Rational Bubbles and Middlemen*

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Abstract

This paper develops a finite-period model of rational bubbles where trade of an asset takes place through a chain of middlemen. We show that there exists a unique equilibrium, and a bubble can occur due to higher-order uncertainty. Under reasonable assumptions, the equilibrium price is increasing and accelerating during bubbles although the fundamental value is constant over time. Bubbles may be detrimental to the economy; however, bubble-bursting policies affect agents' beliefs and it turns out that they have no effect on welfare. We also demonstrate that the possibility that middlemen obtain more information leads to larger bubbles.

Keywords: Rational bubbles; Middlemen; Higher-order uncertainty; Asymmetric information; Flippers

JEL Classification Numbers: D82, D83, D84, G12, G14

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1 Introduction

Bubbles refer to asset prices that exceed the fundamental value of an asset. Famous historical examples are the Dutch tulip mania (1634-7), the Mississippi Bubble (1719-20), the South Sea Bubble (1720), and the Roaring Twenties followed by the 1929 crash. A recent example is the housing bubble that preceded the 2008 financial crisis. During the time of asset market fluctuations, continuous price increases, interrupted by a sudden market crash, often occur through chains of intermediaries. These intermediaries, or middlemen, are engaged in flipping, i.e., purchasing an asset at a low price and quickly reselling it at a higher price.

This paper develops a finite-period model of rational bubbles where trade of an asset takes place through a chain of middlemen. We assume a simple network of agents, or a simple form of search frictions, where agents are located on a straight line and each agent can meet and trade only with his nearest neighbors. An agent located on the one end of the network is the initial owner of an indivisible asset in fixed supply, whereas an agent located on the other end is the final user of this asset. Between the owner and the user, there exist middlemen who do not consume the asset but have storage technologies that allow for the circulation of this asset through the network. Everyone can produce and consume a divisible good, which can be used to buy the asset.

In this setup, we consider the following information structure: all parameters describing utilities, costs, etc., are common knowledge, except for the consumption value of the asset for the final user. We focus on states where the consumption value is zero, in a model where the consumption value is positive in another state. Prior to trade, all agents except for the final user observe the consumption value. When the consumption value is zero, with some probability, the final user receives a signal that tells him that the asset is worthless, and then the information that the final user knows that the asset is worthless spreads from the final user to the initial owner, but it is subject to loss between any two agents. When the final user receives the signal, there is a situation where everyone knows that the consumption value is zero, but this is not common knowledge—one may not know if others know that the asset is worthless. This opens room for a bubble—one acquires an asset, knowing it is overpriced, in hopes of finding a greater fool who believes that he can find an even greater fool.

We show that there exists a unique equilibrium. In our model, agents are rational and share a common prior distribution, but a bubble can occur in equilibrium due to higher-order uncertainty. A middleman buys the asset only if he believes that he will be able to find a greater fool who also expects to find an even greater fool. In this process, despite the fundamental value of zero, the asset is exchanged for a positive amount of the divisible good, and hence a bubble is occurring. However, if one encounters another who is pessimistic about finding a greater fool, he refuses to buy the asset, and then the bubble bursts. Note that, if the fact that the fundamental value of the asset is zero were common knowledge, then bubbles would not occur. The key here is that each middleman cares less about the fundamental value of the asset, but more about how much the other agents value it. Therefore, there is room for higher-order uncertainty to play a role for the occurrence of bubbles. Middlemen are essential for bubbles in the sense that bubbles do not occur without middlemen. Hence, this suggests that they can be a source of fragility in the economy.

Under reasonable assumptions, the equilibrium price has the following properties. First, it is increasing over time during bubbles, and hence middlemen actually act as flippers. This is because each middleman always faces risk that he cannot find a greater fool. The price not only increases but also accelerates. This is because the probability that one can find a greater fool decreases over time. In other words, middlemen who trade in later periods are exposed to bigger risk.

Models with rational agents permit the use of standard tools to analyze welfare in the underlying economy. We show that bubbles are beneficial to the economy when agents enjoy sufficiently high utility from consuming the divisible good (just like during deflation) but detrimental otherwise (during inflation). Based on this result, we discuss policy implications. “Irrational exuberance” is the phrase used by Alan Greenspan in a 1996 speech, “The Challenge of Central Banking in a Democratic Society.” The speech was given during the dot-com bubble, and the Tokyo market moved down sharply by his speech and other markets followed. Hence, his speech was interpreted as a warning that assets were overpriced. In our model, the central bank considers a policy that deflates overpriced assets by revealing information about this overpricing. We assume that the central bank knows that the asset is worthless only when every agent knows this, and then it announces the information before trade takes place. Then, its inaction can affect agents' beliefs and hence, prices of the asset because it reveals the information that the final user does not know that the asset is worthless. This induces a “side effect” when bubbles are detrimental, and we show that the side effect offsets the welfare gain of the bubble-bursting policy, and as a result the policy has no effect on welfare. Therefore, if the central bank has to pay some cost to announce the information, it should not employ the policy, or should keep the information secret.

We also investigate the relationship between the size of bubbles and the amount of information. In our baseline environment, with some probability, middlemen obtain information that the final user knows that the asset is worthless. Comparing the environments with and without such possibility, we demonstrate that information increases the size of bubbles. In other words, if there is the possibility that middlemen can obtain more information about the underlying economy, then prices deviate more from the fundamental value of the asset. This is because, when each middleman does not receive information, he calculates his expected utility based on the probability that he can find a greater fool conditional on the event that he does not receive information. Hence, this result suggests that the development of information technology for financial intermediaries may make the economy more fragile as long as it is incomplete in the sense that they may not receive information.

Related Literature

There are several strands of models in the literature of bubble. First, monetary models by Samuelson (1958), Tirole (1982), and others study rational bubbles in an economy with symmetric information, and thereby they need an infinite horizon to show the occurrence of bubbles.[1] Second, the models building on Allen, Morris, and Postlewaite (1993) consider rational bubbles in an economy with asymmetric information and show the occurrence of bubbles even in a finite horizon. Third, bubbles due to limited arbitrage are examined, for example, by DeLong, Shleifer, Summers, and Waldmann (1990) and Abreu and Brunnermeier (2003). Fourth, Harrison and Kreps (1978), Scheinkman and Xiong (2003), and others investigate heterogeneous-beliefs bubbles. In our model, agents are rational and asymmetrically informed, and share a common prior distribution. Hence, our model is included in the second strand.

Asymmetric information creates a lemons problem, and thus we need some motivation for agents to trade assets. The models starting from Allen, Morris, and Postlewaite (1993) through Liu and Conlon (2018) assume risk-sharing as the motive for trade, and Liu and White (2018) employ intertemporal consumption-smoothing as the motive for trade. In our model, since the consumption value of the asset is zero for all agents except for the final user, there are gains from trade from getting the asset to the final user. When middlemen obtain the asset, they are subject to risk that each of them cannot sell the asset to the next agent, and then the gains from trade are necessary to overcome buyers' risk of being greater fools. In this sense, the motivation for trade in our model is similar to that in the risk-shifting models studied by Allen and Gorton (1993), Allen and Gale (2000) and Barlevy (2014) among others. We study the relationship between bubbles and higher-order uncertainty. In this respect, our paper is also related to Morris, Postlewaite, and Shin (1995), Abreu and Brunnermeier (2003), Conlon (2004, 2015),

[1] Search-theoretic models of money in the line of Kiyotaki and Wright (1989) are also included in this strand. See Lagos, Rocheteau, and Wright (2017) for a recent survey on these models.

Doblas-Madrid (2012, 2016), and Matsushima (2013).[2]

Our contribution to the literature of bubble can be summarized as follows. First, since we derive the uniqueness of equilibrium, we rationalize the occurrence of bubbles in a strong sense. Second, under reasonable assumptions, we show generally that the equilibrium price is accelerating during bubbles although there is no irrational agent and the fundamental value is constant over time. Third, in our model, bubbles may be detrimental to the economy, and we obtain a novel policy implication.[3]

Since the seminal work by Rubinstein and Wolinsky (1987), models of middlemen have been developed to study the role of middlemen not only in goods markets (e.g., Biglaiser (1993) and Lizzeri (1999)) but also in financial markets (e.g., Duffie, Gârleanu, and Pedersen (2005)). Our environment is akin to Wright and Wong (2014), who develop a model of intermediation chains. They show that there is a bubble in their model only when there are an infinite number of middlemen, or time is infinite. Our innovation is to provide a different information structure from theirs by relaxing the assumption of common knowledge and show that there is a bubble in a finite number of periods. The latter result is in line with a recent work by Davis et al. (2019) who use a very different approach than ours and show that money can exist even with a finite horizon setup. Moreover, we find that, for economies with middlemen to be active, the asset in question does not need to have a positive fundamental value. There are papers that propose models of assets traded in intermediation chains or networks and study how intermediaries affect prices and efficiency (e.g., Gofman (2014), Glode and Opp (2016), Choi, Galeotti, and Goyal (2017), Condorelli, Galeotti, and Renou (2017), Farboodi (2017), Malamud and Rostek (2017), Manea (2018), and Hugonnier, Lester, and Weill (2019)).[4] In contrast to these papers, we study the role of intermediaries for bubbles

[2]See Brunnermeier and Oehmke (2013) for a recent survey on bubbles.

[3]Grossman and Yanagawa (1993), Miao and Wang (2014), and Guerron-Quintana, Hirano, and Jinnai (2019) show that bubbles may reduce welfare in models of endogenous growth, but their bubbles are included in a different strand from ours.

[4]See the references of Wright and Wong (2014) to find more papers on middlemen. We also refer

in finite intermediation chains and show that they are essential for the occurrence of bubbles. For empirical studies documenting the importance of intermediation chains in over-the-counter financial markets, see Hollifield, Neklyudov, and Spatt (2017) and Li and Schürhoff (2019).

Hirshleifer (1971) studies the idea that it may be optimal to keep information secret. The idea is also examined by Diamond and Verrecchia (1991), Kaplan (2006), Andolfatto and Martin (2013), Andolfatto, Berentsen, and Waller (2014), Dang, Gorton, Holmström, and Ordoñez (2017), and Monnet and Quintin (2017). Our model has a case where the central bank should keep the information that the asset is worthless secret. Hence, we add a new model to this literature as well.

The Structure of the Paper

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 shows the existence and uniqueness of equilibrium. Section 4 derives implications for price changes and welfare, and discusses policy implications. Section 5 investigates the relationship between the size of bubbles and the amount of information. Section 6 provides two examples with different prior distributions. Section 7 concludes.

2 The Model

In this section, we describe the economic environment in the first subsection and the knowledge structure of agents in the second subsection.

to Allen and Babus (2009) for a survey on networks in financial markets and Condorelli and Galeotti (2016) for a survey on strategic intermediation in networks.

2.1 The Environment

There are N agents $A_1, A_2, \cdots, A_N$ where $2 < N < \infty$. They are spatially separated in the following fashion: A_n can meet and hence, trade with A_{n-1} and A_{n+1} but no one else. Therefore, trade between A_{n-1} and A_{n+1} must go through A_n. We assume that trade is sequential, A_n and A_{n+1} trade in period n, and A_n exits the economy after trading with A_{n+1}. Hence, time is discrete and continues for $N-1$ periods from 1 to $N-1$.

There are two objects in this economy. One is an indivisible asset x in fixed supply, and the other is a divisible good y that every agent can produce at unit cost. Only A_1 is endowed with x, that is, A_1 is the initial owner of x. He can try to trade it to A_2 in exchange for some amount of y, say y_1. We assume the consumption value of x for A_1 is zero. More generally, if A_n acquires x from A_{n-1}, he can try to trade it to A_{n+1} for y_n, which generates a payoff

$$U(y_n) = \kappa y_n$$

where κ is some positive constant.[5] The consumption value of x for A_n is 0 for each $n < N$. The value for the final user A_N is $v > 0$ with some probability and 0 with the remaining probability.[6] For simplicity, agents do not discount utilities between any two periods. Middlemen $A_2, A_3, \cdots, A_{N-1}$ are a necessary part of the process of getting x from the initial owner A_1 to the final user A_N. For simplicity, we employ generalized

[5]If $\kappa > 1$, without any restrictions, agents have incentives to produce an infinite amount of y. However, there are situations where $\kappa > 1$ is legitimate. First, consider the situation where A_n and A_{n+1} cannot obtain any utility from consuming y produced by A_n. Then, A_n produces y only if he trades x with A_{n-1}. In other words, there are $N-1$ divisible goods, and agents have specialized tastes and technologies. Second, consider the situation where there is an upper bound $\overline{y}$ such that $U(y) = \overline{y}$ for each $y \geq \overline{y}$ or the cost of producing y is infinite if $y \geq \overline{y}$. Then, agents produce at most $\overline{y}$ units of y and have incentives to enter the economy as long as κ is not too big.

[6]Even if the consumption value of x for A_n is $f \geq 0$ for each $n < N$, and the value for A_N is $f + v$ with some probability and f with the remaining probability, all the results hold as long as v is sufficiently large, f is sufficiently small, or κ is not too small. Just for simplicity, we assume $f = 0$.

Nash bargaining to determine the terms of trade $(y_n)_{n=1}^{N-1}$, where agents' utilities are zero if they disagree to trade. In trade between A_n and A_{n+1}, let θ be the bargaining power of A_n with $0 < \theta < 1$.[7]

2.2 Knowledge

All parameters describing utilities, costs, etc., are common knowledge, *except* for the consumption value of x for A_N. We will consider states of the world where the consumption value is zero, and all agents including A_N know this, but this is *not* common knowledge—one may not know if others know that x is worthless. This opens room for a bubble—one acquires an asset, knowing it is overpriced, in hopes of finding a greater fool who believes that he can find an even greater fool. We employ a model of knowledge reminiscent of Rubinstein's (1989) Email game.

Prior to trade, all agents except for A_N observe the consumption value of x for A_N, that is, the initial owner A_1 and middlemen $A_2, \cdots, A_{N-1}$ are experts. When it is zero, then A_N receives a signal with some probability. Otherwise, A_N does not receive any signals. Thus, if he receives a signal, then A_N is sure that the consumption value is zero, and in this event, every agent knows that x is worthless. Moreover, if A_N receives a signal, he (non-strategically) sends a signal (email in the terminology of Rubinstein (1989)) to A_{N-1}. The signal reaches A_{N-1} with some probability but is lost with the remaining probability. Thus, if A_{N-1} receives a signal, he is sure that A_N knows that the consumption value is zero. Similarly, if A_{N-1} receives a signal from A_N, he (non-strategically) sends a signal to A_{N-2}. The signal reaches A_{N-2} with some probability but is lost with the remaining probability. This process continues until a signal is lost

[7] When $\theta = 0$, it will turn out that the terms of trade are zero, and hence we assume $\theta > 0$. When $\theta = 1$, agents are indifferent on whether they buy the asset x or not. Thus, to ensure the uniqueness of equilibrium, we assume $\theta < 1$. However, if we assume that agents buy whenever they are indifferent, all the results hold even with $\theta = 1$. In general, our all results are robust to the other forms of bargaining protocol.

between some two agents or the initial owner A_1 receives a signal. In words, the signal (rumor) that A_N knows that x is worthless spreads from A_N to A_1, but it is subject to loss between any two agents.[8] We assume all these signals occur *prior to trade.*

To describe the above situation formally, we introduce $N + 2$ states of the world. The consumption value of x for A_N is $v > 0$ at state ω_v and 0 at the other states. When the state is ω_ϕ, no agent receives a signal although x is worthless. On the other hand, for each $n = 1, \cdots, N$, the state ω_n corresponds to the case where all the agents $A_N, A_{N-1}, \cdots, A_n$ receive signals, while the others do not. Hence, the set of the states is

$$\Omega = \{\omega_v, \omega_\phi, \omega_N, \omega_{N-1}, \cdots, \omega_1\}$$

Let μ be the common prior distribution over Ω, and assume that $\mu(\omega) > 0$ for each $\omega \in \Omega$.

We represent agents' knowledge by partitions of Ω. Agent A_N's partition is

$$\mathcal{P}_N = \{\{\omega_v, \omega_\phi\}, \{\omega_N, \omega_{N-1}, \cdots, \omega_1\}\}$$

The first element, $\{\omega_v, \omega_\phi\}$, corresponds to the case where A_N does not receive a signal and hence, does not know whether x is worthless. The second element, $\{\omega_N, \omega_{N-1}, \cdots, \omega_1\}$, corresponds to the case where A_N receives a signal and knows that x is worthless. For each $n < N$, agent A_n's partition is

$$\mathcal{P}_n = \{\{\omega_v\}, \{\omega_\phi, \omega_N, \omega_{N-1}, \cdots, \omega_{n+1}\}, \{\omega_n, \omega_{n-1}, \cdots, \omega_1\}\}$$

The first element, $\{\omega_v\}$, corresponds to the case where the consumption value of x for A_N is v. The second element, $\{\omega_\phi, \omega_N, \omega_{N-1}, \cdots, \omega_{n+1}\}$, corresponds to the case where

[8] For simplicity, we say that A_n "non-strategically" sends a signal to A_{n-1} when A_n receives a signal. However, A_n does not actually care whether A_{n-1} receives the signal. It will turn out that, if A_{n-1} receives the signal, the only effect on A_n is that A_n does not receive an offer from A_{n-1}, but it is always optimal for A_n to reject the offer given that A_n has already received the signal. Hence, A_n does not care whether he sends a signal to A_{n-1}, and might therefore use a mixed strategy, or even just let the information randomly leak out.

the consumption value is zero, but A_n does not receive a signal. The third element, $\{\omega_n, \omega_{n-1}, \cdots, \omega_1\}$, corresponds to the case where the consumption value is zero and A_n receives a signal. An agent can distinguish any two states if those states belong to a different element of his partition, but cannot otherwise.

Since we are interested in a bubble, we focus on cases where the economy is at state $\omega \neq \omega_v$. Then, if A_N does not receive a signal, the posterior probability that the consumption value of x for A_N is v is $\mu(\omega_v)/[\mu(\omega_v) + \mu(\omega_\phi)]$, and hence the expected value is

$$v_e = \frac{\mu(\omega_v)}{\mu(\omega_v) + \mu(\omega_\phi)} v > 0$$

It will be useful to calculate the probability ψ_n that A_{n+1} does not receive a signal conditional on the event that A_n does not receive a signal. The probability is as follows: for $N-1$,

$$\psi_{N-1} = \frac{\mu(\omega_\phi)}{\mu(\omega_\phi) + \mu(\omega_N)}$$

and for each $n = 2, \cdots, N-2$,

$$\psi_n = \frac{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+2})}{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+2}) + \mu(\omega_{n+1})}$$

Note that $0 < \psi_n < 1$ for each $n = 2, \cdots, N-1$ because $\mu(\omega) > 0$ for each $\omega \in \Omega$. It will turn out that ψ_n is the probability that A_n can find a greater fool.

3 Equilibrium

In this section, we derive a *sequential* equilibrium (henceforth, equilibrium) of the economy, and argue it is unique. In the first subsection, we display the equilibrium and in the second subsection, we provide a proof.

3.1 Life of a Bubble

We will show the existence and uniqueness of equilibrium. To this end, define a sequence $(\hat{y}_n)_{n=1}^{N-1}$ as follows: for $N-1$,

$$\hat{y}_{N-1} = \theta v_e$$

and for each $n = 1, \cdots, N-2$,

$$\hat{y}_n = \theta \kappa \psi_{n+1} \hat{y}_{n+1}$$

Note that $\hat{y}_n > 0$ for each $n = 1, \cdots, N-1$ because $\theta > 0$, $v_e > 0$, $\kappa > 0$, and $\psi_{n+1} > 0$ for each $n = 1, \cdots, N-2$. We obtain the following characterization of equilibrium.

Lemma 1. *Assume $\omega \neq \omega_v$. In equilibrium, if agent A_{n+1} receives a signal, agent A_{n+1} does not trade with agent A_n, or $y_n = 0$; if agent A_{n+1} does not receive a signal, agent A_{n+1} trades with agent A_n and obtains x in exchange for $y_n = \hat{y}_n$. Moreover, this outcome is unique.*

We say that

Definition 1. *A bubble occurs* if all agents know that the asset x is worthless, but it is traded for a positive amount of the good y.

If $\omega \in \{\omega_1, \omega_2\}$, a bubble does not occur because A_2 receives a signal and does not trade with A_1. If $\omega = \omega_\phi$, trade takes place, but this is simply because A_N does not know that x is worthless. If $\omega \in \{\omega_N, \omega_{N-1}, \cdots, \omega_3\}$, this lemma describes a life of a bubble.

To see this, suppose the economy is at ω_{n^*} where $n^* > 2$. Then, agents $A_N, \cdots, A_{n^*}$ receive signals, while the others do not. Given this realization, every agent knows that x is worthless, and hence the fundamental value of x is zero. Yet, the asset x is exchanged for a positive amount of the good y for $n^* - 2$ periods. In this sense, a bubble is occurring. Obviously, if the fact that the fundamental value of x is zero were common knowledge, then x would not be traded. At period $n^* - 1$, agent A_{n^*} refuses to trade with agent A_{n^*-1}, and then the bubble bursts. We summarize this result as follows.

Theorem 1. *The equilibrium is unique. In the equilibrium, bubble occurs when $\omega \in \{\omega_N, \omega_{N-1}, \cdots, \omega_3\}$. Moreover, a bubble bursts for sure.*

Consider a fictitious situation where there is no middleman. Then, the initial owner may not be able to trade with the final user without middlemen. Even if they can trade directly, the final user refuses to trade with the initial owner because the final user receives a signal and knows that the asset is worthless. Hence, middlemen are essential for the occurrence of a bubble.

During a bubble, agent A_{n+1} "buys" the asset x at the "price" $\hat{y}_n$ and tries to "resell" it at the "price" $\hat{y}_{n+1}$. Each agent is exposed to risk that he may be the greatest fool when he buys the asset. However, the final user A_N is the only "real" greater-fool in the sense that A_N can be the only agent who buys the asset from an agent who has more pessimistic information than A_N himself has. In trade among the other agents, if the seller knows that the asset is worthless, then the buyer also knows that it is worthless. Hence, buyers' risk is not that they are buying from "bad" sellers, who knows that buyers will be hurt, but that the next potential buyer may know that the next next buyer knows that ... that the final user A_N knows that the asset is worthless.[9]

This feature automatically makes our model robust to small changes in most parameters. In the models building on Allen, Morris, and Postlewaite (1993), buyers do not know whether they are buying from "good" or "bad" sellers, and hence it is important for the two types of sellers behave the same way. This depends on a coincidence, which means that it is hard to make the models robust.[10] In contrast, in our model, there are no good or bad sellers except when A_{N-1} is selling to A_N. Even in that case, agent A_{N-1}

[9]When the state is ω_v, middlemen know that the final user does not receive any signals. Hence, there is no risk for middlemen, and as a result prices are different from those in the other states. Therefore, we implicitly assume that the final user cannot observe trade from period 1 to period $N-2$ because, otherwise, he can learn the consumption value of the asset from observing prices in previous trade, and a bubble does not occur.

[10]See, e.g., Conlon (2015) and Liu and Conlon (2018) for the discussion of the robustness. We basically need a continuum of states to make the models robust.

behaves the same way whether or not the asset is worthless because the consumption value of x for A_{N-1} is always zero.

Wright and Wong (2014) show the following two results on intermediation bubbles. First, bubbles occur if there are an infinite number of middlemen, or time is infinite, and the utility function U is nonlinear. Second, bubbles never occur if $U(y_n) = y_n$. They need positive potential gains from trade in terms of y to make bubbles occur, and hence bubbles do not occur with the linear utility function. Hence, our innovation is not only relaxing the assumption of common knowledge and showing the occurrence of bubbles in a finite number of periods but also showing that there is a bubble even when the potential gains from trade are zero or negative. In Section 4.2, we will see that this leads us to a different welfare implication from theirs.

3.2 Proof of Lemma 1

Proof is by backward induction.

Trade between A_{N-1} and A_N:

If A_N receives a signal, A_N knows that x is worthless, and hence A_N does not trade with A_{N-1}, or $y_{N-1} = 0$.

If A_N does not receive a signal, his expected value of x is v_e. Then, A_N and A_{N-1} negotiate the terms of trade:

$$\max_{y_{N-1}} (\kappa y_{N-1})^{\theta} (v_e - y_{N-1})^{1-\theta}$$

subject to incentive constraints: $\kappa y_{N-1} \geq 0$ and $v_e - y_{N-1} \geq 0$. Note that, if they disagree to trade, they do not obtain any utility. The solution is

$$\hat{y}_{N-1} = \theta v_e$$

Hence, A_N obtains x in exchange for $\hat{y}_{N-1}$.

Trade between A_{N-2} and A_{N-1}:

If A_{N-1} receives a signal, A_{N-1} knows that A_N receives a signal. Then, as we have shown above, A_{N-1} knows that A_N will not trade with A_{N-1}, and hence A_{N-1} does not trade with A_{N-2}, or $y_{N-2} = 0$.

If A_{N-1} does not receive a signal, there are exactly two possibilities:

1. both A_{N-1} and A_N do not receive signals; and

2. A_N receives a signal, but A_{N-1} does not.

In the first case, A_N trades with A_{N-1}. In the second case, however, A_N does not trade with A_{N-1}. The first case occurs with probability ψ_{N-1} given that A_{N-1} does not receive a signal, and the second case occurs with the remaining probability. Hence, A_{N-1}'s expected utility of obtaining x is $\psi_{N-1}\kappa\hat{y}_{N-1}$. Then, A_{N-1} and A_{N-2} negotiate the terms of trade:

$$\max_{y_{N-2}} (\kappa y_{N-2})^{\theta} (\psi_{N-1}\kappa\hat{y}_{N-1} - y_{N-2})^{1-\theta}$$

subject to incentive constraints: $\kappa y_{N-2} \geq 0$ and $\psi_{N-1}\kappa\hat{y}_{N-1} - y_{N-2} \geq 0$. The solution is

$$\hat{y}_{N-2} = \theta\kappa\psi_{N-1}\hat{y}_{N-1}$$

Therefore, A_{N-1} obtains x in exchange for $\hat{y}_{N-2}$.

Induction Hypothesis:

Suppose that

1. if A_{n+1} receives a signal, A_{n+1} does not trade with A_n, or $y_n = 0$;

2. if A_{n+1} does not receive a signal, A_{n+1} trades with A_n and obtains x in exchange for $y_n = \hat{y}_n$.

Then, we will show that

1. if A_n receives a signal, A_n does not trade with A_{n-1}, or $y_{n-1} = 0$;

2. if A_n does not receive a signal, A_n trades with A_{n-1} and obtains x in exchange for $y_{n-1} = \hat{y}_{n-1}$.

Trade between A_{n-1} and A_n:

If A_n receives a signal, A_n knows that A_{n+1} receives a signal. Then, by induction hypothesis, A_n knows that A_{n+1} will not trade with A_n, and hence A_n does not trade with A_{n-1}, or $y_{n-1} = 0$.

If A_n does not receive a signal, there are exactly two possibilities:

1. both A_n and A_{n+1} do not receive signals; and

2. A_{n+1} receives a signal, but A_n does not.

In the first case, by induction hypothesis, A_{n+1} trades with A_n and obtains x in exchange for $\hat{y}_n$. In the second case, however, A_{n+1} does not trade with A_n again by induction hypothesis. The first case occurs with probability ψ_n given that A_n does not receive a signal, and the second case occurs with the remaining probability. Hence, A_n's expected utility of obtaining x is $\psi_n \kappa \hat{y}_n$. Then, A_{n-1} and A_n negotiate the terms of trade:

$$\max_{y_{n-1}} (\kappa y_{n-1})^{\theta} (\psi_n \kappa \hat{y}_n - y_{n-1})^{1-\theta}$$

subject to incentive constraints: $\kappa y_{n-1} \geq 0$ and $\psi_n \kappa \hat{y}_n - y_{n-1} \geq 0$. The solution is

$$\hat{y}_{n-1} = \theta \kappa \psi_n \hat{y}_n$$

Therefore, A_n obtains x in exchange for $\hat{y}_{n-1}$.

Uniqueness:

In each trade, both agents obtain positive expected utilities because $0 < \theta < 1$. Moreover, when A_{n+1} does not trade with A_n, A_{n+1} has strict incentives to refuse the trade, or choose $y_n = 0$. Hence, there is no indifference among choices of each agent, which implies the uniqueness of equilibrium.

Note also that in each round we are only eliminating conditionally dominated strategies (Shimoji and Watson, 1998).

4 Implications

In this section, we investigate the properties of equilibrium. In the first subsection, we study how the equilibrium price changes over time; in the second subsection, we derive welfare implications; in the third subsection, we discuss policy implications.

4.1 Price Changes

We will show that $\hat{y}_n$, the price that A_{n+1} has to pay to obtain the asset x, is not only *increasing* but also *accelerating* in n during a bubble under reasonable assumptions. To this end, we show the following technical lemma.

Lemma 2. *If $\mu(\omega_2) \leq \cdots \leq \mu(\omega_N)$, the probability ψ_n is decreasing in n, that is,*

$$\psi_{n+1} - \psi_n < 0$$

Proof. For $n = 1, \cdots, N-3$, letting $\mathcal{M} = \mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+3})$,

$$\begin{aligned}
\psi_{n+1} - \psi_n &= \frac{\mathcal{M}}{\mathcal{M} + \mu(\omega_{n+2})} - \frac{\mathcal{M} + \mu(\omega_{n+2})}{\mathcal{M} + \mu(\omega_{n+2}) + \mu(\omega_{n+1})} \\
&= \frac{\mathcal{M}[\mu(\omega_{n+1}) - \mu(\omega_{n+2})] - [\mu(\omega_{n+2})]^2}{[\mathcal{M} + \mu(\omega_{n+2})][\mathcal{M} + \mu(\omega_{n+2}) + \mu(\omega_{n+1})]} \\
&< 0
\end{aligned}$$

The last inequality holds since we have $\mu(\omega_{n+1}) \leq \mu(\omega_{n+2})$ for each $n = 1, \cdots, N-3$ and $\mu(\omega) > 0$ for each $\omega \in \Omega$ by assumption. A similar argument holds for the case between $N-1$ and $N-2$. $\square$

Now, we obtain the following result on price changes.

Proposition 1. *Assume $\theta\kappa \leq 1$. Then, $\hat{y}_n$ is increasing in n, that is,*

$$\hat{y}_{n+1} - \hat{y}_n > 0$$

Moreover, if $\mu(\omega_2) \leq \cdots \leq \mu(\omega_N)$, it is accelerating in n, that is,

$$\hat{y}_{n+2} - \hat{y}_{n+1} > \hat{y}_{n+1} - \hat{y}_n$$

Proof. To see that $\hat{y}_n$ is increasing, we obtain

$$\begin{aligned} \hat{y}_{n+1} - \hat{y}_n &= \hat{y}_{n+1} - \theta\kappa\psi_{n+1}\hat{y}_{n+1} \\ &= (1 - \theta\kappa\psi_{n+1})\hat{y}_{n+1} \\ &> 0 \end{aligned}$$

Note that the first equality follows from the definition of $\hat{y}_n$.

To see that $\hat{y}_n$ is accelerating, we obtain

$$\begin{aligned} & (\hat{y}_{n+2} - \hat{y}_{n+1}) - (\hat{y}_{n+1} - \hat{y}_n) \\ = \ & (1 - \theta\kappa\psi_{n+2})\hat{y}_{n+2} - (1 - \theta\kappa\psi_{n+1})\hat{y}_{n+1} \\ = \ & \hat{y}_{n+2} - \hat{y}_{n+1} + \theta\kappa(\psi_{n+1}\hat{y}_{n+1} - \psi_{n+2}\hat{y}_{n+2}) \end{aligned}$$

Here, we have

$$\begin{aligned} \psi_{n+1}\hat{y}_{n+1} - \psi_{n+2}\hat{y}_{n+2} &= \psi_{n+1}\hat{y}_{n+1} - \psi_{n+2}\hat{y}_{n+1} + \psi_{n+2}\hat{y}_{n+1} - \psi_{n+2}\hat{y}_{n+2} \\ &= (\psi_{n+1} - \psi_{n+2})\hat{y}_{n+1} - \psi_{n+2}(\hat{y}_{n+2} - \hat{y}_{n+1}) \end{aligned}$$

Combining these,

$$\begin{aligned} & (\hat{y}_{n+2} - \hat{y}_{n+1}) - (\hat{y}_{n+1} - \hat{y}_n) \\ = \ & (1 - \theta\kappa\psi_{n+2})(\hat{y}_{n+2} - \hat{y}_{n+1}) + \theta\kappa(\psi_{n+1} - \psi_{n+2})\hat{y}_{n+1} \end{aligned}$$

The first term is positive because $\hat{y}_n$ is increasing in n as we have shown above, and $\theta\kappa \leq 1$. The second term is also positive because the probability ψ_n is decreasing in n by Lemma 2. Therefore, $\hat{y}_n$ is accelerating. $\square$

The fact that $\hat{y}_n$ is increasing follows because each middleman may be the greatest fool with positive probability, $1 - \psi_n$. During a bubble, middlemen "flip"—agent A_{n+1} buys the asset x at the price $\hat{y}_n$ and tries to resell it at the price $\hat{y}_{n+1} > \hat{y}_n$. The fact that $\hat{y}_n$ is accelerating follows because the probability that one can find a greater fool, ψ_n, is decreasing over time. In other words, flippers who trade in later periods are exposed to bigger risk, and the prices are determined in such a way that they are compensated for the risk.

We need a sufficiently small κ for this result. If $\kappa \leq 1$, the assumption is necessarily satisfied. Otherwise, if κ is big, there are a lot of gains from trade in terms of y, and hence each agent is willing to produce a large amount of y to obtain x. For price acceleration, we impose an additional assumption, which is that a state where more agents receive signals is realized with a smaller probability. In Section 6, we will provide simple examples with different prior distributions that satisfy this assumption.

4.2 Welfare

Before discussing welfare implications, to make our analysis meaningful, we consider agents' incentives to participate in the economy. To this end, we will demonstrate that expected utility of each agent is nonnegative in both ex ante and interim stages. The ex ante stage is before the state is determined, the interim stage is after the state is determined but before trade takes place, and the ex post stage is after trade takes place. From the equilibrium prices, we can see that the interim expected utilities are positive for agents who do not receive signals and zero for the other agents. Hence, every agent has incentives to participate in the economy at the interim stage.[11] This further implies that the ex ante utility of each agent is positive, and thus every agent has incentives to

[11] When the state is ω_v, agents $A_1, \cdots, A_{N-1}$ know that the consumption value of the asset x for the final user A_N is $v > 0$, and the final user A_N does not know the fact since he never receives a signal. Then, each trade takes place between any two adjacent agents, and each agent enjoys positive expected utility at the interim stage.

participate in the economy at the ex ante stage as well. Therefore, before trade takes place, it is optimal for each agent to participate in the economy.

Now, we derive welfare implications. Our welfare criterion is utilitarian, that is, welfare is the sum of all agents' utilities. Note that, in trade between A_n and A_{n+1}, the gains from trade are

$$\kappa y_n - y_n = (\kappa - 1)y_n$$

because we normalized the production cost of y to be 1. Thus, when the economy is at ω_{n^*} with $n^* > 2$, ex post welfare is

$$(\kappa - 1)\sum_{n=1}^{n^*-2} \hat{y}_n$$

This is because a bubble continues for $n^* - 2$ periods, and the terms of trade are $y_n = \hat{y}_n$ during a bubble and $y_n = 0$ after period $n^* - 2$. We obtain the following result on welfare.

Proposition 2. *Bubbles are beneficial to the economy if $\kappa > 1$ but detrimental if $\kappa < 1$.*

Consider a fictitious situation where the asset x is traded at its fundamental value, that is, zero. In this case, ex post welfare is zero. Hence, whether bubbles are beneficial or detrimental depends on whether $\kappa > 1$ or $\kappa < 1$. Consider another fictitious situation where there is no middleman. Then, bubbles do not occur, and as a result ex post welfare is zero. Hence, we can reinterpret this result as follows: When the state belongs to $\{\omega_N, \omega_{N-1}, \cdots, \omega_3\}$, middlemen are beneficial to the economy if $\kappa > 1$ but detrimental if $\kappa < 1$.

When $\kappa > 1$, there are positive gains from trade in terms of the good y, but those gains from trade can only be realized if the asset x has a positive price. Hence, bubbles are beneficial to the economy in this case.

When $\kappa < 1$, there are efficiency losses from trading the good y. For example, if there are some transaction costs that sellers must pay when they sell the asset, the ex post welfare is negative because of the costs. Another interpretation is inflation. Consider the good y as money and suppose that there is another market where agents can buy some goods using "money" y. Suppose further that each agent can enter the market whenever he wants, but agent A_n can use y_n units of money to buy goods in the market at period $n+1$ when A_n obtains y_n from A_{n+1} at period n, because, for example, it takes one period for A_n to send the asset x to A_{n+1} and cannot enter the market at period n. Due to inflation, as time passes from period n to period $n+1$, the value of y depreciates at the rate of $1-\kappa$, and A_n can buy goods whose value is κy_n evaluated at period n by using y_n units of money. Hence, when A_n obtains y_n units of money from A_{n+1} at period n, its value for A_n is κy_n. Therefore, when we have inflation, bubbles are detrimental to the economy. Similarly, the case of $\kappa > 1$ could be considered as the case of deflation.

4.3 Policy

The central bank considers a policy that deflates overpriced assets by revealing information about this overpricing. We assume that the central bank knows that the asset is worthless only when each agent knows that the asset is worthless, and then it announces the information before trade takes place. More precisely, the knowledge of the central bank is the same as the final user's:

$$\mathcal{P}_c \equiv \{\{\omega_v, \omega_\phi\}, \{\omega_N, \omega_{N-1}, \cdots, \omega_1\}\} = \mathcal{P}_N$$

Under this policy, when the state is ω_v, the central bank does not announce the information, and the final user A_N still does not know that the consumption value of the asset x is $v > 0$. When the state is ω_ϕ, the central bank again does not announce the information, and in this case, all agents except for A_N knows that A_N does not receive a signal because the central bank would announce the information if A_N received a signal. In other words, the inaction of the central bank affects agents' beliefs. More precisely, for each $n < N$, agent A_n's partition is changed by the inaction to

$$\mathcal{P}'_n = \{\{\omega_v\}, \{\omega_\phi\}, \{\omega_N, \omega_{N-1}, \cdots, \omega_{n+1}\}, \{\omega_n, \omega_{n-1}, \cdots, \omega_1\}\}$$

Then, there is no risk that each middleman cannot sell the asset to the next agent, and thus the terms of trade increase. More specifically, the terms of trade are changed to $(\hat{y}'_n)_{n=1}^{N-1}$ defined as follows: for $N-1$,

$$\hat{y}'_{N-1} \equiv \theta v_e = \hat{y}_{N-1}$$

and for each $n = 1, \cdots, N-2$,

$$\hat{y}'_n \equiv \theta\kappa\hat{y}'_{n+1} = \frac{1}{\psi_{N-1}\psi_{N-2}\cdots\psi_{n+1}}\hat{y}_n > \hat{y}_n$$

When the state belongs to $\{\omega_N, \omega_{N-1}, \cdots, \omega_1\}$, the central bank announces the information, and trade does not take place. That is, the policy is bursting bubble, in the sense of Definition 1. At state ω_ϕ, asset x still is overpriced, but given that agent A_N does not know the fact, we do not call this as bubble.

Suppose $\kappa < 1$. Then, bubbles are detrimental to the economy, and hence the central bank wants to burst bubbles. Since it can burst bubbles when the state belongs to $\{\omega_N, \omega_{N-1}, \cdots, \omega_3\}$, the welfare gain of this policy is

$$-\sum_{n^*=3}^{N} \mu(\omega_{n^*}) \sum_{n=1}^{n^*-2} (\kappa - 1)\hat{y}_n = (1-\kappa) \sum_{n^*=3}^{N} \mu(\omega_{n^*}) \sum_{n=1}^{n^*-2} \hat{y}_n$$

However, there is a "side effect." This policy increases the terms of trade at state ω_ϕ, and thus the welfare loss is

$$-\mu(\omega_\phi) \sum_{n=1}^{N-1} (\kappa - 1)(\hat{y}'_n - \hat{y}_n) = (1-\kappa)\mu(\omega_\phi) \sum_{n=1}^{N-2} \left(\frac{1}{\psi_{N-1}\psi_{N-2}\cdots\psi_{n+1}} - 1 \right) \hat{y}_n$$

Hence, if we have

$$\sum_{n^*=3}^{N} \mu(\omega_{n^*}) \sum_{n=1}^{n^*-2} \hat{y}_n > \mu(\omega_\phi) \sum_{n=1}^{N-2} \left(\frac{1}{\psi_{N-1}\psi_{N-2}\cdots\psi_{n+1}} - 1 \right) \hat{y}_n$$

the central bank should employ the bubble-bursting policy. When $\kappa > 1$, the opposite happens, that is, it should employ the policy if the above inequality is reversed. However, regardless of the value of κ, it turns out that the side effect offsets the welfare gain.

Proposition 3. *The bubble-bursting policy has no effect on ex ante welfare.*

Proof. Since, for each $n = 1, \cdots, N-2$,

$$\psi_{N-1}\psi_{N-2}\cdots\psi_{n+1} = \frac{\mu(\omega_\phi)}{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+2})}$$

we have

$$\begin{aligned}
& \mu(\omega_\phi) \sum_{n=1}^{N-2} \left(\frac{1}{\psi_{N-1}\psi_{N-2}\cdots\psi_{n+1}} - 1 \right) \hat{y}_n \\
= \; & \mu(\omega_\phi) \sum_{n=1}^{N-2} \left[\frac{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+2})}{\mu(\omega_\phi)} - 1 \right] \hat{y}_n \\
= \; & \sum_{n=1}^{N-2} [\mu(\omega_N) + \cdots + \mu(\omega_{n+2})] \hat{y}_n \\
= \; & \sum_{n^*=3}^{N} \mu(\omega_{n^*}) \sum_{n=1}^{n^*-2} \hat{y}_n
\end{aligned}$$

The last equality holds by the following argument. In the sum $\sum_{n=1}^{n^*-2} \hat{y}_n$, we have $\hat{y}_n$ if and only if $n \leq n^* - 2$, or $n^* \geq n + 2$. Thus, in the sum $\sum_{n^*=3}^{N} \mu(\omega_{n^*}) \sum_{n=1}^{n^*-2} \hat{y}_n$, the coefficient of $\hat{y}_n$ is $[\mu(\omega_N) + \cdots + \mu(\omega_{n+2})]$, which is the same as that in the sum $\sum_{n=1}^{N-2} [\mu(\omega_N) + \cdots + \mu(\omega_{n+2})] \hat{y}_n$. $\square$

Even though the side effect occurs only at state ω_ϕ, it is enough large to offset the welfare gain. Therefore, if the central bank has to pay some cost to announce the information that the asset is worthless, it should not employ the bubble-bursting policy, or should keep the information secret. Conlon (2015) studies the same sort of bubble-bursting policies and shows that its effect on ex ante welfare is ambiguous, that is, there are cases where it improves ex ante welfare and other cases where it worsens ex ante welfare.[12]

[12]Conlon (2015) also considers the case where the central bank knows that the asset is worthless whenever it is worthless. We can study this case as well. When the state is ω_v, the central bank announces the information that the consumption value of the asset is $v > 0$, and hence all agents

5 Bubbles and Information

In this section, we investigate the relationship between the size of bubbles and the amount of information. Suppose that a state ω belongs to $\Omega \setminus \{\omega_v, \omega_\phi\}$. Then, the final user A_N receives a signal and hence, knows that the asset is worthless. In the model that we have studied so far, the information that A_N knows that the asset is worthless spreads from A_N to A_1 although it is subject to loss between any two agents. What if there is no possibility that middlemen $A_2, \cdots, A_{N-1}$ know that A_N knows that the asset is worthless?

As before, prior to trade, all agents except for A_N observe the consumption value of x for A_N. When it is zero, then A_N receives a signal with some probability. Otherwise, A_N does not receive any signals. Thus, if he receives a signal, then A_N is sure that the consumption value is zero, and in this event, every agent knows that x is worthless. However, now, A_N is the only agent who receives a signal. In words, the signal (rumor) that A_N knows that x is worthless *never* spreads.

To describe the difference between the information structures formally, we consider the three states that we have used before. The set of the states is

$$\Omega^0 = \{\omega_v, \omega_\phi, \omega_N\}$$

In other words, states $\omega_{N-1}, \cdots, \omega_1$ do not exist in the economy with Ω^0. Let μ^0 be the common prior distribution over Ω^0, and assume that $\mu^0(\omega) > 0$ for each $\omega \in \Omega^0$. Agent A_N's partition is

$$\mathcal{P}_N^0 = \{\{\omega_v, \omega_\phi\}, \{\omega_N\}\}$$

For each $n < N$, agent A_n's partition is

$$\mathcal{P}_n^0 = \{\{\omega_v\}, \{\omega_\phi, \omega_N\}\}$$

including A_N know that the consumption value is $v > 0$. Then, the terms of trade are increased by this policy because A_N's expected utility increases. When the state is $\omega \neq \omega_v$, the central bank announces the information that the asset is worthless, and thus trade does not take place. Even in this case, we can show that the policy has no effect on ex ante welfare.

Note that middlemen $A_2, \cdots, A_{N-1}$ never know that the final user A_N knows that the asset is worthless.[13]

Assume the economy is at state $\omega \neq \omega_v$. If A_N does not receive a signal, the expected value of x is

$$v_e^0 = \frac{\mu^0(\omega_v)}{\mu^0(\omega_v) + \mu^0(\omega_\phi)} v > 0$$

For A_{N-1}, the probability that A_N does not receive a signal is

$$\psi_{N-1}^0 = \frac{\mu^0(\omega_\phi)}{\mu^0(\omega_\phi) + \mu^0(\omega_N)}$$

The other middlemen $A_2, \cdots, A_{N-2}$ are not exposed to risk that each of them cannot find a greater fool. Define a sequence $(\hat{y}_n^0)_{n=1}^{N-1}$ as follows: for $N-1$,

$$\hat{y}_{N-1}^0 = \theta v_e^0$$

for $N-2$,

$$\hat{y}_{N-2}^0 = \theta \kappa \psi_{N-1}^0 \hat{y}_{N-1}^0$$

and for each $n = 1, \cdots, N-3$,

$$\hat{y}_n^0 = \theta \kappa \hat{y}_{n+1}^0$$

Then, by a similar argument to Lemma 1, we can show the following.

Lemma 3. *Assume $\omega \neq \omega_v$. In equilibrium, if agent A_N receives a signal, he does not trade with agent A_{N-1}, or $y_{N-1}^0 = 0$; if agent A_N does not receive a signal, he trades with agent A_{N-1} and obtains x in exchange for $y_{N-1}^0 = \hat{y}_{N-1}^0$. For $n = 1, \cdots, N-2$, agent A_{n+1} always trades with agent A_n and obtains x in exchange for $y_n^0 = \hat{y}_n^0$. Moreover, the equilibrium is unique.*

Now, we compare two economies with the different information structures. To make a fair comparison between the two economies with Ω and Ω^0, we assume $\mu(\omega_v) =$

[13] The economy with Ω^0 is equivalent to the one with Ω if $\mu(\omega) > 0$ for each $\omega \in \{\omega_v, \omega_\phi, \omega_N\}$ and $\mu(\omega) = 0$ for each $\omega \in \Omega \setminus \{\omega_v, \omega_\phi, \omega_N\}$.

$\mu^0(\omega_v)$ and $\mu(\omega_\phi) = \mu^0(\omega_\phi)$ in the following result. This assumption means that the probabilities that the consumption value of the asset x is $v > 0$ are the same across the two economies and the probabilities that the final user A_N knows that the asset x is worthless are also the same across the two economies.

Proposition 4. *Assume $\mu(\omega_v) = \mu^0(\omega_v)$ and $\mu(\omega_\phi) = \mu^0(\omega_\phi)$. Then, $\hat{y}_n$ and $\hat{y}^0_n$ satisfy the following: for $N-1$,*

$$\hat{y}_{N-1} = \hat{y}^0_{N-1}$$

and for each $n = 1, \cdots, N-2$,

$$\hat{y}_n > \hat{y}^0_n$$

Proof. For $N-1$, we have

$$\hat{y}_{N-1} = \theta v_e = \theta \frac{\mu(\omega_v)}{\mu(\omega_v) + \mu(\omega_\phi)} v = \theta \frac{\mu^0(\omega_v)}{\mu^0(\omega_v) + \mu^0(\omega_\phi)} v = \theta v^0_e = \hat{y}^0_{N-1}$$

For $N-2$, we have

$$\begin{aligned} \hat{y}_{N-2} - \hat{y}^0_{N-2} &= \theta\kappa\psi_{N-1}\hat{y}_{N-1} - \theta\kappa\psi^0_{N-1}\hat{y}^0_{N-1} \\ &= v_e\theta^2\kappa(\psi_{N-1} - \psi^0_{N-1}) \end{aligned}$$

Note that

$$\psi_{N-1} = \frac{\mu(\omega_\phi)}{\mu(\omega_\phi) + \mu(\omega_N)} > \frac{\mu(\omega_\phi)}{\mu(\omega_\phi) + \mu^0(\omega_N)} = \frac{\mu^0(\omega_\phi)}{\mu^0(\omega_\phi) + \mu^0(\omega_N)} = \psi^0_{N-1}$$

Hence, we obtain $\hat{y}_{N-2} > \hat{y}^0_{N-2}$.

For $n = 3, \cdots, N-1$, we have

$$\begin{aligned} \hat{y}_{N-n} - \hat{y}^0_{N-n} &= \theta\kappa\psi_{N-n+1}\hat{y}_{N-n+1} - \theta\kappa\hat{y}^0_{N-n+1} \\ &= v_e\theta^n\kappa^{n-1}(\psi_{N-n+1}\psi_{N-n+2}\cdots\psi_{N-1} - \psi^0_{N-1}) \end{aligned}$$

Note that

$$\begin{aligned} \psi_{N-n+1}\psi_{N-n+2}\cdots\psi_{N-1} - \psi^0_{N-1} &= \frac{\mu(\omega_\phi)}{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{N-n+2})} - \frac{\mu(\omega_\phi)}{1 - \mu(\omega_v)} \\ &> 0 \end{aligned}$$

Hence, we obtain $\hat{y}_{N-n} > \hat{y}^0_{N-n}$. □

This result means that, during bubbles, the deviation of prices from the fundamental value of the asset is greater in the economy with Ω than in the economy with Ω^0 although the ex ante expected prices are the same across the two economies. Hence, in states where some middlemen do not know that the final user knows that the asset is worthless, the possibility that middlemen know that the final user knows that the asset is worthless increases the size of bubbles. In the economy with Ω, agent A_{N-1} considers the probability that agent A_N does not receive a signal conditional *not only* on the event that the consumption value of x for A_N is zero *but also* on the event that A_{N-1} does not receive a signal. On the other hand, in the economy with Ω^0, agent A_{N-1} considers the probability that agent A_N does not receive a signal conditional *only* on the event that the consumption value of x for A_N is zero. Hence, when A_{N-1} does not know that A_N knows that the asset is worthless, A_{N-1}'s expected utility of obtaining x is greater in the economy with Ω. Note that, in the economy with Ω^0, all middlemen except for A_{N-1} are not exposed to risk that each of them may be the greatest fool, which raises prices. However, the effect by A_{N-1}'s expectation outweighs the effect by the reduction of the risk for all middlemen except for A_{N-1}, and hence we have $\hat{y}_n > \hat{y}_n^0$ for each $n = 1, \cdots, N-2$. This result suggests that the development of information technology for financial intermediaries may make the economy more fragile as long as it is incomplete in the sense that they may not receive information.

In Section 4.3, middlemen interpret the inaction of the central bank as an implicit endorsement of the asset prices, which raises the prices. In other words, middlemen obtain more information from observing the inaction, and prices increase. On the other hand, in this section, we demonstrated that the *possibility* that middlemen obtain more information raises the asset prices. These results are related but induced by different mechanisms.

A bubble occurs with probability $\mu(\omega_N)+\mu(\omega_{N-1})+\cdots+\mu(\omega_3)$ in the economy with Ω and with probability $\mu^0(\omega_N)$ in the economy with Ω^0. Hence, under the assumption that $\mu(\omega_v) = \mu^0(\omega_v)$ and $\mu(\omega_\phi) = \mu^0(\omega_\phi)$, a bubble occurs with higher probability in the economy with Ω^0 by $\mu(\omega_2) + \mu(\omega_1)$.

6 Examples

In this section, we provide two simple examples with different prior distributions. Both examples satisfy the assumptions that we have imposed in the previous sections.

Example 1. The distribution μ is uniform, that is, for each $\omega \in \Omega$,

$$\mu(\omega) = \frac{1}{N+2}$$

It is obvious that $\mu(\omega) > 0$ for each $\omega \in \Omega$ and $\mu(\omega_2) \leq \cdots \leq \mu(\omega_N)$. The probability ψ_n is

$$\psi_n = \frac{N-n}{N-n+1}$$

and the price $\hat{y}_n$ is

$$\hat{y}_n = \frac{1}{N-n}\theta^{N-n}\kappa^{N-n-1}v_e$$

where

$$v_e = \frac{1}{2}v$$

A bubble occurs when the state belongs to $\{\omega_N, \omega_{N-1}, \cdots, \omega_3\}$, and hence the probability that a bubble occurs is

$$1 - [\mu(\omega_v) + \mu(\omega_\phi) + \mu(\omega_2) + \mu(\omega_1)] = 1 - \frac{4}{N+2}$$

The probability is increasing in N and converges to one as $N \to \infty$. In words, a bubble occurs with an *arbitrarily* high probability. Finally, when $\mu(\omega_v) = \mu^0(\omega_v)$ and $\mu(\omega_\phi) = \mu^0(\omega_\phi)$, the price $\hat{y}_n^0$ is as follows: for $N-1$

$$\hat{y}_{N-1}^0 = \theta v_e$$

and for each $n = 1, \cdots, N-2$,

$$\hat{y}_n^0 = \frac{1}{N+1}\theta^{N-n}\kappa^{N-n-1}v_e$$

See Figure 1 for the graph of $\hat{y}_n$ and $\hat{y}_n^0$. □

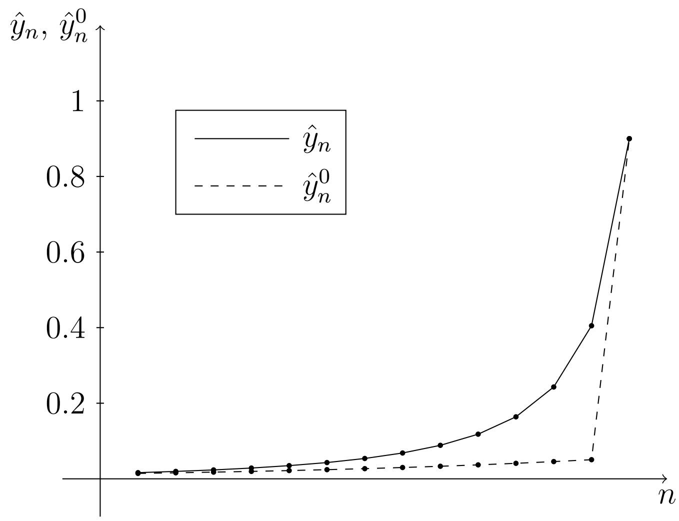

Figure 1: Graph of $\hat{y}_n$ and $\hat{y}_n^0$ for Example 1 ($N = 15$, $\theta = 0.9$, $\kappa = 1$, and $\upsilon = 2$)

Example 2. Consider a situation where the signal to A_N is lost with probability ε, and moreover, between any two adjacent agents, each signal is lost with the same probability, ε. In this case, we have

$$\mu(\omega_\phi) = [1 - \mu(\omega_\upsilon)]\varepsilon$$

for each $n = 2, \cdots, N$,

$$\mu(\omega_n) = [1 - \mu(\omega_\upsilon)](1 - \varepsilon)^{N-n+1}\varepsilon$$

and for $n = 1$,

$$\mu(\omega_1) = [1 - \mu(\omega_\upsilon)](1 - \varepsilon)^N$$

We have $\mu(\omega_\phi) = [1 - \mu(\omega_\upsilon)]\varepsilon$ because the final user A_N does not receive any signals with probability ε given that the asset is worthless. For each $n = 2, \cdots, N$, we have $\mu(\omega_n) = [1-\mu(\omega_\upsilon)](1-\varepsilon)^{N-n+1}\varepsilon$ because the signal is not lost between any two adjacent agents until A_n receives it, but it is lost between A_n and A_{n-1}. We have $\mu(\omega_1) = [1 - \mu(\omega_\upsilon)](1 - \varepsilon)^N$ because the signal is not lost between any two adjacent agents. Assume $0 < \mu(\omega_\upsilon) < 1$ and $0 < \varepsilon < 1$. Then, it is again obvious that $\mu(\omega) > 0$ for each

$\omega \in \Omega$ and $\mu(\omega_2) \leq \cdots \leq \mu(\omega_N)$. The probability ψ_n is

$$\psi_n = \frac{1-(1-\varepsilon)^{N-n}}{1-(1-\varepsilon)^{N-n+1}}$$

and the price $\hat{y}_n$ is

$$\hat{y}_n = \frac{\varepsilon}{1-(1-\varepsilon)^{N-n}} \theta^{N-n} \kappa^{N-n-1} v_e$$

where

$$v_e = \frac{\mu(\omega_v)}{\mu(\omega_v) + [1-\mu(\omega_v)]\varepsilon} v$$

A bubble occurs when the state belongs to $\{\omega_N, \omega_{N-1}, \cdots, \omega_3\}$, and hence the probability that a bubble occurs is

$$1 - [\mu(\omega_v) + \mu(\omega_\phi) + \mu(\omega_2) + \mu(\omega_1)] = [1-\mu(\omega_v)](1-\varepsilon)[1-(1-\varepsilon)^{N-2}]$$

The probability is increasing in N. Moreover, the probability is increasing in ε if $\varepsilon < \overline{\varepsilon}$ and decreasing in ε if $\varepsilon > \overline{\varepsilon}$ where

$$\overline{\varepsilon} = 1 - \left(\frac{1}{N-1}\right)^{\frac{1}{N-2}}$$

Finally, when $\mu(\omega_v) = \mu^0(\omega_v)$ and $\mu(\omega_\phi) = \mu^0(\omega_\phi)$, the price $\hat{y}^0_n$ is as follows: for $N-1$

$$\hat{y}^0_{N-1} = \theta v_e$$

and for each $n = 1, \cdots, N-2$,

$$\hat{y}^0_n = \varepsilon \theta^{N-n} \kappa^{N-n-1} v_e$$

See Figure 2 for the graph of $\hat{y}_n$ and $\hat{y}^0_n$. □

7 Conclusion

We developed a finite-period model of intermediaries and, assuming neither irrational agents nor heterogeneous priors, showed that a bubble and a burst can occur in a

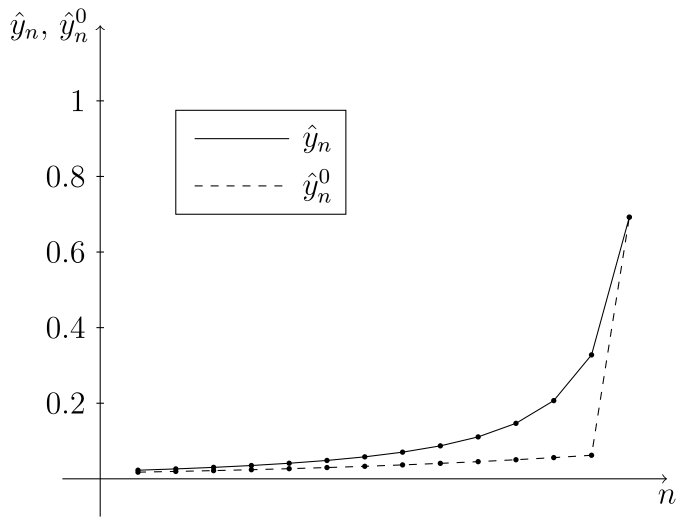

Figure 2: Graph of $\hat{y}_n$ and $\hat{y}_n^0$ for Example 2 ($N = 15$, $\theta = 0.9$, $\kappa = 1$, $v = 2$, $\mu(\omega_v) = 1/17$, and $\varepsilon = 0.1$)

unique equilibrium. The equilibrium price is increasing and accelerating during bubbles although the fundamental value of the asset is constant at zero over time. Bubbles may be detrimental to the economy; however, it turned out that the bubble-bursting policy has no effect on welfare. Moreover, we investigated the relationship between the size of bubbles and the amount of information and showed that the possibility that agents obtain more information about the underlying economy increases the size of bubbles. We focused on the simple network, bilateral trade, and bargaining. It would be interesting to extend our model to more complicated networks, different matching technologies, and different pricing mechanisms. We leave these as future works.

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