- Gov’t conducts monetary policy, that decides the nominal interest rates: $\{i_{t+j}\}_{j=0}^{\infty}$,

  - $i_t$ satisfies $i_t \geq 0$
Household takes $z_t$ as given, where $z_t = \{y_{t+j}, \tau_{t+j}, i_{t+j}, P_{t+j}\}_{j=0}^{\infty}$

Household solves the following problem, given $z_t$ and $b_t$, where $b_t = \frac{B_t}{P_t}$ and $B_t$ is the nominal bond holdings at the beginning of $t$:

$$V(b_t; z_t) = \max_{c_t, b_{t+1}} U(c_t) + \beta W(b_{t+1}; z_{t+1}),$$

subject to

$$c_t + (1 + \pi_{t+1}) b_{t+1} \leq y_t + (1 + i_t) b_t + \tau_t$$

where $W(b_{t+1}; z_{t+1})$ is the generation $t$’s expectation on the generation $t + 1$’s lifetime utility.

Equilibrium conditions

$$c_t \leq y_t,$$

$$b_{t+1} \leq b_{t+1}^s.$$
Intergenerational rationality

- Intergenerational rationality: consistency of value functions for all generations

\[ W(b; z) = V(b; z) \]

- Given the generation \( t \)'s expectation that \textit{generation} \( t+1 \)'s \textit{lifetime utility is} \( V(b_{t+1}; z_{t+1}) \), the lifetime utility of generation \( t \) becomes \( V(b_t; z_t) \).

- Intergenerationaly-rational household solves, given \( z_t \) and \( b_t \),

\[
V(b_t; z_t) = \max_{c_t, b_{t+1}} U(c_t) + \beta \quad V(b_{t+1}; z_{t+1}) ,
\]

\[ \text{my utility} \quad \text{my expectation on my child's utility} \]

\[ \text{s. t.} \quad c_t + (1 + \pi_{t+1})b_{t+1} \leq y + (1 + i_t)b_t + \tau_t \]

- This is a standard \textbf{Bellman equation}!
Standard recursive macroeconomics

Sequential problem:

\[(SP) \quad \max \sum_{j=0}^{\infty} \beta^j U(c_{t+j}), \]

\[\text{s. t.} \quad c_t + (1 + \pi_{t+1})b_{t+1} \leq y_t + (1 + i_t)b_t + \tau_t,\]

Recursive problem

\[(RP) \quad V(b_t; z_t) = \max_{\text{my utility at } t} U(c_t) + \beta V(b_{t+1}; z_{t+1}),\]

\[\text{s. t.} \quad c_t + (1 + \pi_{t+1})b_{t+1} \leq y_t + (1 + i_t)b_t + \tau_t.\]

- The nature decides the objective function: \(\sum_{j=0}^{\infty} \beta^j U(c_{t+j}).\)

- Usual interpretation: (SP) is the original problem. (RP) is reformulation for convenience.

- TVC is necessary for the optimality in (SP)

\[\text{TVC:} \quad \lim_{t \to \infty} \beta^t \lambda_t b_t = 0.\]
New interpretation as intergenerational problem

\begin{align*}
\text{(RP)} \quad & V(b_t; z_t) = \max U(c_t) + \beta V(b_{t+1}; z_{t+1}), \\
& \text{my utility} \quad \text{my expectation on my child's utility} \\
\text{s. t.} \quad & c_t + (1 + \pi_{t+1}) b_{t+1} \leq y_t + (1 + i_t) b_t + \tau_t.
\end{align*}

\begin{align*}
\text{(SP)} \quad & \max \sum_{j=0}^{\infty} \beta^j U(c_{t+j}), \\
\text{s. t.} \quad & c_t + (1 + \pi_{t+1}) b_{t+1} \leq y_t + (1 + i_t) b_t + \tau_t.
\end{align*}

- Our interpretation:
  - \text{(RP)} is the original problem. \text{(SP)} is not true description of the economy.
  - TVC need not to be satisfied
  - Expectation $V(b_{t+1}; z_{t+1})$ is endogenous, based not only on the nature but also on the social norm or social convention.
Definition: Intergenerationally-Rational Expectations Equilibrium (IREE)

A set of quantities \( \{c_t, b_{t+1}\} \) and prices \( \{P_t\} \) that satisfies

1. quantities solve (RP), given \( b_t = b^s_t \) and \( z_t \),
2. value function \( V(b; z) \) is well-defined,
3. resource constraints are satisfied.

\[(RP) \quad V(b_t; z_t) = \max_{c_t, b_{t+1}} U(c_t) + \beta V(b_{t+1}; z_{t+1}),\]
\[\text{s. t.} \quad c_t + (1 + \pi_{t+1})b_{t+1} \leq y_t + (1 + i_t)b_t + \tau_t. \quad (\lambda_t)\]

We will show:

- The solution to (RP) may or may not satisfy TVC, \( \lim_{t \to \infty} \beta^t \lambda_t (1 + i_t)b_t = 0 \)
- In equilibrium,
  - for some policy schedule \( \{\tau_{t+j}, i_{t+j}\}_{j=0}^{\infty} \),
    TVC is satisfied and the equilibrium is the usual rational expectations equilibrium (REE). It is also IREE.
  - for other policy schedule \( \{\tau_{t+j}, i_{t+j}\}_{j=0}^{\infty} \),
    TVC is not satisfied and the equilibrium is IREE, but not REE.
The solution to the Bellman equation

\[(RP) \quad V(b_t; z_t) = \max_{c_t, b_{t+1}} U(c_t) + \beta V(b_{t+1}; z_{t+1}),\]

s. t. \[c_t + (1 + \pi_{t+1})b_{t+1} \leq y_t + (1 + i_t)b_t + \tau_t \] \( (\lambda_t) \)

- Constraints and conditions for the household:
  \[ c_t = y_t + (1 + i_t)b_t - (1 + \pi_{t+1})b_{t+1} + \tau_t, \]
  \[ \lambda_t = U'(c_t), \]
  \[ (1 + \pi_{t+1})\lambda_t = \beta V'(b_{t+1}; z_{t+1}), \]
  \[ V'(b_t; z_t) = (1 + i_t)\lambda_t. \]

which implies

\[(1 + \pi_{t+1})\lambda_t = \beta(1 + i_{t+1})\lambda_{t+1}. \] \( (1) \)

- Given \( \{i_t, \tau_t, \pi_t\}_{t=0}^{\infty}, \) the household decides \( \{c_t, b_{t+1}\}_{t=0}^{\infty}. \)

- Equilibrium conditions:
  \[ c_t = y_t, \]
  \[ b_t = b_t^s. \]

- The equilibrium inflation \( \{\pi_{t+j}\}_{j=1}^{\infty} \) is determined by \( (1) \) and \( c_t = y_t. \)
The fundamental solution to the Bellman equation

- The fundamental solution \( V^f(b; z) \) is given as the solution to
  \[
  V^f(b_t; z_t) = \max_b U(y_t + (1 + i_t)b_t - (1 + \pi_t+1)b + \tau_t) + \beta V^f(b; z_{t+1}).
  \]
- Define
  \[
  V_t^* = \sum_{j=0}^{\infty} \beta^j U(y_{t+j}).
  \]
- We know that the fundamental equilibrium such that \( V(b; z) = V^f(b; z) \), \( c_t = y_t \), and \( b_t = b^s_t \), exists if
  \[
  \lim_{t \to \infty} \beta^t \tilde{\lambda}_t (1 + i_t)b^s_t = 0,
  \]
  where \( \tilde{\lambda}_t = U'(y_t) \).
- If the fundamental equilibrium exists, then
  \[
  V^f(b^s_t; z_t) = V_t^*.
  \]
The bubbly solution to the Bellman equation

\[(RP) \quad V(b_t; z_t) = \max_{c_t, b_{t+1}} U(c_t) + \beta V(b_{t+1}; z_{t+1}),\]

\[\text{s. t.} \quad c_t + (1 + \pi_{t+1}) b_{t+1} \leq y_t + (1 + i_t) b_t + \tau_t \quad (\lambda_t)\]

can guess and verify the value function with a bubble term:

\[V(b_t; z_t) = V^b(b_t; z_t) \equiv V^*_t + (1 + i_t) \bar{\lambda}_t b_t + \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} \tau_{t+j} = V^*_t + (1 + i_t) \bar{\lambda}_t (b_t - b^*_t) + \beta^{-t} X^s,\]

\[b_{t+1} = B(b_t; z_t) \equiv \frac{(1 + i_t) b_t + \tau_t}{1 + \pi_{t+1}},\]

\[c_t = C(b_t; z_t) \equiv y_t.\]

where \(\bar{\lambda}_t \equiv U'(y_t)\) and \(X^s = \lim_{t \to \infty} \beta^t \bar{\lambda}_t b^*_t.\)

We can verify the above guess: Given that the FOC (1) is satisfied,

\[V^b(b_t; z_t)\] satisfies all FOCs and envelope condition,

it is shown that \(V^b(b_t; z_t) = U(y_t) + \beta V^b(B(b_t, z_t); z_{t+1}),\)

\(V^b(b_t; z_t)\) is well-defined

TVC can be violated: \(\lim_{t \to \infty} \beta^t \bar{\lambda}_t b^*_t \neq 0.\)
The economy was initially in the zero inflation steady state.

**Government commitment:**
The government make the following commitment in period 0.

- As long as $\pi_t \leq 0$, the government sets $i_t = \tau_t = 0$,
- if $\pi_t > 0$, the government sets

  $$i_t = i^* = \beta^{-1}(1 + \pi^*) - 1,$$

  $$\tau_t = \tau^* = -i^* \hat{b}.$$ 

- The policy $(i^*, \tau^*)$ is consistent with the inflation target $\pi^* > 0$. 
Standard prediction: immediate inflation

- Suppose that the equilibrium should be REE.
- The inflation target is immediately attained.

Given the government commitment, $P_0$ cannot be $P_{-1}$.

- Suppose $P_0 = P_{-1}$. Then, $\pi_0 = 0$.
- Then, the government chooses $i_0 = 0$.
- Then, $\pi_1 = \beta(1 + i_0) - 1 = \beta - 1 < 0$. Thus, the government chooses $i_1 = 0$.
- By induction, $i_t = 0$ and $\pi_t = \beta - 1$ for all $t \geq 1$.
- Then, the TVC is violated: $\lim_{t \to \infty} \beta^t b_t = \lim_{t \to \infty} \beta^t \frac{B_0}{(1+\pi)^t} = B_0$.
- As TVC should be satisfied in REE, $P_0$ cannot be $P_{-1}$.

- The only possible equilibrium is $P_0 \geq (1 + \pi^\ast)P_{-1}$ and $\pi_t = \pi^\ast$ for all $t \geq 0$. 
Suppose that the equilibrium is IREE, not REE.

In this case, the steady state with permanent deflation can be an equilibrium.

- $P_0 = P_{-1}$
- $i_t = 0$ for all $t \geq 0$.

In the steady state, $c_t = y$, $\lambda_t = \bar{\lambda}_t = U'(y)$, and \( \frac{1+i}{1+\pi_{t+1}} = \beta^{-1} \). (Fisher equation)

Real value of nominal bond evolves by $b_t = \beta^{-t} b_0 \to \infty$.

If the nominal rate is zero, $i = 0$, then $\frac{P_{t+1}}{P_t} = 1 + \pi_t = \beta < 1$. (Deflation)

TVC is violated:

\[
\lim_{t \to \infty} \beta^t \lambda_t b_t = \lim_{t \to \infty} \beta^t \times U'(y) \beta^{-t} b_0 = U'(y) b_0 > 0.
\]
Deflationary equilibrium: Policy implications

- IREE: The TVC can be violated in equilibrium
  - TVC is the key for the usual logic to escape from deflation.
    1. If deflation continues under monetary easing ($i = 0$), then TVC will be violated
    2. TVC must be satisfied in equilibrium
    3. Therefore, deflation will stop, only if monetary easing continues

  - IREE indicates this logic may not be correct.
    Deflation can continue under $i_t = 0$ in the IREE.

- Extreme monetary easing ($i = 0$) may induce persistent deflation, as
  \[ 1 + \pi_t = (1 + i_t)\beta \]

- Government debt can grow indefinitely
  - Along this equilibrium path, TVC is violated.
Two notes on the bubbly solution

1. The same argument holds for the model with capital, $k_t$.
   - There should be unique equilibrium path $\{k_t\}_{t=0}^{\infty}$ that satisfies the resource constraints $c_t \geq 0$ and $k_t \geq 0$ for all $t$.
   - There is the unique fundamental value function: $V^f(k_t; z_t)$
   - Define
     $$V^b(k_t, b_t; z_t) = V^f(k_t; z_t) + (1 + i_t)\bar{\lambda}_t(b_t - b^s_t) + \beta^{-t}X^s.$$  
     Then, $V^b(k_t, b_t; z_t)$ is the bubbly solution.

2. Two types households can coexist in equilibrium:
   - Type-F whose value is $V^f(b_t; z_t)$ and Type-B whose value is $V^b(b_t; z_t)$
     - They can coexist, as long as the measure of Type-F is not too large.
     - Type-F consume more and Type-B consume less.
     - TVC is satisfied for Type-F.
     - TVC is not satisfied for Type-B.
Sequential formulation of bubbly solution

- We will consider a sequential problem, the solution to which is

\[
V^b(b_t; z_t) = V_t^* + (1 + i_t)\bar{\lambda}_t b_t + \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} \tau_{t+j} = (1 + i_t)\bar{\lambda}_t b_t + \sum_{j=0}^{\infty} \beta^j [U(y_{t+j}) + \bar{\lambda}_{t+j} \tau_{t+j}]
\]

- Suppose that, given \( b_t \), the generation \( t \) chooses \( c_t \) and \( \varepsilon_t = (1 + \pi_{t+1})b_{t+1} - (1 + i_t)b_t \) to maximize

\[
(1 + i_t)\bar{\lambda}_t b_t + \sum_{j=0}^{\infty} \beta^j [U(c_{t+j}) + \bar{\lambda}_{t+j} \varepsilon_{t+j}]
\]

- \( V^b(b; z) \) is the solution to the sequential problem with intertemporal budget:

\[
(1 + i_t)\bar{\lambda}_t b_t + \max_{c_{t+j}, \varepsilon_{t+j}} \sum_{j=0}^{\infty} \beta^j [U(c_{t+j}) + \bar{\lambda}_{t+j} \varepsilon_{t+j}],
\]

s. t. \( \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} (c_{t+j} + \varepsilon_{t+j}) \leq \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} (y_{t+j} + \tau_{t+j}) \)

with the equilibrium conditions: \( c_t \leq y_t \) and \( \varepsilon_t \leq \tau_t \).
Sequential formulation of bubbly solution

- (1) implies
  \[(1 + i_t)\bar{\lambda}_t b_t + \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} \varepsilon_{t+j} = \lim_{n \to \infty} \beta^n \bar{\lambda}_{t+n} (1 + i_{t+n}) b_{t+n},\]

  \[(1 + i_t)\bar{\lambda}_t b^s_t + \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} \tau_{t+j} = \lim_{n \to \infty} \beta^n \bar{\lambda}_{t+n} (1 + i_{t+n}) b^s_{t+n}.\]

- Define \(X \equiv \lim_{t \to \infty} \beta^t \bar{\lambda}_t (1 + i_t) b_t\) and \(X^s \equiv \lim_{t \to \infty} \beta^t \bar{\lambda}_t (1 + i_t) b^s_t\).

- Note that \(X = X(b_0; z_0)\) is the PDV of the remaining debt in the infinite future, which is evaluated in period 0.

- Then, \(V^b(b, z)\) is the solution to the bubbly sequential problem:

\[
\text{(SP: B)} \quad \max_{c_{t+j}, X} \left\{ \sum_{j=0}^{\infty} \beta^j U(c_{t+j}) \right\} + \beta^{-t} X,
\]

subject to

\[
\sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} c_{t+j} + \beta^{-t} X \leq \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} y_{t+j} + \beta^{-t} X^s + (1 + i_t)\bar{\lambda}_t (b_t - b^s_t),
\]

with the equilibrium conditions: \(c_t \leq y_t\) and \(X \leq X^s\).
Consider Recursive problem (RP), Bubbly problem (SP: B), Fundamental problem (SP: F).

(RP) \[ V(b_t; z_t) = \max_{c_t, b_{t+1}} U(c_t) + \beta V(b_{t+1}; z_{t+1}), \]
\[ \text{s. t.} \quad c_t + (1 + \pi_{t+1})b_{t+1} \leq y_t + (1 + i_t)b_t + \tau_t \quad (\lambda_t) \]

(SP: B) \[ \max_{c_{t+J}, X} \left\{ \sum_{j=0}^{\infty} \beta^j U(c_{t+j}) \right\} + \beta^{-t} X, \]
\[ \text{s. t.} \quad \left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} c_{t+j} \right\} + \beta^{-t} X \leq \left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} y_{t+j} \right\} + \beta^{-t} X^s + (1 + i_t)\bar{\lambda}_t (b_t - b_t^s), \]

(SP: F) \[ \max_{c_{t+J}, X} \left\{ \sum_{j=0}^{\infty} \beta^j U(c_{t+j}) \right\}, \]
\[ \text{s. t.} \quad \left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} c_{t+j} \right\} + \beta^{-t} X \leq \left\{ \sum_{j=0}^{\infty} \beta^j \bar{\lambda}_{t+j} y_{t+j} \right\} + \beta^{-t} X^s + (1 + i_t)\bar{\lambda}_t (b_t - b_t^s), \]

with the equilibrium conditions, \( X \leq X^s \) and \( c_t \leq y_t \).
Intergenerational rationality: redux

- Intergenerational rationality: consistency of value function $V(b_t; z_t)$ for all generations $t$
- Objective function for the bubbly problem (SP: B):
  \[
  \left\{ \sum_{j=0}^{\infty} \beta^j U(c_{t+j}) \right\} + \beta^{-t}X,
  \]
  where $X$ is the PDV of the remaining debt in the infinite future.
- Intergenerational economy: $X > 0$ is possible, because
  - generation $t$
    - has the belief that “only the last generation of infinite future obtains utility $\lim_{t \to \infty} \beta^{-t}X$ from remaining debt.”
    - $\beta^{-t-1}X$ in expectation is given not by the nature, but social norm or social convention.
  - This belief is not irrational: $\forall t (< \infty)$, generation $t$ cannot refute the belief.
    - Similar to Abreu and Gul (2000)
3 Secular Stagnation in a New Keynesian economy

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4 Conclusion
A reduced-form New Keynesian model with government bond

Intergenerational problem with labor $l_t$ and bond $b_t$.

Cashless economy with no capital:

\[
\text{(RP)} \quad V(b_t; z_t) = \max_{c_t, l_t, b_{t+1}} U(c_t, l_t) + \beta V(b_{t+1}; z_{t+1}),
\]

s. t. \quad c_t + (1 + \pi_{t+1})b_{t+1} \leq w_t l_t + (1 + i_t)b_t + \tau_t + d_t, \quad (\lambda_t)

- Supply side of the economy is given by
  - production technology (PT): $y_t = Al_t$
  - New Keynesian Phillips Curve (NKPC): $\pi_t = \beta \pi_{t+1} + \kappa(y_t - y^f)$

**Definition:** IREE in a NK model

Set of allocations $\{c_t, y_t, l_t, b_t\}_{t=0}^{\infty}$ and prices $\{p_t, w_t\}_{t=0}^{\infty}$ that satisfies

1. $\{c_t, l_t, b_t\}_{t=0}^{\infty}$ is the solution to (RP). $V(b_t; z_t)$ is well-defined.
2. prices $\{p_t, w_t\}_{t=0}^{\infty}$ clears the goods market, labor market, and bond market
3. (PT) and (NKPC) are satisfied.
Stationary equilibrium

- All variables are pinned down by $\pi$ or $i$, where $1 + \pi = \beta(1 + i)$

$$P_t = (1 + \pi)^t P_0$$

$$c(\pi) = y(\pi) = y^f + \frac{(1 - \beta)\pi}{\kappa},$$

$$l(\pi) = \frac{y(\pi)}{A},$$

$$w = -\frac{U_l(y(\pi), l(\pi))}{U_c(y(\pi), l(\pi))}$$

- Can guess and verify that

$$V(b_t, z_t) = \hat{V}(\pi) + \lambda(\pi)(1 + i) b_t + C^b_t$$

is well-defined and solves (RP), satisfying all constraints, where $\hat{V}(\pi) = \frac{1}{1-\beta} U(y(\pi), l(\pi))$, $\lambda(\pi) = U_c(y(\pi), l(\pi))$, and

$$C^b_t = \sum_{j=0}^{\infty} \beta^j \lambda(\pi) \tau_{t+j} < +\infty,$$ as $\tau_t$ is bounded.
Equilibrium path is given by \( \{y_t, \pi_{t+1}\} \), which is determined by

\[
\text{(NKPC)} \quad \pi_t = \beta \pi_{t+1} + \kappa(y_t - y^f),
\]

\[
\text{(FOCs)} \quad 1 + \pi_{t+1} = (1 + i)\beta \frac{U_c(y_{t+1}, \frac{y_{t+1}}{A})}{U_c(y_t, \frac{y_t}{A})}
\]

Phase diagram


Deflationary stagnation with \( i = 0 \)

Suppose that central bank set \( i = \tau = 0 \)

Then, \( 1 + \pi = \beta(1 + i) = \beta \)

\[ P_t = \beta^t P_0, \quad \text{(Deflation)} \]

\[ c = y = y_z \equiv y^f - \frac{(1 - \beta)^2}{\kappa}, \quad \text{(Stagnation)} \]

Value function is

\[ V(b_t) = \hat{V}(\pi) + \underbrace{\lambda b_t}_{\text{bubble term}} \]

TVC is violated in the IREE with \( i = 0 \) and growing \( b_t \)

\[
\lim_{t \to \infty} \beta^t U_c(c_t, l_t)b_t = \lim_{t \to \infty} \beta^t U_c(y_z, l_z)b_0 = U_c(y_z, l_z)b_0 > 0
\]

The equilibrium is not REE, but it is IREE
1 Introduction

2 Baseline model

3 Secular Stagnation in a New Keynesian economy

4 Conclusion
Summary

We show deflation with growing debt can be an equilibrium

- Intergenerational altruism: assets are bequeathed indefinitely
- (Bubbly) expectations: current generation is happy with money as they believe the future generation will be happy with money
  - The expectations on the future generation is irrefutable, as long as they are consistent with the current generation.
- TVC is not necessarily satisfied in equilibrium

Extreme monetary easing may not be effective in fighting deflation

- Same as the Neo-Fisherian: $1 + \pi_t = (1 + i_t)\beta$ with $i_t = 0$ implies $\pi_t < 0$.
- Secular stagnation can be a steady state generated by the policy $i_t = 0$.

Government debt, as a bubble, can grow indefinitely in equilibrium

- The bubble may collapse $\Rightarrow$ Sudden inflation (i.e., debt crisis)
Future Research

1. Theoretical implications
   - Asset price bubbles
   - Endogenous heterogeneity in preferences and beliefs
   - Further study on TVC and bubbles

2. Empirical and quantitative implications
   - Consistency with the data on
     - the money demand during deflationary period
     - the government debt
     - intergenerational altruism
Appendix:

Alternative interpretation of the intergenerational rationality

- Suppose that $\varepsilon_t - \bar{\varepsilon}_t$ gives the utility as a social status, where $\bar{\varepsilon}_t$ is the social level of $\varepsilon_t$. (Cole, Mailath, and Postlewaite 1992)
- We assume that the utility of the social status is $\lambda_t(\varepsilon_t - \bar{\varepsilon}_t)$.
- (SP: B) is equivalent to:

\[ (1 + i_t)\lambda_t b_t + \max_{c_{t+j}, \varepsilon_{t+j}} \sum_{j=0}^{\infty} \beta^j [U(c_{t+j}) + \lambda_{t+j}(\varepsilon_{t+j} - \bar{\varepsilon}_{t+j})] , \]

s. t. \[ \sum_{j=0}^{\infty} \beta^j \lambda_{t+j}(c_{t+j} + \varepsilon_{t+j}) \leq \sum_{j=0}^{\infty} \beta^j \lambda_{t+j}(y_{t+j} + \tau_{t+j}) . \]

- It is rewritten as

\[
(\text{SP: B}) \max_{c_{t+j}, X} \left\{ \sum_{j=0}^{\infty} \beta^j U(c_{t+j}) \right\} + \beta^{-t} (X - \bar{X}) , \]

s. t. \[
\left\{ \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} c_{t+j} \right\} + \beta^{-t} X \leq \left\{ \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} y_{t+j} \right\} + \beta^{-t} X^s + (1 + i_t) \lambda_t (b_t - b^s) ,
\]

with the equilibrium conditions: $c_t \leq y_t$ and $X \leq X^s$, where $\bar{X}$ is the social level of $X$. 
