Optimal Timing of College Subsidies

Enrollment, Graduation and the Skill Premium

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Motivation

- The skill premium has been rising in the US from 50% in 1980 to 90% now.
- The skill premium is determined by "the race between education and technology."
- Krueger and Ludwig (2016) suggest college subsidies to increase enrollment.
- But almost half of the college enrollees in the US drop out.
- Current subsidies might increase dropout and be inefficient.
- It is important to consider college subsidies targeting graduation.

This paper

- College subsidy scheme varying with year of college: year-dependent subsidies
- Year-dependent subsidies will have differential impacts on enrollment & graduation.
  - Individuals enroll based on high school GPA.
  - Some college enrollees learn their college GPA is low.
  - Drop out if they don't like studying or graduating with a low GPA.

$\rightarrow$ Back-loaded subsidies would decrease enrollment and increase graduation.

- Questions: What timing of subsidies will maximize the number of college graduates? What will maximize social welfare?

What I do

- A quantitative OLG model with endogenous enrollment/graduation decisions.
  - based on empirical findings (ex, Stinebrickner and Stinebrickner (2014)).
- Examine the effect of year-dependent subsidies on the skill premium and welfare.
  - Focus on the relative sizes across years in college (slope).
  - I fix the total budget of college subsidies from now on.

Summary of results

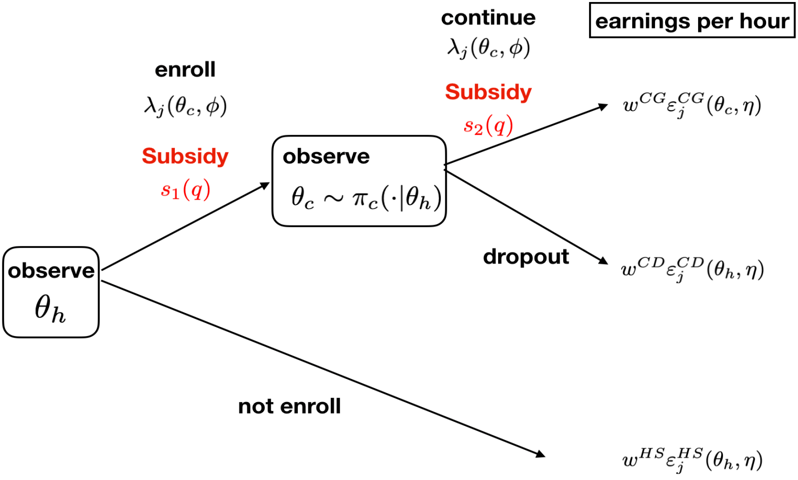
- Back-loaded subsidies maximize the number of college graduates and welfare.
- increases the number of college graduates and decreases the skill premium
  - as much as the case with increasing the total budget by 50%.
  - The skill premium decreases from 91% to 83%.

Outline

Building Blocks

1. Year-dependent subsidies.
2. Endogenous enrollment and graduation with learning ability.
3. GE with imperfect substitution between skilled and unskilled labor.
4. OLG with endogenous labor, transfer of assets to children, and retirement.
    - I focus on steady state from now on.
    - OLG: each individual has one offspring living with them until independence.
    - One period is two years.

Big Picture of Educational Decisions



Full Model: Preferences

$$\mathbb{E}_1 \left[\sum_{j=1}^{J} \tilde{\beta}_{j-1} u(c_j, \ell_j) - \sum_{j=1}^{2} \tilde{\beta}_{j-1} d_j(\boldsymbol{s}_j^c) \lambda_j(\theta_c, \phi) + \tilde{\beta}^{j_b - 1} \nu V_0 \right]$$

1. The expected discounted sum where

$$u(c, \ell) = \frac{(c^{\mu} \ell^{1-\mu})^{1-\gamma}}{1-\gamma}$$

   and c_j denotes consumption and ℓ_j is leisure at age j.

2. Expected psychic cost of college attendance:

$$\lambda_j(\theta_c, \phi) = \lambda + \lambda^{\theta} \theta_c + \lambda_j^{\phi} \phi$$

   Psychic cost depends on college ability θ_c and college taste ϕ.

3. Parental altruism: They enjoy their children's lifetime utility with a weight ν.

Education stage: Enrollment

$$V_0(a, \theta_h, \eta, q, \phi) = \max[\underbrace{V_1^c(a, \theta_h, \eta, q, \phi)}_{\text{enrolling}}, \underbrace{V_1(a, HS, \theta_h, \eta)}_{\text{not enrolling}}]$$

- Initial asset a endogenously transferred by parents
- High school ability θ_h correlated with parent's ability
- Idiosyncratic transitory productivity $\eta \sim \Pi^{HS}$
- family income q affecting college subsidies $s_j(q)$ (need-based)
- Taste $\phi \sim N(0, 1)$

High School and College Ability

- College ability θ_c is correlated with high school ability θ_h.

$$\theta_c = \theta_h + \epsilon_c \text{ and } \epsilon_c \sim N(0, \sigma_c^2)$$

- I assume enrollees are overoptimistic on college abilities.

$$\theta_c^p = \underbrace{\mu_c(\theta_h)}_{\text{bias}} + \underbrace{\theta_h + \epsilon_c}_{\text{actual ability}} \text{ and } \epsilon_c \sim N(0, \sigma_c^2), \quad \text{(Perceived law of motion)}$$

where

$$\mu_c(\theta_h) = \mu_{c0} + \mu_{c1}\theta_h$$

Education stage: First half of college

$$V_1^c(a,\theta_h,\eta,q,\phi) = \max_{c,h,a',y} u(c,1-h-\bar{h}) - \mathbb{E}_{\theta_c|\theta_h}\lambda_1(\theta_c,\phi)$$
$$+ \beta \mathbb{E}_{\theta_c^p|\theta_h}\mathbb{E}_{\eta'} \max[\underbrace{V_2^c(a',\theta_c^p,\eta',q,\phi)}_{\text{continue}}, \underbrace{V_2(\tilde{a}(a'),CD,\theta_h,\eta')}_{\text{dropout}}]$$

subject to

$$c + a' + p_e = a + y + s_1(q) - T(c,a,y)$$

$$y = w^{HS}\varepsilon_1^{HS}(\theta_h,\eta)h, \;\; a' \geq -\underline{A}_1^c \;\; c \geq 0, \;\; 0 \leq h \leq 1-\bar{h}$$

$$\theta_c^p = \theta_h + \mu_c(\theta_h) + \epsilon_c, \;\; \epsilon_c \sim N(0,\sigma_c^2), \;\; \eta' \sim \Pi^{CD}$$

- They can work as high school graduates.
- Going to college requires a fraction $\bar{h}$ of time.
- At the beginning of $j=2$, they observe θ_c and η' and make a dropout decision. ▸ $\tilde{a}$

After the first half of college

- The second half is similar except
  - subsidies $s_2(q)$
  - enrollees can work as college dropouts during the second half
  - graduate and draw $\eta' \sim \Pi^{CG}$ at the end of the period
- After education, individuals face a standard problem with borrowing limit $\underline{A}^e$.
  Working Stage
- At j_b, individuals transfer asset to offspring after observing their high school ability.
  Tranfer
- I assume retirees offer no labor, receive pension, and have no access to loans.
  Retirement Stage

Goods Sector

- A representative firm produces final good from capital K and aggregate labor H:

$$Y = F(K, H) = K^{\alpha} H^{1-\alpha}$$

- H is composed of two skills: skilled labor H^S and unskilled labor H^U:

$$H = (a(H^S)^{\rho} + (1-a)(H^U)^{\rho})^{\frac{1}{\rho}}$$

where ρ is calibrated to match the elasticity of substitution 1.41.

  - CG work as skilled labor: $w^{CG} = w^S$
  - HS and CD work as unskilled labor: $w^{HS} = w^{CD} = w^U$

Government

- The government collects tax $T(c, a, y)$ and spend the revenues on
  - college subsidies
    $$G_e = \sum_{j=1,2} \int_{S_j^c} s_j(q) d\mu_j^c$$
  - other government consumption
  - retirement benefits
- The tax function is assumed to be

$$T(c, a, y) = \tau_c c + \tau_k r a \mathbf{1}_{a \geq 0} + \tau_l y - d \frac{Y}{N}$$

Outline

Calibration Strategy

There are two sets of parameters:

- Those estimated outside of the model or fixed based on literature
- The remaining parameters to match moments given the first set of parameter values.

Labor Productivity

- I assume labor productivity

$$\ln \epsilon_j^e(\theta, \eta) = \ln \epsilon^e + \ln \psi_j^e + \epsilon_\theta^e \theta + \ln \eta$$

- Normalize $\epsilon^{HS} = \epsilon^{CG} = 1$ and calibrate ϵ^{CD} to match the college dropout premium.
- ψ_j^e is the age profile of workers at age j estimated from PSID. ▸ PSID
- For ϵ_θ^e, after filtering out ψ_j^e, I regress hourly wages on ln AFQT80 from NLSY79. ▸ Ability

	HS	CD	CG
log AFQT	.61	.74	1.31
	(.32)	(.32)	(.24)

Transitory Labor Productivity Process

- I assume $\pi^e_\eta(\eta'|\eta)$ is a two-state Markov chain approximating ▸ Markov Chain

$$\ln \eta' = \rho^e \ln \eta + \epsilon^e_\eta, \quad \epsilon^e_\eta \sim N(0, \sigma^{e2}_\eta)$$

- Minimum Distance Estimator separately for each education level.

	HS	CD	CG
ρ^e	0.94	0.95	0.95
σ^{e2}_η	0.017	0.021	0.025

Intergenerational Ability Transmission

- New independent individuals draw their high school abilities θ'_h.

$$\theta'_h \sim N(m + m_\theta \theta, \sigma_h^2)$$

- I regressed children's ability on parents' ability to get $m_\theta = 0.46$.

Subsidies and Loans

q	family income	subsidies to students	subsidies to colleges	total $\bar{s}(q)$
1	- \$30,000	\$2,820	\$10,477	\$13,297
2	\$30,000 - \$80,000	\$668	\$10,477	\$11,145
3	\$80,000 -	\$143	\$10,477	\$10,620

- The government subsidizes the education sector $\$10,477$ in the data.
- In the model, students receive all subsidies but pay the full cost of education.
- In the current system, college subsidies are constant and $s_1(q) = s_2(q) = \bar{s}(q)$.
- Students' interest rate is the prime rate plus $\iota^s = 2.3\%$, annual.
- The loan limit for the first half $\underline{A}_1^c$ is \$6,125 $(= \$2,625 + \$3,500)$ from Stafford loan.
- The loan limit for the second half $\underline{A}_2^c$ is \$23,000.

The Remaining Parameters

Parameter	Description	Value
μ_c^0	college ability bias intercept	0.190
μ_c^1	college ability bias slope	-0.409
λ	psychic cost intercept	23.2
λ^θ	psychic cost slope	-241
λ_1^ϕ	first period college taste	64.1
λ_2^ϕ	second half college taste	41.3
a^S	productivity of skilled labor	0.457
ϵ^{CD}	productivity of CD	1.02
σ_c	s.d. of college ability	0.340
κ	education cost	0.226
μ	consumption share of preference	0.418
β	time discount rate	0.938
v	altruism	0.0948
d	lump-sum transfer ratio	0.125
ι	borrowing wedge ($r^- = r + \iota$)	18.0%
m	intergenerational ability transmission intercept	-0.0471
σ_h	intergenerational ability transmission s.d.	0.171

▸ Parameters

Matched Moments

Moment	Model	Data
Expected/Actual graduation rate −1	0.431	0.433
Enrollment rate of ability quartile	(figure)	(figure)
Graduation rate of ability quartile	(figure)	(figure)
Enrollment rate of family income quartile	(figure)	(figure)
Graduation rate of family income quartile	(figure)	(figure)
Skill premium for CG	90.8%	90.2%
Skill premium for CD	19.6%	19.9%
Education cost/mean income at 48	0.320	0.33
Hours of work	33.8%	33.3%
K/Y	1.298	1.325
Transfer/mean income at 48	67.0%	66%
Log pre-tax/post-tax income	61.2%	61%
Borrowers	6.59%	6.8%
Mean of AFQT	-0.0135	0
Standard deviation of AFQT	0.217	0.213

Remarks on Some Calibrated Values

- College Ability Bias: μ_c^0 and μ_c^1
  - $\mu_c^0 > 0$ and 48% of the s.d. of college ability.
  - $\mu_c^1 < 0$: Low ability students are more optimistic, which is consistent with data.
- Psychic Cost: λ and λ^θ
  - $\lambda > 0$: college leads to psychic cost ($208,880 monetary value)
  - $\lambda^1 < 0$: psychic cost is smaller for high-ability agents
- Uncertainty of college ability: σ_c
  - S.D. of college ability is 90% greater than S.D. of high school ability

Model Fit

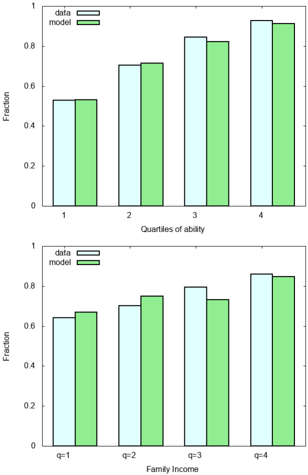


Figure: Enrollment rates

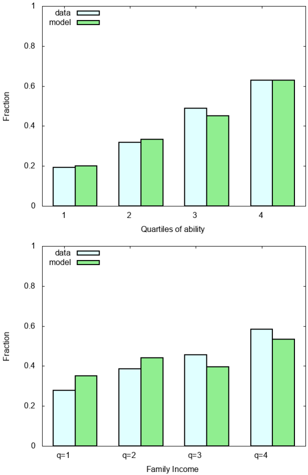


Figure: Graduation rates

▸ Model Fit

Validation 1: Partial Equilibrium Effect of Year-Invariant subsidies

- I simulate the change in enrollment to an \$1,000 increase in subsidies evenly.
  - The additional subsidies are given to only one generation.
  - All the prices and the distribution of initial state are fixed (PE).
- Increases by 1.05 percentage points in the simulation, which is broadly in the range.
  - The fraction of college graduates increases by 0.45 percentage points.
  - The fraction of college dropouts increases by 0.60 percentage points.

Validation 2: Sluggish increase in college graduates

- The number of college graduates increased sluggishly despite the skill premium.
- Derive the two steady states imitating 1980 and 2000 skill premiums.
- Compare the changes of the numbers of college graduates and dropouts with data.

	1980	2000	change (model)	change (data)
college graduate premium	46.2%	90.9%	44.7pp	43.2pp
college dropout premium	12.1%	19.6%	7.5pp	7.4pp
share of college graduates	28.0%	32.9%	4.9pp	4.98pp
share of college dropouts	42.8%	41.3%	-1.5pp	2.41pp

Outline

Main Exercises

- Exercise 1: Keep total spending fixed but choose subsidies by year (year-dependent subsidies) to maximize the number of college graduates in stationary equilibrium.

- Exercise 2: Keep total spending fixed and choose subsidies to maximize welfare in stationary equilibrium.

Exercise 1: Year Dependent Subsidies That Maximize College Graduates

$$\max_{g_1>0, g_2>0, \tau_\ell} \int_{S_2^{CG}} d\mu_2^{CG}$$

subject to

$$g_1 \int_{S_1^c} \bar{s}(q) d\mu_1^c + g_2 \int_{S_2^c} \bar{s}(q) d\mu_2^c = G_e$$

and the government budget constraint where $s_j(q) = g_j \bar{s}(q)$.

$s_j(q)$	year-invariant $\bar{G}_e$	year-dependent $\bar{G}_e$
$s_1(1)$	\$13,599	\$4
$s_1(2)$	\$11,447	\$4
$s_1(3)$	\$10,922	\$3
$s_2(1)$	\$13,599	\$42,436
$s_2(2)$	\$11,447	\$35,720
$s_2(3)$	\$10,922	\$34,082

- Back-loaded

The Effect of Year-dependent Subsidies

year-invariant/dependent	invariant $\bar{G}_e$	dependent $\bar{G}_e$	invariant $1.5\bar{G}_e$
enrollment rate	74.2%	68.7%	77.2%
share of college graduates	32.9%	34.5%	34.2%
skill premium	90.9%	82.6%	82.8%

- Share of college graduates increases more than increasing the total budget by 50%.
- Skill premium decreases more than increasing the total budget by 50%.
- College graduates increase and enrollment decreases (different directions).
- You don't need to increase tax.

Mechanism

- In the current system, increasing enrollment encourages more people who are more likely to drop out.
- The enrollment margin is not so important from the perspective of getting people to graduate.
- It is easier to create incentives for the marginal dropout to finish than to create incentives for the marginal non-enrollee to enroll and finish.
- Decreasing subsidies for the first period serves mainly to discourage people who are unlikely to graduate from enrolling.
- The higher subsidies for the second period encourages marginal dropouts to finish.
- In addition, we can shift subsidies away from college dropouts to college graduates.

Exercise 2: Year Dependent Subsidies That Maximize Welfare of Newborns

$$\max_{g_1>0, g_2>0, \tau_\ell} \sum_j N_j \left(\int V_j(\boldsymbol{s}_j) d\bar{\mu}_j(\boldsymbol{s}_j) + \int V_j^c(\boldsymbol{s}_j^c) d\bar{\mu}_j(\boldsymbol{s}_j^c) \right)$$

subject to

$$g_1 \int_{S_1^c} \bar{s}(q) d\mu_1^c + g_2 \int_{S_2^c} \bar{s}(q) d\mu_2^c = G_e$$

and the government budget constraint where $s_j(q) = g_j \bar{s}(q)$.

- The government recalculates the lifetime values with rational expectation.

	Current state	Optimal
$s_1(1)$	\$13,599	\$10,721
$s_1(2)$	\$11,447	\$9,025
$s_1(3)$	\$10,922	\$8,611
$s_2(1)$	\$13,599	\$19,858
$s_2(2)$	\$11,447	\$16,716
$s_2(3)$	\$10,922	\$15,949

- Optimal subsidies are back-loaded.

Aggregates

	Current state	Optimal
share of college enrollees	74.2%	73.8%
share of college graduates	32.9%	33.6%
skill premium	90.9%	87.3%
welfare gain		+0.15%

	Total	Level	Uncertainty	Inequality
Optimal	+0.07%	+0.15%	+0.04%	−0.09%

- Back-loaded subsidies improve welfare.
- The level effect is positive while inequality at the initial state increases.

Welfare

	$q = 1$	$q = 2$	$q = 3$
$\theta = 1$	$+0.6\%$	$+0.1\%$	$+0.5\%$
$\theta = 2$	$+0.2\%$	-0.4%	$+0.5\%$
$\theta = 3$	-0.8%	-0.3%	$+0.5\%$
$\theta = 4$	-0.9%	-0.0%	$+0.4\%$

- High-ability poor-family enrollees lose welfare.

No Optimism

Conclusion

- Back-loaded subsidies maximize the number of college graduates and social welfare.
- The number of college graduates increases and the skill premium decreases as much as the case with increasing the total budget by 50%.
- Enrollment decreases despite an increase in college graduates. Policies increasing enrollment might be misguided.

Outline

Correcting bias

- If we can correct bias, do we still need year-dependent subsidies?

- Find the new optimal year-dependent subsidies given that government eliminates bias costlessly.

	Current state	Optimal
$s_1(1)$	\$20,344	\$21,750
$s_1(2)$	\$17,124	\$18,308
$s_1(3)$	\$16,339	\$17,469
$s_2(1)$	\$20,344	\$17,808
$s_2(2)$	\$17,124	\$14,990
$s_2(3)$	\$16,339	\$14,302

- Front-loaded subsidies are optimal when correcting bias.

Correcting Bias

	Total	Level	Uncertainty	Inequality
Correct bias	$+1.69\%$	-2.77%	$+3.57\%$	-1.34%
Correct bias (Optimal)	$+2.05\%$	-2.31%	$+3.51\%$	-1.37%

	Current state	Correcting bias	Optimal
share of college enrollees	74.2%	45.5%	45.8%
share of college graduates	32.9%	26.2%	26.0%
skill premium	90.9%	124%	125%
welfare gain		-9.28%	-9.25%

Back

- Correcting bias reduces welfare significantly.
- Enrollment is excessively low due to no insurance on college ability.

Demography

- I focus on steady state from now on.
- OLG: each individual has one offspring living with them until independence.
- New independent individuals make an enrollment decision.
- One period is two years and completing college requires 2 periods.
- At the beginning of the 2nd period of college, enrollees make a dropout decision.
- Once an individual finishes their schooling, they will be high school graduates ($e = HS$), college dropouts (CD), or college graduates (CG).
- After that, they face a standard life cycle problem with income risk.

Education stage: Second half of college

$$V_2^c(a, \theta_c, \eta, q, \phi) = \max_{c,h,a',y} u(c, 1 - h - \bar{h}) - \lambda_2(\theta_c, \phi) + \beta \mathbb{E}_{\eta'} V_3(\tilde{a}(a'), CG, \theta_c, \eta)$$

subject to

$$c + a' + p_e - s_2(q) - y + T(c, a, y) = \begin{cases} (1+r)a & \text{if } a \geq 0 \\ (1+r^s)a & \text{if } a < 0 \end{cases}$$

$$y = w^{CD} \varepsilon_2^{CD}(\theta_c, \eta) h, \;\; a' \geq -\underline{A}_2^c \;\; c \geq 0, \;\; 0 \leq h \leq 1 - \bar{h}$$

- They can work as college dropouts.
- At the end of the period, one completes college and draws η' from Π^{CG}.
- Financial Market

Student Loan Transformation

- The fixed payment to repay full debt for 20 years (10 periods) d is given by

$$a' = \sum_{t=0}^{9} \frac{d}{(1+r^s)^t} = \frac{d}{1+r^s} \frac{1-(1+r^s)^{-10}}{1-(1+r^s)^{-1}} = d\frac{1-(1+r^s)^{-10}}{r^s}$$

- To have the same payment schedule d with interest r^-, the initial balance has to be

$$\tilde{a}(a') = \sum_{t=0}^{9} \frac{d}{(1+r^-)^t} = \frac{d}{1+r^-} \frac{1-(1+r^-)^{-10}}{1-(1+r^-)^{-1}} = d\frac{1-(1+r^-)^{-10}}{r^-}$$

- As a result,

$$\tilde{a}(a') = a' \times \frac{r^s}{1-(1+r^s)^{-10}} \times \frac{1-(1+r^-)^{-10}}{r^-}$$

- Back

Working Stage

$$V_j(a, e, \theta, \eta) = \max_{c,h,a',y} u\left(\frac{c}{1 + \mathbf{1}_{\mathcal{J}_f}\zeta}, 1 - h\right) + \beta \mathbb{E}_{\eta'|\eta} V_{j+1}(a', e, \theta, \eta')$$

subject to

$$c + a' - y + T(c, a, y) = \begin{cases} (1 + r)a & \text{if } a \geq 0 \\ (1 + r^-)a & \text{if } a < 0 \end{cases}$$

$$y = w^e \varepsilon_j^e(\theta, \eta)h, \;\; a' \geq -\underline{A}^e \;\; c \geq 0, \;\; 0 \leq h \leq 1, \;\; \eta' \sim \pi^e(\cdot|\eta)$$

where $\mathbf{1}_{\mathcal{J}_f}$ is an indicator function which is one when the individual lives with its children ($j \in [j_f, j_b - 1]$). Back

Transfer

$$V_j(a,e,\theta,\eta) = \max_{c(\theta_h'),h(\theta_h'),a'(\theta_h'),y(\theta_h')} \mathbb{E}_{\theta_h'|e,\theta}\{u(c(\theta_h'),1-h(\theta_h')) + \tilde{V}_{j_b+1}(a',\theta,\theta_h',e,\eta)\}$$

subject to

$$c(\theta_h') + a'(\theta_h') - y(\theta_h') + T(c(\theta_h'),a(\theta_h'),y(\theta_h')) = \begin{cases}(1+r)a & \text{if } a \geq 0\\ (1+r^-)a & \text{if } a < 0\end{cases}$$

$$y(\theta_h') = w^e\varepsilon_j^e(\theta,\eta)h(\theta_h'),\ \ a' \geq -\underline{A}^e\ \ c(\theta_h') \geq 0,\ \ 0 \leq h(\theta_h') \leq 1,\ \ \eta' \sim \pi^e(\cdot|\eta)$$

where

$$\tilde{V}_{j_b+1}(a,\theta,\theta_h',e,\eta) = \max_{b\in[0,a]} \beta\mathbb{E}_{\eta'|\eta}V_{j_b+1}(a-b,e,\theta,\eta') + \nu\mathbb{E}_{\eta'',\phi}V_0(b,\theta_h',\eta'',\tilde{q}(w^e\varepsilon_j^e(\theta,\eta)),\phi)$$

for all θ_h'.

- Individuals can make parental transfers b to their children only at this age.
- Before making any decisions, individuals observe only their children's high school ability θ_h' from $\pi_\theta(\theta_h'|e,\theta)$.
- Back

Family income level

- Family income level

$$\tilde{q}(w^e \varepsilon_j^e(\theta,\eta)) = \begin{cases} 1 & \text{if } w^e \varepsilon_j^e(\theta,\eta) \times 0.35 \in [0, q_1] \\ 2 & \text{if } w^e \varepsilon_j^e(\theta,\eta) \times 0.35 \in [q_1, q_2] \\ 3 & \text{else} \end{cases}$$

  where q_1 and q_2 correspond to \$30,000 and \$80,000.

- Back

Retirement Stage

$$V_j(a,e,\theta) = \max_{c,a'} u(c,1) + \beta\varphi_{j+1} V_{j+1}(a',e,\theta)$$

subject to

$$c + a' = (1+r)\varphi_j^{-1} a + p(e,\theta) - T(c, \varphi_j^{-1} a, 0)$$

$$a' \geq 0 \;\; c \geq 0$$

- The sources of income is asset earnings and retirement benefits $p(e,\theta)$.
- The asset inflated by φ_j^{-1} reflects that assets of expiring households are distributed within cohorts (perfect annuity market).
- Back Social Security

Social Security

- The average life time income is

$$\hat{y}(e,\theta) = \frac{\sum_{j=j_a+2}^{j_r-1} w^e \varepsilon_j^e(\theta,1)\bar{h}}{j_r - 2}$$

- The pension formula is given by

$$p(e,\theta) = \begin{cases} s_1\hat{y}(e,\theta) & \text{for } \hat{y}(e,\theta) \in [0, b_1) \\ s_1 b_1 + s_2(\hat{y}(e,\theta) - b_1) & \text{for } \hat{y}(e,\theta) \in [b_1, b_2) \\ s_1 b_1 + s_2(b_2 - b_1) + s_3(\hat{y}(e,\theta) - b_2) & \text{for } \hat{y}(e,\theta) \in [b_2, b_3) \\ s_1 b_1 + s_2(b_2 - b_1) + s_3(b_3 - b_2) & \text{for } \hat{y}(e,\theta) \in [b_3, \infty) \end{cases}$$

  where $s_1 = 0.9$, $s_2 = 0.32$, $s_3 = 0.15$, $b_1 = 0.22\bar{y}$, $b_2 = 1.33\bar{y}$, $b_3 = 1.99\bar{y}$, $\bar{y} = \$28,793$ annually.

- Back

Financial Market

- There is no insurance market and individuals can self-insure using only risk-free assets.
- Borrowing wedge:
  - Overseeing cost ι for workers: $r^{-} = r + \iota$
  - Overseeing cost $\iota + \iota^{s}$ for enrollees: $r^{s} = r^{-} + \iota^{s}$
- Borrowing limit:
  - $\underline{A}^{e}$ for workers with education e
  - $\underline{A}_{j}^{c}$ for enrollees at age j
- Back

College

- A representative college requires κ units of skilled labor to provide education.

$$p_e E - w^S \kappa E$$

where E is the measure of college enrollees and p_e is tuition.

- I assume colleges are competitive and there is free entry: $p_e = w^S \kappa$

Government Budget

- Government Budget Constraint

$$G_c + G_e + \sum_{j=j_r}^{J} \int_{S_j} p(e,\theta) d\mu_j = \sum_{j=1,2} \int_{S_j^c} T(c_j^c(\mathbf{s}_j^c), a_j^c(\mathbf{s}_j^c), y_j^c(\mathbf{s}_j^c)) d\mu_j^c$$
$$+ \sum_j \int_{S_j} T(c_j(\mathbf{s}_j), a_j(\mathbf{s}_j^s), y_j(\mathbf{s}_j^s)) d\mu_j^s$$

  where

$$G_c = gF(K, H)$$
$$G_e = \sum_{j=1,2} \int_{S_j^c} s_j(q,\theta) d\mu_j^c$$

- ▸ Back

Market clearing

- Aggregate labor

$$H^S + \kappa E = H^{CG}$$
$$H^U = H^{HS} + H^{CD}$$

where

$$H^{CG} = \sum_{j=3}^{j_r-1} \int_{S_j^{CG}} \epsilon_j^{CG}(\theta,\eta) h_j(\mathbf{s}_j) d\mu_j^{CG}$$

$$H^{CD} = \sum_{j=2}^{j_r-1} \int_{S_j^{CD}} \epsilon_j^{CD}(\theta,\eta) h_j(\mathbf{s}_j) d\mu_j^{CD} + \int_{S_2^c} \epsilon_2^{CD}(\theta,\eta) h_2^c(\mathbf{s}_2^c) d\mu_2^c$$

$$H^{HS} = \sum_{j=1}^{j_r-1} \int_{S_j^{HS}} \epsilon_j^{HS}(\theta,\eta) h_j(\mathbf{s}_j) d\mu_j^{HS} + \int_{S_1^c} \epsilon_1^{HS}(\theta,\eta) h_1^c(\mathbf{s}_1^c) d\mu_1^c$$

- Capital

$$K = \sum_{j=1}^{j_r-1} \int_{S_j} a_j'(\mathbf{s}_j) d\mu_j + \sum_{j=1,2} \int_{S_j^c} a_j'^c(\mathbf{s}_j^c) d\mu_j^c$$

- Education

$$E = \sum_{j=1,2} \int_{S_j^c} d\mu_j^c$$

- Back

Equilibrium

Definition

A stationary equilibrium is a list of value functions of workers and college enrollees $\{V_j(\mathbf{s}_j), V_j^c(\mathbf{s}_j^c)\}$, *decision rules of enrollment* $d_0(\mathbf{s}_0)$ *and graduation* $d_1(\mathbf{s}_1^c)$, *decision rules of consumption, asset holdings, labor, output, parental transfers of workers* $\{c_j(\mathbf{s}_j), a_j'(\mathbf{s}_j), h_j(\mathbf{s}_j), y_j(\mathbf{s}_j), b(\mathbf{s}_j)\}$, *decision rules of college enrollees* $\{c_j^c(\mathbf{s}_j^c), a_j'^c(\mathbf{s}_j^c), h_j^c(\mathbf{s}_j^c), y_j^c(\mathbf{s}_j^c)\}$, *aggregate enrollees, capital, and labor inputs* $\{E, K, H^S, H^U\}$, *prices* $\{r, w^S, w^U, p_e\}$, *policies* τ_ℓ, *measures* $\boldsymbol{\mu} = \{\mu_j^c(\mathbf{s}_j^c), \mu_j(\mathbf{s}_j), \mu_j^e(\mathbf{s}_j^e)\}$ *such that*

1. *Taking prices and policies as given, value functions* $\{V_j^c(\mathbf{s}_j^c), V_j(\mathbf{s}_j)\}$ *solve the household Bellman equation*s and* $d_0(\mathbf{s}_0)$, $d_1(\mathbf{s}_1^c)$, $\{c_j(\mathbf{s}_j), a_j'(\mathbf{s}_j), h_j(\mathbf{s}_j), y_j(\mathbf{s}_j), b(\mathbf{s}_j)\}$, $\{c_j^c(\mathbf{s}_j^c), a_j'^c(\mathbf{s}_j^c), h_j^c(\mathbf{s}_j^c), y_j^c(\mathbf{s}_j^c)\}$ *are associated decision rules.*
2. *Taking prices and policies as given,* K, H^{HS}, H^{CG} *solve the optimization problem of the good sector and* E *solves the optimization problem of the education sector.*
3. *The government budget is balanced.*
4. *Human capital, asset, and education markets clear.*
5. *Measures* $\boldsymbol{\mu}$ *are reproduced for each period.*

- ▸ Market Clearing

Labor Productivity Process Estimation

- PSID: SRC sample, only people with 8 or more individual-year observations
- keep only positive hours of labor aged 25-63
- eliminate extreme changes in earnings
- quadratic ages are separately estimated by education group with year dummies
- Back

	HS	CD	CG
Age	.0530181	.0684129	.0955783
	(.0030501)	(.0040353)	(.0036997)
Age^2	-.0005314	-.0006872	-.0009521
	(.0000356)	(.0000474)	(.0000429)

Labor Productivity

- For high school graduates, $\theta = \theta_h$ which is approximated by ln AFQT80.

- For college dropouts and college graduates, I use high school ability $(\theta_c = \theta_h + \epsilon_c)$.

$$\ln \epsilon^e + \ln \psi_j^e + \epsilon_\theta^e \theta_c + \ln \eta = \ln \epsilon^e + \ln \psi_j^e + \epsilon_\theta^e \theta_h + (\ln \eta + \epsilon_\theta^e \epsilon_c)$$

because θ_h is uncorrelated with $\ln \eta + \epsilon_\theta^e \epsilon_c$.

Markov Chain Approximation

- Two state Markov chain with education-specific states for $\{-\sigma_e, \sigma_e\}$ and transition matrix

$$\Pi = \begin{bmatrix} \pi_e & 1-\pi_e \\ 1-\pi_e & \pi_e \end{bmatrix}$$

where

$$\rho^{e2} = 2\pi_e - 1$$

$$\sigma_e = \frac{\sigma_\eta^e}{\sqrt{1-\rho^{e2}}}$$

- Back

Minimum Distance Estimator

- The residual process is assumed to be

$$y_{ia} = \alpha_i + z_{ia} + u_{ia}$$

where

$$z_{ia} = \rho z_{ia-1} + \epsilon_{\eta ia}, \quad \epsilon_{\eta ia} \sim N(0, \sigma_\eta^2)$$

- Then

$$cov(y_{ia}, y_{ia-d}) = \sigma_\alpha^2 + \rho^d \frac{1-\rho^{2a}}{1-\rho^2} \sigma_\eta^2 + \mathbf{1}_{d=0} \sigma_u^2$$

- Back

Parameters Determined outside the Model

Parameters	Interpretation	Value
γ	Coef of relative risk aversion	4
$\bar{h}$	Study time	0.25
ζ	Adult equivalence scale	0.3
α	Capital share	33.3%
δ	Depreciation (annual)	7.55%
ρ	Elasticity of substitution in production 1.41	0.2908
ι^s	Stafford interest premium (annual)	2.3%
$\underline{A}_1^c$	Borrowing constraint for 1st half (Stafford loan)	\$6,125
$\underline{A}_2^c$	Borrowing constraint for 2nd half (Stafford loan)	\$23,000
$\underline{A}^{HS}$	Borrowing constraint, HS (SCF)	\$17,000
$\underline{A}^{CD}$	Borrowing constraint, CD (SCF)	\$20,000
$\underline{A}^{CG}$	Borrowing constraint, CG (SCF)	\$34,000
τ_c	Consumption tax rate	7%
τ_k	Capital income tax rate	27%
g	Gov cons to GDP ratio	17.1%

Back

Monetary Value of Psychic Cost

$$\sum_{j=1}^{J} \tilde{\beta}^{j-1} u(\bar{C} - c_\lambda, \bar{L}) = \sum_{j=1}^{J} \tilde{\beta}^{j-1} u(\bar{C}, \bar{L}) + \lambda(0,0) + \beta\lambda(0,0) \tag{1}$$

- Derive c_λ where $\bar{C}$, $\bar{L}$, and $\lambda(0,0)$ are the average consumption, leisure, and psychic cost.
- Take the present value of c_λ.

Exercise 0: Year Invariant Subsidies

G_e	0.75 $\bar{G}_e$	$\bar{G}_e$	$1.5\bar{G}_e$	$2\bar{G}_e$
enrollment rate	72.7%	74.2%	77.2%	77.8%
share of college graduates	32.1%	32.9%	34.2%	35.0%
skill premium	95.0%	90.9%	82.8%	78.3%

- As the budget changes, enrollment and college graduates move in the same direction.
- You have to increase tax to increase the share of college graduates.

Welfare

	Current state	Optimal
Y	0.318	0.318
K	0.413	0.413
C	0.211	0.211
w^S	0.355	0.352
w^U	0.405	0.408
std c	0.129	0.129
std a	0.478	0.475
std h	0.0834	0.0833
std wage	0.544	0.540

	$q = 1$	$q = 2$	$q = 3$
$\theta = 1$	$+0.6\%$	$+0.1\%$	$+0.5\%$
$\theta = 2$	$+0.2\%$	-0.4%	$+0.5\%$
$\theta = 3$	-0.8%	-0.3%	$+0.5\%$
$\theta = 4$	-0.9%	-0.0%	$+0.4\%$

- High-ability poor-family enrollees lose welfare.

▸ No Optimism

Responding to the consumption loss at the first period

	% of subsidy loss
Subsidies	-100%
Labor income	+24%
(Price of an hour of working)	+13%
(Leisure)	(-0.061)
Transfer from parents	+0.03%
Reducing savings	+65%
Less tuition	+4%
Consumption	-7%

- Consumption at the first period does not decrease much because:
  - The wage of college enrollees increases due to a smaller skill premium.
  - They work for longer hours.
  - Parents increase transfer.

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No Optimism

- In this paper, optimism is a key factor for college dropouts.
- A different approach to explain college dropouts: High option value due to high uncertainty of college ability.
- I assume that the standard deviations of college ability can vary across high school ability.

$$\sigma_c(\theta_h) = \sigma_c \exp(\sigma_c^\theta \theta_h)$$

No Optimism: The Remaining Parameters

Parameter	Description	Value
λ	psychic cost intercept	-16.6
λ^{θ}	psychic cost slope	287
λ_1^{ϕ}	first period college taste	-68.8
λ_2^{ϕ}	second half college taste	-40.0
a^S	productivity of skilled labor	0.435
ϵ^{CD}	productivity of CD	0.985
σ_c	s.d. of college ability intercept	0.721
σ_c^{θ}	s.d. of college ability slope	0.158
κ	education cost	0.422
μ	consumption share of preference	0.422
β	time discount rate	0.931
v	altruism	0.0630
d	lump-sum transfer ratio	0.131
ι	borrowing wedge ($r^- = r + \iota$)	18.7%
m	intergenerational ability transmission intercept	-0.0384
σ_h	intergenerational ability transmission s.d.	0.0764

No Optimism: Matched Moments

Moment	Model	Data
Enrollment rate of ability quartile	(figure)	(figure)
Graduation rate of ability quartile	(figure)	(figure)
Enrollment rate of family income quartile	(figure)	(figure)
Graduation rate of family income quartile	(figure)	(figure)
Skill premium for CG	90.7%	90.2%
Skill premium for CD	20.1%	19.9%
Education cost/mean income at 48	0.308	0.33
Hours of work	33.3%	33.3%
K/Y	1.241	1.325
Transfer/mean income at 48	67.2%	66%
Log pre-tax/post-tax income	60.5%	61%
Borrowers	6.07%	6.3%
Mean of AFQT	0.0880	0
Standard deviation of AFQT	0.204	0.213

No Optimism: Model Fit

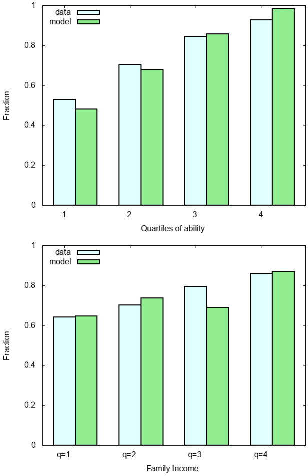

Figure: Enrollment rates

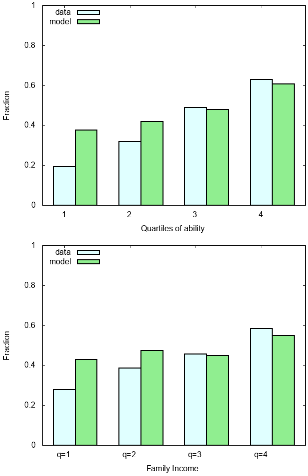

Figure: Graduation rates

No Optimism: Optimal Policy

	Current state	Optimal
$s_1(1)$	\$13,600	\$14,153
$s_1(2)$	\$11,448	\$11,913
$s_1(3)$	\$10,923	\$11,367
$s_2(1)$	\$13,600	\$12,478
$s_2(2)$	\$11,448	\$10,503
$s_2(3)$	\$10,923	\$10,021

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