Herding Cycles

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Motivation

• Many recessions are preceded by booming periods of frenzied investment after introduction of new technology (“boom-bust cycle”)
  ▶ IT-led boom in late 1990s

• While standard practice in business cycle analysis is to treat them separately, another view is that booms and busts are two sides of the same coin
  ▶ “booms sow the seeds of the subsequent busts” (Schumpeter)
  ▶ extent and magnitude of expansion cause and determine depth of downturn

• Our objective is to develop a theory of (quasi-)endogenous boom-and-bust cycles
• We embed herding features into a business cycle framework
  ▶ Social learning: people collectively fool themselves into thinking they’re into a boom
  ▶ We explore the ability of such models to generate economic booms followed by sudden crashes
  ▶ Under multidimensional uncertainty, agents may attribute observations to wrong causes, with possibility of quick reversals in beliefs

• Preview of results:
  ▶ Model has predictions on when booms-and-busts arise and when they collapse
  ▶ Since cycle is endogenous, policy can be powerful in eliminating such cycles
  ▶ Quantitatively, even with rational agents, booms-and-bust may arise with reasonably high probability (≈15%)
The Story

• Boom-bust cycles as false-positives:
  ▶ Technological innovations arrive exogenously with uncertain qualities
  ▶ Agents have private information and observe aggregate investment rates
  ▶ Importantly, we assume that there is common noise in private signals
    • Correlation of beliefs due to agents having similar sources of information
    • Allows for average beliefs to be away from true fundamentals
  ▶ High investment indicates either:
    • state with good technology, or
    • state with bad technology but where agents hold optimistic beliefs.
Development of a **boom-bust cycle**: 

- Unusually large realizations of noise may send the economy on **self-confirming boom** where:
  - agents mistakenly attribute high investment to technology being good
  - leads agents to take actions that seemingly confirm their assessment
  - investment rises...
- However, agents are rational and information keeps arriving, so probability of false-positive state rises
  - at some point, most pessimistic agents stop investing
  - suddenly, high beliefs are no longer confirmed by experience
  - sharp reversal in beliefs and collapse of investment \( \Rightarrow \) **bust**
  - truth is learned in the end
Related Literature

• News/noise-driven cycle literature
  ▶ Shares the view of boom-bust cycles as false-positives
  ▶ Can view our contribution as endogenizing the information process for news cycles

• Herding literature
  ▶ Relax certain assumption of early herding models:
    • Rely crucially on agents moving sequentially and making binary decisions
    • Boom-busts only arrive for specific sequence of events and particular ordering of people
  ▶ In our model, agents move simultaneously and learn from aggregates
    • Do not rely on a specific ordering of agents to generate cycle, but instead on the endogenous evolution of beliefs in the presence common noise
    • Closest to Avery and Zemsky (1998) for herding with multidimensional uncertainty

• This paper:
  ▶ Boom-busts cycles arise endogenously after a single impulse shock
  ▶ Application to business cycles and policy analysis
Plan

1. Simplified learning model
2. Business-cycle model with herding
Plan

1. Simplified learning model
2. Business-cycle model with herding
Learning Model

- Simple, abstract model
- Time is discrete $t = 0, 1, \ldots, \infty$
- Unit continuum of risk neutral agents indexed by $j \in [0, 1]$
Learning Model: Technology

- Agents choose whether to invest or not, \( i_{jt} = 1 \) or 0
  - Investing requires paying the cost \( c \)
- Investment technology has common return

\[
R_t = \theta + u_t
\]

with:
- Permanent component \( \theta \in \{\theta_H, \theta_L\} \) with \( \theta_H > \theta_L \), drawn once-for-all
- Transitory component \( u_t \sim \text{iid } F^u \)
• Agents receive a private signal \( s_j \)
  
  ▶ Example:
  \[
  s_j = \theta + \xi + v_j \text{ where } v_j \sim \mathcal{N} \left( 0, \sigma_v^2 \right)
  \]
  
  ▶ \( \xi \) is some common noise drawn from cdf \( F^\xi \)
    
    • captures the fact that agents learn from common sources (media, govt)

• More generally, \( s_j \) is drawn from distributions with pdf \( f_{\theta+\xi}^s (s_j) \)
  
  ▶ denote CDFs by \( F_{\theta+\xi}^s (s_j) \) and complementary CDFs by \( F_{\theta+\xi}^c (s_j) \)
  
  ▶ assume that \( F_x^s \) satisfies monotone likelihood ratio property (MLRP), i.e.,
  \[
  \text{for } x_2 > x_1, s_2 > s_1, \quad \frac{f_{x_2}^s (s_2)}{f_{x_1}^s (s_2)} \geq \frac{f_{x_2}^s (s_1)}{f_{x_1}^s (s_1)} \quad (\text{MLRP})
  \]
  
  ▶ **Intuition:** a higher \( s \) signals a higher \( \theta + \xi \)
In addition, all agents observe public signals

- return on investment $R_t$
- measure of investors $m_t$ (social learning)

Measure of investors is given by

$$m_t = \int_0^1 i_{jt}dj + \varepsilon_t$$

where $\varepsilon_t \sim iid \ F^m$ captures informational noise or noise traders

⇒ learning from endogenous non-linear aggregator of private information
Learning Model: Timing

Simple timing:

- At date 0: $\theta$, $\xi$ and the $s_j$’s are drawn once and for all
- At date $t \geq 0$,
  1. Agent $j$ chooses whether to invest or not
  2. Production takes place
  3. Agents observe $\{R_t, m_t\}$ and update their beliefs
• Beliefs are heterogeneous

• Denote public information to an outside observer at beginning of period $t$

\[ \mathcal{I}_t = \{ R_{t-1}, m_{t-1}, \ldots, R_0, m_0 \} \]
\[ = \{ R_{t-1}, m_{t-1} \} \cup \mathcal{I}_{t-1} \]

• The information set of agent $j$ is

\[ \mathcal{I}_{jt} = \mathcal{I}_t \cup \{ s_j \} \]
• Multiple sources of uncertainty so must keep track of joint distribution for public beliefs:
\[ \pi_t(\tilde{\theta}, \tilde{\xi}) = \Pr(\theta = \tilde{\theta}, \xi = \tilde{\xi} | I_t) \]

• Heterogeneous beliefs so keep track of distribution of individual beliefs \( \{\pi_{jt}\}_j \)

• Fortunately, heterogeneity is one-dimensional and constant:
  
  ▶ Distribution of private beliefs can be reconstructed anytime from public beliefs
Learning Model: Characterizing Beliefs

- For ease of exposition, simplify aggregate uncertainty to three states (slides only)

\[ \omega = (\theta, \xi) \in \left\{ (\theta_L, 0), (\theta_H, 0), (\theta_L, \Delta) \right\} \text{ with } \theta_L < \theta_L + \Delta < \theta_H \]

- \( \omega = (\theta_L, \Delta) \) is the false-positive state: technology is low, but agents receive unusually positive news

- Just need to keep track of two state variables \((p_t, q_t)\):

\[ p_t \equiv \pi_t (\theta_H, 0) \text{ and } q_t \equiv \pi_t (\theta_L, \Delta) \]
Learning Model: Characterizing Beliefs

- Private beliefs \((p_{jt}, q_{jt})\) are given by Bayes’ law:

\[
p_{jt} \equiv p_j (p_t, q_t, s_j) = \frac{p_t f_{\theta H}^s (s_j)}{p_t f_{\theta H}^s (s_j) + q_t f_{\theta L+\Delta}^s (s_j) + (1 - p_t - q_t) f_{\theta L}^s (s_j)}
\]

\[
q_{jt} \equiv q_j (p_t, q_t, s_j) = \ldots
\]

- Under MLRP, individual beliefs \(p_j\) are monotonic in \(s_j\)

\[
\frac{\partial p_j}{\partial s_j} (p_t, q_t, s_j) \geq 0
\]
• Agents invests iff

\[ E_{jt} [R_t | I_{jt}] \geq c \]

that is, whenever \( p_{jt} \geq \hat{\rho} \) where

\[ \hat{\rho} \theta_H + (1 - \hat{\rho}) \theta_L = c \]

• The optimal investment decision takes the form of a cutoff rule \( \hat{s}(p_t, q_t) \)

\[ i_{jt} = 1 \iff s_j \geq \hat{s}(p_t, q_t) \text{ with } p_j(p_t, q_t, \hat{s}_t) = \hat{\rho} \]
Learning Model: Endogenous Learning

- The measure of investing agents is

\[ m_t = \bar{F}_{\partial+\xi} (\hat{s}(p_t, q_t)) + \varepsilon_t \]

- Since \( \hat{s}(p_t, q_t) \) is known by all agents, \( m_t \) is a noisy signal about \( \theta + \xi \)
- \( \bar{F}_{\partial} \) is known, so inference problem is tractable

- In the 3-state example, only three measures \( m_t \) are possible (up to \( \varepsilon_t \)):
• As in early herding model, markets stop revealing info for extreme public beliefs
  ▶ For high/low $p_t$, only agents with extreme private signals go against the crowd
  ▶ There are few of them, so hard to detect if $m_t$ is noisy
  ▶ “Smooth” information cascade $\Rightarrow$ persistent “bubble” situation
Simulations

• Parametrization
  ▶ Fundamentals: $\theta_h = 1.0$, $\theta_l = 0.5$, $\Delta = 0.4$, $c = 0.75$
  ▶ Priors: $P(\theta_h, 0) = 0.25$, $P(\theta_l, \Delta) = 0.05$, $P(\theta_l, 0) = 0.7$
  ▶ Signals: Gaussian, e.g.:

$$s_j = \theta + \xi + v_j \text{ with } v_j \sim \mathcal{N}(0, \sigma^2_v)$$

with $\sigma_v = 0.4$ (private), $\sigma_\varepsilon = 0.2$ ($m_t$), $\sigma_u = 2.5$ ($R_t$)
Simulations: False Positive \((\theta_l, \Delta)\)

- **Boom phase:**

- **Mechanism:**
  - High investment rates quickly exclude low state \((\theta_l, 0) \Rightarrow p \text{ and } q\) rise progressively
  - For initial \(q_0\) sufficiently low, \(p\) picks up more strongly
Simulations: False Positive \((\theta_l, \Delta)\)

- **Information Cascade**

- **Mechanism:**
  - \(p\) is so high that almost everyone invests, releasing close to no information
  - because information not exactly 0, \(q\) slowly rises in the background
Simulations: False Positive \((\theta_I, \Delta)\)

- **Bursting**

- **Mechanism:**
  - when \(q\) high enough, some investors leave the market, releasing more information
  - early exit of investors incompatible with high state \(\Rightarrow\) quick collapse of investment
Simulations: Continuous $\xi$

- Previous simulations may look knife-edge
  - require state $(\theta_I, \Delta)$ to be infrequent and resemble $(\theta_H, 0)$
- We now allow $\xi$ to take a continuum of values
- Take-away:
  - small shocks ($<1$ SD) are quickly learned,
  - but unusually large shocks lead to boom-bust pattern
Simulations: Continuous $\xi$

- True fundamental ($\theta_I = 0, \xi = \text{multiple of } \sigma_\xi$)

![Graph showing mass of investors and beliefs over time for different values of $\xi$.]
Additional Results

Proposition

For $F_{\theta+\xi}$ unbounded or $\sigma_u < \infty$ (public info), there always exists a large enough $\xi$ such that $\xi \geq \xi$ triggers a boom and bust episode.

• Asymmetry: slow boom and sudden crash?
  ▶ We extend to continuous arrival of private information
  ▶ Initially, with little public information, distribution of private beliefs fans out, slowing the boom
  ▶ Crash remains sudden because it arises later when public signals have accumulated and beliefs are less dispersed

• Intensive margin: robustness?
  ▶ mechanism survives as long as individual investment displays concavity in beliefs
    (Straub and Ulbricht, 2018)
  ▶ Ex.: binding budget or borrowing constraints...
• **Information externality:** agents do not internalize how investment affects the release of information

• **We study the social planning problem**
  - Optimal policy **leans against the wind** to maximize collect of information
  - Implementation with investment tax/subsidy

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### Graphs

- **Mass m** over **Time t**
  - Equilibrium planner
  - Mass m decreases over time, reaching a steady state.

- **Beliefs p** over **Time t**
  - Equilibrium planner
  - Beliefs decrease over time, approaching a steady state.
Plan

1. Learning model
2. Business-cycle model with herding
• We want a model in which rising beliefs cause a boom, then a recession when beliefs collapse
  ▶ Key difficulty is to generate comovement in absence of technology shock
    • Wealth effect reduces labor and output
    • For risk aversion greater than 1 (IES<1), want to move resources from rich to poor states: investment declines before realization of productivity

• Build on the news-driven business cycle literature
• Parsimonious NK DSGE model with:
  1. Dynamic arrival of new technologies and technology choice
  2. Two types of capital: Traditional (T) and IT
     • Investment is required to enjoy the new technology
  3. Nominal rigidities (Lorenzoni, 2009)
     • Without, large spike in interest rate which lowers consumption and investment
     • With nominal rigidities, interest rate response is muted, consumption rises (wealth effect)

• Key mechanism:
  ▶ Each period, entrepreneurs choose their technology and agents learn from measure of tech adopters
  ▶ Learning akin to previous simplified model
• Agents:
  ▶ Households
  ▶ Retailers and monetary authority
  ▶ Entrepreneurs

• Three sectors: entrepreneur sector, retail sector and final good
  ▶ **Entrepreneur sector**: technology choice, no nominal rigidities
  ▶ **Retail sector**: buys the bundle of goods from entrepreneurs, subject to nominal rigidities
  ▶ **Final good**: bundle of retail goods used for consumption and investment
Business Cycle Model: Entrepreneurs

- Unit measure of entrepreneurs indexed by $j \in [0, 1]$
  - monopolistic producers of a single variety
- At any date, there is a traditional technology ("old") to produce varieties
  \[
  Y_{jt}^o = A^o \left( \omega_o \left( K_o^T \right)^{\frac{\zeta-1}{\zeta}} + (1 - \omega_o) \left( K_o^T \right)^{\frac{\zeta-1}{\zeta}} \right) \frac{\alpha}{\zeta-1} \left( L_{jt}^o \right)^{1-\alpha}
  \]
- With probability $\eta$, an innovative technology arrives ("new")
  \[
  Y_{jt}^n = A^n_t \left( \omega_n \left( K_n^T \right)^{\frac{\zeta-1}{\zeta}} + (1 - \omega_n) \left( K_n^T \right)^{\frac{\zeta-1}{\zeta}} \right) \frac{\alpha}{\zeta-1} \left( L_{jt}^n \right)^{1-\alpha}
  \]
  where
  \[
  \omega_n > \omega_o
  \]
• The new technology needs to mature to become fully productive

\[ A_t^n = \begin{cases} 
A^o & \text{before maturation} \\
\theta & \text{after} 
\end{cases} \]

• The new technology matures with probability \( \lambda \) per period

• The true productivity \( \theta \) is high or low \( \theta \in \{\theta_H, \theta_L\} \) with \( \theta_H > \theta_L \)
• Each period, entrepreneurs choose which technology to use
  ▶ for simplicity, assume no cost of switching so problem is static
  ▶ denote $m_t$ the measure of entrepreneurs that adopt the new technology
• A fraction $\mu$ of entrepreneurs is clueless when it comes to technology adoption
  ▶ “noise entrepreneurs”
  ▶ random fraction $\varepsilon_t$ adopts the new technology
• At $t = 0$, all entrepreneurs receive a private signal about $\theta$ from pdf $f_{\theta+\xi}$
  ▶ same assumptions as before (MLRP, etc.)

• Social learning takes place through economic aggregates which reveal

$$m_t = (1 - \mu) F_{\theta+\xi}(\hat{s}_t) + \mu \varepsilon$$

• Assume public signal $S_t = \theta + u_t$ which capture media, statistical agencies, etc.

• No additional uncertainty, hence information evolves identically to learning model
## Calibration: Standard Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>.36</td>
<td>Labor share</td>
</tr>
<tr>
<td>$\beta$</td>
<td>.99</td>
<td>4% annual interest rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
<td>risk averion (log)</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>.75</td>
<td>1 year price duration</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>10</td>
<td>Markups of about 11%</td>
</tr>
<tr>
<td>$\phi_Y$</td>
<td>.125</td>
<td>Clarida, Gali and Gertler (2000)</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>1.5</td>
<td>Clarida, Gali and Gertler (2000)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>9.11</td>
<td>Schmitt-Grohe and Uribe (2012)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>2</td>
<td>Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>1.71</td>
<td>Elas. between types of K (Boddy and Gort, 1971)</td>
</tr>
</tbody>
</table>
Calibration: Non-Standard Parameters

Objective: target moments from the late 90s Dot com bubble

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0$</td>
<td>.34</td>
<td>IT invest in GDP pre-1995 (2.86%)</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>.36</td>
<td>IT investment post-2005 (3.56%)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1/10</td>
<td>Duration of NASDAQ boom-bust 1998Q4-2001Q1</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>1.045</td>
<td>SPF’s highest growth forecast over 1998-2001</td>
</tr>
<tr>
<td>$\theta_l$</td>
<td>.95</td>
<td>SPF’s lowest growth forecast over 1998-2001</td>
</tr>
<tr>
<td>$s_j$</td>
<td>$N (0, .137)$</td>
<td>SPF’s avg. dispersion in forecasts over 1998-2001</td>
</tr>
<tr>
<td>$\mu$</td>
<td>5%</td>
<td>Fraction of noise traders</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Beta(2, 2)</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\xi$</td>
<td>$N \left(0, \sigma_\xi^2 \right)$</td>
<td>See below</td>
</tr>
</tbody>
</table>

Tricky parameters:

- Noise traders $\mu$ and $\varepsilon$: little guidance in the literature (David, et al. 2016)
  - Sensitivity $\mu \in [0.02, 0.15]$: agents learn too fast if $\mu < 0.02$, too slowly if $\mu > 0.15$ (no quick collapse)
- Common noise $\xi$: little information without a large sample of such crises
  - We trace out the probability of boom-bust cycles as we vary $\sigma_\xi$
  - Trade-off: high $\sigma_\xi \Rightarrow$ large $\xi$ quickly detected, low $\sigma_\xi \Rightarrow$ boom-bust have low proba
IRF to False-Positive

True state: \((\theta, \xi) = (\theta_l, 0.95 (\theta_h - \theta_l))\)
Summary of results

- **Quantitative:**
  - Endogenous boom-bust with positive comovement between $c$, $i$, $h$ and $y$
  - But boom-bust cycles arise with fairly high probability $\simeq 16\% \gg 10^{-6}$ (Avery and Zemsky, 1998)
  - Peak-to-trough is $\sim 1.5\%$, less than 2-3\% in the data (standard pb with news shocks)

- **Policy:**
  - *Leaning-against-the-wind* monetary policy dampens magnitude of cycle
  - Investment tax/subsidy can virtually eliminate false-positives at the cost of slowing “good booms”
Policy Analysis

• Govt policies are powerful in this setup:
  ▶ Learning externality: agents do not internalize that investment affects release of info
  ▶ Since cycle is endogenous, policies can partially eliminate boom-busts

• We show two examples of leaning-against-the-wind policies:
  ▶ Monetary policy rule:
    \[ r_t = \phi_\pi \pi_t + \phi_y y_t + \phi_m m_t \]
  ▶ A direct tax on using the new technology
    \[ t_t = c_0 + c_p p_t + c_q q_t \]

• Optimal policy: in the making...
In this simple framework, monetary policy:

- dampens the cycle but inefficient at fighting the information cascade
  - barely affects the technology choice, only the magnitude of boom and bust
  - at the additional cost of slowing down true booms
• **Tech-specific tax** policy can effectively affect the technology choice
  ▶ may eliminate some of the boom-bust cycles
  ▶ trade-off in slowing down true booms and maximizing collection of information
• Introduce herding phenomena as a potential **source of business cycles**

• We have proposed a business cycle model with herding
  ▶ people can collectively fool themselves for extended period of time
  ▶ endogenous boom-bust cycles patterns after unusually large noise shocks
  ▶ the model has predictions on the **timing and frequency** of such phenomena

• Quantitatively, such crises can arise with relatively **high probability** despite fully rational agents

• Provides rationale for **leaning-against-the-wind** policies which can substantially dampen fluctuations
• After observing $m_t$, public beliefs are updated

$$ p_{t+1} = \frac{p_t f^m (m_t - F_{\theta_H}^s (\hat{s}_t))}{\Omega} $$

and

$$ q_{t+1} = \frac{q_t f^m (m_t - F_{\theta_L}^s + \Delta (\hat{s}_t))}{\Omega} $$

where $\Omega = p_t f^m (m_t - F_{\theta_H}^s (\hat{s}_t)) + q_t f^m (m_t - F_{\theta_L}^s + \Delta (\hat{s}_t)) + (1 - p_t - q_t) f^m (m_t - F_{\theta_L}^s (\hat{s}_t))$

• Similar updating rule with exogenous signal $R_t = \theta + u_t$
Simulations: True Negative \((\theta_l, 0)\)

- Mass of investors \(m\)

- Beliefs

\[
p, \quad q, \quad 1 - p - q
\]
Simulations: True Positive \((\theta_h, 0)\)
Continuous Arrival of Private Signals

Mass of investors $m$

Beliefs $p$, $q$, $1 - p - q$

Time $t$
We adopt the welfare criterion from Angeletos and Pavan (2007)

\[ V(p, q) = \max_{\hat{s}} E_{\theta, \xi} \left[ \int_{\hat{s}} E[\theta - c|I_j] \, dj + \gamma V(p', q') |I \right] \]

where \( I \) is public info and \( I_j \) is individual info

- Crucially, the planner understands how \( \hat{s} \) affects evolution of beliefs
• Entry threshold planner vs equilibrium

yellow = less investment in planner, green = same, blue = more
• More information is endogenously released in the efficient allocation

purple = same info in planner, light blue = more, yellow = a lot more
Business Cycle Model: Households

- Households live forever, work, consume and save in capital
- Preferences

\[
E \left[ \sum \beta^t \left( \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \right) \right], \quad \sigma \geq 1, \psi \geq 0,
\]

where \( C_t = \left( \int_0^1 C_{jt} \frac{\sigma-1}{\sigma} dj \right)^{\frac{\sigma}{\sigma-1}} \) is the final good

- Adjustment costs in capital

\[
K_{jt+1} = (1 - \delta) K_{jt} + I_{jt} \left( 1 - S \left( \frac{l_{jt}}{l_{jt-1}} \right) \right), j = o, n
\]

- Budget constraint

\[
C_t + \sum_{j=o,n} I_{jt} + \frac{B_t}{P_t} = W_t L_t + \sum_{j=o,n} R_{jt} K_{jt} + \frac{1 + r_{t-1}}{1 + \pi_t} \frac{B_{t-1}}{P_{t-1}} + \Pi_t
\]
• **Retail sector:**
  - buys the bundle of goods produced by entrepreneurs
  - differentiates it one-for-one without additional cost
  - subject to Calvo-style nominal rigidity → standard NK Phillips curve

• **Monetary authority** follows the Taylor rule

\[ r_t = \phi_\pi \pi_t + \phi_y y_t \]