

# Degree Inflation and Hierarchical Labor Demand\*

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## Abstract

This paper develops a multi-sector job search model that incorporates workers' education choice and firms' hierarchical labor demand to identify and quantify the underlying channel of 'degree-inflation' measured by the share of highly educated workers employed in unskilled positions. Highly educated workers being assumed to not always be qualified for skilled tasks gives rise to a vicious circle whereby (i) the information and communication technology (ICT) shock prompts the creation of vacancies for the highly educated rather than general workforce in both skill-intensive and (general) labor-intensive sectors, (ii) a significant proportion of skilled workers eventually perform unskilled tasks in the latter sectors, (iii) the resulting cross-skill matches crowd out the general workforce and suppress unskilled wages, and (iv) the college premium escalates and the college enrollment rate is self-reinforced. Numerical experiments based on Canadian data from early 1980s to 2000s suggest that without degree inflation, the unskilled, skilled, and overall wages in the Canadian labor market would be higher by (upto) 46, 43, and 56 percent, respectively, in early 2000s. For this extent, resolving the degree inflation problem, for example, through improving the quality of higher education, would induce substantial welfare gains to not only skilled workers but also unskilled workers.

Keywords: Degree Inflation, Cross-skill Match, Returns to Education

JEL Classification: I25, J31, J64, O41

## 1 Introduction

Both college attainment and college premium have increased steadily over the past few decades, especially in such developed economies as those of Canada, Japan, South Korea, and the United States. This is contrary to standard demand and supply analysis whereby returns to college education are diminished as the supply of college graduates increases. Skill biased technological change (hereafter, SBTC), that is, a shift in production technology that favors skilled over unskilled workers by increasing relative productivity and therefore labor demand, was until the late 1980s considered one of the most plausible explanations for the concurrent growth of college attainment and college premium.<sup>1</sup> Numerous studies provided empirical evidence for the shift-out of demand for skilled workers consequent to rapid advances in information and communications technology (ICT) and integration of the world economy during the Cold War.

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<sup>1</sup>In this paper, we use the terms skilled workers and college graduates interchangeably.

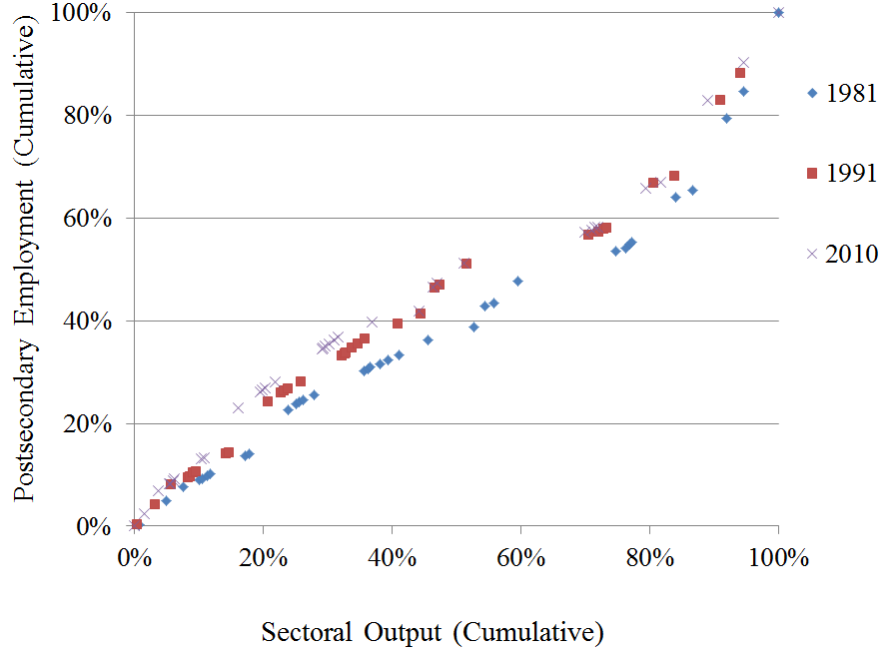


Figure 1: Sectoral Output and Skilled Worker Ratio

Note: The figure plots the ratio of college graduates within each sector and orders them, using LFS micro-data, left to right, from lowest to highest ratio.

Detailed examination of sectoral resource allocation, especially in Canada, presents a puzzle, however, because the significant joint input and output growth in ‘skill-intensive sectors’ posited by SBTC has been absent in Canada from the late 1980s through the early 2000s. A remarkable increase in the number of college graduates during this period, concentrated instead in ‘(general) labor-intensive sectors,’ has not necessarily been followed by concomitant growth of output in those sectors. Indeed, the ‘(general) labor-intensive sectors,’ which employed fewer college graduates in the 1980s, subsequently experienced a greater increase in the proportion of educated workers than the ‘skill-intensive sectors,’ even as the output share of the former sectors significantly declined.

The disproportionate increase in the number of skilled workers in the (general) labor-intensive, relative to the skill-intensive, sectors is depicted in Figure 1, which sorts, and orders left to right from lowest to highest ratio, 43 sectors. Fishing and hunting, agriculture, mining, accommodation and food services, and transport equipment (i.e., the (general) labor-intensive sectors) are situated to the left, finance, computers and electronics, science and technology, and education services (i.e., the skill-intensive sectors) to the right, by this sorting strategy. The cumulative sectoral output share is plotted on the horizontal, the cumulative highly educated employees share on the vertical, axis. College graduates, who previously were not employed in large numbers by the (general) labor-intensive sectors, were between 1981 and 2001 hired in greater numbers by those sectors than by the skill-intensive sectors. Although SBTC might have increased demand for skilled workers in the (general) labor-intensive sectors, it is unlikely, output in those sectors not having grown proportionately, that all college graduates employed in those sectors occupy skilled positions.

These observations lead us to consider that SBTC is not necessarily the dominant driver of the concurrent elevation of college premium and enrollment rate, at least since late 1980s. For these periods, we posit the additional, if not alternative, explanation of so called ‘degree inflation,’ that is, the increasing occupation by college graduates of unskilled positions previously

held by less educated workers.<sup>2</sup> We exploit the example of the Canadian experience to shed light on through what channels, and to what extent the enhanced supply of, college degree holders crowd out less educated labor and raise the college premium, thereby self-reinforcing college attainment and incurring a concomitant adverse effect on welfare.

This paper accounts for ‘degree inflation’ by developing a two-sector job search model that incorporates workers’ endogenous education decision and firms’ hierarchical labor demand. On the labor-supply side, whereas unskilled workers can perform only unskilled tasks (i.e., tasks without skill requirements), skilled workers can, commensurate with their qualifications, perform either skilled or unskilled tasks. On the labor-demand side, the (general) labor-intensive sectors’ (endogenously determined) lower, relative to the skill-intensive sectors, qualifications for skilled tasks results in a smaller share of matches between skilled tasks and workers. Given this hierarchical structure of skills, firms in the (general) labor-intensive sector respond to increases in the supply of the effective units of skills due to positive ICT shocks by creating more vacancies for college graduates than for less educated workers. Because not all skilled workers are qualified for all skilled tasks, the measure of college graduates in unskilled positions (hereafter, cross-skill match) in the (general) labor-intensive sector and, thus, the entire economy expands, crowding out less educated workers, suppressing unskilled wages, and increasing returns to college education. The college premium and college attainment rate are, in turn, self-reinforced, and the latter hindered from contributing fully to output growth by degree inflation, gauged by the measure of cross-skill matches. We calibrate the model using Canadian labor market data that reveals strong growth patterns in college attainment, college premia, and cross-skill matches.

The observation that many countries are oversupplied with college graduates is not novel. [Hecker \(1992\)](#), in research for the Bureau of Labor Statistics, found there to be more job seekers with college degrees than job openings requiring degree holders. Contrasting findings were reported by [Gottchalk and Hansen \(2003\)](#), who found the proportion of college-educated workers in the United States employed in unskilled occupations to have declined from the mid-1980s to the mid-1990s, and no evidence to support the notion that the increasing proportion of overqualified college graduates was being forced to accept noncollege jobs. Recently, [Wolff \(2006\)](#), through detailed examination of employment data, reported a significant disconnect between growth in the number of highly educated workers and jobs requiring higher level skills. How the incidence of highly educated workers occupying lower-skilled jobs is pushing unskilled workers down the occupation ladder, and ultimately out of the labor force altogether, has also been examined by [Beaudry, Green, and Sand \(2013\)](#). They argue that during the post-2000 period, the U.S. economy, consequent to an endogenously generated boom and bust cycle precipitated by pre-2000 SBTC, experienced a significant demand reversal for skilled workers. Building on these and other prior studies, we explore as a possible channel that reinforces the educated work force’s downward shift on the occupation ladder the phenomenon of degree inflation.

The paper is organized as follows. In Section 2 we discuss the relevant Canadian data and labor market patterns as well as the model we developed to motivate the empirical exercise. Our calibration protocol and results, and a comparative static analysis and its results, are explained in Section 3. We briefly discuss policy implications and concludes in Section 4.

## 2 The Model

### 2.1 Environment

We consider an economy with two sectors (denoted by  $i = 1, 2$ ) populated by a continuum of entrepreneurs and workers. Sector 1 is skill-, sector 2 less skill-intensive. Workers’ schooling

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<sup>2</sup>This phenomenon is not unique to Canada. According to [Quintini \(2011\)](#), although there is significant variation across countries and socio-demographic groups, as much as 35 percent of the workforce in OECD countries is considered to be overeducated with respect to their jobs.

decisions determine the skills they acquire before entering the labor market, which is subject to search and matching friction, as in [Mortensen and Pissarides \(1994\)](#). Entrepreneurs create jobs with and without skill requirements, as in [Albrecht and Vroman \(2002\)](#). There are in each sector three types of matches: unskilled matches ( $j = u$ ) of unskilled workers and jobs, skilled matches ( $j = s$ ) of skilled workers and jobs, and cross-skill matches ( $j = c$ ) of skilled workers and unskilled jobs. Throughout the paper, we focus on the steady state equilibrium in a continuous time framework with interest rate  $r$ .

**Final Goods** Final goods, denoted by  $Y$ , are assembled from two sectoral intermediate goods using the following production technology,

$$Y = (y_1^{\frac{\sigma-1}{\sigma}} + y_2^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}, \quad (1)$$

where  $(y_1, y_2)$  represent each sector's quantity of intermediate goods, and  $\sigma (> 1)$  is the constant elasticity of substitution. Final goods are consumed or used as inputs to the production process for sectoral intermediate goods through the perfectly competitive final goods market. Let  $p_i$  and  $P$  be the price of the sectoral intermediate goods  $i$  and final goods, respectively. For each  $i \in \{1, 2\}$ , demand for each of the intermediate goods required to produce one unit of final goods is characterized by

$$y_i = p_i^{-\sigma} P^{\sigma-1}, \quad \text{where } P = (p_1^{1-\sigma} + p_2^{1-\sigma})^{\frac{1}{1-\sigma}}. \quad (2)$$

Final goods are treated as *numéraire*, their price normalized to be one, *i.e.*  $P = 1$  in the steady state equilibrium of interest. There being a unit measure of homogenous entrepreneurs in each sector, aggregate supply of each sectoral product is given by  $y_i$ . Equating aggregate supply and demand and reordering yields the following market clearing condition. For each  $i \in \{1, 2\}$ ,

$$\frac{p_i y_i}{p_1 y_1 + p_2 y_2} = \frac{p_i^{1-\sigma}}{p_1^{1-\sigma} + p_2^{1-\sigma}}, \quad (3)$$

which implies that the price ratio reflects the revenue ratio of each sector. Together with the normalization assumption  $P = 1$ , equation (3) determines  $(p_1, p_2)$ .

**Intermediate Goods** Intermediate goods are produced by the representative entrepreneur in each sector. Production technologies are given by

$$y_i = \alpha_i [\beta_{is} k_i^{\gamma_i} h_{is}^{\kappa_i} + \beta_{iu} (h_{ic} + h_{iu})^{\kappa_i}]^{1/\kappa_i} \quad (4)$$

where  $(k_i, h_{is}, h_{ic}, h_{iu})$  represent, respectively, the capital stock and number of skilled, cross-skilled, and unskilled matches in sector  $i \in \{1, 2\}$ . Parameter  $\alpha_i$  captures the productivity of sector  $i$  and  $k_i^{\gamma_i}$  reflects the capital-skill complementarity, borrowed from [Krusell, Ohanian, Rios-Rull, and Violante \(2000\)](#), that skilled workers are more productive than unskilled workers with respect to capital. Parameters  $(\beta_{is}, \beta_{iu})$  represent the intensity of each factor, and  $\beta_{is} + \beta_{iu} = 1$ . The parameters that govern the elasticity of substitution in each sector satisfy  $0 < \gamma_i < \kappa_i < 1$ .

The representative entrepreneur in sector  $i \in \{1, 2\}$  purchases investment goods  $x_i$  at fixed price  $p_x$  through the international financial market. In the steady state equilibrium, capital depreciation is set equal to the investment:

$$\lambda k_i = x_i, \quad (5)$$

where  $\lambda_i$  is the depreciation rate.

Let  $(v_{is}, v_{iu})$  be the number of skilled and unskilled vacancies, respectively. Unskilled vacancies are filled by unskilled job searchers at rate  $q(\theta_u)$ , skilled vacancies contacted by skilled job searchers at rate  $q(\theta_s)$  and filled with qualification probability  $\chi_i$  per contact. Skilled candidates who prove unqualified for skilled tasks are assigned to unskilled positions (cross-skill matches).

The contact accruing rent due to search friction, the entrepreneur has no incentive to dissolve the contact. Matches are dissolved by exogenous separation shocks at rate  $\delta$  or retirement shocks at rate  $\rho$ . In steady state, newly hired workers take positions previously held by workers who have left the entrepreneur.

$$(\delta + \rho)h_{ij} = \begin{cases} \chi_i q(\theta_s)v_{is} & \text{if } j = s \\ (1 - \chi_i)q(\theta_s)v_{is} & \text{if } j = c \\ q(\theta_{ut})v_{iu} & \text{if } j = u \end{cases} \quad (6)$$

The left-hand side of equation (6) captures workers separated from their jobs, the right-hand side newly hired employees in steady state. The profit flow of each firm is given by

$$\pi_i = p_i y_i - p_x x_i - \sum_{j \in \{s, c, u\}} w_{ij} h_{ij} - \eta v_{is} - \eta v_{iu}, \quad (7)$$

where  $w_{ij} h_{ij}$  represents the wage payment to all  $j$ -type employees and  $(\eta v_{is}, \eta v_{iu})$  represent the cost of creating each type of vacancy. Using the profit flow, the entrepreneur consumes the final goods.

Denote by  $J_{ij}(k_i, h_{is}, h_{ic}, h_{iu})$  the marginal value of a  $j$ -type match to the entrepreneur in sector  $i$  having  $(k_i, h_{is}, h_{ic}, h_{iu})$ , which is given by

$$r J_{ij}(k_i, h_{is}, h_{ic}, h_{iu}) = \max_{x_i, v_{is}, v_{iu}} \frac{\partial \pi_i(k_i, h_{is}, h_{ic}, h_{iu})}{\partial h_{ij}} - (\delta + \rho) J_{ij}(k_i, h_{is}, h_{ic}, h_{iu}), \quad (8)$$

subject to (5) and (6). Lemma 1 says that the entrepreneur chooses  $(x_i, v_{is}, v_{iu})$  to equate the marginal cost and the present value of the marginal benefit flow.

**Lemma 1** *The entrepreneur in sector  $i \in \{1, 2\}$  makes investment and vacancy creation decisions such that*

$$p_x = \int_s^\infty e^{-(r+\lambda_i)(\tau-s)} \left( \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} \right) d\tau, \quad (9)$$

$$\eta = q(\theta_{ut}) \int_t^\infty e^{-(r+\delta+\rho)(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial h_{iu\tau}} d\tau = q(\theta_{ut}) J_{iut}, \quad \text{and} \quad (10)$$

$$\eta = q(\theta_{st}) \int_t^\infty e^{-(r+\delta+\rho)(\tau-t)} \left[ \chi_i \frac{\partial \pi_{i\tau}}{\partial h_{is\tau}} + (1 - \chi_i) \frac{\partial \pi_{i\tau}}{\partial h_{ic\tau}} \right] d\tau = q(\theta_{st}) J_{ist}, \quad (11)$$

where  $y_i$  is given by (4).

*Proof.* See Appendix A.

**Workers** Newly-born workers decide whether or not to acquire skills through advanced schooling. Those who opt for higher education begin their careers as skilled workers. Those who do not, enter the labor market as unskilled workers. The individual cost of higher education  $\varepsilon$  is randomly drawn from the logistic distribution *Logistic*( $\varepsilon, \zeta$ ). The probability of newly-born workers acquiring skills is given by

$$s = [1 + \exp[-(V_s - V_u - \varepsilon)/\zeta]]^{-1}, \quad (12)$$

where  $V_s$  is the lifetime value of a skilled unemployed worker and  $V_u$  the lifetime value of an unskilled unemployed worker. We borrow the choice probability function from [McFadden \(1974\)](#) and [Rust \(1987\)](#).

Unemployed workers collect  $b$  per instant through non-market activity until they retire (or die) or find a job. Let  $W_{iu}$  be the lifetime value of the unskilled worker employed in an unskilled

position in sector  $i$ . Denote by  $\ell_{ij} = v_{ij}/(v_{1j} + v_{2j})$  the proportion of the total  $j$ -type vacancies in sector  $i$ . The Hamilton-Jacobi-Bellman (hereafter, HJB) equation for the unskilled unemployed worker in the steady state is given by

$$rV_u = b - \rho V_u + f(\theta_u)[(\ell_{1u}W_{1u} + \ell_{2u}W_{2u}) - V_u], \quad (13)$$

where  $f(\theta_u)$  is the job finding rate of an unskilled worker (discussed in detail below). The left-hand side of equation (13) can be interpreted as the opportunity cost of holding the asset, unskilled unemployment, at every instant. The terms on the right-hand side represent the benefit flow of holding the asset  $V_u$ , which consists of the dividend flow, potential loss from retirement, and potential gains from job finding. The asset value equation for skilled unemployed workers  $V_s$  in the steady state is as follows:

$$rV_s = b - \rho V_s + f(\theta_s)\left[\sum_{i=1,2} \ell_{is}(\chi_i W_{is} + (1 - \chi_i)W_{ic}) - V_s\right], \quad (14)$$

where  $W_{is}$  and  $W_{ic}$  represent worker value in skilled and cross-skill matches, respectively. We will see later that skilled workers, because they receive a higher wage in a skilled than in an unskilled position, prefer the former. In our paper, an advanced degree plays a role as an imperfect signal, but is different from the signaling device in Spence (1973) in the sense that it increases the uncertainty of the degree holder's qualifications. For each  $j \in \{s, c, u\}$ ,  $w_{ij}$  represents the wage payment at the  $ij$ -type match. Employed workers are laid off consequent to separation shock at rate  $\delta$ . The HJB equation for employed workers is

$$rW_{ij} = w_{ij} - \rho W_{ij} + \delta(V_j - W_{ij}), \quad \text{for each } j \in \{s, c, u\}, \quad (15)$$

where it is assumed that  $V_j = V_s$  when  $j = c$ .

In the presence of labor market friction, the entrepreneur creates vacancies and waits for job searchers. Let  $u_s$  and  $u_u$  be the measures of skilled unemployed and unskilled unemployed workers, respectively. The vacancies and unemployed workers matched at any given time are randomly selected by the constant returns to the scale matching function of  $(v_{1j} + v_{2j}, u_j)$  in each submarket. Define labor market tightness as  $\theta_j := (v_{1j} + v_{2j})/u_j$  for each  $j \in \{s, u\}$  at every instant. Given the constant return to scale property of the matching technology, the job-finding and vacancy-filling rates are denoted entirely by functions of the market tightness  $\theta_j$  such that

$$f(\theta_j) = \theta_j q(\theta_j), \quad \text{for each } j \in \{u, s\}. \quad (16)$$

The economy is populated by a unit measure of workers. Let  $\{H_{is}, H_{ic}, H_{iu}\}_{i=1,2}$  be the total employed workers in skilled, cross-skilled, and unskilled positions, respectively. To abstract from the fertility decision, we assume the measure of newly-born workers to be that of retirees at every moment. In the steady state equilibrium, the mass of each of the matches is summarized in Lemma 2.

**Lemma 2** *Suppose that  $(\theta_s, \theta_u, \ell_{1s}, \ell_{2s})$  are given in steady state. The steady state measure of newly-born college graduates is given by*

$$s = \left[1 + \exp\left[-\frac{1}{\zeta}\left(\frac{\phi\eta(\theta_s - \theta_u)}{(1 - \phi)(r + \rho)} - \varepsilon\right)\right]\right]^{-1}. \quad (17)$$

For each  $i \in \{1, 2\}$ , the steady state measures of workers are obtained by

$$H_{is} = \frac{\chi_i \ell_{is} f(\theta_s)}{\rho + \delta} u_s, \quad H_{ic} = \frac{(1 - \chi_i) \ell_{is} f(\theta_s)}{\rho + \delta} u_s, \quad \text{and} \quad H_{iu} = \frac{\ell_{iu} f(\theta_u)}{\rho + \delta} u_u, \quad (18)$$

where  $(u_s, u_u)$  are given by

$$u_s = \frac{\rho(\rho + \delta)s}{(\rho + \delta)(\rho + f(\theta_s)) - \delta f(\theta_s)} \quad \text{and} \quad u_u = \frac{\rho(\rho + \delta)(1 - s)}{(\rho + \delta)(\rho + f(\theta_u)) - \delta f(\theta_u)}. \quad (19)$$

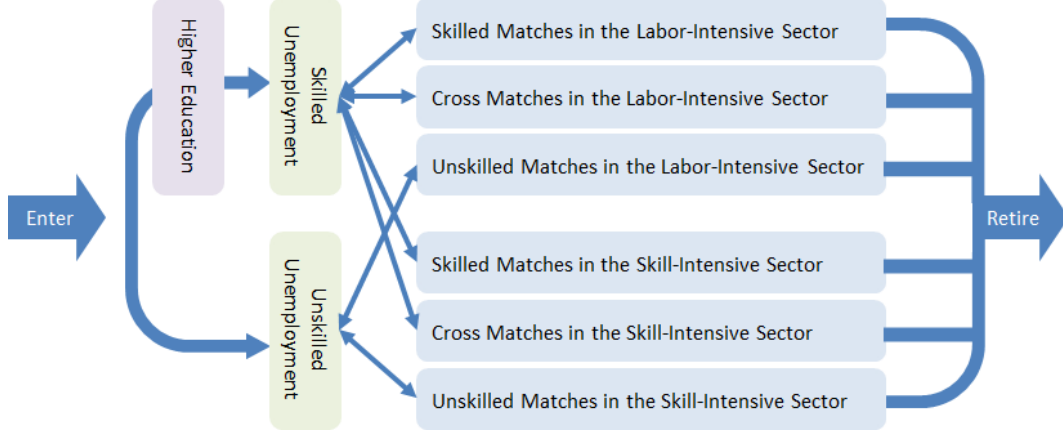


Figure 2: Worker Flow

**Wage Determination** Wages are determined by the internal bargaining mechanism proposed by [Stole and Zwiebel \(1996\)](#). Let  $\phi \in (0, 1)$  be the share of the marginal surplus (i.e., the bargaining power) that accrues to the worker at each match. The entrepreneur keeps  $(1 - \phi)$  of the marginal surplus, that is,

$$(1 - \phi)(W_{ij} - V_j) = \phi J_{ij}, \quad (20)$$

where  $V_c = V_s$ . There is an implicit restriction such that in any equilibrium  $W_{ij} - V_j \geq 0$  and  $J_{ij} \geq 0$  for each  $j \in \{s, c, u\}$ . Guess  $w_{ij} = A_i p_i (\partial y_i / \partial l_{ij}) + B_{ij}$  for each  $j \in \{s, c, u\}$ . Combining (8), (13), (14), (15) with (20) yields differential equations. Plugging the guess form into the differential equations and applying the undetermined coefficient methods results in the following wage formula.

**Lemma 3** *Suppose that  $(p_1, p_2, \theta_s, \theta_u)$  are given. Then,*

$$w_{ij} = \begin{cases} \phi p_i \frac{\partial y_i}{\partial h_{is}} + (1 - \phi)b + \eta \phi \theta_s & \text{if } j = s \\ \phi p_i \frac{\partial y_i}{\partial h_{ic}} + (1 - \phi)b + \eta \phi \theta_s & \text{if } j = c \\ \phi p_i \frac{\partial y_i}{\partial h_{iu}} + (1 - \phi)b + \eta \phi \theta_u & \text{if } j = u \end{cases} \quad (21)$$

The first terms on the right-hand side of equation (21) indicate that the wage payment is proportional to the marginal product of labor; the rest capture the labor market condition.

**Equilibrium** We conclude this section with a characterization of the equilibrium of interest. Our model is portrayed by the following summary definition.

**Definition** *A steady state equilibrium consists of choice rules  $\{x_i, v_{is}, v_{iu}\}_{i=1,2}$ , a labor market tightness parameter  $\{\theta_s, \theta_u\}$ , value equations  $\{W_{1s}, W_{1c}, W_{1u}, W_{2s}, W_{2c}, W_{2u}, V_s, V_u\}$ , and measures  $\{H_{1s}, H_{1c}, H_{1u}, H_{2s}, H_{2c}, H_{2u}, u_s, u_u\}$  such that:*

- (i) *Newly born workers optimally choose their schooling level.*
- (ii) *Each representative entrepreneur creates the optimal number of vacancies at every moment.*
- (iii) *Aggregate consistency requires that the vacancy creation decision be consistent with the definition of market tightness  $\{\theta_s, \theta_u\}$ .*
- (iv) *The wage setting rule in (21) determines the wage payment for each type of match, the market clearing condition in (3) the price of each of the intermediate goods.*



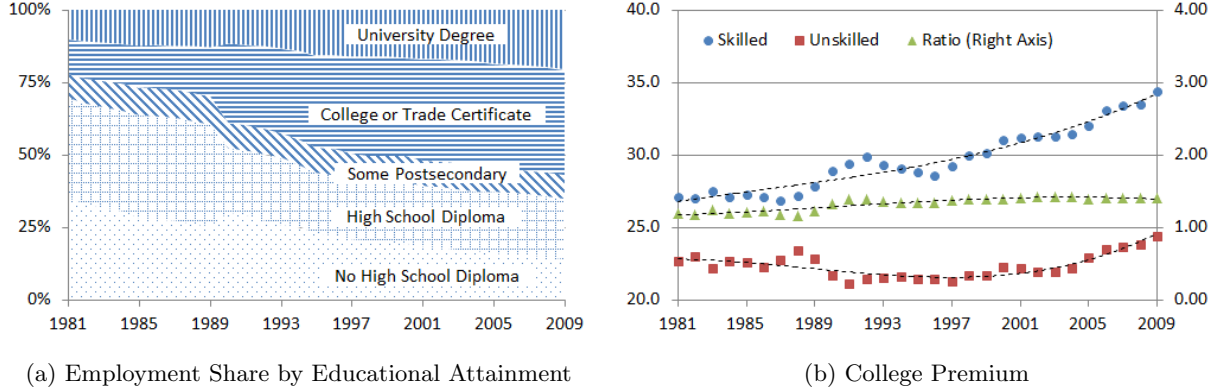


Figure 3: Employment Share by Educational Attainment and College Premium in Canada

Note: Panel (a) is based on LFS micro-data, panel (b) on the data constructed by Statistics Canada, which combines information from Census and LFS. Mean real hourly wages are constructed by dividing annual labor compensation by annual hours worked for each education category (both business and non-business sectors are included). There are two education groups: high school graduates or less; and post-secondary education or more. The real wage is calculated by deflating nominal wages using the all-items CPI (CANSIM Table 326-0021).

- (v) *The evolution of the system is recursively governed by the law of motions of (13)-(15) and (18)-(19).*

**Lemma 4** *Suppose that  $(p_1, p_2, \theta_s, \theta_u, \ell_{1s}, \ell_{2s})$  are given in steady state. Then, the capital stock in each sector is uniquely determined by*

$$k_i^{\frac{1-\gamma_i}{1-\kappa_i}} = \left[ \frac{(1-\phi)p_i\alpha_i\gamma_i\beta_{is}h_{is}^{\kappa_i}}{\kappa_i(r+\lambda_i)p_x} \right]^{\frac{\kappa_i}{1-\kappa_i}} [\beta_{is}k_i^{\gamma_i}h_{is}^{\kappa_i} + \beta_{iu}(h_{ic} + h_{iu})^{\kappa_i}], \quad (22)$$

where  $(h_{is}, h_{ic}, h_{iu})$  are given by Lemma 3.

Because  $\kappa_i > \gamma_i$ , the slopes are steeper on the left-hand than on the right-hand side of (22). The left-hand side being less than the right-hand side at  $k_{i=0}$ , and the former being larger than the latter at sufficiently large  $k_i$ , equation (22) has a unique solution. The following lemma shows the steady state to be pinned to the six-dimensional system of equations.

**Lemma 5** *There exists a steady state equilibrium if and only if  $(p_1, p_2, \theta_s, \theta_u, \ell_{1s}, \ell_{2s})$  solve for*

$$p_i^\sigma = (p_1y_1 + p_2y_2)y_i^{-1}, \quad (23)$$

$$\eta = \frac{q(\theta_s)}{r + \delta + \rho} \left[ \chi_i(1-\phi)p_i \frac{\partial y_i}{\partial h_{is}} + (1-\chi_i)(1-\phi)p_i \frac{\partial y_i}{\partial h_{ic}} - (1-\phi)b - \eta\phi\theta_u \right], \quad \text{and} \quad (24)$$

$$\eta = \frac{q(\theta_u)}{r + \delta + \rho} \left[ (1-\phi)p_i \frac{\partial y_i}{\partial h_{iu}} - (1-\phi)b - \eta\phi\theta_u \right], \quad \text{for each } i \in \{1, 2\}, \quad (25)$$

together with Lemmas 2 and 4.

## 3 Calibration

### 3.1 Canadian Labor Market

This section presents a quantitative assessment of degree inflation in Canada using data from 1981 to 2000. We calibrate our model using Canadian data on ICT investment, college enroll-



Table 1: Parameters Exogenously Given

Parameter	Value	Interpretation
$r$	0.05	Discount Rate
$\rho$	0.025	Retirement Rate
$\sigma$	3.8	Elasticity of Substitution in Preference
$\delta$	0.335	Separation Rate
$(\nu_s, \nu_u)$	(0.46, 0.46)	Elasticity Parameter of Matching Function
$\phi$	0.46	Bargaining Power of Workers
$\eta$	1.0	Vacancy Creation Cost
$\lambda$	0.320	Capital Depreciation Rate
$p_k$	1.00	Price of ICT Goods in 1981-1985
	0.23	Price of ICT Goods in 1996-2000

ment rate, college premium, sectoral value added, sectoral employment share, and so on. The Canadian economy is assumed to be in a pre-shock steady state during 1981-1985 and a post-shock steady state during 1996-2000, over the course of which the price of ICT goods dropped dramatically by 23 percent. We use our calibrated model of comparative statics to assess the impact of degree inflation on the labor market. A summary discussion of the Canadian economy is followed by a brief description of our basic parameterization and identification strategy in subsection 3.2. We present, in subsection 3.3, the outcome of the calibration that matches the Canadian labor market and the results of the comparative statics analysis, which reveals how degree inflation matters to labor market outcomes and educational attainment.

Panel (a) in Figure 3, which is depicted based on Labor Force Survey micro-level household data (LFS micro-data), shows attainment of higher (at least some post-secondary) education to have increased steadily, and the fraction of employees without a high school diploma to have dropped significantly, in Canada.<sup>3</sup> The approximately 65.4 percent of Canadian workers who had trade certificates, college diplomas, or university degrees in 2009 is more than double the 30.4 percent of Canadian workers who had any sort of post-secondary degree in 1981. Panel (b) in Figure 3 exploits the data set constructed by Statistics Canada, which combines the information from the LFS and Census, to determine the real hourly wages of highly educated (skilled) and less educated (unskilled) workers in Canada. The increased supply of highly educated workers observed in panel (a) notwithstanding, the college (skill) premium measured by ‘gap’ has been increasing, and the college premium measured by ‘ratio’ risen significantly, from the 1980s (20 percent, on average) to the early 1990s (36 percent, on average) due to a gradual decline in unskilled wages. Although the average unskilled wage in the denominator has increased since the late 1990s, the premium measured by the ratio has, owing to increases in the price of oil, remained stable in the Canadian labor market in contrast to the United States and other developed countries.

### 3.2 Calibration Strategy

Parameters independently estimated or borrowed from outside the model are determined in advance as follows. The base unit of time interval is normalized to be one year, which sets the discount rate to  $r = 0.05$ . The retirement rate, exogenously fixed at  $\rho = 0.025$ , implies that an individual worker is expected to remain in the labor market for 40 years. Due to the recoverability

<sup>3</sup>The accelerated growth of highly educated employees since the 1990s is partially due to a definitional change in the LFS’s education question. See “Labour and Household Surveys Analysis Division Staff Report: The Impact of the 1990 Changes to the Education Questions on the Labour Force Survey” by Statistics Canada.

Table 2: The Target Moments

Variable	Value in 1981-85	Value in 1996-2000	Interpretation
$p_{1t}y_{1t}/(p_{1t}y_{1t} + p_{2t}y_{2t})$	0.497 (0.528)	0.438 (0.474)	GDP Share of Labor-intensive Sector
$(\sum_{j=s,c,u} H_{1jt})/(\sum_{i=1,2} \sum_{j=s,c,u} H_{ijt})$	0.567 (0.542)	0.514 (0.491)	Employment Share of Labor-intensive Sector
$(\sum_{j=s,c} H_{1jt})/(\sum_{i=1,2} \sum_{j=s,c} H_{ijt})$	0.366 (0.370)	0.403 (0.391)	College Graduate Employment Share of Labor-intensive Sector
$(\sum_{i=1,2} \sum_{j=s,c} H_{ijt})/(\sum_{i=1,2} \sum_{j=s,c,u} H_{ijt})$	0.335 (0.325)	0.590 (0.604)	Proportion of College Graduate Employees in Total Employment
$u_{st}/(u_{st} + \sum_{i=1,2} \sum_{j=s,c} H_{ijt})$	NA (0.085)	0.064 (0.077)	Unemployment Rate of College Graduates
$u_{ut}/(u_{ut} + \sum_{i=1,2} H_{iut})$	NA (0.094)	0.111 (0.086)	Unemployment Rate of Non-college Graduates
$(p_{kt}x_{1t})/(p_{1t}y_{1t})$	0.006 (0.005)	0.016 (0.010)	ICT Investment-Value Added Ratio in Labor-Intensive Sector
$(p_{kt}x_{2t})/(p_{2t}y_{2t})$	0.035 (0.020)	0.046 (0.031)	ICT Investment-Value Added Ratio Skill-Intensive Sector
$\frac{(\sum_{i=1,2} \sum_{j=s,c} w_{ijt} H_{ijt})/(\sum_{i=1,2} \sum_{j=s,c} H_{ijt})}{(\sum_{i=1,2} w_{iut} H_{iut})/(\sum_{i=1,2} H_{iut})} - 1$	0.201 (0.268)	0.374 (0.315)	College Premium
$\frac{b}{(\sum_{i=1,2} \sum_{j=s,c,u} w_{ijt} H_{ijt})/(\sum_{i=1,2} \sum_{j=s,c,u} H_{ijt})}$	0.600 (0.644)	0.600 (0.556)	Replacement Ratio

Note: Values without parentheses are from the data, values within parentheses from the model. The data is from Statistics Canada.

issue, the discount and retirement rates are fixed rather than estimated, following practice in the literature. The elasticity of substitution in household preference is fixed at  $\sigma = 3.8$ , as estimated by [Bernard, Eaton, Jensen, and Kortum \(2003\)](#) using U.S. plant-level manufacturing data. The capital depreciation rate is fixed at  $\lambda = 0.320$ , which corresponds to the average depreciation rate of computer, electronic product, and software from 1981 to 2000. The price of ICT investment goods from 1981 to 1985 is fixed at one for normalization, and from 1996 to 2000 set to 0.23, which reflects the ratio of average prices between these periods.

With regard to labor market parameters, such as the bargaining power of workers, separation rate, and elasticity parameter of the matching technology, we follow [Zhang \(2008\)](#). The arrival rate of separation shock  $\delta$  is set at 0.335, which yields a total annual separation rate of 0.36 jointly with retirement. The elasticity parameter of the matching function and bargaining power parameter of workers are set at 0.46, following common practice in the literature. [Zhang \(2008\)](#) equalizes those parameters by invoking the rule in [Hosios \(1990\)](#). Absent any data on vacancy creation, and it not being identified separately from the efficiency of matching technology, we set the vacancy creation cost at  $\eta = 1.0$ . [Table 1] lists the parameters exogenously fed into the model and their values.

The remaining parameters are calibrated to minimize the sum of the squared distance between the target moments in the data and corresponding statistics in the model. All parameters being assumed to be constant across the aforementioned periods, the price of ICT investment goods alone causes the transition from the pre- to post-shock steady state. We set the target

Table 3: Parameters Endogenously Determined

Parameter	Value	Interpretation
$\alpha_2/\alpha_1$	0.709	TFP Ratio
$\beta_{1s}$	0.708	Skill Intensity in the Labor-Intensive Sector
$\beta_{2s}$	0.641	Skill Intensity in the Skill-Intensive Sector
$\kappa$	0.891	elasticity of substitution
$\chi_1$	0.263	Qualification Probability in the Labor-Intensive Sector
$\chi_2$	1.000	Qualification Probability in the Skill-Intensive Sector
$\varepsilon$	1.175	Average Cost of Education
$\zeta$	4.508	Sensitivity of Education Choice
$\mu_s$	9.542	Scale of Matching Technology for College Graduates
$\mu_u$	12.225	Scale of Matching Technology for Non-college Graduates
$\gamma$	0.075	Capital Contribution Parameter
$b$	0.217	Value of Unemployment

moments by assigning industries to the skill-intensive or (general) labor-intensive sectors on the basis of their employment share of college graduates. Industries for which the share of college graduate employees is higher (lower) than the national average of the share in 1990 are assigned to the skill-intensive ((general) labor-intensive) sector.<sup>4</sup> [Table 2] reports the model’s target moments and corresponding statistics. The GDP share of the (general) labor-intensive sector captures the TFP ratio  $\alpha_2/\alpha_1$  across sectors. The productivity parameter of the general labor-intensive sector,  $\alpha_1$ , is fixed at one for normalization, the counterpart of the skill-intensive sector endogenously estimated. Employment allocations, such as the proportion of college graduate employees in total employment, share of the (general) labor-intensive sector in college graduate employment, and share of the (general) labor-intensive sector in total employment, capture skill intensity ( $\beta_1, \beta_2$ ) and the parameter of technical substitution is set to be equal across sectors (i.e.,  $\kappa_1 = \kappa_2$ ). The time series behavior of employment allocations determines qualification probability ( $\chi_1, \chi_2$ ) by capturing complementarity and substitutability between workers with and without postsecondary education. The level and dynamic behavior of the college premium determine the education cost  $\varepsilon$  and sensitivity parameter of the education decision  $\zeta$ . The unemployment rates for the skilled and unskilled labor forces are exploited to fix the scale of matching technology ( $\mu_s, \mu_u$ ), the investment-value added ratio to determine the capital contribution parameter  $\gamma$  in the production technology. The replacement ratio, which is set at 0.6 following Zhang (2008), determines the value of the unemployment benefit  $b$ .

[Table 3] shows the parameter values finalized by the calibration exercise. The qualification probability, that is, the probability that a college graduate can effectively fill a position with skill requirements, is much smaller in the (general) labor-intensive sector, making that sector less intensive in the employment of college graduates even though the parameter values of ( $\beta_{1s}, \beta_{2s}$ ) show skill-intensity to be little less than for the skill-intensive sector (why the labor-intensive sector’s skilled-labor demand is sensitive to the ICT price drop is discussed in the comparative statics below). Although the values of ( $\mu_s, \mu_u$ ) suggest that college graduates use less efficient technology to search for jobs, their unemployment rate is smaller, as firms create more vacancies

<sup>4</sup>Industry definitions, with certain exceptions, are based primarily on two-digit NAICS codes; three-digit codes are used in manufacturing and some two-digit industries are treated as a single industry. Industries assigned to the skill-intensive sector include Utilities (NAICS Code 22), Petro/Coal Products (Code 324), Chemical Manufacturing (Code 325), Machinery Manufacturing (Code 333), Computer/Electronic Product Manufacturing (Code 334), Finance/Insurance/Management/Administration (Codes 52, 53, 55, and 56, treated as a single industry in the dataset), Professional/Scientific/Technical Services (Code 54), Educational Services (Code 61), Health Care/Social Assistance (Code 62), Information/Culture/Recreation (Codes 51 and 71, treated as a single industry), Other Services (Code 81), and Public Administration (Code 91). The other industries are assigned to the (general) labor-intensive sector.

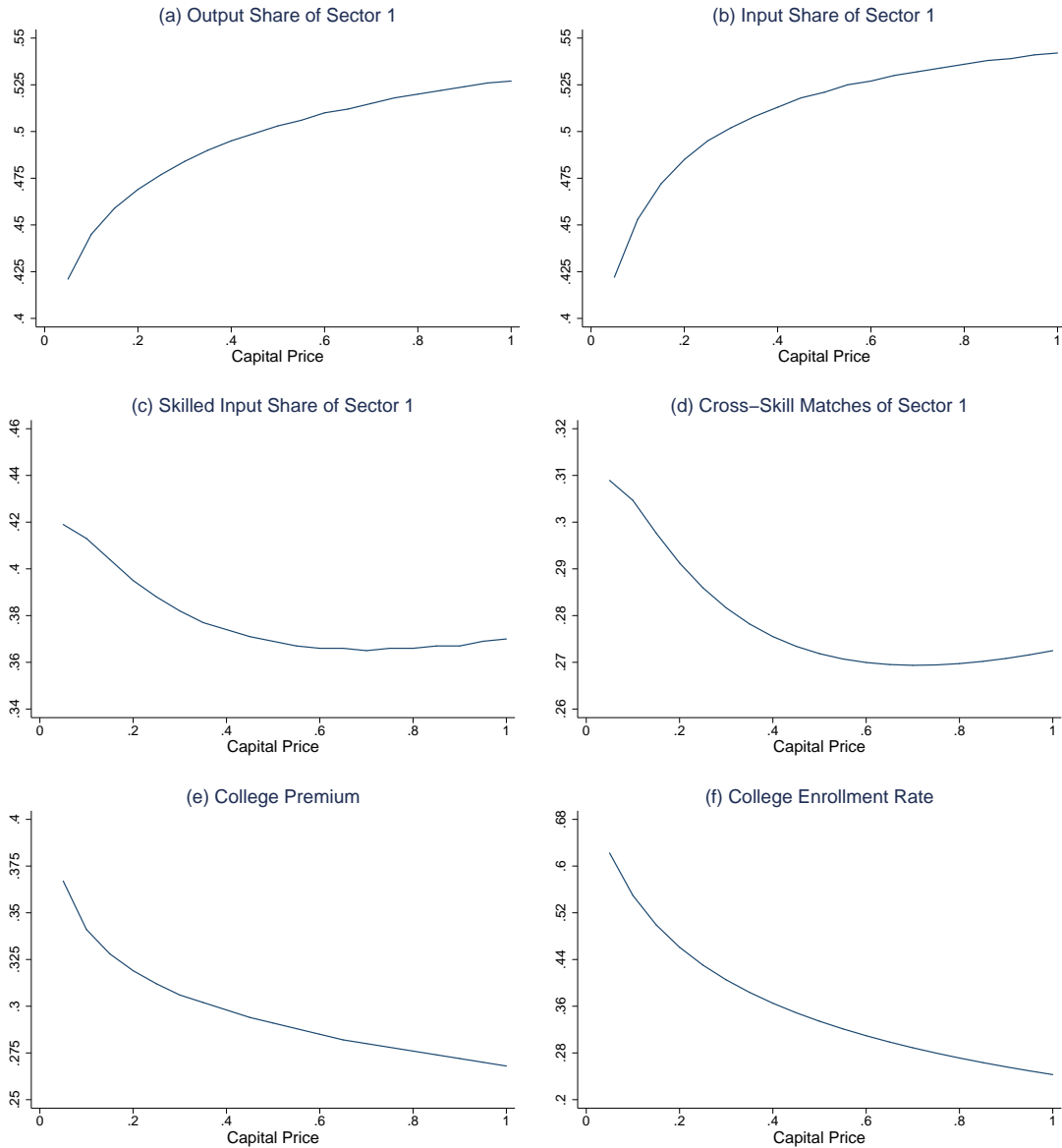


Figure 4: Steady State Equilibrium Outcomes across Price Levels of the ICT Goods

Note: These figures draw the loci of the steady state equilibrium outcomes. The horizontal axis in each panel represents the price level of the ICT technology goods.

for them. The average education cost of 1.175 is roughly 350 percent of the average annual income of non-college graduates, and not so much different from the education cost in reality if opportunity cost as well as tuition is taken into account.<sup>5</sup>

<sup>5</sup>Average annual income is around 0.3 in the calibrated model, while education cost is around 1.1. Our choice of education cost is equivalent to approximately 3.5 years of income of noncollege workers. Given the actual cost (mostly tuition) and opportunity cost ('no labor income') during schooling, 3.5 years of income is a plausible number.

Table 4: Steady State Equilibrium Outcomes across Qualification Rates

Qualification Probability	Weighted-Average MPL of			Average Wage of		
	Non-College	College	Overall	Non-College	College	Overall
0.263	0.340	0.452	0.407	0.327	0.430	0.389
0.289	0.340	0.459	0.421	0.328	0.437	0.402
0.316	0.342	0.466	0.435	0.329	0.444	0.415
0.342	0.343	0.474	0.449	0.330	0.451	0.428
1.000	0.497	0.648	0.638	0.478	0.619	0.609

Note: MPL stands for marginal product of labor, and is measured in value added.

### 3.3 Calibration Results

Our calibration exercises on the degree inflation problem fueled by the price drop of the ICT products are based on the comparative statics analysis with different price levels of ICT goods. [Figure 4] shows the key variables with different price levels from 0.05 to 1.00 on the horizontal axis. For example, the positive ICT shock in Canada after 1980s reduced the ICT capital price from 1.00 (normalized) to 0.23, corresponding to the movements from right to left in each panel. Sector 1 in [Figure 4] corresponds to the sum of all (general) labor-intensive sectors.

Panels (a), (b), (e), and (f) present the straightforward interpretation. In the (general) labor-intensive sectors, because the ICT products contribute less in production, it benefits less from the price drop of those ICT capital products, and consequent to the price drop shrinks in terms of both output and employment share of the sector as shown in panels (a) and (b) (from right to left). The endogenous increase in skill intensity occasioned by the price of ICT capital goods drop precipitates increases in both college premium and college enrollment rate as shown in panels (e) and (f). These patterns reconcile very well the actual experience in Canada (and other developed countries).

Panels (c) and (d) in the middle show the (general) labor-intensive sectors' share of total employment of college graduates and percentage of college graduates in cross-skill matches. The non-monotone behaviors of those key variables imply the conflicting effects of the ICT goods price level. When the impact of the positive ICT shock is moderate, the skill-intensive sectors respond to it more sensitively than the other sectors, which makes the skilled input share of (general) labor-intensive sectors decline. However, when the impact of the shock is sufficiently large, the vicious circle is triggered, in which labor-intensive sector strategically creates more vacancies for the skilled workers and a substantial portion of the skilled employees to perform unskilled tasks. It makes the skilled input share of (general) labor-intensive sectors paradoxically rises together with the positive ICT shock as shown in Panel (c). Consistently, Panel (d) reports a similar non-monotone behavior of the measure of cross-skill matches in the (general) labor-intensive sectors.

Although it can be hardly imagined any decline in actual output or welfare given significant and rapid positive ICT shock, it should not be passed over the opportunity loss for potential growth and welfare improvement. [Table 4] uncovers the potential loss due to degree inflation by investigating the change in productivity and wages along the qualification rate of the (general) labor-intensive sectors. In this model, the qualification probability governs the rate at which skilled workers in the (general) labor-intensive sector are matched to skilled tasks. Whereas the baseline calibration exercise fixes the qualification probability of those (general) labor-intensive sectors at 0.263, [Table 4] increases from 0.263 to 1.00 as a thought experiment for inducing policy implications. Simply speaking, if the government can improve the quality of higher education and, hence, the qualification probability of the skilled workers for skilled positions, the share of cross-skill matches declines. The results show wages for both skilled and unskilled workers improve monotonically as the cross-skill matches decline (i.e., market segmentation). These patterns correspond to the monotonically increasing patterns of the marginal product

of labor. In short, without degree inflation (qualification rate reaches one), productivity of unskilled (non-college), skilled (college), and overall Canadian workers would have been higher by 46, 43 and 56 percent during the period. The wage payments would have been larger by the almost same percent according to [Table 4].

## 4 Conclusion

This paper develops a job search model with endogenous education choice and hierarchical labor demand to analyze so-called ‘degree-inflation.’ The role of imperfect signal of individual workers’ qualifications is played by the academic degree, the acquisition of which can be matched to skilled tasks with expectation of a positive probability. Skilled workers who prove unqualified for the skilled tasks for which they are hired are assigned to unskilled tasks (cross-skill matches) previously performed by workers without degrees. As the ICT shock stimulates an increase in the numbers of degreed workers performing unskilled tasks, unskilled wages decline and workers without academic degrees are crowded out by degree holders. Notwithstanding the positive probability of being assigned to unskilled positions by virtue of the cross-skill match, academic degrees are increasingly sought in response to the ‘expected’ rise in the college premium (i.e., ex-ante, it still makes sense to obtain a degree). Without significant improvement in the overall qualifications of degree holders, enhanced college attainment will find more degree holders in unskilled jobs, precipitating a vicious circle.

We match labor market development and educational attainment trends in Canada over the past two decades, during which both college premium and attainment rate have steadily increased and average real wages for both groups continuously declined. We show the positive technology shock engendered by recent advances in ICT to have initiated and aggravated the vicious circle by forcing substantial numbers of degree holders into unskilled jobs in the (general) labor-intensive sectors (e.g., agriculture, fishing, mining, and so on), which previously did not hire degree holders in large numbers. Comparative statics results suggest that eliminating the degree inflation problem would have increased the wages (and marginal productivity) of unskilled, skilled, and overall workers by 46, 43, and 57 (46, 43, and 56) percent, respectively. Clearly, the positive ICT shock cannot be the only shock during the period and our approach simplifies the shortage of labor demand and slow growth, the numerical exercise show that the degree inflation causes a substantial wage and productivity loss of workers despite positive ICT shock.

Seemingly overqualified degree holding workers filling unskilled positions are a growing concern not only in Canada but also in other well-developed countries. In the OECD, approximately one in four workers, on average, are reported to be a mismatch in terms of academic degree and job description. To reverse the degree inflation trend, it will be necessary to achieve the separating equilibrium without cross-skill matches through appropriate policy interventions. One channel implied by our model, improvement of career counseling and internship programs, could be helpful in reducing cross-skill matches by improving the efficiency of matching technology among college students. Alternatively, improving the quality of higher education can reduce cross-skill matches (as firms create corresponding vacancies). Since designing better policies and evaluating those are more complex and out of the scope of our paper, we leave those topics for future research. Instead, the present paper identifies the mechanisms that underlie ‘degree-inflation’ with simple extensions of a standard search model, and provides a framework for further policy analysis aimed at resolving the resulting unfortunate vicious cycle.

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## A Mathematical Proofs

**Proof of Lemma 1** We proceed with the optimal control by the entrepreneur having in sector  $i$ . The entrepreneur having  $(\bar{k}_i, \bar{h}_{is}, \bar{h}_{ic}, \bar{h}_{iu})$  chooses the schedule of  $(x_{i\tau}, v_{is\tau}, v_{iu\tau})$  at every  $\tau \in [t, \infty)$  to maximize

$$\int_t^\infty e^{-r(\tau-t)} \pi_{i\tau} d\tau \tag{A1}$$

subject to (6) and the initial condition  $(k_{it}, h_{ist}, h_{ict}, h_{iut}) = (\bar{k}_i, \bar{h}_{is}, \bar{h}_{ic}, \bar{h}_{iu})$ . The Hamiltonian for the above problem is given by

$$\mathcal{H} = e^{-r(\tau-t)} \pi_{i\tau} - \mu_k [\lambda_i k_{it} - x_{it}] - \mu_s [(\delta + \rho) h_{is\tau} - \chi_i q(\theta_{s\tau}) v_{is\tau}]$$



$$-\mu_c[(\delta + \rho)h_{ic\tau} - (1 - \chi_i)q(\theta_{s\tau})v_{is\tau}] - \mu_u[(\delta + \rho)h_{iu\tau} - q(\theta_{u\tau})v_{iu\tau}]. \quad (\text{A2})$$

The maximum principle implies that

$$x_{i\tau} : e^{-r(\tau-t)} \left[ \frac{\partial \pi_{it}}{\partial x_{it}} \right] = \mu_k, \quad (\text{A3})$$

$$v_{is\tau} : e^{-r(\tau-t)} \eta = \mu_s \chi_i q(\theta_{s\tau}) + \mu_c (1 - \chi_i) q(\theta_{s\tau}), \quad (\text{A4})$$

$$v_{iu\tau} : e^{-r(\tau-t)} \eta = \mu_u q(\theta_{u\tau}), \quad (\text{A5})$$

$$k_{i\tau} : \dot{\mu}_k = -e^{-r(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} + \mu_k \lambda_i, \quad (\text{A6})$$

$$h_{is\tau} : \dot{\mu}_s = -e^{-r(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial h_{is\tau}} + \mu_s (\delta + \rho), \quad (\text{A7})$$

$$h_{iu\tau} : \dot{\mu}_u = -e^{-r(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial h_{iu\tau}} + \mu_u (\delta + \rho), \quad \text{and} \quad (\text{A8})$$

$$h_{ic\tau} : \dot{\mu}_c = -e^{-r(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial h_{ic\tau}} + \mu_c (\delta + \rho). \quad (\text{A9})$$

From (A6),

$$\begin{aligned} e^{-\lambda_i(s-t)} \dot{\mu}_k - \lambda_i e^{-\lambda_i(s-t)} \mu_k &= -e^{-(r+\lambda_i)(s-t)} \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} \\ \iff \mu_k &= e^{\lambda_i(s-t)} \int_s^\infty e^{-(r+\lambda_i)(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} d\tau + A_{ik} e^{\lambda_i(s-t)}. \end{aligned}$$

Since the shadow price  $\mu_k$  cannot diverge as  $s \rightarrow \infty$ ,  $A_{ik} = 0$ . Thus, we get

$$\mu_k = e^{-r(s-t)} \int_s^\infty e^{-(r+\lambda_i)(\tau-s)} \left( \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} \right) d\tau. \quad (\text{A10})$$

Plugging (A10) into (A3) yields

$$\frac{\partial \pi_{is}}{\partial x_{is}} = \int_s^\infty e^{-(r+\lambda_i)(\tau-s)} \left( \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} \right) d\tau = p_x. \quad (\text{A11})$$

By the same reasoning, solving differential equations (A7)-(A9) yields that

$$\mu_j = \int_\tau^\infty e^{-(r+\delta+\rho)(\hat{\tau}-\tau)} \frac{\partial \pi_{i\hat{\tau}}}{\partial l_{ij\hat{\tau}}} d\hat{\tau} + C_{ij} e^{(\delta+\rho)(\tau-t)}, \quad \text{for each } j \in \{s, u, c\}. \quad (\text{A12})$$

Since  $\mu_j$  cannot diverge, the integral constant  $C_{ij}$  should be zero as well. Plugging (A12) into (A4), and (A5) yields

$$\eta = q(\theta_{ut}) \int_t^\infty e^{-(r+\delta+\rho)(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial h_{iu\tau}} d\tau = q(\theta_{ut}) J_{iut}, \quad \text{and} \quad (\text{A13})$$

$$\eta = q(\theta_{st}) \int_t^\infty e^{-(r+\delta+\rho)(\tau-t)} \left[ \chi_i \frac{\partial \pi_{i\tau}}{\partial h_{is\tau}} + (1 - \chi_i) \frac{\partial \pi_{i\tau}}{\partial h_{ic\tau}} \right] d\tau = q(\theta_{st}) J_{ist}. \quad \square \quad (\text{A14})$$

**Proof of Lemma 2** From (13),

$$\begin{aligned} (r + \rho)V_u &= b + f(\theta_u)[(\ell_{1u}W_{1u} + \ell_{2u}W_{2u}) - V_u] = b + \frac{f(\theta_u)\phi}{1 - \phi}[(\ell_{1u}J_{1u} + \ell_{2u}J_{2u})] \\ &= b + \frac{\theta_u\phi}{1 - \phi}[(\ell_{1u}\eta + \ell_{2u}\eta)] = b + \frac{\theta_u\phi\eta}{1 - \phi}. \end{aligned} \quad (\text{A15})$$

The second equality follows from (20) and the third from (A13). By the same reasoning, we obtain that

$$(r + \rho)V_s = b + \frac{\phi\eta\theta_s}{1 - \phi}. \quad (\text{A16})$$

Subtracting (A15) from (A16) and plugging it into (12) yields

$$s = \left[ 1 + \exp \left[ -\frac{1}{\zeta} \left( \frac{\phi \eta (\theta_s - \theta_u)}{(1-\phi)(r+\rho)} - \varepsilon \right) \right] \right]^{-1}. \quad (\text{A17})$$

Since the outflow from and inflow into each type of unemployment are equalized on steady state, we obtain that

$$(\rho + f(\theta_u))u_u = \delta(H_{1u} + H_{2u}) + \rho(1-s) \quad \text{and} \quad (\text{A18})$$

$$(\rho + f(\theta_s))u_s = \delta(H_{1s} + H_{1c} + H_{2s} + H_{2c}) + \rho s. \quad (\text{A19})$$

By the same reasoning, we we get

$$H_{is} = \frac{\chi_i \ell_{is} f(\theta_s)}{\rho + \delta} u_s, \quad H_{ic} = \frac{(1-\chi_i) \ell_{is} f(\theta_s)}{\rho + \delta} u_s, \quad \text{and} \quad H_{iu} = \frac{\ell_{iu} f(\theta_u)}{\rho + \delta} u_u. \quad (\text{A20})$$

Plugging equations in (A20) into (A18) and (A19) yields

$$u_s = \frac{\rho(\rho + \delta)s}{(\rho + \delta)(\rho + f(\theta_s)) - \delta f(\theta_s)} \quad \text{and} \quad u_u = \frac{\rho(\rho + \delta)(1-s)}{(\rho + \delta)(\rho + f(\theta_u)) - \delta f(\theta_u)}. \quad \square \quad (\text{A21})$$

**Proof of Lemma 3** Combining (8), (13), (14), (15), and (20) yields

$$w_{ijt} + \phi \frac{\partial w_{ist}}{\partial h_{ijt}} h_{ist} + \phi \frac{\partial w_{ict}}{\partial h_{ijt}} h_{ict} + \phi \frac{\partial w_{iut}}{\partial h_{ijt}} h_{iut} = \phi p_{it} \frac{\partial y_{it}}{\partial h_{ijt}} + (1-\phi)b + \eta \phi \theta_{jt}, \quad (\text{A22})$$

where  $j \in \{s, c, u\}$  and  $\theta_{ct} = \theta_{st}$ . Since the differential equations in (A22) should be true for all  $t \in [0, \infty)$ , the ‘guess and verify’ method yields the wage formula of

$$w_{ijt} = \phi p_{it} \left[ \frac{\partial y_{it}}{\partial h_{ijt}} \right] + (1-\phi)b + \eta \phi \theta_{st}, \quad w_{ijt} = \phi p_{it} \left[ \frac{\partial y_{it}}{\partial h_{ijt}} \right] + (1-\phi)b + \eta \phi \theta_{st}, \quad \text{and} \\ w_{iut} = \phi p_{it} \left[ \frac{\partial y_{it}}{\partial h_{iut}} \right] + (1-\phi)b + \eta \phi \theta_{ut}. \quad (\text{A23})$$

It completes the proof.  $\square$

**Proof of Lemma 4** Plugging wage formulas (A23) into (A11) and applying the steady state condition yields

$$p_x = \frac{1}{r + \lambda_i} \frac{\partial \pi_i}{\partial k_i} = \frac{(1-\phi)p_i}{r + \lambda_i} \frac{\partial y_i}{\partial k_i} \\ = \frac{(1-\phi)p_i \alpha_i \gamma_i \beta_{is} k_i^{\gamma_i - 1} h_{is}^{\kappa_i}}{(r + \lambda_i) \kappa_i} [\beta_{is} k_i^{\gamma_i} h_{is}^{\kappa_i} + \beta_{iu} (h_{ic} + h_{iu})^{\kappa_i}]^{1/\kappa_i - 1}. \quad (\text{A24})$$

Reordering (A24) yields (22).  $\square$

**Proof of Lemma 5** Plugging wage formulas (A23) into (A13) and (A14) and applying the steady state condition yields that

$$\eta = \frac{q(\theta_u)}{r + \delta + \rho} \left[ (1-\phi)p_i \frac{\partial y_i}{\partial h_{iu}} - (1-\phi)b - \eta \phi \theta_u \right], \quad \text{and} \quad (\text{A25})$$

$$\eta = \frac{q(\theta_s)}{r + \delta + \rho} \left[ \chi_i (1-\phi)p_i \frac{\partial y_i}{\partial h_{is}} + (1-\chi_i)(1-\phi)p_i \frac{\partial y_i}{\partial h_{ic}} - (1-\phi)b - \eta \phi \theta_u \right], \quad (\text{A26})$$

for each  $i \in \{1, 2\}$ . We get six equations together with conditions (23) for six unknowns  $(p_1, p_2, \theta_s, \theta_u, \ell_{1s}, \ell_{1u})$ .  $\square$