Small and orthodox fiscal multipliers at the zero lower bound.*†‡

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Abstract

Does fiscal policy have large and qualitatively different effects on the economy when the nominal interest rate is zero? An emerging consensus in the New Keynesian literature is that the answer is yes. New evidence provided here suggests that the answer is often no. For a broad range of empirically relevant parameterizations of the Rotemberg model of costly price adjustment, the government purchase multiplier is about one or less and the response of hours to a labor tax increase is either negative or close to zero.

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1 Introduction

The recent experiences of Japan, the United States, and Europe with zero/near-zero nominal interest rates have raised new questions about the conduct of monetary and fiscal policy in a liquidity trap. A large and growing body of new research has emerged that provides answers using New Keynesian (NK) frameworks that explicitly model the zero bound on the nominal interest rate. The zero bound on the nominal interest rate is particularly important in the NK model because the interest rate policy of the monetary authority plays a central role in stabilizing the economy. Very low nominal interest rates constrain the ability of monetary policy to respond to shocks and this may result in macroeconomic instability.

One conclusion that has emerged is that fiscal policy has very different effects on the economy when the nominal interest rate is zero. Eggertsson (2011) finds that hours worked increase sharply in response to an increase in the labor tax, a property that he refers to as the "paradox of toil." Christiano, Eichenbaum, and Rebelo (2011), Woodford (2011) and Erceg and Lindé (2010) find that the size of the government purchase multiplier is close to two or even larger. These findings stand in stark contrast to the properties of the NK model outside the zero bound where the government purchase multiplier is typically about one or less and a higher labor tax has the orthodox property that hours worked fall. The result that fiscal policy is most potent in precisely those situations where monetary policy is least effective is a powerful argument for more fiscal stimulus.

New evidence provided in this paper reveals that the properties of fiscal policy in the NK model at the zero bound are often not all that different from the properties of fiscal policy away from the zero bound. We formulate and estimate a tractable model with Rotemberg (1982) costly price adjustment and find that for a broad range of empirically relevant parameterizations of the model the government purchase multiplier is about 1 or less. Moreover, the response of hours to a higher labor tax is either negative or, when positive, so small that the conclusion is that hours are inelastic to change in the labor tax rate. These results obtain when parameterizing the model to reproduce either the U.S. Great Recession or the U.S. Great Depression.

Why are our results so different from the previous findings? Focusing on parameterizations of the model that can reproduce the declines in output and inflation observed during either the U.S. Great Recession or the U.S. Great Depression is important. When this is done, small government purchase multipliers of about one or less and orthodox responses of hours to an increase in the labor tax are commonplace. We show that these empirical restrictions impose tight restrictions on two parameters: the slope of the New Keynesian Philips curve and the expected duration of zero interest rates. This allows us to provide simple intuition for our results that is robust to some of the specific details of our model and to relate our findings to previous findings in the literature.

Another reason why our findings are so different pertains to the solution method. Previous results are based on a solution method that models the nonlinearity induced by the zero bound on the nominal interest rate but that loglinearizes the other equilibrium conditions about a stable price level. We show that this solution method can break down in several distinct ways when analyzing the zero bound.

One breakdown arises from the fact that the loglinearized solution rules out local dynamics that obtain in the true nonlinear model at the zero bound. Perhaps the most significant example of this point is that the loglinearized solution does not admit conventional responses of both hours and inflation to a higher labor tax or of both output and inflation to an improvement in technology when the nominal interest rate is zero. We show that the true nonlinear NK model, in contrast, can and does exhibit conventional responses at the zero bound using empirically relevant parameterizations of the model.

Loglinearized solutions can also distort the parameters of the model that are needed to hit a pre-specified set of targets and this in turn can result in distorted inferences about the effects of fiscal policy. This problem is most severe when parameterizing the model to hit targets from the Great Depression. For instance, we find that the size of the government purchase multiplier drops from 1.8 to 1.2 and that the paradox of toil disappears when we use an exact solution instead of a loglinearized solution to hit the same targets.

Our analysis does not rule out the possibility of very large fiscal multipliers. Previous research by Woodford (2011) and Carlstrom, Fuerst, and Paustian (2012) has found that fiscal multipliers can exhibit asymptotes at the zero bound using loglinearized solutions. In the neighborhood of an asymptote, the response of GDP to a small change in government purchases, for instance, can be arbitrarily large and positive or arbitrarily large and negative. Asymptotes also occur in our nonlinear NK model. They are located in a region of the parameter space that is small but of empirical interest. Depending on the details of the specification the asymptote occurs when the expected duration of zero interest rates ranges from 7 to 25 quarters.

We make these points in a nonlinear stochastic NK model with quadratic price adjustment costs as in Rotemberg (1982). Rotemberg costly price adjustment has been used by Benhabib, Schmitt-Grohe, and Uribe (2001) to analyze multiplicity of equilibria due to the zero bound on interest rates; Evans, Guse, and Honkapohja (2008) to model learning in a similar environment; Aruoba and Schorfheide (2013) to compare zero bound fundamentals and sunspot equilibria during the Great Recession; Gust, Lopez-Salido, and Smith (2012) to assess the relative importance of demand and technology shocks during the Great Recession; and Eggertsson, Ferrero, and Raffo (2013) to consider the effects of structural reforms in a liquidity trap. Our model also delivers the same loglinearized equilibrium conditions as the Calvo model with firm specific labor of Eggertsson (2011) and the Calvo model with homogeneous labor of Woodford (2011).

A distinct advantage of Rotemberg as compared to Calvo price adjustment is tractability. There are no endogenous state variables in our setup and the equilibrium conditions for hours and inflation can be reduced to two nonlinear equations when the zero bound is binding. These equations are the nonlinear analogues of what Eggertsson and Krugman (2012) refer to as "aggregate demand" (AD) and "aggregate supply" (AS) schedules. This structure allows us to provide intuition for our results and makes it possible to characterize some properties of equilibrium analytically. When analytical results are not available, it is straightforward to compute the various zero bound equilibria by finding the roots of a nonlinear equation in the inflation rate and then checking the other equilibrium conditions. The resulting numerical solutions are accurate up to the precision of the computer.

Rotemberg costly price adjustment is more tractable than Calvo price setting, yet it is rich enough to capture a common element of both models of price adjustment that is absent from the loglinearized equilibrium conditions. Both models of costly price adjustment have the property that price adjustment absorbs resources whose size changes endogenously with the inflation rate. Abstracting from this resource cost has important consequences for the properties of the model in the zero bound state. For instance, our finding that the loglinearized equilibrium conditions rule out conventional local dynamics that obtain in the true model can be attributed to the fact that the resource cost of price adjustment is zero in the loglinearized solution.

Our research is closest to research by Christiano and Eichenbaum (2012) which is, in part, a response to an earlier version of this paper which demonstrated the possibility of multiple zero bound equilibria. They consider a similar setup to ours and confirm the possibility of multiple zero bound equilibria. They go on to show that imposing a particular form of e-learnability rules out one of the two equilibria that occur in their model and find that the qualitative properties of the remaining equilibrium are close to the loglinearized solution under their baseline parameterization. But the parameterization they consider does not reproduce the declines in output and inflation during the Great Recession documented by Christiano, Eichenbaum, and Rebelo (2011). When we adjust their parameterization to hit these targets we find that the government purchase multiplier in their favored equilibrium falls from 2.18 to 1.08. More importantly, our main conclusions about the size and sign of fiscal multipliers do not rely on multiplicity of zero bound equilibrium. In fact, some of our strongest results occur in regions of the parameter space where equilibrium is unique. We also describe some problems with applying their E-learning equilibrium selection strategy in our setting. It does not omit all forms of multiplicity that we encounter and selects equilibria that are not empirically relevant while ruling out equilibria that reproduce the data targets.

Our research is also related to but distinct from recent work by Gust, Lopez-Salido, and Smith (2012) and Aruoba and Schorfheide (2013). Gust, Lopez-Salido, and Smith (2012) compare and contrast the role of technology and demand shocks in the Great Recession and find that demand shocks are more important. Aruoba and Schorfheide (2013) building on previous work by Mertens and Ravn (2010) contrast the view that the Great Recession was induced by an exogenous switch in expectations (a sunspot equilibrium) with an alternative view that it was driven by shocks to technology, demand and monetary policy. We also model shocks to technology. They make it possible to reconcile parameterizations of the model that imply a large slope of the New-Keynesian Phillips curve with outcomes from the Great Recession and Great Depression. Both events exhibited large output declines but relatively small declines in the inflation rate. We don't analyze exogenous sunspot equilibria here. However, some of the zero bound equilibria that reproduce the Great Recession occur in regions of the parameter space where there is another equilibrium with a positive interest rate. It follows that exogenous sunspot equilibria can be constructed that fluctuate back and forth between positive interest rates and zero interest rates. We also encounter multiple zero bound equilibria which opens the door to a new type of exogenous sunspot equilibrium that fluctuates between two different zero bound equilibria.

A major advantage of our model is that it is relatively straightforward to determine whether a nonlinear equation has multiple roots and, if so, how many of those are zero bound equilibria. Ascertaining the presence of multiple equilibria and computing them is a much more challenging task in the versions of the Rotemberg model considered by Gust, Lopez-Salido, and Smith (2012) and Aruoba and Schorfheide (2013) or the model with Calvo pricing considered by Fernandez-Villaverde, Gordon, Guerron-Quintana, and Rubio-Ramirez (2012). More generally, our findings of multiple zero bound equilibria, asymptotes in fiscal multipliers and changes in the model's local dynamics away from the asymptotes is a matter of some concern because it raises the possibility that these same phenomena are possible in these other models. Our results offer guidance about the regions of the parameter/shock space where these issues are most likely to arise.

Finally, our research is related to previous work by Braun and Waki (2010) who find that the government purchase multipliers are generally well above one under both Rotemberg and

Calvo price setting in a nonlinear perfect nonlinear foresight model with capital accumulation. Their parameterization of the model is not calibrated. One message of our paper is that government purchase multipliers are much smaller when one uses a parameterization of the NK model that reproduces targets from either the Great Recession or the Great Depression.

The remainder of our analysis proceeds in the following way. In Section 2 we describe the model and equilibrium concept. Then in Section 3 we provide a characterization of zero bound equilibria in the loglinearized economy. Section 4 contains our analytical results for the nonlinear economy. The parametrization of our model is discussed in Section 5. Sections 6 and 7 report numerical results and Section 8 concludes.

2 Model and equilibrium

We consider a stochastic NK model with Rotemberg (1982) quadratic costs of price adjustment faced by intermediate goods producers. Monetary policy follows a Taylor rule when the nominal interest rate exceeds its lower bound of zero. The equilibrium analyzed here is the Markov equilibrium proposed by Eggertsson and Woodford (2003).

2.1 The model

Households. The representative household chooses consumption c_t, hours h_t and bond holdings b_t to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big(\prod_{j=0}^{t} d_j \Big) \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\nu}}{1+\nu} \right\} \tag{1}$$

subject to

$$b_t + c_t = \frac{b_{t-1}(1+R_{t-1})}{1+\pi_t} + w_t h_t (1-\tau_{w,t}) + T_t. \tag{2}$$

The preference discount factor from period t to $t+1$ is βd_{t+1}, and d_t is a preference shock. We assume that the value of d_{t+1} is revealed at the beginning of period t. The variable T_t includes transfers from the government and profit distributions from the intermediate producers. The optimality conditions for consumption and labor supply choices are

$$c_t^{\sigma} h_t^{\nu} = w_t (1-\tau_{w,t}), \tag{3}$$

and

$$1 = \beta d_{t+1} E_t \left\{ \frac{1+R_t}{1+\pi_{t+1}} \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} \right\}. \tag{4}$$

Final good producers. Perfectly competitive final good firms use a continuum of intermediate goods $i \in [0, 1]$ to produce a final good with the technology: $y_t = [\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}$. The profit maximizing input demands for final goods firms are

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} y_t, \tag{5}$$

where $p_t(i)$ denotes the price of the good produced by firm i and P_t the price of the final good. The price of the final good satisfies $P_t = [\int_0^1 p_t(i)^{1-\theta} di]^{1/(1-\theta)}$.

Intermediate goods producers. Producer of intermediate good i uses labor to produce output, using the linear technology: $y_t(i) = z_t h_t(i)$, where z_t is the common productivity. Labor market is not firm-specific and thus for all firms real marginal cost equals w_t/z_t. Producer i sets prices to maximize

$$E_0 \sum_{t=0}^{\infty} \lambda_{c,t} \left[(1+\tau_s) p_t(i) y_t(i) - P_t \frac{w_t}{z_t} y_t(i) - \frac{\gamma}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 P_t y_t \right] / P_t \tag{6}$$

subject to the demand function (5). Here $\lambda_{c,t}$ is the stochastic discount factor and is equal to $\beta^t (\prod_{j=0}^{t} d_j) c_t^{-\sigma}$ in equilibrium. We assume that the intermediate goods producers receive a sales subsidy τ_s such that $(1+\tau_s)(\theta-1) = \theta$, i.e. zero profits in the zero inflation steady state. The last term in the brackets is a quadratic cost of price adjustment. This is assumed to be proportional to the aggregate production y_t, so that the share of price adjustment costs in the aggregate production depends only on the inflation rate. The first order condition for this problem in a symmetric equilibrium is:

$$0 = \frac{w_t}{z_t} - 1 - \frac{\gamma}{\theta} \pi_t (1+\pi_t) + \beta d_{t+1} E_t \left\{ \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} \frac{y_{t+1}}{y_t} \frac{\gamma}{\theta} \pi_{t+1} (1+\pi_{t+1}) \right\} \tag{7}$$

where $\pi_t = P_t/P_{t-1} - 1$ is the net inflation rate, and we have $y_t(i) = y_t$ and $h_t(i) = h_t$.

Monetary policy. Monetary policy follows a Taylor rule that respects the zero lower bound on the nominal interest rate

$$R_t = \max(0, r_t^e + \phi_\pi \pi_t + \phi_y \hat{gdp}_t), \tag{8}$$

where $r_t^e \equiv 1/(\beta d_{t+1}) - 1$ and $\hat{gdp}_t$ is the log deviation of GDP from its steady state value.[1]

[1] The assumption that monetary policy responds directly to variations in d_t is made to facilitate comparison with other papers in the literature.

The aggregate resource constraint is given by

$$c_t = (1 - \kappa_t - \eta_t) y_t, \tag{9}$$

where $\kappa_t \equiv (\gamma/2)\pi_t^2$ is the resource cost of price adjustment and government purchases $g_t = \eta_t y_t$. Gross domestic product in our economy, gdp_t, is

$$gdp_t \equiv (1 - \kappa_t) y_t = c_t + g_t. \tag{10}$$

This definition of GDP assumes that the resource costs of price adjustment are intermediate inputs and are consequently subtracted from gross output when calculating GDP.

The distinction between GDP and output, $y_t (= z_t h_t)$, plays a central role in the analysis that follows. We will show below that loglinearizing equation (10) about a steady state with a stable price level can introduce some large biases when analyzing the properties of this economy in a liquidity trap. To see why this is the case, observe first that GDP falls when the term κ_t increases, even when output is unchanged. It follows that both the sign and magnitude of movements in GDP can differ from output. However, when equation (10) is loglinearized about a steady state with a stable price level, the term κ disappears, and GDP and output are always equal. In the neighborhood of the steady state, κ is so small that it can safely be ignored. Ignoring κ is not innocuous if a large shock moves the economy outside of this neighborhood . One situation where the distinction between GDP and output can be important is when one or more shocks drive the nominal interest rate to zero. The distinction between GDP and output is not specific to our model of price adjustment. An analogous term to κ also appears in the resource contraint under Calvo price setting. In that setting the term governs the resource cost of price dispersion (see Yun (2005)) and loglinearizing about a stable price level creates the same type of bias.

2.2 Stochastic equilibrium with zero interest rates

To facilitate comparisons with the previous literature, we consider the Markov equilibrium proposed by Eggertsson and Woodford (2003). The state of the economy s_t is assumed to follow a two-state, time-homogeneous Markov chain with state space $\{L, N\}$ (low and normal) and initial condition $s_0 = L$. Its transition probability from the L state to the L state is $p < 1$ and that from the N state to the N state is 1. We assume that the preference shock d_{t+1}, technology shock z_t, and fiscal policies $(\tau_{w,t}, \eta_t)$ change if and only if s_t changes: $(d_{t+1}, z_t, \tau_{w,t}, \eta_t)$ equals $(d^L, z^L, \tau_w^L, \eta^L)$ when $s_t = L$, and $(1, z, \tau_w, \eta)$ when $s_t = N$.

We focus exclusively on a Markov equilibrium with state s. Such an equilibrium is characterized by two distinct values for prices and quantities: one value obtains when $s_t = L$ and the other obtains when $s_t = N$. We will use the superscript L to denote the former value and no subscript to indicate the latter value. We assume further that the economy is in a steady state with zero inflation in the N state, i.e. $h = \{(1-\tau_w)/(z^{\sigma-1}(1-\eta)^{\sigma})\}^{1/(\sigma+\nu)}$ and $\pi = 0$ if $s_t = N$. This implies that in the N state the zero bound doesn't bind, i.e. it is not the "unintended" zero interest rate steady state in Benhabib, Schmitt-Grohe, and Uribe (2001) and Bullard (2010). When the zero bound binds in the L state, we call such an equilibrium a *zero bound equilibrium*. We limit attention to situations where this type of equilibrium exists and assume $p\beta d^L < 1$, to guarantee that utility is finite. Although this structure is limited in the sense that it only has two states, it is rich, for it encapsulates a case with preference shock only, a case with technology shock only, a case with sunspot shock only $((d^L, z^L, \tau_w^L, \eta^L) = (1, z, \tau_w, \eta))$, and any combinations of these (with perfect correlation).

2.3 Equilibrium hours and inflation in a zero bound equilibrium

An attractive feature of our framework is that the equilibrium conditions for hours and inflation in the L state can be summarized by two equations in these two variables. These equations are nonlinear versions of what Eggertsson and Krugman (2012) refer to as "aggregate supply" (AS) and "aggregate demand" (AD) schedules. In what follows, we adopt the same shorthand when referring to these two equations.

The AS schedule is an equilibrium condition that summarizes intermediate goods firms' price setting decisions, the household's intratemporal first order condition, and the aggregate resource constraint. To obtain the AS schedule, we start with (7), substitute out the real wage using (3), and then use (9) to replace consumption with hours

$$\begin{aligned} 0 =& \frac{(1-\kappa_t-\eta_t)^{\sigma}}{(1-\tau_{w,t})z_t^{1-\sigma}} h_t^{\nu+\sigma} - 1 - \frac{\gamma}{\theta}\pi_t(1+\pi_t) \\ &+ \beta d_{t+1} E_t \left\{ \left(\frac{1-\kappa_t-\eta_t}{1-\kappa_{t+1}-\eta_{t+1}} \right)^{\sigma} \left(\frac{z_t h_t}{z_{t+1} h_{t+1}} \right)^{\sigma-1} \frac{\gamma}{\theta}\pi_{t+1}(1+\pi_{t+1}) \right\}. \end{aligned} \tag{11}$$

The AD schedule summarizes the household's Euler equation and the resource constraint. It is obtained by substituting consumption out of the household's intertemporal Euler equation (4) using the resource constraint (9). The resulting AD schedule is

$$1 = \beta d_{t+1} E_t \left\{ \frac{1+R_t}{1+\pi_{t+1}} \left(\frac{(1-\kappa_t-\eta_t)z_t}{(1-\kappa_{t+1}-\eta_{t+1})z_{t+1}} \right)^{\sigma} \left(\frac{h_t}{h_{t+1}} \right)^{\sigma} \right\}. \tag{12}$$

In a zero bound Markov equilibrium, the AD and AS schedules become

$$1 = p\left(\frac{\beta d^L}{1+\pi^L}\right) + (1-p)\beta d^L \left(\frac{\left(1-\kappa^L-\eta^L\right) z^L h^L}{(1-\eta) z h}\right)^{\sigma} \tag{13}$$

$$\pi^L(1+\pi^L) = \frac{\theta}{\gamma(1-p\beta d^L)}\left[\frac{\left(1-\kappa^L-\eta^L\right)^{\sigma}(h^L)^{\sigma+\nu}}{(1-\tau_w^L)(z^L)^{1-\sigma}} - 1\right] \tag{14}$$

where $\kappa^L = (\gamma/2)(\pi^L)^2$ is the price adjustment costs in the L state.[2]

We have chosen to express the aggregate demand and supply schedules in terms of labor input rather than GDP. As pointed out above, labor input and GDP can move in independent directions in our model. Promoting employment is one of the mandates of U.S. monetary policy and our choice helps one to understand how labor input responds to fiscal shocks at the zero bound and why it responds in a particular way. For instance, our assumption facilitates understanding when the response of labor input/employment to a change in the labor tax rate is positive and when it is negative and why. This does not mean that GDP is not also important. For completeness we document properties of both variables when we report our results.

3 A characterization of zero bound equilibria and fiscal multipliers using the loglinearized equilibrium conditions

In this section we show that there are two types of zero bound Markov equilibria using the loglinearized equilibrium conditions and that the fiscal multipliers have different signs and magnitudes in each type of equilibrium. These results are an important reference point for the global analysis of the nonlinear model that follows and are also of independent interest.

[2]Other equilibrium objects are recovered as $y^L = z^L h^L$, $gdp^L = (1-\kappa^L)y^L$, $c^L = \left(1-\kappa^L-\eta^L\right)y^L$, etc. Strictly speaking, not all pairs (h^L, π^L) that solve this system are equilibria: $\left(1-\kappa^L-\eta^L\right) \geq 0$ and $\hat{r}_L^e + \phi_\pi \pi^L + \phi_y \hat{y}^L \leq 0$ must also be satisfied.

3.1 Type 1 and Type 2 equilibria

Following the literature we start by log-linearizing (13) and (14) about the zero inflation steady state

$$\pi^L = \frac{1-p}{p}\sigma\left[\hat{h}^L + \hat{z}^L\right] + \frac{1}{p}[-\hat{r}^e_L - (1-p)\sigma\frac{\hat{\eta}^L}{1-\eta}], \tag{15}$$

$$\pi^L = \frac{\theta(\sigma+\nu)}{(1-p\beta)\gamma}\hat{h}^L + \frac{\theta}{(1-p\beta)\gamma}\Big[-\sigma\frac{\hat{\eta}^L}{1-\eta} + \frac{\hat{\tau}^L_w}{1-\tau_w} - (1-\sigma)\hat{z}^L\Big], \tag{16}$$

where we have used the fact that $\theta = (\theta-1)(1+\tau_s)$, and where $\hat{z}^L = \ln(z^L/z)$, $\hat{r}^e_L = 1/\beta - 1 - \hat{d}^L$, $\hat{\eta}^L = \eta^L - \eta$ and $\hat{\tau}^L_w = \tau^L_w - \tau_w$.[3] The other log-linearized equilibrium objects are given by $\hat{y}^L = \hat{z}^L + \hat{h}^L$, $\hat{gdp}^L = \hat{y}^L$ and $\hat{c}^L = \hat{y}^L - \eta/(1-\eta) \times \hat{\eta}^L$.[4] Observe that the resource costs of price adjustment have disappeared from the aggregate resource constraint. This property of the loglinearized equilibrium conditions plays a central role in our subsequent analysis. Another property pertains to the signs of the coefficients on $\hat{h}^L$ in (15) and (16) which we subsequently refer to as *slope*(AD) and *slope*(AS). They are both positive. It follows that there are two types of zero bound equilibria and neither of them exhibits a conventional configuration of demand and supply. We now provide conditions for each type of equilibrium. To facilitate the exposition suppose that $\hat{\eta}^L = \hat{\tau}^L_w = 0$.

Proposition 1 Existence of a zero bound equilibrium. *Suppose* $\hat{\eta}^L = \hat{\tau}^L_w = 0$, $(\phi_\pi, \phi_y) \geq (p, 0)$, $\hat{d}^L \geq 0$, $\hat{z}^L \leq 0$, $0 < p \leq 1$, $\sigma \geq 1$, *and that the AD schedule and the AS schedules are not identical. Then there exists a unique zero bound equilibrium with deflation and depressed labor input,* $(\pi^L, \hat{h}^L) < (0,0)$, *if*

1a) $\left(slope(AD) - slope(AS)\frac{\sigma-1}{\sigma+\nu}\right)\hat{z}^L - \frac{\hat{r}^e_L}{p} > 0$ *and*

1b) $slope(AD) > slope(AS)$,

or

2a) $\left(slope(AD) - slope(AS)\frac{\sigma-1}{\sigma+\nu}\right)\hat{z}^L - \frac{\hat{r}^e_L}{p} < 0$ *and*

2b) $slope(AD) < slope(AS)$.

If the parameters do not satisfy either both 1a) and 1b) or alternatively both 2a) and 2b), then there is no zero bound equilibrium with depressed labor input $\hat{h}^L < 0$.[5]

[3] Note that $\hat{r}^e_t$ is the linearized version of $r^e_L = 1/(\beta d^L) - 1$.

[4] In addition, $1 - \kappa^L - \eta^L \geq 0$ and $\hat{r}^e_L + \phi_\pi \pi^L + \phi_y \hat{y}^L \leq 0$ must also be satisfied.

[5] Proofs for this and all other propositions are found in Appendix A.

We say that a zero bound equilibrium is of Type 1 when 1a) and 1b) are satisfied, and that it is of Type 2 when 2a) and 2b) are satisfied. We also refer to configurations of parameters that satisfy 1*a*) and 1*b*) as "Type 1 parameterizatons" and those that satisfy 2*a*) and 2*b*) as "Type 2 parameterizations."

The final statement in Proposition 1 leaves open the possibility that a zero bound equilibrium with $\hat{h}^L \geq 0$ exists for parameterizations that satisfy 1a) and 2b) (or 1b) and 2a)). This is a serious possibility because if z^L is sufficiently low, labor input will be above its steady state level in state L. The nominal interest rate could still be zero if there is sufficient deflation due to high d^L and/or GDP is lower than the steady state, i.e. $\hat{z}^L + \hat{h}^L < 0$.[6] It is our view that zero bound equilibria with high employment and deflation are not relevant for understanding events such as the U.S. Great Recession or Great Depression. Both events were associated with large declines in employment. Proposition 1 tells us that if we limit attention to this type of situation, then all zero bound equilibria are either of Type 1 or Type 2.

Most of the recent literature on the zero bound has focused exclusively on Type 1 equilibria. Type 1 parameterizations have the attractive property that equilibrium is globally unique (See Proposition 4 in Braun, Körber, and Waki (2012)). A smaller literature including Bullard (2010), Mertens and Ravn (2010) and Aruoba and Schorfheide (2013) has considered zero bound sunspot equilibria. There is a close relationship between our results and this second literature and it is most transparent when $\hat{z}^L = 0$. Under this assumption, all Type 2 equilibria are sunspot equilibria. This is because there is a second equilibrium with a positive nominal interest rate: $(\hat{h}^L, \pi^L, R^L) = (0, 0, \hat{r}^e_L)$. Previous research has focused on pure sunspot equilibria which is a Type 2 equilibrium with $d^L = 1$. By focusing on the Type 2 zero bound equilibrium we are implicitly assuming that d^L plays two roles: it affects the preference discount factor and serves as a coordination device for agents beliefs. This is sometimes referred to as an endogenous sunspot equilibrium. The exogenous sunspot equilibria considered by Mertens and Ravn (2010) and Aruoba and Schorfheide (2013) set $d^L = 1$ and posit an expectations shock that coordinates expectations on state L.[7] Another example of a Type 2 equilibrium is the deflationary steady-state described in Benhabib, Schmitt-Grohe, and Uribe (2001). Their deflationary steady state is found by setting $d^L = 1$ and $p = 1$ in equations (16)–(15). The next section shows that the Type 2 equilibrium in our loglinearized model has the same implications for fiscal policy as the nonlinear sunspot equilibrium

[6] If it is assumed that $\hat{z}^L = 0$, then the final clause can be removed and any *any* ZLB equilibrium must satisfy both (1a) and (1b) or both (2a) and (2b). For further details see Braun, Körber, and Waki (2012).

[7] Aruoba and Schorfheide (2013) also allow for shocks to fundamentals but they are orthogonal to the sunspot shock.

considered by Mertens and Ravn (2010). It follows that one important determinant of the properties of the fiscal multipliers is the parameterization of the model.

3.2 Fiscal policy in the loglinearized model

Fiscal policy has different effects on output and inflation in the two types of equilibria.

Proposition 2 The effects of fiscal policy in the loglinear model

a) *In a Type 1 equilibrium, a labor tax increase increases hours and inflation. The government purchase output multiplier is above 1.*

b) *In a Type 2 equilibrium a labor tax increase lowers hours and inflation.*

c) *Suppose parameters are such that an increase in η results in an increase in government purchases, then in a Type 2 equilibrium the government purchase output multiplier is less than 1.*[8]

4 Analytical results on the global properties of zero bound equilibria

The local effects of fiscal policy in the nonlinear Rotemberg model also depend on the local slopes of the AS and AD schedules. However, a much richer set of configurations of the AD and AS schedules can be supported as a zero bound equilibrium. In particular, the model can exhibit configurations of AD and AS that are impossible using the loglinearized equilibrium conditions including a conventional configuration with $slope(AD) < 0 < slope(AS)$.

4.1 Slopes of the AS and AD schedules in the nonlinear model

Since the slopes of the AD and AS schedules vary with (h^L, π^L) in the nonlinear economy, we derive conditions for their *local* slopes at a given point (h^L, π^L). Before documenting these properties, we wish to remind the reader that we can restrict our attention to the following range of π^L, since equilibrium consumption and hours must be positive: $1 - \kappa^L - \eta^L > 0$, $1 - p\beta d^L/(1 + \pi^L) > 0$, and $(1 - p\beta d^L)\gamma\pi^L(1 + \pi^L) + \theta > 0$.

Proposition 3 below establishes a condition under which the nonlinear AD schedule is *locally* downward sloping.

[8] In the proof we provide the exact condition in terms of parameters under which an increase in η results in an increase in government purchases.

Proposition 3 *The AD schedule is downward sloping at (h^L, π^L) if and only if*

$$\frac{1}{\sigma}\frac{p\beta d^L/(1+\pi^L)^2}{1-p\beta d^L/(1+\pi^L)} + \frac{(\kappa^L)'}{1-\kappa^L-\eta^L} < 0, \tag{17}$$

where $(\kappa^L)' = \gamma\pi^L$. It is upward-sloping (vertical) if and only if the left hand side is positive (zero).

To understand these results, recall that the AD schedule is obtained by combining the Euler equation and the aggregate resource constraint. The Euler equation, in isolation, creates a positive relationship between inflation and consumption because higher (expected) inflation lowers the real interest rate. In the true model this gets balanced against the aggregate resource constraint, which creates a negative relationship between inflation and the hours-consumption ratio when the inflation rate is negative. An increase in inflation reduces the resource costs of price adjustment, thereby reducing the gap between hours and consumption. Loglinearizing the aggregate resource constraint at $\pi = 0$ loses this second channel. The hours-consumption ratio becomes independent of inflation in the aggregate resource constraint and the slope of the AD schedule is unambiguously positive. Observe next that the first term in (17) is the inflation elasticity of consumption implied by the Euler equation and the second term is the inflation elasticity of the hours-consumption ratio implied by the aggregate resource constraint. Since $h = c \times (h/c)$, their sum is the inflation elasticity of hours, and its reciprocal is naturally the local slope of the AD schedule. The first term is always non-negative, and the second term is negative when $\pi^L < 0$. Loglinearizing about a stable price level imposes $(\kappa^L)' = \kappa^L = 0$ and the second term disappears from (17). Thus, the presence of price adjustment costs in the aggregate resource constraint is responsible for the possibility of a downward sloping AD schedule in the nonlinear model.

Using these results we can document one breakdown in the loglinearized solution. Suppose $p = 0$ and $\pi^L < 0$. Under these assumptions the slope of the AD schedule is unambiguously negative. The loglinearized equilibrium, in contrast, has the property that all zero bound equilibria have a positively sloped AD schedule. This breakdown occurs in *any* zero bound equilibrium and does not depend on the size of the shocks beyond the requirement that they be large enough to induce a zero nominal interest rate. We will show in the numerical results that follow that this type of breakdown can occur for values of p that are as large as 0.86 and that it has direct implications for the sign of the response of hours to a change in the tax rate.

The next proposition characterizes the local slope of the AS schedule.

Proposition 4 *The AS schedule is downward sloping at (h^L, π^L) if and only if*

$$\frac{\gamma(1+2\pi^L)(1-p\beta d^L)}{\gamma\pi^L(1+\pi^L)(1-p\beta d^L)+\theta}+\frac{\sigma(\kappa^L)'}{1-\kappa^L-\eta^L}<0. \tag{18}$$

It is upward-sloping (vertical) if and only if the left hand side is positive (zero).

To provide some intuition for these results combine the household's first-order condition and the aggregate resource constraint to get: $(h^L)^{\sigma+\nu} = w^L(1-\tau_w^L)(z^L)^{1-\sigma}/(1-\kappa^L-\eta^L)^\sigma$. It follows that the inflation elasticity of hours is proportional to the sum of the inflation elasticity of the real wage and that the inflation elasticity of $1/(1-\kappa^L-\eta^L)^\sigma$. The first term in (18) is the inflation elasticity of the real wage implied by the New Keynesian Phillips curve,[9] while the second term is the inflation elasticity of $1/(1-\kappa^L-\eta^L)^\sigma$. The former term is positive when $\pi^L > -1/2$, and the latter term is negative when $\pi^L < 0$. When inflation rate is negative, an increase in inflation increases consumption for given labor input, through the resource constraint. This reduces the household's incentive to supply labor and labor input may fall even if the real wage rises. The AS schedule is unambiguously upward sloping, however, when the model is loglinearized around a stable price level. This is because the second term in (18) drops out ($(\kappa^L)' = 0$).

One implication of these propositions is that the AD and AS schedules are downward sloping when the term $(\kappa^L)'$ is negative and sufficiently large in absolute value. Since $(\kappa^L)' = \gamma\pi^L$, this occurs when the price adjustment cost parameter γ is sufficiently large and/or the equilibrium rate of deflation is sufficiently large. Note however that this term doesn't have to be sizeable - it only has to be larger than the first term in (17) or (18), which can be small. In Section 7 we show that this configuration of the AD and the AS arises when we calibrate the model to reproduce the declines in output and inflation during the Great Depression.

A final point that we wish to mention pertains to the shape of the AD and AS schedules. In the loglinearized economy their intersection is unique if it exists, except for the non-generic case where they coincide, while in the nonlinear economy there may be multiple intersections. Our empirical results below indicate that it is not unusual to encounter multiple zero bound equilibria.

4.2 Fiscal policy in the nonlinear model

Labor tax In our empirical results, we encounter four distinct patterns of the AD and AS schedules. For each of these patterns, Proposition 5 summarizes how hours and inflation respond to an increase in the labor tax.

[9] We are specifically referring to: $\gamma\pi^L(1+\pi^L) = \theta(w^L-1) + p\beta d^L\gamma\pi^L(1+\pi^L)$.

Proposition 5 Response of hours and inflation to a change in the labor tax

a) If both the AS and AD schedule is downward sloping and the AS schedule is steeper **or** *if the AS schedule is upward sloping and the AD schedule is downward sloping, then a higher labor tax lowers hours and rises inflation.*

b) If both the AS and the AD schedule are upward sloping and the AD schedule is steeper, then a higher labor tax increase rises hours and inflation. Consumption also increases if $\pi^L < 0$.

c) If both the AS and the AD schedule are upward sloping and the AS schedule is steeper, then a higher labor tax lowers hours and inflation. Consumption also falls if $\pi^L < 0$.

Case a) is unique to the nonlinear model. Moreover, it implies a conventional response of hours to labor income tax; hours fall in response to a labor tax increase. Cases b) and c) are generalizations of Proposition 2 a) and b).

Government spending multiplier We have explored the possibility of deriving analytical responses in the nonlinear economy and found that the magnitude of the response of GDP and other variables to a change in government purchases is difficult to pin down in the nonlinear economy. Instead we will use numerical solutions to investigate this question which requires us to assign particular values to the model's parameters.

5 Parameterization of the model

We divide this discussion into two parts. We start by describing how we assign values to the parameters that are not specific to the zero bond. Then we discuss how we assign values to parameters that pertain to the zero bound.

5.1 Parameters that are not specific to the zero bound

We will demonstrate that some of the properties of the model can be linked to the size of the slope of the conventional loglinear New Keynesian Phillips Curve (NKPC). In our model this coefficient is given by $slope(NKPC) \equiv \theta(\sigma + \nu)/\gamma$. There are many different choices of parameters that can make this number big or small. Here we describe how we come up with reference values for these parameters that are used to produce our baseline results.

Our reference value of σ is one. This corresponds to the case of log preferences over consumption, which is a common reference point in the DSGE literature. The parameter θ

is set to 7.67, which implies a markup of 15%. We found that θ and γ are not individually identified by our estimation procedure. Given the central role played by γ in the NK model, we decided to fix θ and estimate γ. Our choice of θ lies midway between previous estimates from disaggregated and aggregate data. Broda and Weinstein (2004) find that the median value of this parameter ranges from 3 to 4.3 using 4-digit level industry level data for alternative country pairs. Denes, Eggertsson, and Gilbukh (2013) estimate θ to be about 13 in a representative agent NK model. It is also well known that β is not well identified in DSGE models. Consequently, β is fixed at 0.997 which implies an annual preference rate of time preference of 1.2%.

We also fix the government purchase share parameter η at 0.2 and the labor tax τ_w at 0.2. The remaining two parameters ν and γ are estimated using a strategy that is similar to the one used by policy institutions such as the Bank of England. They estimate structural parameters via Bayesian MLE over a sample period that ends before the nominal interest rate falls to zero (see e.g. Burgess, Fernandez-Corugedo, Groth, Harrison, Monti, Theodoridis, and Waldron (2013)). We estimate ν and γ via Bayesian MLE using quarterly U.S. data on inflation, the output gap and the Federal Funds rate over a sample period that extends from 1985:I through 2007:IV.[10] Complete details on our estimation procedure are reported in the Online Appendix.[11] Here we briefly comment on the resulting estimates of γ and ν. The estimated posterior mode of γ is 458.4 and 90% of its posterior mass lies between 315 and 714. This value is larger than Ireland (2003) who estimates a value of 162, and Gust, Lopez-Salido, and Smith (2012) who estimate $\gamma = 94$. If we use a tighter prior on γ, we can produce estimates of γ that lie in the range of 100 to 150. However, most posterior mass is centered well outside of the prior. Relaxing the prior on this parameter yields diagnostics that suggest that the resulting estimates are less influenced by the prior in the sense that there is more overlap of the prior and posterior densities for γ and by the fact that the peaks of the likelihood function and posterior mode are close. The posterior mode for ν, which governs the curvature of the disutility of work, is 0.28 with 90% of its posterior mass lying in the interval 0.08 and 0.63. A complete set of estimation results including priors is provided in the Online Appendix. These estimates in conjunction with our choices for θ and σ imply that $slope(NKPC) = 0.0214$ which is very close to the value of 0.024 estimated by Rotemberg and Woodford (1997).

[10] Our measure of the output gap uses the Congressional Budget Office measure of potential GDP.

[11] The Online Appendix is available at: https://sites.google.com/site/yuichirowaki/research.

5.2 Parameters and shocks that are specific to the zero bound

We consider two specific episodes with zero interest rates: the U.S. Great Recession (GR) and the U.S. Great Depression (GD). To facilitate a comparison of our results with those in the previous literature, we consider the two state Markov zero bound equilibria we described in Section 2.2. This structure also allows us to provide a transparent and intuitive analysis of our results, and to identify the possibility of multiple equilibria. To pursue this strategy, we need to specify p, which determines the expected duration of zero interest rates, and the size of the shocks that produce a zero nominal interest rate. Rather than using a specific p, we report results for a range of values of p. For each given value of p, we choose d^L and z^L to reproduce targeted declines on output and the annualized inflation rate, assuming that the fiscal policy in the L state is the same as in the steady state, i.e. $(\tau_w^L, \eta^L) = (\tau_w, \eta)$. The Online Appendix explains how these targets uniquely pin down d^L and z^L. We then measure the fiscal policy effects by computing how an equilibrium in the L state changes when (τ_w^L, η^L) changes from (τ_w, η).

Our targets for the GR are taken from Christiano, Eichenbaum, and Rebelo (2011). They provide empirical evidence that the U.S. financial crisis that ensued after the collapse of Lehman Brothers in the third quarter of 2008 produced a peak decline in GDP of 7% and a peak decline in the inflation rate of 1%. The GD targets are taken from Eggertsson (2011) who posits a 30% decline in GDP and a 10% decline in the annualized inflation rate.

6 Numerical results for the Great Recession

In this section we describe the fiscal multipliers that emerge from our model using our estimated parameterization and the GR targets. We then relate our findings to other results in the literature.

6.1 Our results

Fiscal multipliers are reported in Figure 1 for alternative values of p that range from 0.05 to 0.95. For each choice of p the shocks to d^L and z^L have been adjusted so that the model produces a 7% decline in output and a 1% decline in the inflation rate. Figure 1 is divided into three panels. The upper and middle panels display the responses of GDP and hours respectively to a one unit change in government purchases. The lower panel shows the response of hours in percentage points to a one percentage point increase in the labour tax. Each panel reports results for three subintervals $p \in [0.05, 0.8]$, $p \in [0.8, 0.86]$

and $p \in [0.865, 0.95]$. Results using the exact nonlinear solution of the model are labeled as NL and results using the loglinearized solution are labeled as LL. In some cases there are two nonlinear zero bound equilibria. The label "NL (target)" refers to the equilibrium that reproduces the calibration targets, while "NL (non-target)" refers to the nonlinear equilibrium that doesn't hit the targets.

One of the main objectives of this paper is to demonstrate that there are a broad range of empirically plausible parameterizations of the NK model that deliver small government purchase multipliers at the zero bound. Figure 1 shows that the government purchase multipliers are small over two distinct intervals of p. Results in the leftmost chart of this figure indicate that the government purchase GDP multiplier is smaller than 1.15 when $p \leq 0.8$. It ranges from a high of 1.14 when $p = 0.8$ to a low of 1.00 when $p = 0.05$. The government purchase hours multiplier has a similar pattern. It falls from 1.15 to 0.95 as p is reduced from 0.8 to 0.05. From the rightmost charts of Figure 1, one can see that the government purchase multiplier is also small when $p \geq 0.88$ in the NL (targeted) equilibrium. In this interval the responses of GDP and hours to a one unit change in government purchases are both less than one. For instance, when $p = 0.9$ the value of the government purchase GDP multiplier is 0.62 and the hours multiplier is 0.72 and when $p = 0.95$, the two multipliers are respectively 0.79 and 0.86.

Figure 1 also reports response of hours in percentage points to a one percentage point increase in the labor tax. It shows that the sign of the labor tax hours multiplier is orthodox in two distinct regions. Hours fall in response to an increase in the labor tax when $p < 0.6$ and when $p \geq 0.865$. Even when the response of hours to a tax increase is positive, its magnitude is generally very small. The labor tax multiplier never exceeds 0.2% when $p < 0.75$. which we interpret to mean that hours worked are highly inelastic to changes in the labor tax.

There are two qualifications to these results. The first qualification is clearest in the middle and right charts of Figure 1. At a value of $p \approx 0.86$ the fiscal multipliers have an asymptote. In the neighborhood of the asymptote the fiscal multipliers become very large and their signs flip on either side of the asymptote. Woodford (2011) and Carlstrom, Fuerst, and Paustian (2012) have previously documented asymptotes in fiscal multipliers using loglinearized solutions. Our results indicate that they also arise in nonlinear NK models. The second qualification can be seen in the right panels of Figure 1. When $p \in [0.87, 0.89]$, there is a second zero bound equilibrium of the model. This equilibrium is not targeted and for most values of p it produces responses of inflation and/or output that are very far from the targets. The non-targeted zero bound equilibrium also occurs in a very small interval to the left of the asymptote.

A disturbing aspect of our results is that the asymptote occurs at a value of p that is close to values maintained in previous research. Denes, Eggertsson, and Gilbukh (2013) report an estimate of $p = 0.86$ and Christiano and Eichenbaum (2012) posit a value of $p = 0.775$. Small perturbations of p in this region have large effects on the magnitudes and/or signs of the fiscal policy multipliers.

High values of $p > 0.9$, which imply expected durations of zero interest rates that exceed 10 quarters, are most consistent with the long episodes of nearly zero interest rates experienced by the U.S., some European countries and Japan (See rightmost charts in Figure 1). For instance, the U.S. and Japan have experienced interest rates that are close to zero since the fourth quarter of 2008.[12]

Figure 1 also reports fiscal multipliers computed from the loglinearized solution. For $p \geq 0.865$, the linear structure of this solution implies that it cannot discover the presence of the other zero bound equilibrium. When $p \geq 0.9$, there is a single zero bound equilibrium in the true model and the loglinear solution approximates this solution well. Both solutions have the properties that the government purchase GDP is less than one and that hours fall in response to an increase in the labor tax. A range of NK models including models with Calvo pricing have the same loglinear structure as our model. If the loglinear solution works as well in these other models as it does here, then these results for $p \geq 0.9$ also obtain in a larger class of NK models.

Biases in the government purchase multiplier are somewhat larger for values of p that are to the left of the asymptote. For instance, when $p < 0.4$ the government purchase GDP multiplier is less than one using the loglinearized solution but slightly above one using the exact solution. Biases for the labor tax multiplier are larger too. For values of $p \leq 0.57$ the loglinearized solution predicts that the response of hours to a tax increase is positive when in fact it is negative.

The largest biases occur in the neighborhood of the asymptote. The loglinearized solution understates the true multipliers for GDP, hours and inflation immediately to the left of the asymptote and overstates the size of these multipliers just to the right of the asymptote.

Table 1 reports other properties of the model for selected values of p that help provide more intuition for our results. It is immediately clear from inspection of the table that the asymptote occurs at a point where the slopes of the AD and AS schedules are parallel. To the right of the asymptote, the targeted nonlinear equilibrium has the property that $slope(AS) > slope(AD) > 0$ and it follows immediately from Proposition 5 that the sign of the labor tax multiplier must be negative. The loglinearized solution is consistent with this

[12] Hayashi and Koeda (2013) who identify two other episodes with low interest rates: March 1999 – July 2000 and March 2001 – June 2006.

property of the model since it produces a Type 2 equilibrium. Immediately to the left of the asymptote, both solutions have the property that $slope(AD) > slope(AS) > 0$ and there is a paradox of toil.

We have seen that the Rotemberg model has a second zero bound equilibrium to the right of the asymptote. The properties of this second equilibrium vary with the value of p. It has the property that $slope(AS) < slope(AD) < 0$ when $p \in [0.867, 0.89]$ and $0 < slope(AS) < slope(AD)$ when $p \in [0.86, 0.865]$.

In Section 3.1, we discussed the possibility of multiple equilibria and sunspot equilibria when the loglinearized solution is a Type 2 equilibrium. Table 1 indicates that the loglinearized solution is a Type 2 equilibrium when $p = 0.9$. In fact, the equilibrium is Type 2 when $p \geq 0.865$. In this region there are two indeed equilibria. One equilibrium is the zero bound equilibrium displayed in Figure 1 and the other equilibrium has a positive interest rate, a positive inflation rate and high hours worked. In the positive interest rate equilibrium, the government purchase multiplier is less than one and hours fall in response to a labor tax increase since $slope(AD) < 0 < slope(AS)$. The true model also has a positive interest rate equilibrium when $p \geq 0.86$ and inflation and hours are also above their steady-state values in this equilibrium. The government purchase multiplier is less than one but $slope(AD) > slope(AS) > 0$ and the response of hours to a tax increase is positive.

Given that there are multiple equilibria, if the intent is to select the equilibrium that reproduces the GR targets, then the shocks to fundamentals must also act to coordinate beliefs on that equilibrium. In this sense the (targeted) zero bound equilibrium is a type of endogenous sunspot equilibrium. We do not pursue this possibility here but one could construct other sunspot equilibria that alternate from positive interest rates to zero interest rates as in Mertens and Ravn (2010) or Aruoba and Schorfheide (2013). When $p \in [0.86, 0.89]$, it is also possible to entertain sunspot equilibria that switch between the two zero bound equilibria with distinct local dynamics and an equilibrium with a positive interest rate.

Christiano and Eichenbaum (2012) have proposed that one use an E-learning criteria to rule out multiple zero bound equilibria in nonlinear NK models. Applying that criterion here does not resolve the issue of multiplicity of equilibrium for $p \in [0.86, 0.865]$ in our nonlinear model. This is because both the positive interest rate equilibrium and the non-targeted zero bound equilibrium are E-learnable. More importantly, neither of these equilibria reproduce the GR targets. For instance, the E-learnable zero bound equilibrium produces a 13% decline in GDP and an annualized rate of deflation of 5% when $p = 0.865$ and the positive interest rate equilibrium has the property that GDP falls but only by 0.6%. Hours, in contrast, are 5% above their steady state level. When p exceeds 0.865, the E-learning criterion rules out all zero bound equilibria in the Rotemberg model and selects the positive interest rate equilibrium. However, the positive interest rate equilibrium continues to predict a GDP response of about zero. Moreover, hours are even higher and increase with p. We do not believe that it is useful to use an equilibrium selection criterion that rejects zero bound equilibria that exceed seven quarters and reproduce the GR facts and instead selects equilibria that are inconsistent with the fact that the U.S. Great Recession has been accompanied by zero nominal interest rates, depressed GDP, depressed hours and a depressed inflation rate.

Next consider results in Table 1 for $p \leq 0.8$. In Figure 1, we documented a paradox of toil using the loglinearized solution and an orthodox response of hours using the exact solution when $p \leq 0.57$. A possible explanation for this finding was provided in Section 4.1 where we found that the loglinearized solution produces the wrong slope of the AD schedule in *any* zero bound equilibrium when p is in the neighborhood of zero. It was not possible, however, to establish the size of this neighborhood using analytical arguments. Results in Table 1 indicate that this neighborhood can be very large. For instance, when $p = 0.5$, the slopes of AD and AS have their conventional signs in the Rotemberg model which in turn implies from Proposition 5 that there is no paradox of toil. The slope of the AD schedule using the loglinearized solution is positive and exceeds the slope of the AS schedule. This is why the loglinearized solution incorrectly predicts a paradox of toil in this region of the parameter space. More generally, the two solutions are different when $p \leq 0.57$ or in words when the expected duration of zero interest rates is less than two quarters.

The evidence we discussed above suggests that two quarters is a very short expected duration of zero interest rates. However, short expected durations cannot be completely ruled out on a priori grounds. Short expected durations of zero interest rates occur in empirical analyses of the zero bound such as Aruoba and Schorfheide (2013) which require a long sequence of negative monetary policy shocks to account for the fact that the U.S. policy rate has been about zero since 2008. It is also not difficult to find alternative parameterizations of the model that hit the GR targets and where this same type of breakdown occurs with higher values of p. We will present results for another parameterization below where the true model produces a conventional configuration of AD and AS for p as large as 0.857.

Table 1 also reports the resource cost of price adjustment expressed as a ratio of output. The magnitude of the price adjustment cost in the Rotemberg model is 0.14% of output or less for all equilibria that reproduce the GR targets. It is difficult to directly measure the overall magnitude of the resource cost of price adjustment but a rough idea of the potential magnitude of this cost is provided by Levy, Bergen, Dutta, and Venable (1997) who find that menu costs constitute 0.7% of revenues of supermarket chains.

Recognizing the resource cost of price adjustment plays a central role in obtaining a conventional configuration of the slopes of AD and AS. Our analytical results in Section 4 offered some insight into their role. Another way to understand the role of the resource cost of price adjustment is to observe that if it was absent, the response of hours and GDP would be identical. Thus, evidence that hours and GDP are responding differently to a shock illustrates that resource costs matter. This is the case for $p \in [0.215, 0.57]$.[13] In this interval higher labor taxes reduce hours and increase GDP. It follows that the positive response of GDP stems from the fact that the reduction in resource costs is larger than the decline in hours.

6.2 Relating our results to the literature

We now consider how our results relate to the literature which has found that government purchase GDP multipliers are large and positive and that hours fall in response to a tax cut when the interest rate is zero. This discussion also provides us with an opportunity to document the robustness of our conclusions to the parameterization of the model. In what follows we will consider parameterizations in which the value of γ is as low as 100, values of θ that range from one to 13 and a value of ν that is as high as 1.69.

6.2.1 Differences in the shocks: Preference shock only

Following Eggertsson and Woodford (2003), many analyses of the zero bound consider scenarios where a single demand shock drives the nominal interest rate to zero. One possible explanation for the difference between our results and these previous results is that we allow two shocks to do this. We now adopt their assumption and set $z^L = z = 1$. With a single shock it is no longer possible to hit both Great Recession targets for general p, and we need to adjust one or more parameters from their baseline values. We choose to adjust θ and keep the other parameters fixed.[14]

Fiscal multipliers using the exact and loglinearized solutions are reported in Figure 2 for values of p that range from 0.05 to 0.95. Other properties of the model are reported for selected values of p in Table 2.

Observe that the GDP multiplier is now less than 1.1 when $p \leq 0.9$. Most empirical evidence suggests that $\theta \geq 2$ (see e.g. Broda and Weinstein (2004)). If we limit attention to values of $\theta \geq 2$ and thus $p \leq 0.84$, the government purchase GDP multiplier is 1.04 or less and the labor tax multiplier is 0.18 or less. Note further that for values of $p \leq 0.9$, the

[13] The GDP response to a higher labor tax are reported in our Online Appendix.

[14] The Online Appendix describes how d^L and θ are calibrated to hit the two GR targets.

loglinearized solution for both government purchase multipliers is reasonably accurate. From Table 2, we see that the equilibrium is of Type 1 (slope(AD)>slope(AS)>0) using either solution when $0.6 \leq p \leq 0.9$. In spite of this fact the government purchase GDP multipliers are small and very close to their theoretical lower bound of 1 (see Proposition 2). We know from our analytical results that for p in this interval, there is a paradox of toil. However, the lower panel of Figure 2 indicates that the magnitude is very small. The labor tax multiplier is 0.11 or less when $0.6 \leq p \leq 0.9$

The differences between the exact and loglinearized solutions are generally smaller when $z^L = z = 1$. There are still two regions where the loglinearized solution breaks down. When $p < 0.58$, the exact solution exhibits a conventional configuration of AD and AS and thus no paradox of toil. The loglinearized solution, in contrast, has the counterfactual property that slope(AD)>slope(AS)>0. The second region where the loglinearized solution breaks down is in the neighborhood of $p = 0.96$. In this region, there is an asymptote in the fiscal multipliers as the slope of the AD schedule shifts from being steeper than AS to flatter.

Taken together, these results indicate that our findings do not depend on our previous assumption about the number and type of shocks. The government purchase multipliers are small not only when we allow for two shocks but also when we specify a single shock to d^L. This is not to say that allowing for $z^L < z$ is not important. It is a challenge for the Rotemberg model to account for the GR targets with low markups and long expected durations of the zero bound when there is a single shock to d^L. Using our maintained value of $\theta = 7.67$, p must be close to 0.4 if that specification is to reproduce the output and inflation targets with a single shock to demand. A negative technology shock allows the model to reproduce the GR targets with much larger values of p.

6.2.2 Differences in the parameterization: slope of the New Keynesian Phillips Curve

We have seen above that the loglinearized solution works well in some regions of the parameter space. This is convenient because it turns out that the loglinearized equilibrium conditions offer some valuable insights into the roles of the various structural parameters in producing our results as well as other results in the literature. In particular, working with the loglinearized equilibrium conditions allows us to derive an upper bound on the size of $slope(NKPC)$ that is consistent with the GR and joint restrictions that the GR imposes on p and $slope(NKPC)$. We are also able to derive joint restrictions on $slope(NKPC)$ and p that determine the range of Type 1 and Type 2 equilibria. Since a variety of models map into the same loglinearized reduced form, these results may be robust to some of the particular modeling assumptions that we have made.

We first show how our GR targets restrict the plausible range of $slope(NKPC)$. For expositional purposes, suppose that the only shock that induces the nominal rate to fall to zero is a discount factor shock.[15] Then it follows from the loglinearized AS equation (16) that

$$1 - \beta p = slope(NKPC) \frac{\hat{h}^L}{\pi^L} \tag{19}$$

Next observe that our GR targets imply $\hat{h}^L/\pi^L = (-0.07)/(-0.0025) \approx 28$ when $\hat{z}^L = 0$. Combining these two pieces of information and setting $p = 0$ implies that the largest value of $slope(NKPC)$ that is consistent with the GR targets is $1/28 \approx 0.036$.

More generally, equation (19) shows that reproducing the GR imposes joint restrictions on p and $slope(NKPC)$. These restrictions have some empirical content. For instance, if one conditions on a value of $slope(NKPC) \approx 0.02$, which is the value implied by our estimates, then $p \approx 0.4$. Entertaining a value of $p \geq 0.9$ requires that $slope(NKPC) = \theta(\sigma + \nu)/\gamma \leq 0.005$. In the preference-shock-only case, we accomplished this by lowering θ.[16]

The loglinearized model can also be used to relate the conditions for Type 1 and Type 2 zero bound equilibria to the values of $slope(NKPC)$. Depending on whether the left hand side of

$$\frac{(1-p)(1-p\beta)}{p} \gtreqless \frac{slope(NKPC)}{\sigma} \tag{20}$$

is larger/smaller than the right hand side, the AD schedule is steeper/flatter than the AS schedule. The left hand side of equation (20) is declining in p. An implication of this condition is that producing a Type 1 zero bound equilibrium when p is large requires a very small value of $slope(NKPC)/\sigma$. For instance, using our reference parameterization ($\sigma = 1$ and $\beta \approx 1$) producing a type 1 equilibrium with $p > 0.9$ requires that $slope(NKPC) < 0.011$.

More generally, if the objective is to produce a large region where the equilibrium is of Type 1, then very small values of $slope(NKPC)$ and/or small values of p are desirable. A small value of $slope(NKPC)$ is associated with a large value of γ or alternatively small values of θ, σ and/or ν. To make the range of Type 2 equilibria large instead, it is desirable to use configurations that make $slope(NKPC)$ large and/or p large.

These results imply that the parameters p, θ, σ, ν and γ are of particular significance. In

[15] We discuss the role of technology shocks in the Online Appendix.

[16] We show in the Online Appendix that a negative technology shock increases the value of p implied by (16) for given $slope(NKPC)$, and that is why we are able to hit the GR target without lowering $slope(NKPC)$ in that specification.

what follows, we will focus on particular combinations of these parameters that have been used in the previous literature.

6.2.3 Accounting for the Great Recession with the parameterization of Christiano and Eichenbaum (2012)

We next turn to conduct a more detailed comparison of our results with those of Christiano and Eichenbaum (2012). They find that the government purchase multiplier exceeds 2 in a nonlinear Rotemberg model that is very close to ours.[17] Following their paper, we set the preference discount factor $\beta = 0.99$, the coefficient of relative risk aversion for consumption $\sigma = 1$ and the curvature parameter for leisure $\nu = 1$. The technology parameter θ that governs the substitutability of different types of goods is set to three, the adjustment costs of price adjustment $\gamma = 100$, and the conditional probability of exiting the low state $p = 0.775$. Our model has three fiscal parameters. The labor tax $\tau_w = 0.2$, government purchases share in output $\eta = 0.2$, and the subsidy to intermediate goods producers τ_s is set so that steady-state profits are zero. Finally, the coefficients on the Taylor rule are $\phi = 1.5$ and $\phi_y = 0$. With this parameterization our loglinearized system is identical to the one reported in Christiano and Eichenbaum (2012).[18]

Row 1 of Table 3 reproduces their baseline results using our nonlinear model and the loglinearized solution.[19] The loglinear solution yields a government purchase multiplier of 2.77 implying that fiscal stimulus is highly effective at the zero bound. Note next that there is a paradox of toil. This follows from the observation that the equilibrium is of Type 1. The magnitude is very large with hours increasing by 4.84% to a one percentage point increase in the labor tax. For purposes of comparison, it is about five times as large as the response reported in Eggertsson (2011) for a similar model that is estimated to hit targets from the Great Depression.

Consider next results using the exact solution. Our nonlinear model is slightly different from theirs but it delivers the same fiscal multipliers as theirs for this parameterization of shocks.[20] The size of the government purchase multiplier is about 20% smaller when using the exact solution. There are much larger gaps between the two solutions for the government

[17] Christiano, Eichenbaum, and Rebelo (2011) report similar results but it is easier for us to compare our results with the results of Christiano and Eichenbaum (2012) because they also posit Rotemberg adjustment costs.

[18] To be precise, they fix the level of government purchases as opposed to its share in output. However, the loglinearized systems are equivalent when one considers the same type of fiscal policy shock.

[19] Formally, we set the discount factor shock d^L so that the loglinearized solution reproduces a 7% decline in GDP and then solve our nonlinear model using the same sized shock.

[20] We assume that the resource costs of price adjustment apply to gross production $(\gamma\pi_t^2 y_t)$ whereas they assume that the resource costs of price adjustment only apply to GDP $(\gamma\pi_t^2(c_t + g_t))$.

purchases hours multiplier. It is 1.71 using the nonlinear solution or about 40% smaller than the loglinear solution. The reason for this difference between the government purchase hours and GDP multipliers is that in the nonlinear model the resource costs of price adjustment κ drive a wedge between the hours and GDP response. The larger GDP response partly reflects a savings associated with lower resource costs of price adjustment. These properties of the model are lost when the model is loglinearized about a zero inflation rate. Finally, the equilibrium has the property that slope(AD)>slope(AS)>0 and it follows that hours increase in response to an increase in the tax rate. Hours increase by 1.32% in response to a one percentage point increase in the labor tax which is about 1/4 the size of this multiplier using the loglinearized solution.

Unfortunately, their parameterization does not reproduce the GR targets. Their loglinear solution produces a 7% decline in output and a 7% decline in the annualized inflation rate. This second response is seven times as large as our GR target which is taken from Christiano, Eichenbaum, and Rebelo (2011). To understand why the inflation response is so large, observe that their parameterization implies $slope(NKPC) = 0.06$.[21] which exceeds the upper bound of 0.036 that we established above using our GR targets. It follows that no choice of p allows this parameterization of the loglinearized model to reproduce the GR output and inflation targets.

When we adjust their parameterization to reproduce our GR targets, the fiscal multipliers fall to levels consistent with our previous results. Row 2 of Table 3 reports results from this adjustment. We first tried to recalibrate their model by adjusting the size of d^L and θ so that the exact solution reproduced the GR inflation and output targets. We found, however, that the required value of θ is less than one whereas this parameter only makes economic sense if it is at least one in magnitude. So we then increased γ to 300. This choice of γ in conjunction with $\theta = 1.24$ allows the nonlinear model to hit the two targets. Changing the parameterization in this way has a big impact on the results. On the one hand, the gap between the solutions is much smaller. On the other hand, the substance of the results is quite different. Both solutions continue to have the property that slope(AD)>slope(AS)>0. However, the government purchase multiplier is now much smaller. It is 1.05 using the loglinearized solution and 1.06 using the exact solution. There is also a sharp large decline in the labor tax hours multiplier. Hours now rise by 0.087% using the loglinear solution and 0.058% using the nonlinear solution. To put it more succinctly the response of hours to a one percentage point change in the labor tax rate is about zero.

The model of Christiano and Eichenbaum (2012) differs from our model in several respects.

[21] This result holds when we fix the share of government purchases in output. If instead the level of government purchases is held fixed, $slope(NKPC)$ is 0.0675.

These differences, however, do not affect the substance of our finding. If we solve for the exact solution of their model using the same parameters and shocks reported in Row 2 of Table 3, it produces a government purchase GDP multiplier of 1.04, a government purchase labor input multiplier of 1.03 and a response of hours to a one percentage point change in the labor tax of 0.049%.[22]

6.2.4 Accounting for the Great Recession with the parameterization of Denes, Eggertsson, and Gilbukh (2013)

We now consider the parameterization of Denes, Eggertsson, and Gilbukh (2013). Their results are interesting for four reasons. First, their estimates lie in a different region of the parameter space from the ones that we have considered so far. In particular, their estimated parameterization implies that $slope(NKPC)$ is very small (0.0075). Second, their fiscal multipliers are relatively small: the government purchase multiplier is 1.2 and the labor tax multiplier is 0.1 for a shock that reduces output by 10% and the annualized inflation rate by 2%. Third, we find that this type of parameterization produces conventional configurations of the AD and AS schedules (slope (AD)<0<slope(AS)) in the Rotemberg model. Fourth, this type of parameterization illustrates a new type of breakdown of the loglinearized solution.

They use an overidentified Quasi-Bayesian method of moments procedure to estimate a loglinearized NK model with a single shock to d^L. All of their model's parameters are estimated using two moments that they associate with the GR: an output decline of 10% and an annualized value of π^L of -2%. Their estimates of structural parameters are: $(p, \theta, \beta, \sigma, \nu) = (0.857, 13.23, 0.997, 1.22, 1.69)$. Their fiscal parameters are set in the same way that we have assumed up to now.

The loglinearized representations of their model and our model are identical, for a suitable choice of parameter values. However, their underlying nonlinear model of price setting is different from ours. They posit Calvo price adjustment and firm specific labor markets. Their GR targets are also different from ours. To facilitate comparison with our other results, we use all of their estimated parameters except the Calvo parameter in our nonlinear model and adjust d^L and γ to reproduce our GR inflation and output targets using the exact solution to our model. These adjustments have only a very small effect on $slope(NKPC)$. It rises from 0.0075 in their estimated parameterization to 0.0086 using our calibrated values of d^L and γ. Results for this parameterization of our model are reported in Row 3 of Table 3.

This parameterization of the Rotemberg model exhibits a conventional configuration of

[22]One difference between our model and the nonlinear model of CE (2012) is that equilibrium is unique for our model but that there are two equilibria in their model. The second equilibrium in their model has the property that $slope(AS) > slope(AD) > 0$.

AD and AS as slope(AD)<0<slope(AS). It follows that the response of hours (and of inflation) to a higher labor tax is orthodox. In terms of magnitude, hours fall by -0.2% when the labor tax is increased by one percentage point. Note also that the GDP government purchase multiplier is 1.08 and thus close to one.

Using the loglinearized equilibrium conditions produces results that are close to the results reported in Denes, Eggertsson, and Gilbukh (2013). The tax multiplier using our calibration is 0.11 as compared to their value of 0.1 and the government purchase multiplier using our calibration is 1.0,8 as compared to their value of 1.2. From this we see that the equilibrium is of Type 1 in both cases. It follows that this parameterization of our nonlinear model has different qualitative properties from the loglinearized solution. In the nonlinear model, an increase in the labor tax lowers hours and inflation and an improvement in technology increases output while the loglinearized solution predicts the opposite responses.[23]

Our previous discussion implies that $slope(NKPC)$ must be small to produce a Type 1 equilibrium with a value of $p = 0.857$. Indeed, this is the case here. Our calibration implies that $slope(NKPC) = 0.0086$, a value which is less than half the size of our estimate of 0.021. The associated value of γ is 6345, which is very large. To understand why this is the case, observe that their estimates of θ, σ, and ν are all much higher than our estimates. These parameters together with γ determine $slope(NKPC)$ in our model. The fact that these other parameters are so large when combined with their high value of p implies that γ must be very large if $slope(NKPC)$ is to be small enough to reproduce the GR targets.

A high value of γ implies that the resource costs of price adjustment are much larger here as compared to our previous results. They now account for about 2% of output. We have explored ways of reducing γ and thereby lowering the resource costs of price adjustment to a more reasonable level while keeping $\theta = 13.2$. Row 4) of Table 3 provides an example of such a parameterization. This parameterization of the model allows for a simultaneous negative shock to technology $z^L = 0.95$ and sets $(\nu, p) = (0.28, 0.7)$. These choices result in a value of $\gamma = 720$ which implies that the resource costs of price adjustment are 0.23% of output. The other parameters are set to the estimated values reported in Denes, Eggertsson, and Gilbukh (2013). These adjustments increase $slope(NKPC)$ to 0.0275. Using the exact solution the GDP government purchase multiplier is 1.04, the AD and AS schedules have their conventional slopes and hours fall by -0.016% in response to a one percentage increase in the labor tax. Using the loglinearized solution one concludes instead that the government purchase multiplier is 1.09, the equilibrium is of Type 1, and that hours increase by 0.18% in response to one percentage point increase in the labor tax.

[23]Formulas for the responses of output and inflation to a perturbation in the technology shock in the Online Appendix.

Finally, we wish to point out that a new type of breakdown in the loglinearized solution occurs using this parameterization of the model. Recall that for the baseline parameterization of the model, both the exact and loglinearized solutions had a positive interest rate equilibrium when $p \geq 0.865$. For this parameterization of the model, in contrast, there are a broad range of values of p where there are both zero bound and positive interest rate equilibria in the true model and a unique zero bound equilibrium using the loglinearized solution. For instance, using the Denes, Eggertsson, and Gilbukh (2013) parameterization reported in Row 3 of Table 3, there is also a positive interest rate equilibrium in the nonlinear model for all values of $p \geq 0.79$. The loglinearized solution, in contrast, produces a unique Type 1 equilibrium for these same choices of p.

7 Numerical results for the Great Depression

We now turn to consider the properties of the Rotemberg model of price adjustment for much larger shocks that are of the magnitude to reproduce output and inflation declines from the Great Depression (GD).

7.1 Our Results

We start by considering our estimated parameterization of the model with $\theta = 7.67$. As before, we choose d^L and z^L to reproduce the declines in GDP and inflation using the exact nonlinear solution. Item 1 of Table 4 reports results for values of p that range from 0.7 to 0.8.[24] Two distinct configurations of the AD and AS schedules occur. For values of $p \geq 0.74$, slope(AS)$<$ slope(AD)$<$0, and for values of $p \leq 0.73$, slope(AD)$<$0 and slope(AS)$>$0. For both configurations, hours fall in response to an increase in the labor tax. The response of hours to a one percentage point increase in the labor tax is largest (-1.50%) when $p = 0.8$ and then gradually falls to -0.81% when $p = 0.7$. The government purchase GDP multiplier ranges from a high of 1.13 when $p = 0.80$ to a low of 1.034 when $p = 0.7$. The shift in the slope of the AS schedule from negative to positive does not have a big effect on these multipliers. Although not reported here due to space considerations, we wish to point out that the implications for π^L are quite different in the two regions of the parameter space.[25] When $p \geq 0.74$, an increase in government purchases lowers the inflation rate. In the other region higher government purchases are associated with higher inflation.

[24] We omit $p = 0.9$ because this choice has the property that hours in the low state is higher than steady state hours.

[25] Inflation responses to a change in government purchases are reported in the Online Appendix.

Consider next the loglinearized solution. This solution exhibits a very large bias in both the sign and magnitude of the labor tax multiplier. It is 0.72 when $p = 0.8$ and 0.19 when $p = 0.7$. The reason for this difference is that the loglinearized solution has the property that slope(AD)>slope(AS)>0 for values of p in this interval. The biases in the size of the government purchase multiplier are much smaller but they can be either positive or negative. Note for instance that the government purchase multiplier using the loglinearized model understates the true one when $p = 0.7$. In fact, the multiplier is less than one using the loglinearized solution. This may appear to be puzzling to the reader since the equilibrium satisfies slope(AD)>slope(AS)>0. However, this is not a violation of Proposition 1 because that Proposition limits attention to the case where $z^L < 1$, while we have $z^L > 1$ here. For this parameterization of the model, the deflationary effects due to $z^L > 1$ offset the inflationary effects of higher government purchases. This acts to dampen the response of the markup much as monetary policy does when $R > 0$. The same factor is at work for higher values of p and this helps to explain why the government purchase multipliers under the two solutions are so close.

7.2 Accounting for the Great Depression with the parameterization of Eggertsson (2011)

Our finding that the government purchase multiplier is small in the GD using the loglinearized solution differs from the result of Eggertsson (2011) who has found that the GDP multiplier is over 1.8 and that a one percent increase in the labor tax increases labor input by one percentage point using a loglinearized solution to a NK model with Calvo price setting and firm specific labor. That model has the same loglinearized representation as our model. It is thus interesting to investigate the properties of our loglinear and nonlinear solutions using Eggertsson's parameterization. Row 2 reproduces those results and reports results using the exact solution for our model. The settings of the other model parameters are $\sigma = 1.16$, $\nu = 1.69$ and $\gamma = 4059.8$.[26] For this parameterization, the government purchase GDP multiplier rises to 1.82 using the loglinearized solution but is only 1.2 using the exact solution. Observe next that the configuration of the schedules once again differs across the two solutions with the loglinearized solution yielding a Type 1 configuration and the nonlinear solution producing $slope(AS) < slope(AD) < 0$.

It follows that the finding of a paradox of toil using the loglinearized solution is due to the choice of solution method and not a property of the true model. Note also that the implied resource costs of price adjustment using the loglinearized solution are so large that

[26] This choice γ pins down the correct value of $slope(NKPC)$.

they exceed output!

The exact solution understates the decline in both output and inflation when using the same configuration of parameters and demand shock that allow the loglinearized model to reproduce the two GD targets. In the nonlinear model, output falls by -24% and the annualized inflation rate falls by only -0.03%. Thus, if one uses the exact solution is to reproduce the GD instead, the resulting configuration of shocks and parameters will be very different.

To illustrate this point, we considered several strategies for calibrating the exact model to reproduce a 30% decline in GDP and a 10% decline in inflation using the same values of structural parameters including γ. We found that no combination of d^L and p worked. In fact, even if we allowed for d^L, p and $z^L < z$, the Rotemberg model still could not hit these two targets. We next tried holding fixed p and lowering γ instead and this worked. Results for that calibration of the model are reported in row 3 of Table 4.

For this parameterization of the model there are two zero bound equilibria. The targeted equilibrium has the property that the government purchase GDP multiplier is 1.26 which is the largest value of that multiplier we have encountered using the exact solution. This large and positive GDP multiplier is due to lower resource costs of price adjustment. This can be discerned from the fact that the hours government purchases multiplier is negative. This difference in the sign of the responses stems from the fact that higher government purchases increase the inflation rate. The resulting savings associated with lower resource costs of price adjustment are so large that GDP increases even though labor input falls.

In the targeted equilibrium, the labor tax multiplier has a negative sign– a higher labor tax lowers labor supply– and the magnitude of the response is rather strong. A one percentage point increase in the labor tax lowers labor input by -0.87%.

Observe next that the non-targeted equilibrium satisfies slope(AS)>slope(AD)>0. It is not empirically relevant. GDP falls by 1.9% but the annualized inflation rate increases to 5.6%.

The bias in the loglinearized solution is particularly severe. According to the loglinearized equilibrium conditions, there is no zero bound equilibrium for this parameterization of the model. Yet it is precisely this parameterization that allows the true model to reproduce the GD targets.

A common feature of all of these parameterizations of the model is that the resource costs of price adjustment are exceptionally large. Using our estimates with $\gamma = 458.4$, they amount to 15.5% of output. For the Eggertsson (2011) parameterization of the model they are even higher accounting for over 40% of output. The most direct way to reduce the size of these resource costs is to lower γ from its estimated value of 458.4. That estimate was based on a sample period extending from 1985:I-2007:IV and may not be representative of conditions that prevailed at the time of the GD. If we lower γ, other parameters and shocks need be adjusted so that we can continue to hit the GD targets. The natural candidates are θ, p , and z^L. For instance, if we set $\gamma = 100$ and $\theta = 5$, then we can reproduce the two GD targets with values of $p = 0.8$ and $z^L = 0.8$. This results in a government purchases GDP multiplier of 0.52 and a labor tax multiplier of -1.05. However, the resulting value of the resource costs of price adjustment are 3.4% of output. This figure is still very large.

7.3 Robustness

We close by considering two issues related to the resource costs of price adjustment. Are the large resource costs of price adjustment that we have found using the GD targets specific to our model of price adjustment or do they hold up under Calvo price adjustment as well? We present some evidence that indicates that this issue is not special to Rotemberg costly price adjustment but also a feature of models with Calvo price adjustment. We then consider the robustness of our results to an alternative specification of the Rotemberg model with real wage rigidities. This model relies less on nominal rigidities and can reproduce the GD targets when γ is as small as 25.

Calvo price adjustment Braun and Waki (2010) compare and contrast Rotemberg and Calvo price adjustment and find that the size of the resource costs of price dispersion are smaller but also very large under Calvo price adjustment. For instance, an annualized shock to d^L of 5% results in resource costs of price adjustment of 3.50% of output under Rotemberg price adjustment and resource costs of price dispersion of 1.9% of output under Calvo price adjustment which is already very large. A shock of this size produces a 6% decline in output in the Calvo model and about a 13% decline in annualized inflation. A much larger shock to d^L would be required to reproduce a 30% decline in output.

Real wage rigidities We have seen above that to entertain low values of γ and hit the inflation and output targets, we also had to adjust θ downwards. The lower bound on θ of 1 limits how far we can reduce γ. We would like to alter the tradeoff between γ and θ so that we can consider lower values of γ for a given value of θ. One way to do this is to assume that real wages are rigid as in Blanchard and Galí (2007). This allows the Rotemberg model to reproduce the GD inflation and output targets with a smaller amount of nominal price rigidity. Rows 4 and 5 of Table 4 illustrate two such parameterizations.[27] This simulation

[27] Appendix B contains a complete description of this version of the model.

uses our baseline values of (σ, ν, θ), and a much lower value for γ of 25. Production now has diminishing returns in labor, $y = zh^{1-\alpha}$. The curvature parameter α affects the strength of real rigidity: a lower value of α means a stronger real rigidity. We chose α to target our baseline value of $slope(NKPC) = 0.021$ which yields $\alpha = 0.064$. The government purchase and labor tax multipliers also have an asymptote in this version of the model at $p \approx 0.835$. We avoid the asymptote and only report results for $p = 0.9$ and $p = 0.75$. This version of the model produces resource costs of price adjustment of 0.8% of output. A second feature of this model is that with rigid real wages much smaller shocks to technology are needed to hit the GD targets.

One difference between this model and the previous model is that the interval of p where the government purchase GDP multiplier is negative is now much larger. It is negative for all values of $p \geq 0.84$. For values of p that are smaller than 0.835, the sign is once again positive. Now the government purchase multiplier falls below 1.15 when $p = 0.75$ in the targeted equilibrium. For purposes of comparison, it fell below this level when $p = 0.8$ for the baseline model with flexible real wages.

A second and more striking difference pertains to the labor tax hours multiplier. In this model, households are typically not on their labor supply schedules and labor supply is inelastic to changes in income taxes. So instead we report changes in labor input rented by firms when a payroll tax is perturbed from a value of one. This multiplier is massive. A one percentage point change in the payroll tax reduces hours worked by -24.8% when $p = 0.9$. This multiplier continues to be very large and negative for larger values of p. This multiplier is -18.2 when $p = 0.95$. The converse is true for small p that lie below the asymptote. For instance, the payroll tax hours multiplier is 8.43 when $p = 0.75$. The reason why these multipliers are so large stems from the fact that wages are rigid and do not respond to shocks. This in turn reduces the response of the markup to shocks and produces much larger responses in quantities as compared to the model with flexible real wages. In other words, AS shifts more in response to a shock of a given size.

Another property of this model is that the range of multiplicity is larger and falls in a different region of the parameter space. There are multiple equilibria for $p \in [0.75, 0.835]$. One of the equilibria has slope(AD)$>$slope(AS)$>$0 and the other has slope(AS)$>$slope(AD)$>$0. Moreover, there are no other equilibria with a positive interest rate in this region of the parameter space.

Finally, note that the loglinearized solution has similar properties for these two alternative choices of p. As we found before in Table 1, the biases in this solution are small for $p \geq 0.9$. Although not reported due to space considerations, the nonlinear model also exhibits a conventional configuration of demand and supply when $p \leq 0.3$.[28] In this interval, the loglinear solution exhibits a Type 1 equilibrium instead. The only difference is that the size of the bias in the labor tax multiplier is now larger for p in this range.

Overall, introducing real wage rigidities allows us to entertain smaller values of γ and thus smaller resource costs of price adjustment. Government purchase multipliers are still small or negative for a broad range of values of p. The major shortcoming of the model with rigid wages is that the labor tax multipliers are now extraordinarily large. In addition, the region with multiple zero bound equilibria is larger.

8 Conclusion

In this paper we have documented the global properties of the Rotemberg model at the zero bound and found that for a broad range of empirically relevant parameters the government purchase multiplier is about one or less and there is no paradox of toil. Our results do not rule out the possibility of large fiscal multipliers. As we have shown they can be arbitrarily large and positive or arbitrarily large and negative when the slopes of the AD and AS schedules are sufficiently close.

In the course of making these points, we have described the global properties of the Rotemberg model of price adjustment. This model has a rich set of dynamics when interest rates are zero. At least four distinct configurations of AD and AS schedules occur including the conventional configuration with slope(AD)$<$0$<$slope(AS). In addition, there can be multiple zero bound equilibria. These properties of the Rotemberg model arise in empirically relevant ranges of the space of parameters and shocks.

Our analysis has focused on the properties of fiscal multipliers when the nominal rate of interest is zero. It is clear, though, that our findings are relevant for a broad range of other analyses of the zero bound. The properties of an optimal monetary policy are of particular interest to us. The results on optimal monetary policy in e.g. Eggertsson and Woodford (2003) and Werning (2011) are derived in models that respect the nonlinearity in the Taylor rule but loglinearize the remaining equilibrium conditions about a steady state with a stable price level. In our future work, we plan to consider the properties of monetary policy in a nonlinear framework along the lines of the one we have considered here.

[28] A more complete set of results for this model is reported in the Online Appendix.

Appendix A Proofs

Proof of Proposition 1 The AD and the AS schedules are

$$\pi^L = [slope(AD)\hat{z}^L - \frac{\hat{r}^e_L}{p}] + slope(AD)\hat{h}^L, \text{and } \pi^L = slope(AS)\frac{\sigma-1}{\sigma+\nu}\hat{z}^L + slope(AS)\hat{h}^L.$$

They are upward-sloping, for both $slope(AD)$ and $slope(AS)$ are positive.

First, assume (1a) and (1b). They imply that the AD schedule is strictly steeper than the AS schedule, and that the intercept term is strictly higher for the AD schedule than for the AS schedule. It follows that at the intersection $\hat{h}^L < 0$. Solving for π^L, we obtain

$$\begin{aligned}\pi^L &= \frac{1}{slope(AS) - slope(AD)}\left[slope(AS)\{slope(AD)\hat{z}^L - \frac{\hat{r}^e_L}{p}\} - slope(AD)slope(AS)\frac{\sigma-1}{\sigma+\nu}\hat{z}^L\right] \\ &= \frac{slope(AS)}{slope(AS) - slope(AD)}\left[slope(AD)\hat{z}^L + slope(AD)\frac{\sigma-1}{\sigma+\nu}(-\hat{z}^L) - \frac{\hat{r}^e_L}{p}\right].\end{aligned}$$

Since $slope(AS) - slope(AD) < 0$, π^L is negative at the intersection if and only if the terms in the square brackets are positive.

$$\begin{aligned}& slope(AD)\hat{z}^L + slope(AD)\frac{\sigma-1}{\sigma+\nu}(-\hat{z}^L) - \frac{\hat{r}^e_L}{p} \\ \geq\; & slope(AD)\hat{z}^L + slope(AS)\frac{\sigma-1}{\sigma+\nu}(-\hat{z}^L) - \frac{\hat{r}^e_L}{p} \\ & \quad \text{(By assumption, } (\sigma-1)(-\hat{z}^L) \geq 0 \text{ and } slope(AS) - slope(AD) < 0.) \\ >\; & 0. \quad \text{(By condition 1a).)}\end{aligned}$$

Thus, at the intersection of the AD and the AS schedules, $(\pi^L, \hat{h}^L) < (0, 0)$.

What remains to show is that at the intersection, the Taylor rule implies zero nominal interest rate. The linear part of the Taylor rule in (8) prescribes

$$\begin{aligned}& \hat{r}^e + \phi_\pi \pi^L + \phi_y \hat{y}^L \\ <\; & p\left(slope(AD) - slope(AS)\frac{\sigma-1}{\sigma+\nu}\right)\hat{z}^L + \phi_\pi \pi^L + \phi_y \hat{y}^L. \quad \text{(Condition 1a).)}\end{aligned}$$

Since $\hat{y}^L = \hat{z}^L + \hat{h}^L$, we know that $(\hat{z}^L, \pi^L, \hat{y}^L)$ are all negative. Condition 1b) implies that the coefficient on $\hat{z}^L$ is strictly positive. Together with the assumption $(\phi_\pi, \phi_y) \geq 0$, it follows that the RHS of the above inequality is strictly negative, and the nominal interest rate at the intersection is zero.

Next, assume (2a) and (2b). Proof is almost the same as that in the case with (1a) and

(1b). The only difference is that ϕ_π needs to be sufficiently large to have $\hat{r}^e_L + \phi_\pi \pi^L + \phi_y \hat{y}^L < 0$. Since the AD schedule is upward sloping and $\hat{h}^L < 0$, π^L is smaller than the intercept of the AD schedule, $\mathit{slope}(AD)\hat{z}^L - \hat{r}^e_L/p$. Thus, the assumption $\phi_\pi \geq p$ implies

$$\hat{r}^e_L + \phi_\pi \pi^L + \phi_y \hat{y}^L \leq \hat{r}^e_L + p\pi^L \leq \hat{r}^e_L + p\{\mathit{slope}(AD)\hat{z}^L - \hat{r}^e_L/p\} \leq p \times \mathit{slope}(AD)\hat{z}^L \leq 0.$$

The nominal interest rate is thus zero.

Finally, suppose 1a) holds but 1b) doesn't. Then the AD is no steeper than the AS, and the intercept of the AD is larger than the intercept of the AS. When the AD and the AS are parallel but their intercepts differ, then there is no intersection and thus no equilibrium with $R = 0$. When the AS is strictly steeper than the AD, then their intersection satisfies $\hat{h}^L > 0$, and there is no ZLB equilibrium with $\hat{h}^L \leq 0$. The same argument goes through for the case where 2a) holds but 2b) doesn't.□

Proof of Proposition 2 We first solve for $(\hat{h}^L, \pi^L)$, allowing non-zero $\hat{\eta}^L$ and τ^L_w. Define the intercepts of the AS and AD schedules as

$$\begin{aligned}
\mathit{icept}(AS) &= -\mathit{slope}(AS)\frac{\sigma}{\sigma+\nu}\frac{1}{1-\eta}\hat{\eta}^L + \mathit{slope}(AS)\frac{1}{\sigma+\nu}\frac{1}{1-\tau_w}\hat{\tau}^L_w - \mathit{slope}(AS)\frac{1-\sigma}{\sigma+\nu}\hat{z}^L \\
\mathit{icept}(AD) &= \frac{-\hat{r}^e_L}{p} - \mathit{slope}(AD)\frac{1}{1-\eta}\hat{\eta}^L + \mathit{slope}(AD)\hat{z}^L.
\end{aligned}$$

Then

$$\begin{aligned}
\hat{h}^L &= \frac{\mathit{icept}(AD) - \mathit{icept}(AS)}{\mathit{slope}(AS) - \mathit{slope}(AD)} \\
\pi^L &= \mathit{icept}(AS) + \mathit{slope}(AS)\hat{h}^L \quad (= \mathit{icept}(AD) + \mathit{slope}(AD)\hat{h}^L)
\end{aligned}$$

Consider a paradox of toil: by differentiating $(\hat{h}^L, \pi^L)$ by $\hat{\tau}^L_w$, we obtain

$$\frac{d\hat{h}^L}{d\hat{\tau}^L_w} = \frac{-\mathit{slope}(AS)}{\mathit{slope}(AS) - \mathit{slope}(AD)}\frac{1}{\sigma+\nu}\frac{1}{1-\tau_w}, \quad \frac{d\pi^L}{d\hat{\tau}^L_w} = \mathit{slope}(AD)\frac{d\hat{h}^L}{d\hat{\tau}^L_w}.$$

Under the Type 1 parameterization, both hours and inflation increase in response to an increase in labor tax ($d\hat{h}^L/d\hat{\tau}^L_w$ and $d\pi^L/d\hat{\tau}^L_w$ are positive), thereby establishing the paradox of toil. Under the Type 2 parameterization, however, both $d\hat{h}^L/d\hat{\tau}^L_w$ and $d\pi^L/d\hat{\tau}^L_w$ are negative, and both hours and inflation decline in response to an increase in labor tax.

Next, consider a change in government purchase share. Differentiating by $\hat{\eta}^L$, we obtain

$$\frac{d\hat{h}^L}{d\hat{\eta}^L} = \frac{1}{1-\eta}\frac{\mathit{slope}(AS)\frac{\sigma}{\sigma+\nu} - \mathit{slope}(AD)}{\mathit{slope}(AS) - \mathit{slope}(AD)}, \quad \frac{d\pi^L}{d\hat{\eta}^L} = \mathit{slope}(AD)[-\frac{1}{1-\eta} + \frac{d\hat{h}^L}{d\hat{\eta}^L}].$$

Under the Type 1 parameterization, $slope(AD) > slope(AS) > slope(AS)\sigma/(\sigma+\nu) > 0$, and thus $d\hat{h}^L/d\hat{\eta}^L$ is positive and greater than $1/(1-\eta)$. This further implies $d\pi^L/d\hat{\eta}^L$ is positive. An increase in η thus increases both hours and inflation.

Under the Type 2 parameterization, it is either (1) $slope(AD) - slope(AS)\sigma/(\sigma+\nu) > 0$ or (2) $slope(AD) - slope(AS)\sigma/(\sigma+\nu) < 0$. In the first case, both $d\hat{h}^L/d\hat{\eta}^L$ and $d\pi^L/d\hat{\eta}^L$ are negative. In the second case, we have $0 < d\hat{h}^L/d\hat{\eta}^L < 1/(1-\eta)$ and $d\pi^L/d\hat{\eta}^L < 0$.

We are more interested in the response of GDP to a change in government purchases than a change in its share. Suppose for a moment that the government purchase g increases in response to an increase in η. In this case, the government purchase multiplier on GDP is greater than one if and only if consumption increases in response to η. From the resource constraint $c = (1-\eta-\kappa)zh$, we have $d\hat{c}^L/d\eta^L = -1/(1-\eta) + d\hat{h}^L/d\eta^L$. We know from the preceding discussion that under the Type 1 (2) parameterization the right hand side is positive (negative). Thus the government purchase multiplier is greater than one under the Type 1 parameterization, and less than one under the Type 2 parameterization, as long as the government purchases increase in response to an increase in η.

Finally, the government purchases decrease in response to an increase in η only when hours respond negatively. This requires $slope(AS) > slope(AD) > slope(AS)\sigma/(\sigma+\nu)$. From $g = \eta zh$, we have $d\hat{g}^L/d\eta^L = 1/\eta + d\hat{h}^L/d\eta^L$. This is less than zero if and only if

$$(1-\eta+\eta\frac{\sigma}{\sigma+\nu})slope(AS) < slope(AD).$$

Proof of Proposition 3: We loglinearize the AD relationship (13) around a reference point (h^L, π^L) which satisfies (13). Let Δh^L be the log deviation of hours and $\Delta\pi^L$ be the level deviation of inflation rate from this reference point. This yields

$$\Delta h^L = \Big\{\frac{1}{\sigma}\frac{p\beta d^L/(1+\pi^L)^2}{1-p\beta d^L/(1+\pi^L)} + \frac{(\kappa^L)'}{1-\kappa^L-\eta^L}\Big\}\Delta\pi^L. \tag{21}$$

This implies Proposition 3.

Proof of Proposition 4 We loglinearize the AS relationship around a reference point (h^L, π^L) which satisfies (14). Let $(\Delta h^L, \Delta\pi^L)$ be the same as in the proof of Proposition 3. Then it yields

$$(\sigma+\nu)\Delta h^L = \Big\{\frac{(1-p\beta d^L)\gamma(1+2\pi^L)}{(1-p\beta d^L)\gamma\pi^L(1+\pi^L)+\theta} + \frac{\sigma(\kappa^L)'}{1-\kappa^L-\eta^L}\Big\}\Delta\pi^L. \tag{22}$$

Since $\sigma+\nu > 0$, Proposition 4 is implied.

Proof of Proposition 5 To analyze the effect of labor tax changes, we loglinearize the AS (14) with respect to the tax change too. We obtain

$$(\sigma+\nu)\Delta h^L = \left\{\frac{(1-p\beta d^L)\gamma(1+2\pi^L)}{(1-p\beta d^L)\gamma\pi^L(1+\pi^L)+\theta} + \frac{\sigma(\kappa^L)'}{1-\kappa^L-\eta^L}\right\}\Delta\pi^L - \frac{1}{1-\tau_w^L}\Delta\tau_{w,L}, \tag{23}$$

where $\Delta\tau_{w,L}$ is the level deviation from τ_w^L. Thus we have

$$\Delta\pi^L = \mathit{slope}(AD)\Delta h^L = \mathit{slope}(AS)\Delta h^L + \mathit{slope}(AS)\frac{1/(1-\tau_w^L)}{\sigma+\nu}\Delta\tau_w^L$$

Solving for $(\Delta\pi^L, \Delta h^L)$, the policy effect is summarized as

$$(\Delta h^L, \Delta\pi^L) = \left(\frac{\mathit{slope}(AS)/(1-\tau_w^L)}{(\mathit{slope}(AD)-\mathit{slope}(AS))(\sigma+\nu)}\Delta\tau_w^L, \frac{\mathit{slope}(AS)\mathit{slope}(AD)/(1-\tau_w^L)}{(\mathit{slope}(AD)-\mathit{slope}(AS))(\sigma+\nu)}\Delta\tau_w^L\right). \tag{24}$$

From the resource constraint, consumption response is

$$\Delta c^L = \frac{-(\kappa^L)'}{1-\kappa^L-\eta^L}\Delta\pi^L + \Delta h^L.$$

When inflation and hours move in the same direction, consumption also moves in that direction if $\pi^L < 0$.

If either $\mathit{slope}(AS) > 0 > \mathit{slope}(AD)$ or $\mathit{slope}(AS) < \mathit{slope}(AD) < 0$, multiplier on hours is negative while that on inflation is positive. This proves the part a). If $\mathit{slope}(AD) > \mathit{slope}(AS) > 0$, multipliers on hours and inflation are positive. Consumption also rises. This proves the part b). If $\mathit{slope}(AS) > \mathit{slope}(AD) > 0$, multipliers on hours and inflation are negative. Consumption also falls. This proves the part c).

Appendix B Model with real wage rigidities

This Appendix describes the model with Rotemberg costly price adjustment real wage stickiness. Modeling rigid real wages makes it possible to reproduce the Great Depression output and inflation targets with a smaller amount of nominal price rigidity.[29] We consider real wage rigidities along the lines of Blanchard and Galí (2007). Suppose that the real wage paid to one unit of labor is constant at its steady-state level of $w_t = w_{ss}$ and that the production technology is $y_t(i) = z_t h_t(i)^{1-\alpha}$ with $0 < \alpha \leq 1$. It is well known that this production function can also be interpreted as being constant returns-to-scale with a second input (with a

[29]The firm-specific labor market assumption that is frequently made in Calvo models does not generate real rigidities in the Rotemberg model because there is no price dispersion.

fixed rental price) that cannot be adjusted in the very short-run. Following the example of Blanchard-Gali we suppose that equilibrium labor is demand-determined and obeys

$$w_t = (w_{t-1})^{\gamma} (\frac{v'(h_t)}{u'(c_t)(1-\tau_{w,t})})^{1-\gamma}$$

with $\gamma = 1$ and $w_{-1} = w_{ss}$. Firms are on their input demand schedules so this assumption implies that the after-tax real wage will not generally equal the household's marginal rate of substitution.

The cost function for firm i is now $(1+\tau_{P,t})w_{ss}/z_t \times y_t(i)^{\frac{1}{1-\alpha}}$, where $\tau_{P,t}$ is a payroll tax paid by firms. Labor income tax cuts are neutral in this setting so we introduce a payroll tax to analyze the effect of changes in this tax wedge on labor. We set its steady state value to zero.

Under our previous assumption that $\theta = (1+\tau_s)(\theta-1)$, the NKPC is given by:

$$\gamma\pi_t(1+\pi_t) = \frac{\theta}{1-\alpha}(1+\tau_{P,t})\frac{w_{ss}}{z_t}h_t^{\alpha} - \theta + \beta d_{t+1}E_t[\frac{u'(c_{t+1})}{u'(c_t)}\frac{y_{t+1}}{y_t}\gamma\pi_{t+1}(1+\pi_{t+1})],$$

and the AS equation is given by

$$(1-p\beta d^L)\frac{\gamma}{\theta}\pi_L(1+\pi_L) = \frac{w_{ss}}{(1-\alpha)z^L}(1+\tau_P^L)(h^L)^{\alpha} - 1.$$

Note that this is independent of the individual labor income tax rate τ_w, and thus an equilibrium is unaffected by changes in τ_w.

When loglinearized around (h^L, π^L) and $\tau_P^L = \tau_P$,

$$\Delta\pi = \frac{(1-p\beta d^L)\frac{\gamma}{\theta}\pi^L(1+\pi^L)+1}{(1-p\beta d^L)\frac{\gamma}{\theta}(1+2\pi^L)}\left\{\alpha\Delta h + \frac{\Delta\tau^P}{1+\tau_P} - \Delta z\right\}.$$

The resource constraint is

$$c = (1-\kappa-\eta)zh^{1-\alpha}.$$

and thus the loglinearized AD equation around (h^L, π^L) is

$$\Delta\pi = \frac{(1-\frac{p\beta d^L}{1+\pi^L})\sigma}{\frac{p\beta d^L}{(1+\pi^L)^2} + (1-\frac{p\beta d^L}{1+\pi^L})\frac{\sigma\gamma\pi^L}{1-\kappa^L-\eta^L}}\left\{(1-\alpha)\Delta h + \Delta z - \frac{\Delta\eta}{1-\kappa^L-\eta^L}\right\}.$$

It follows that the tax multiplier on hours is given by

$$\frac{slope(AS)}{slope(AD) - slope(AS)}\frac{1}{\alpha(1+\tau^P)}.$$

References

ARUOBA, S. B., AND F. SCHORFHEIDE (2013): "Macroeconomic Dynamics Near the ZLB: A Tale of Two Equilibria," NBER Working Papers 19248, National Bureau of Economic Research, Inc. 1, 3.1, 7, 6.1

BENHABIB, J., S. SCHMITT-GROHE, AND M. URIBE (2001): "Monetary Policy and Multiple Equilibria," *American Economic Review*, 91(1), 167–186. 1, 2.2, 3.1

BLANCHARD, O., AND J. GALÍ (2007): "Real Wage Rigidities and the New Keynesian Model," *Journal of Money, Credit and Banking*, 39(s1), 35–65. 7.3, B

BRAUN, R. A., L. KÖRBER, AND Y. WAKI (2012): "Some unpleasant properties of log-linearized solutions when the nominal interest rate is zero," *Federal Reserve Bank of Atlanta Working Paper 2012-5a*. 3.1, 6

BRAUN, R. A., AND Y. WAKI (2010): "On the Size of the Fiscal Multiplier When the Zero Bound on Nominal Interest Rates is Binding," mimeo. 1, 7.3

BRODA, C., AND D. WEINSTEIN (2004): "Globalization and the gains from variety," Staff Reports 180, Federal Reserve Bank of New York. 5.1, 6.2.1

BULLARD, J. (2010): "Seven Faces of "The Peril"," *Federal Reserve Bank of St. Louis Review*, 92(5), 339–352. 2.2, 3.1

BURGESS, S., E. FERNANDEZ-CORUGEDO, C. GROTH, R. HARRISON, F. MONTI, K. THEODORIDIS, AND M. WALDRON (2013): "The Bank of England's Forecasting Platform: COMPASS, MAPS, EASE and the Suite of Models," *Bank of England working paper*. 5.1

CALVO, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 12(3), 383–398.

CARLSTROM, C., T. FUERST, AND M. PAUSTIAN (2012): "Fiscal Multipliers under an Interest Rate Peg of Deterministic and Stochastic Duration," mimeo. 1, 6.1

CHRISTIANO, L., AND M. EICHENBAUM (2012): "Notes on Linear Approximations, Equilibrium Multiplicity and E-learnability in the Analysis of the Zero Bound," mimeo. 1, 6.1, 6.2.3, 17

Christiano, L., M. Eichenbaum, and S. Rebelo (2011): "When is the Government Spending Multiplier Large?," *Journal of Political Economy*, 119(1), 78–121. 1, 5.2, 17, 6.2.3

Denes, M., G. B. Eggertsson, and S. Gilbukh (2013): "Deficits, Public Debt Dynamics and Tax and Spending Multipliers," *Economic Journal*, 0, 133–163. 5.1, 6.1, 6.2.4

Eggertsson, G. (2011): "What Fiscal Policy is Effective at Zero Interest Rates?," in *NBER Macroeconomics Annual 2010, Volume 25*, NBER Chapters, pp. 59–112. National Bureau of Economic Research, Inc. 1, 5.2, 6.2.3, 7.2

Eggertsson, G., A. Ferrero, and A. Raffo (2013): "Can Structural Reforms Help Europe?," Unpublished manuscript, Brown University. 1

Eggertsson, G., and P. Krugman (2012): "Debt, Deleveraging and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *Quarterly Journal of Economics*, 127(3). 1, 2.3

Eggertsson, G., and M. Woodford (2003): "The Zero Interest Rate Bound and Optimal Monetary Policy," *Brookings Panel on Economic Activity*. 2, 2.2, 6.2.1, 8

Erceg, C., and J. Lindé (2010): "Is There a Fiscal Free Lunch in a Liquidity Trap?," Board of Governors Board of Governors International Finance Discussion Papers No. 1003. 1

Evans, G. W., E. Guse, and S. Honkapohja (2008): "Liquidity traps, learning and stagnation," *European Economic Review*, 52(8), 1438–1463. 1

Fernandez-Villaverde, J., G. Gordon, P. A. Guerron-Quintana, and J. Rubio-Ramirez (2012): "Nonlinear Adventures At The Zero Lower Bound," *NBER Working Paper 18058*. 1

Gust, C., D. Lopez-Salido, and M. E. Smith (2012): "The empirical implications of the interest-rate lower bound," Finance and Economics Discussion Series 2012-83, Board of Governors of the Federal Reserve System (U.S.). 1, 5.1

Hayashi, F., and J. Koeda (2013): "A Regime-Switching SVAR Analysis of Quantitative Easing," CARF F-Series CARF-F-322, Center for Advanced Research in Finance, Faculty of Economics, The University of Tokyo. 12

Ireland, P. N. (2003): "Endogenous money or sticky prices?," *Journal of Monetary Economics*, 50(8), 1623–1648. 5.1

Levy, D., M. Bergen, S. Dutta, and R. Venable (1997): "The Magnitude of Menu Costs: Direct Evidence from Large U. S. Supermarket Chains," *The Quarterly Journal of Economics*, 112(3), 791–824. 6.1

Mertens, K., and M. O. Ravn (2010): "Fiscal Policy in an Expectations Driven Liquidity Trap," CEPR Discussion Papers 7931, C.E.P.R. Discussion Papers. 1, 3.1, 6.1

Rotemberg, J., and M. Woodford (1997): "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," in *NBER Macroeconomics Annual 1997, Volume 12*, NBER Chapters, pp. 297–361. National Bureau of Economic Research, Inc. 5.1

Rotemberg, J. J. (1982): "Sticky Prices in the United States," *Journal of Political Economy*, 90(6), 1187–1211. 1, 2

Werning, I. (2011): "Managing a Liquidity Trap: Monetary and Fiscal Policy," mimeo. 8

Woodford, M. (2011): "Simple Analytics of the Government Expenditure Multiplier," *American Economic Journal: Macroeconomics*, 3(1), 1–35. 1, 6.1

Yun, T. (2005): "Optimal Monetary Policy with Relative Price Distortions," *American Economic Review*, 95(1), 89–109. 2.1

Table 1: Zero Bound Equilibrium Great Recession: ($\gamma = 458.4, \nu = 0.28$)

Specification	d^L	p	z^L	$\kappa^L \times 100$	Slope(AD)	Slope(AS)	$\frac{\Delta GDP}{\Delta G}$	$\frac{\Delta h}{\Delta \tau_w}$
Parameterization 1)	1.008	0.90	0.94					
Exact solution 1				0.14	0.12	0.29	0.62	-1.66
Loglinear solution				0.22	0.11	0.21	0.62	-2.09
Parameterization 2)	1.01	0.86	0.95					
Exact solution 1				0.14	0.193	0.192	5.49	163.2
*Exact solution 2**				0.09	0.182	0.183	6.52	-172.7
Loglinear solution				0.11	0.16	0.15	2.49	11.5
Parameterization 3)	1.015	0.80	0.95					
Exact solution				0.14	0.35	0.13	1.14	0.55
Loglinear solution				0.14	0.25	0.11	1.18	0.72
Parameterization 4)	1.029	0.60	0.98					
Exact solution				0.14	5.65	0.056	1.02	0.010
Loglinear solution				0.14	0.67	0.053	1.04	0.085
Parameterization 5)	1.037	0.50	0.99					
Exact solution				0.14	-2.76	0.043	1.008	-0.015
loglinear solution				0.15	1.000	0.043	1.02	0.044

The size of the demand and technology shocks are chosen so that the exact solution produces a 7% decline in GDP and a 1% annualized decline in the inflation rate. The values of κ^L for the loglinearized solution are imputed.
*This solution does not reproduce the targets.

Table 2: Zero Bound Equilibrium Great Recession: Our estimates ($\gamma = 458.4, \nu = 0.28$) demand shock only.

Specification	d^L	p	θ	$\kappa^L \times 100$	Slope(AD)	Slope(AS)	$\frac{\Delta GDP}{\Delta G}$	$\frac{\Delta h}{\Delta \tau_w}$
Parameterization 1)	1.0076	0.90	1.28					
Exact solution				0.14	0.12	0.036	1.085	0.41
Loglinear solution				0.12	0.11	0.035	1.079	0.45
Parameterization 2)	1.015	0.80	2.55					
Exact solution				0.14	0.35	0.036	1.030	0.11
Loglinear solution				0.13	0.25	0.035	1.029	0.16
Parameterization 3)	1.029	0.60	5.15					
Exact solution				0.14	5.65	0.036	1.010	0.006
Loglinear solution				0.14	0.67	0.036	1.010	0.055
Parameterization 4)	1.037	0.50	6.47					
Exact solution				0.14	-2.76	0.036	1.007	-0.013
Loglinear solution				0.15	1.000	0.036	1.007	0.037

The size of the demand shock and the value of θ are chosen so that the exact solution reproduces a targeted decline in output of 7% and annualized inflation decline of 1%. The values of κ^L for the loglinearized solution are imputed.

Table 3: Zero Bound Equilibria Great Recession

Parameterization	d^L	p	γ	θ	$\kappa^L \times 100$	Slope(AD)	Slope(AS)	$\frac{\Delta GDP}{\Delta G}$	$\frac{\Delta h}{\Delta G}$	$\frac{\Delta h}{\Delta \tau_w}$
1) Christiano and Eichenbaum (CE) (2012)	1.012	0.775	100	3						
Exact solution (ES)					0.7	0.46	0.31	2.18	1.71	1.32
Loglinear solution (LLS)					1.6	0.29	0.26	2.77	2.77	4.95
2) CE (2012) calibrated using ES,	1.0237	0.775	300	1.24						
Exact solution					0.09	0.36	0.034	1.06	1.04	0.066
Loglinear solution					0.09	0.29	0.035	1.05	1.05	0.087
3) Denes et al. (2013) calibrated using ES,	1.0135	0.857	6345	13.23						
Exact solution					2.0	-0.067	0.057	1.08	0.792	-0.20
Loglinear solution					3.0	0.204	0.0416	1.12	1.12	0.11
4) Denes et al. (2013) low γ calibrated using ES,	1.0266	0.70	720	13.23						
Exact solution					0.23	-5.89	0.11	1.04	1.05	-0.016
Loglinear solution					0.19	0.52	0.091	1.09	1.09	0.18

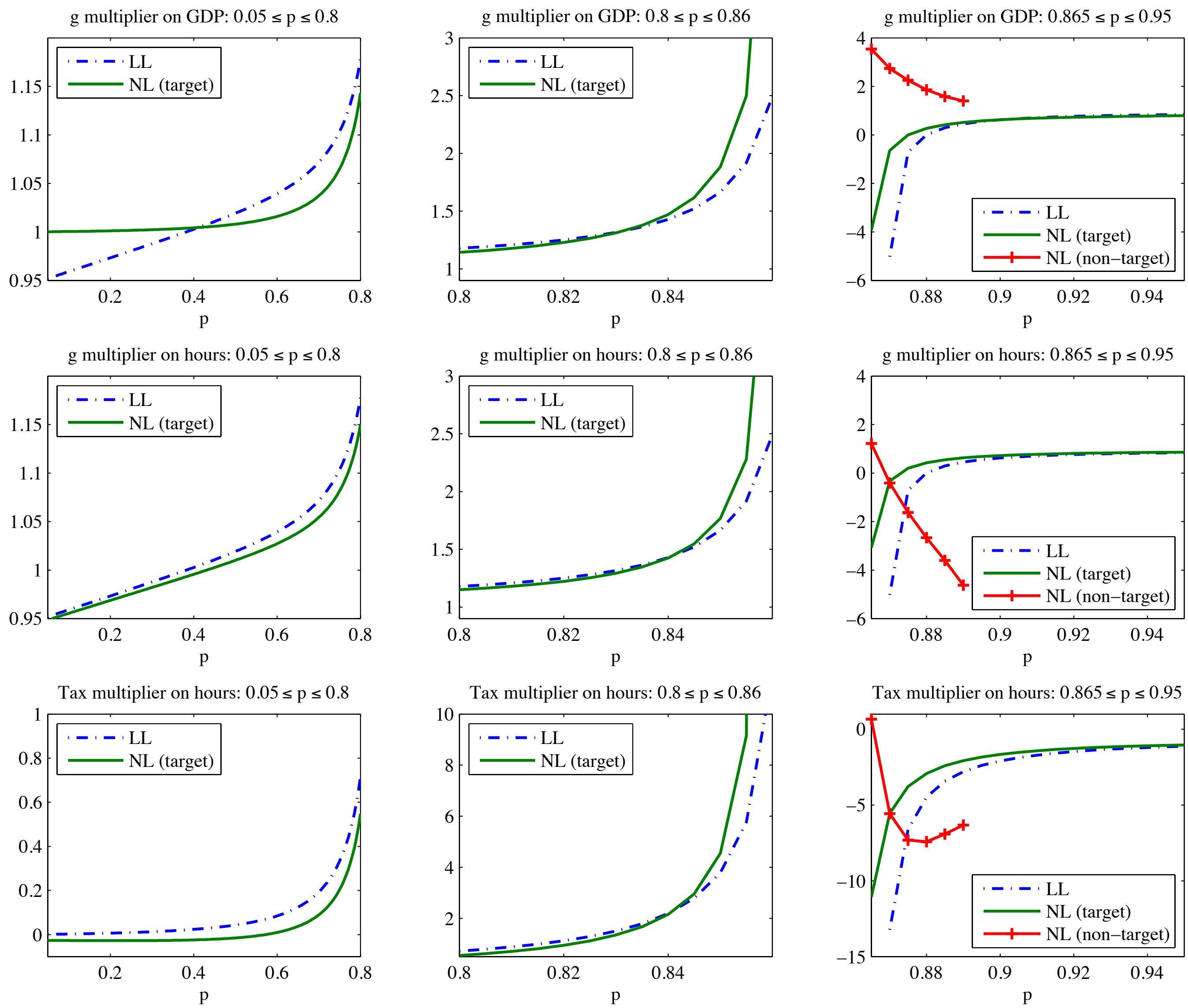
Results labeled "calibrated" have been calibrated to produce a 7% GDP decline and 1% annualized inflation decline using the specified solution. The values of κ^L for the loglinearized solution are imputed.

Table 4: Zero Bound Equilibria Great Depression

Specification	d^L	p	z^L	γ	θ	κ^L	Slope(AD)	Slope(AS)	$\frac{\Delta GDP}{\Delta G}$	$\frac{\Delta h}{\Delta G}$	$\frac{\Delta h}{\Delta \tau_w}$
1) Our estimates calibrated using ES,											
Parameterization a)	1.042	0.80	0.93	458.4	7.67						
Exact solution (ES)						0.155	-0.080	-0.23	1.13	0.91	-1.50
Loglinear solution (LLS)						0.121	0.25	0.11	1.20		0.72
Parameterization b)	1.059	0.75	1.00	458.4	7.67						
Exact solution						0.155	-0.071	-0.99	1.061	0.96	-1.05
Loglinear solution						0.145	0.33	0.085	1.064		0.33
Parameterization c)	1.063	0.74	1.009	458.4	7.67						
Exact solution						0.155	-0.070	-3.92	1.054	0.96	-0.99
Loglinear solution						0.149	0.35	0.082	1.043		0.30
Parameterization d)	1.067	0.73	1.023	458.4	7.67						
Exact solution						0.155	-0.069	1.87	1.047	0.96	-0.94
Loglinear solution						0.153	0.37	0.079	1.023		0.26
Parameterization e)	1.078	0.70	1.070	458.4	7.67						
Exact solution						0.155	-0.067	0.32	1.034	0.95	-0.81
Loglinear solution						0.166	0.43	0.071	0.97		0.19
2) Eggertsson (2011), calibrated using LLS,	1.0134	0.903	1	4059.78	12.77						
Exact solution						0.11	-0.033	-0.22	1.20	0.43	-0.54
Loglinear solution						1.24	0.12	0.086	1.82		1.02
3) Eggertsson (2011), calibrated using ES,	1.0213	0.903	1	1209.2	12.77						
Exact solution 1						0.41	-0.015	-0.032	1.26	-0.24	-0.87
*Exact solution 2**						0.11	0.031	0.077	0.76	0.068	-0.76
Loglinear solution						N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
4) Rigid wage model, calibrated using ES,	1.0051	0.9	0.989	25	7.67						
Exact solution						0.0084	0.078	0.21	-1.00	-0.928	-24.83
Loglinear solution						0.017	0.10	0.19	-2.22		-28.45
5) Rigid wage model, calibrated using ES,	1.0505	0.75	0.998	25	7.67						
Exact solution						0.0084	0.271	0.095	1.48	1.47	8.43
Loglinear solution						0.039	0.312	0.078	1.31		4.33

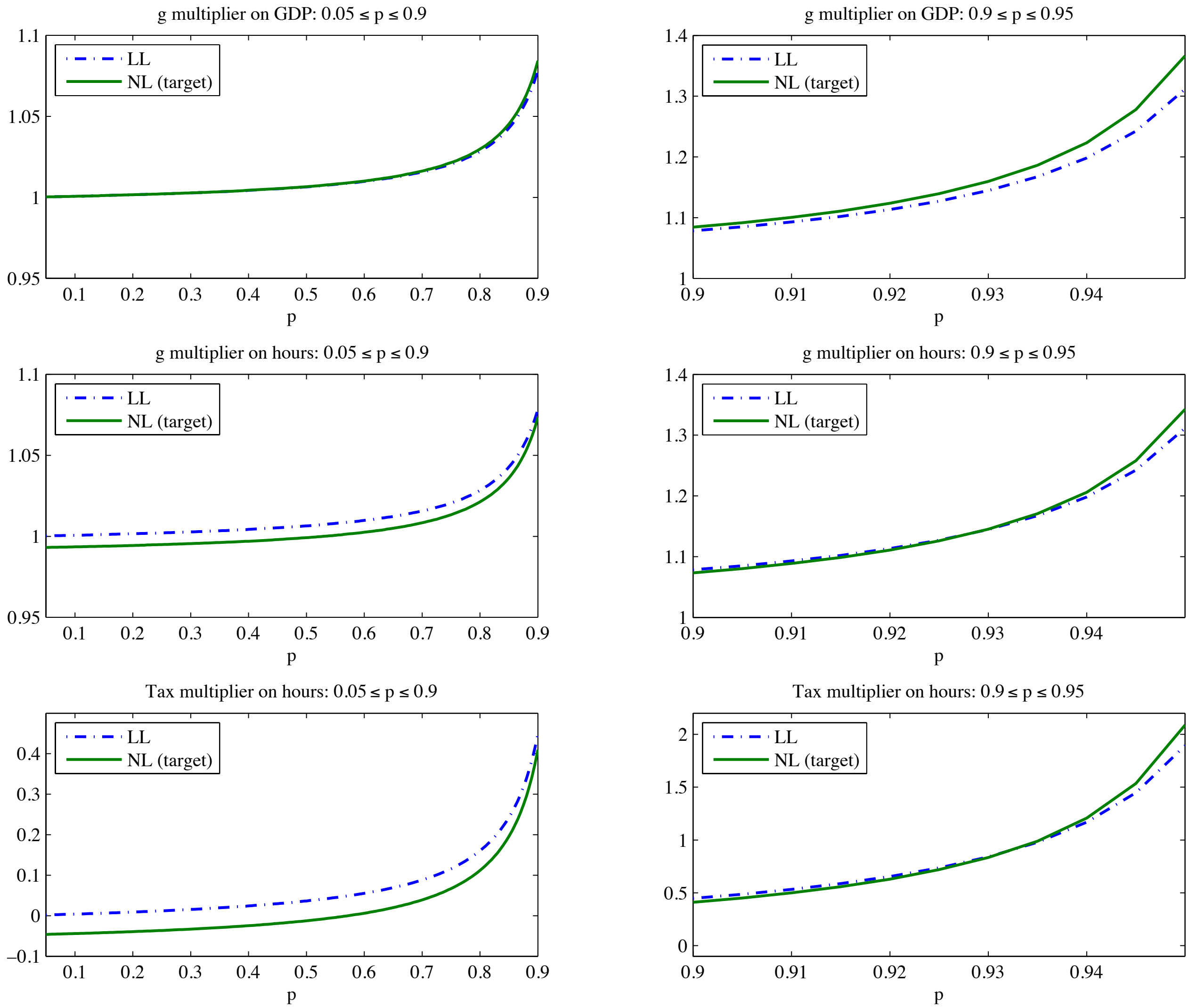
For results labeled "calibrated" model is calibrated to produce a 30% GDP decline and a 10% inflation decline using the specified solution. The values of κ^L for the loglinearized solution are imputed.
* This solution does not hit the targets.

Figure 1: Government purchase multipliers and labor tax hours multipliers.



NL refers to the multipliers using the exact solution and LL refers to multipliers using the loglinearized solution. Scaling: A value of one implies that a one percentage point change in the labor tax results in a one percent increase in hours.

Figure 2: Government purchase GDP multipliers and labor tax hours multipliers in a model with no technology shock.



NL refers to the multipliers using the exact solution and LL refers to multipliers using the loglinearized solution. Scaling: A value of one implies that a one percentage point change in the labor tax results in a one percent increase in hours.