Wealth, Wages, and Employment

Preliminary

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Very Preliminary
We want a theory of the joint distribution of employment, wages, and wealth, where

- Workers are risk averse, so only use self-insurance.
- Employment and wage risk are endogenous.
- The economy aggregates into a modern economy (total wealth, labor shares, consumption/investment ratios)
- Business cycles can be studied.

Such a framework does not exist in the literature.

1. Requires heterogeneous agents.
2. No (search-matching) closed form solutions possible.
3. Wage formation? Nash bargaining not very promising:
   - Wages are an increasing function of worker wealth.
   - Not time-consistent: bargaining with commitment makes no sense.
   - Not numerically well-behaved.

We offer an alternative: competitive job search with commitment to a wage (or wage schedule) while the job lasts.
-- At its core is Aiyagari (1994) meets Moen (1997).


-- Developing empirically sound versions of these ideas compels us to

  -- Add extreme value shocks to transform decision rules from functions into densities to weaken the correlation between states and choices.

  -- Pose quits, on the job search, and explicit role for leisure so quitting is not only to search for better jobs.

  -- Use new potent tools to address the study of fluctuations in complicated economies Boppart, Krusell, and Mitman (2018).
What are the uses?

- The study of Business cycles including gross flows in and out of employment, unemployment and outside the labor force

- Policy analysis where now risk, employment, wealth (including its distribution) and wages are all responsive to policy.

2. **Endogenous Quits**: Higher wage dispersion may arise to keep workers longer (quits via extreme value shocks). But Wealth trumps wages and wage dispersion collapses.
   - Commitment not to wage but to wage schedule $w(z)$.

3. **On the Job Search** workers may get outside offers and take them. (Some in Chaumont and Shi (2017)). Fluctuations.
No (Endogenous) Quits Model
Jobs are created by firms (plants). A plant with capital plus a worker produce one \((z)\) unit of the good.

- Firms pay flow cost \(\bar{c}\) to post a vacancy in market \(\{w, \theta\}\).
- Firms cannot change wage (or wage-schedule) afterwards.
- Think of a firm as a machine programmed to pay \(w\) or \(w(z)\).
- Plants (and their capital) are destroyed at rate \(\delta^f\).
- Workers quit exogenously at rate \(\delta^h\). Typically they do not want to quit (for now, it is a quantitative issue).

Households differ in wealth and wages (if working). There are no state contingent claims, nor borrowing.

- If employed, workers get \(w\) and save.
- If unemployed, workers produce \(b\) and search in some \(\{w, \theta\}\).

General equilibrium: Workers own firms.
1. Households enter the period with or without a job: \( \{e, u\} \).

2. Production & Consumption: Employed produce \( z \) on the job. Unemployed produce \( b \) at home. They choose savings.

3. Firm Destruction and Exogenous Quits:
   Some Firms are destroyed (rate \( \delta^f \)) They cannot search this period. Some workers quit their jobs for exogenous reasons \( \delta^h \). Total job destruction is \( \delta \).

4. Search: Firms and the unemployed choose wage \( w \) and tightness \( \theta \).

5. Job Matching: \( M(V, U) \): Some vacancies meet some unemployed job searchers. A match becomes operational the following period. Job finding and job filling rates \( \psi^h(\theta) = \frac{M(V,U)}{U} \), \( \psi^f(\theta) = \frac{M(V,U)}{V} \).
No Quits Model: Household Problem

- Individual state: wealth and wage
  - If employed: \((a, w)\)
  - If unemployed: \((a)\)

- Problem of the employed: (Standard)

\[
V^e(a, w) = \max_{c, a'} u(c) + \beta \left[ (1 - \delta) V^e(a', w) + \delta V^u(a) \right]
\]

s.t. \(c + a' = a(1 + r) + w, \quad a \geq 0\)

- Problem of the unemployed: Choose which wage to look for

\[
V^u(a) = \max_{c, a', w} u(c) + \beta \left\{ \psi^h[\theta(w)] V^e(a', w) + [1 - \psi^h[\theta(w)]] V^u(a') \right\}
\]

s.t. \(c + a' = a(1 + r) + b, \quad a \geq 0\)

\(\theta(w)\) is an equilibrium object
Firms Post vacancies: Choose wages & filling probabilities

- Value of a job with wage $w$: uses constant $\bar{k}$ capital that depreciates at rate $\delta^k$

$$\Omega(w) = z - \bar{k}\delta^k - w + \frac{1 - \delta^f}{1 + r} \left[(1 - \delta^h) \Omega(w) + \delta^h \bar{k}\right]$$

- Affine in $w$: $\Omega(w) = \left[z + \bar{k} \left(\frac{1-\delta^f}{1+r} \delta^h - \delta^k\right) - w\right] \frac{1+r}{r + \delta^f + \delta^h - \delta^f \delta^h}$

Block Recursivity Applies (firms can be ignorant of Eq)

- Value of creating a firm: $\psi^f[\theta(w)] \Omega(w) + [1 - \psi^f[\theta(w)]] \Omega$

- Free entry condition requires that for all offered wages

$$\bar{c} + \bar{k} = \psi^f[\theta(w)] \frac{\Omega(w)}{1 + r} + [1 - \psi^f[\theta(w)]] \frac{\Omega}{1 + r},$$
A stationary equilibrium is functions \( \{ V^e, V^u, \Omega, g^{le}, g^{lu}, w^u, \theta \} \), an interest rate \( r \), and a stationary distribution \( \pi(x) \) over \( (a, w) \), s.t.

1. \( \{ V^e, V^u, g^{le}, g^{lu}, w^u \} \) solve households’ problems, \( \{ \Omega \} \) solves the firm’s problem.

2. Zero profit condition holds for active markets

\[
\bar{c} + \bar{k} = \psi^f[\theta(w)] \frac{\Omega(w)}{1 + r} + [1 - \psi^f[\theta(w)]] \frac{\bar{k}(1 - \delta - \delta_k)}{1 + r}, \quad \forall w \text{ offered}
\]

3. An interest rate \( r \) clears the asset market

\[
\int a \, dx = \int \Omega(w) \, dx.
\]
CHARACTERIZATION OF A WORKER’S DECISIONS

- Standard Euler equation for savings

\[ u_c = \beta (1 + r) \ E \{ u'_c \} \]

- A F.O.C for wage applicants

\[ \psi^h[\theta(w)] \ V_e^w(a', w) = \psi^h_\theta[\theta(w)] \ \theta_w(w) \ [V^u(a') - V^e(a', w)] \]

- Households with more wealth are able to insure better against unemployment risk.

- As a result they apply for higher wage jobs and we have dispersion
How does the Model Work

Worker’s Wage Application Decision

Wealth

Wage

$w_{\text{apply}}(a)$
How does the Model Work

Worker’s saving decision

![Graph showing worker's saving decision](image-url)
1. Easy to Compute Steady-State with key Properties
   i. Risk-averse, only partially insured workers, endogenous unemployment
   ii. Can be solved with aggregate shocks too
   iii. Policy such as UI would both have insurance and incentive effects
   iv. Wage dispersion small—wealth doesn’t matter too much
   v. ... so almost like two-agent model (employed, unemployed) of Pissarides despite curved utility and savings

2. In the following we examine the implications of a quitting choice
Endogenous Quits
1. Temporary Shocks to the utility of working or not working: Some workers quit.

2. Workers may or may not have an intrinsic taste for leisure.

3. Adds a (smoothed) quitting motive so that higher wage workers quit less often: Firms may want to pay high wages to retain workers.

4. Conditional on wealth, high wage workers quit less often.

5. But Selection (correlation 1 between wage and wealth when hired) makes wealth trump wages and those with higher wages have higher wealth which makes them quite more often: Wage inequality collapses.

6. We end up with a model with little wage dispersion but with endogenous quits that respond to the cycle.
Quit **ning** Model: Time-line

1. Workers enters period with or without a job: \{e, u\}.

2. Production occurs and consumption/saving choice ensues:

3. Exogenous job/firm destruction happens.

4. Quitting:
   - e draw shocks \{e^e, e^u\} and make quitting decision. 
     Joblosers cannot search this period.
   - u draw shocks \{ε^u_1, ε^u_2\}. No decision but same expected means.

5. Search: New or Idle firms post vacancies. Choose \{w, θ\}.
   Wealth is not observable. (Unlike Chaumont and Shi (2017)).
   Yet it is still Block Recursive

6. Matches occur
** Quitting Model: Workers **

- Workers receive i.i.d shocks \( \{\epsilon^e, \epsilon^u\} \) to the utility of working or not.

- Value of the employed right before receiving those shocks:

\[
\hat{V}^e(a', w) = \int \max\{V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u\} \, dF^\epsilon
\]

\( V^e \) and \( V^u \) are values after quitting decision as described before.

- If shocks are Type-I Extreme Value dbtn (Gumbel), then \( \hat{V} \) has a closed form and the ex-ante quitting probability \( q(a, w) \) is

\[
q(a, w) = \frac{1}{1 + e^{\alpha [V^e(a, w) - V^u(a)]}}
\]

higher parameter \( \alpha \to \) lower chance of quitting.

- Hence higher wages imply longer job durations. Firms could pay more to keep workers longer.
Problem of the employed: just change $\hat{V}^e$ for $V^e$

$$V^e(a, w) = \max_{c, a'} u(c) + \beta \left[ (1 - \delta)\hat{V}^e(a', w) + \delta V^u(a) \right]$$

s.t. $c + a' = a(1 + r) + w, \quad a \geq 0$

Problem of the unemployed is like before except that there is an added term $E\{\max[\epsilon^u_1, \epsilon^u_2]\}$

So that there is no additional option value to a job.
**Quitting Model: Value of the Firm**

- $\Omega^j(w)$: Value with $j$-tenured worker.
  Free entry condition requires that for all offered wages $\bar{c} + \bar{k} = \frac{1}{1+r} \left\{ \psi f[\theta(w)] \Omega^0(w) + [1 - \psi f[\theta(w)]] \Omega \right\}$.

- Probability of retaining a worker with tenure $j$ at wage $w$ is $\ell^j(w)$.
  (One to one mapping between wealth and tenure)
  
  $$\ell^j(w) = 1 - q^e[g^{e,j}(a, w), w]$$

  $g^{e,j}(a, w)$ savings rule of a $j$–tenured worker that was hired with wealth $a$

- Firm’s value

  $$\Omega^j(w) = z - \bar{k}\delta^k - w + \frac{1 - \delta^f}{1+r} \left\{ \ell^j(w)\Omega^{j+1}(w) + [1 - \ell^j(w)] \Omega \right\}$$
Quitting Model: Solving forward for the Value of the Firm

\[ \Omega^0(w) = (z - w - \delta^k k) Q^1(w) + (1 - \delta^f - \delta_k)k Q^0(w), \]

\[ Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[ \left( \frac{1 - \delta^f}{1 + r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right], \]

\[ Q^0(w) = \sum_{\tau=0}^{\infty} \left[ \left( \frac{1 - \delta^f}{1 + r} \right)^{1+\tau} \left[ 1 - \ell^{\tau}(w) \right] \left( \prod_{i=0}^{\tau-1} \ell^i(w) \right) \right]. \]

• New equilibrium objects \{Q^0(w), Q^1(w)\}. Rest is unchanged.

• It is Block Recursive because wealth can be inferred from \(w\) and \(j\). (No need to index contracts by wealth (as in Chaumont and Shi (2017)) ).
Value of the Firm as Wage Varies: The Poor

- For the poorest, employment duration increases when wage goes up.
- Firms value is increasing in the wage

![Graph showing the relationship between wage and firm value.](image-url)
VALUE OF THE FIRM AS WAGE VARIES: THE RICH

- For the richest, employment duration increases but not fast enough.
- Firm value is slowly decreasing in wages (less than static profits).
• Large drop from below to above equilibrium wages.
• In Equilibrium wage dispersion COLLAPSES due to selection.

- Related to the Diamond dispersion paradox but for very different reasons.
Effect of Quitting: The Mechanism

- Two forces shape the dispersion of wages

  - Agents quit less at higher paid jobs, which enlarge the spectrum of wages that firms are willing to pay (for a given range of vacancy filling probability).

  - However, by paying higher wages, firms attract workers with more wealth.

- Wealthy people quit more often, shrink employment duration.

- In equilibrium, the wage gap is narrow (disappears?) and the effect of wealth dominates.

- Need to weaken link between wages and wealth but not today (this is achieved via aiming (extreme value) shocks).
• Increasing in Wage (up to Grid calculation): Unique wage.
Quitting Makes a Big Difference

- Job finding prob with Endo

![Graph showing the relationship between wage and quitting probability with Exogenous and Endogenous Quitting models. The graph shows a sharp decrease in probability at a certain wage level for Endogenous Quitting, indicating a significant impact of quitting on job finding.](image-url)
On the Job Search
On the Job Search Model: Time-line

1. Workers enter period with or without a job: $V^e, V^u$.

2. Production & Consumption:

3. Exogenous Separation

4. Quitting? Searching? Neither?: Employed draw shocks $(\epsilon^e, \epsilon^u, \epsilon^s)$ and make decision to quit, search, or neither. Those who quit become $u'$, those who search join the $u$, in case of finding a job become $\{e', w'\}$ but in case of no job finding remain $e'$ with the same wage $w$ and those who neither become $e'$ with $w$. $\hat{V}^E(a', w)$, is determined with respect to this stage.

5. Search: Potential firms decide whether to enter and if so, the market ($w$) at which to post a vacancy; $u$ and $s$ assess the value of all wage applying options, receive match specific shocks $\{\epsilon^w\}$ and choose the wage level $w'$ to apply. Those who successfully find jobs become $e'$, otherwise become $u'$.

6. $\hat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.

7. Match
After saving, the unemployed problem is

\[ \hat{V}^u(a') = \int \max_{w'} \left[ \psi^h(w') V^e(a', w') + (1 - \psi^h(w')) V^u(a') + \epsilon^{w'} \right] dF^e \]

After saving, the employed choose whether to quit, search or neither

\[ \hat{V}^e(a', w) = \int \max \{ V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u, V^s(a', w) + \epsilon^s \} dF^e \]

The value of searching is

\[ V^s(a', w) = \int \max_{w'} \left[ \psi^h(w') V^e(a', w') + [1 - \psi^h(w')] V^e(a', w) + \epsilon^{w'} \right] dF^e \]
On the Job Search: Household choices

- The probabilities of quitting and of searching

\[
q(a', w) = \frac{1}{1 + \exp(\alpha[V^e(a', w) - V^u(a')]) + \exp(\alpha[V^s(a', w) - V^u(a') + \mu^s])},
\]

\[
s(a', w) = \frac{1}{1 + \exp(\alpha[V^u(a') - V^s(a', w)]) + \exp(\alpha[V^e(a', w) - V^s(a', w) - \mu^s])}.
\]

\[\mu^s < 0\] is the mode of the shock \(\epsilon^s\) which reflects the search cost.

- Households solve

\[
V^e(a, w) = \max_{a' \geq 0} u[a(1 + r) + w - a'] + \beta \left[ \delta V^u(a') + (1 - \delta) \hat{V}^e(a', w) \right]
\]

\[
V^u(a) = \max_{c, a' \geq 0} u[a(1 + r) + b - a'] + \beta \hat{V}^u(a')
\]
• The value of the firm is again given like in the Quitting Model

\[ \Omega^0(w) = (z - w - \delta^k k) Q^1(w) + (1 - \delta - \delta_k)k Q^0(w), \]

\[ Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[ \left( \frac{1 - \delta}{1 + r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right], \]

\[ Q^0(w) = \sum_{\tau=0}^{\infty} \left[ \left( \frac{1 - \delta}{1 + r} \right)^{1+\tau} \left[ 1 - \ell^\tau(w) \right] \left( \prod_{i=0}^{\tau-1} \ell^i(w) \right) \right]. \]

• Except that now the probability of keeping a worker after \( j \) periods is

\[ \ell^j(w) = 1 - \int h(w; a) q[g^{e,j}(a, w), w] \, dx^u(a) - \int h(w; a) s[w; g^{e,j}(a, w)] \left[ \int \hat{h}[	ilde{w}; g^{e,j}(a, w), w] \xi \phi^h(\tilde{w}) \, d(\tilde{w}) \right] \, dx^u(a) \]
- The rich pursue often other activities (leisure?)
Extensions:

Wages depend on the Aggregate State
Firms Choose Search Intensity
Wages move some with the Aggregate State of the Economy

- Wages are indexed to the Aggregate state \( z \)

- The firm is hard wired to pay not \( w \) but

\[
w[1 + \gamma(z - 1)]
\]

- It will reduce (depending on \( \gamma \) the incentive to quit and look for another job in an expansion)

- Very easy to implement

- Same steady state
Firms choose Search Intensity

- The number of vacancies posted is chosen by firms
- Easy to implement
- Slightly Different steady state
FREE ENTRY WITH VARIABLE RECRUITING INTENSITY

- Let $\nu(\overline{c})$ be a technology to post vacancies where $\overline{c}$ is the cost paid.

- Then the free entry condition requires that for all offered wages

$$0 = \max_{\overline{c}} \left\{ \nu(\overline{c}) \psi^f[\theta(w)] \left[ \frac{\Omega(w)}{1 + r} + \left[ 1 - \nu(\overline{c}) \psi^f[\theta(w)] \right] \frac{\overline{k}(1 - \delta_k)}{1 + r} - \overline{c} \right] \right\},$$

- With FOC given by

$$\nu(\overline{c}) \left\{ \psi^f[\theta(w)] \left[ \frac{\Omega(w)}{1 + r} - \frac{\overline{k}(1 - \delta_k)}{1 + r} \right] \right\} = 1,$$
How to make it consistent with the current steady state

• If \( v(\overline{c}) = \frac{v_1 \overline{c}^2}{2} + v_2 \overline{c} \), we have

\[
(v_1 \overline{c} + v_2) \left\{ \psi^f[\theta(w)] \left[ \frac{\Omega(w)}{1 + r} - \frac{k(1 - \delta_k)}{1 + r} \right] \right\} = 1,
\]

• By choosing \( v \) so that for the numbers that have now

\[
\left\{ \left[ \frac{v_1 \overline{c}^2}{2} + v_2 \overline{c} \right] \psi^f[\theta(w)] \frac{\Omega(w)}{1 + r} + \left[ 1 - \frac{v_1 \overline{c}^2}{2} - v_2 \overline{c} \right] \psi^f[\theta(w)] \frac{k(1 - \delta_k)}{1 + r} \right\} = \overline{c} + \overline{k}
\]

• Solving for \( \{v_1, v_2\} \) that satisfy both equations given our choice of \( \overline{c} \) we are done
Various Economies

• Limited Comparable Results

• Right now we have three Economies
  1. Only Exogenous Quitting
  2. Endogenous Quitting
  3. 4 On the Job Search With Aiming and Quiting

• Yearly Potential output is Normalized to 1.
Half-Quarterly Calibration

In half quarter units

- $K = 3$, $Y = 1/8$, $r = 0.37\%$
- firm destruction rate $\delta^f = 0.36\%$
- Exogenous Quits rate $\delta^h = 1.07\%$
- capital maintenance rate $\delta^k = 0.8\%$ from $I/Y = 25\%$.
- $\eta = 0.62$
- $\chi = 0.15$ to match $u = 10\%$.
- $\beta = 0.99928$
Steady States $r = 3\%$ 1/2 quarter- Same $\beta$

<table>
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<th>Exogenous Quits</th>
<th>Endogenous Quits</th>
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• A lot more wealth in Endogenous quitting

• Higher wages

• Yet less quits (need to recalibrate to get the same)

• Little wealth in OJS and also lower wages

• Excessive Unemployment duration
Steady States: \( r = 1.5\% \) 1/2 quarter Closed Economies

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<th>No Quits</th>
<th>Endogenous Quits</th>
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<th>Aiming &amp; Quits</th>
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<td>0.212</td>
<td>0.248</td>
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<td>0.045</td>
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<tr>
<td>Job Losers</td>
<td>0.114</td>
<td>0.069</td>
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<td>-</td>
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<tr>
<td>unemployment rate</td>
<td>0.121</td>
<td>0.113</td>
<td>0.072</td>
<td>0.106</td>
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<tr>
<td>std consumption</td>
<td>0.014</td>
<td>0.009</td>
<td>0.014</td>
<td>0.016</td>
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<tr>
<td>std wage</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
<td>0.003</td>
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<tr>
<td>std wealth</td>
<td>3.052</td>
<td>2.876</td>
<td>3.231</td>
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<td>mean-min consumption</td>
<td>2.287</td>
<td>2.306</td>
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<tr>
<td>mean-min wage</td>
<td>1.012</td>
<td>1.001</td>
<td>2.234</td>
<td>2.250</td>
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<tr>
<td>UE transition</td>
<td>0.119</td>
<td>0.084</td>
<td>0.136</td>
<td>0.084</td>
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<tr>
<td>total vacancy</td>
<td>0.581</td>
<td>0.387</td>
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<td>0.612</td>
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<tr>
<td>avg unemp duration</td>
<td>1.008</td>
<td>1.059</td>
<td>0.675</td>
<td>0.943</td>
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<td>avg emp duration</td>
<td>7.354</td>
<td>10.68</td>
<td>6.984</td>
<td>10.73</td>
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</table>
Vacations: Steady States: \( r = 1.5\% \) 1/2 quarter Closed Economies

<table>
<thead>
<tr>
<th></th>
<th>Vacation &amp; Quits</th>
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<tbody>
<tr>
<td>( \beta )</td>
<td>0.990</td>
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<td>interest rate</td>
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<td>avg consumption</td>
<td>0.673</td>
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<td>avg wage</td>
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<td>Stock Market</td>
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<td>quit ratio</td>
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<td>OJS search ratio</td>
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<td>unemployment rate</td>
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<td>wage of newly hired unemployed</td>
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<tr>
<td>std consumption</td>
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<tr>
<td>std wage</td>
<td>0.000</td>
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<tr>
<td>std wealth</td>
<td>1.568</td>
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<tr>
<td>mean-min consumption</td>
<td>2.243</td>
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<tr>
<td>mean-min wage</td>
<td>1.001</td>
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<tr>
<td>mean-min wealth</td>
<td>Inf</td>
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<tr>
<td>UE transition</td>
<td>0.098</td>
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<tr>
<td>EE transition</td>
<td>0.000</td>
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<tr>
<td>total vacancy</td>
<td>0.185</td>
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<tr>
<td>avg unemp duration</td>
<td>1.822</td>
</tr>
<tr>
<td>avg emp duration</td>
<td>8.242</td>
</tr>
</tbody>
</table>
Summary, Closed Economies

- Less wealth in Endogenous quitting
- Higher wages,
- Much higher Consumption
- Yet less quits (need to recalibrate to get the same)
- In endogenous quits, the quits are judicious
Aggregate Fluctuations
**What is needed?**

- **Two steps**
  1. Compute the TRUE impulse response to an MIT Shock
  2. Use this path as a dynamic linear approximation to generate fluctuations (Boppart, Krusell, and Mitman (2018))

- The transition is a large but doable problem:
  - Firms need to know functions \( \{ Q^0_t(w), Q^1_t(w), \psi^f(w) \} \) at each stage (no block recursivity)
  - Households need to know \( \phi^h_t(w) \) job finding probabilities every period.
  - Also need to know sequence of interest rates (not today)

- So it is a second order difference functional equation.
No Quits.  5% TFP Shock ($\rho = .95$)

Wage of Newly Hired Path

- Constant Wage
- Flexible Wage: Dependence=0.5
No Quits. 5% TFP Shock ($\rho = .95$)

Average Wage Path

- Constant Wage
- Flexible Wage: Dependence=0.5
No Quits. 5% TFP Shock ($\rho = .95$)
Summary, Exogenous Quits

- Large Shock creates little employment .15% (out of 5%)
- Also small wage increases if constant (1.5%) larger if adjusted 3%
- Big bottleneck in job market (Curvature of matching function)
- Yet less quits (need to recalibrate to get the same)
- In endogenous quits, the quits are judicious
Endogenous Quitting 5% TFP Shock ($\rho = .95$)

Wage of Newly Hired Path

- Constant Wage
- Flexible Wage: Dependence=0.5
Endogenous Quitting  5% TFP Shock ($\rho = .95$)

Average Wage Path

- Constant Wage
- Flexible Wage: Dependence=0.5
Endogenous Quitting 5% TFP Shock ($\rho = .95$)

Path of Job Finding Prob

- $t = 2$
- $t = 4$
- $t = 7$
- $t = 11$
- $t == T$
**Endogenous Quitting**  5% TFP Shock ($\rho = .95$)

**Quitting Rate Path**

- **Constant Wage**
- **Flexible Wage: Dependence=0.5**
Endogenous Quitting 5% TFP Shock ($\rho = .95$) % Devels

Quitting Rate Path

- Constant Wage
- Flexible Wage: Dependence = 0.5

Period: 0 10 20 30 40 50 60 70 80 90 100
Unemployment Rate Path

- **Constant Wage**
- **Flexible Wage**: Dependence=0.5

**Endogenous Quitting** 5% TFP Shock ($\rho = .95$)
**Endog Quitting**  
5% TFP Shock ($\rho = .95$) % Devs

Unemployment Rate Path

- **Constant Wage**
- **Flexible Wage: Dependence=0.5**
Role of Endog Quits  5% TFP Shock ($\rho = .95$) Fixed Wages % Deviations

Unemployment Rate Path

- Exogen Quits
- Endogen Quits

Percent Deviations

Period
Role of Endog Quits  5% TFP Shock ($\rho = .95$) Partially Adjusted Wages % Deviations

Unemployment Rate Path

Percent Deviations

Exogen Quits
Endogen Quits
Business Cycle Behavior of On the Job Search
Very Preliminary Assessment

Shocks are truncated at $t = 5$
  - Eliminating future shocks reins in the massive initial quits
  - Converge faster and less computational burden

OJS Switches are Pro-cyclical

OJS search amplifies the responses of wages and employment
OJS 5% TFP Shock ($\rho = .9$, truncated at $t=5$) OJS Search Rate, Percent Deviations
OJS 5% TFP Shock ($\rho = .9$, truncated at t=5) Avg Wage, Percent Deviations

Average Wage Path

- Blue line: Aiming & Quitting & OJS
- Red line: Aiming & Quitting & No OJS
OJS 5% TFP Shock ($\rho = .9$, truncated at t=5) Quits, Percent Deviations
OJS 5% TFP Shock ($\rho = .9$, truncated at $t=5$) Unemployment, Percent Deviations

Unemployment Rate Path

Aiming & Quitting & OJS
Aiming & Quitting & No OJS
• Develop tools to get a joint theory of wages, employment and wealth that marry the two main branches of modern macro:
  1. Aiyagari models (output, consumption, investment, interest rates)
  2. Labor search models with job creation, turnover, wage determination, flows between employment, unemployment and outside the labor force.
  3. Add tools from Empirical Micro to generate quits

• Useful for business cycle analysis: We are getting procyclical
  • Quits
  • Employment
  • Investment and Consumption
  • Wages
Conclusions II

- Exciting set of continuation projects:
  1. Endogenous Search intensity on the part of firms
  2. Aiming Shocks to soften correlation between wages and wealth
  3. Efficiency Wages: Endogenous TFP (firms use different technologies with different costs of idleness)
  4. Move towards more sophisticated life cycle movements
References


Appendix
• The model features strong response of investment but insufficient response of employment.
  • We examine the mechanics of this.
• Consider for simplicity the model with aiming shocks but no quitting shocks (ANQ model). For a 1% productivity shock (with persistence 0.7), it generates
  • 1% increase of vacancies
  • 0.2% decrease of unemployment, which translates to only 0.01% increase of employment
  • and 4% increase of investment
ANQ: 1% TFP Shock \((\rho = .7)\) UNEMPLOYMENT AND VACANCIES

Unemployment and Vacancies

Percent Deviations

Period

-0.2
0
0.2
0.4
0.6
0.8
1
1.2

Unemployment and Vacancies

unemployment path
vacancy path
ANQ: 1% TFP Shock ($\rho = .7$) Output, Investment and Consumption
ANQ: 1% TFP Shock ($\rho = .7$) Decomposition of the Investment Path

Investment Path

- total investment
- capital formation
- job posting cost
- capital maintenance

Percent Deviation vs. Period
Why does 1% increase of vacancies $v$ generate 4% increase of investment?

- At the steady state, about 80% of the vacancies are posted by old idle firms and 20% by newly created firms.
- Investment = wage posting cost + capital maintenance cost + new capital formation
- As the shock hits the economy, firstly it only increases the creation of new firms, generating massive movements of investment in the form of capital formation ($ek$).

Why does 1% increase of vacancies $v$ generate only 0.01% increase of employment?

- As an approximation, $\hat{m} = (1 - \eta)\hat{v} + \eta \hat{u}$.
- Upon facing the shock, at first $u$ does not move. So the response of matches depend on the response of $v$ and the parameter $\eta$.
- $\hat{m} \approx (1 - 0.72) \times 1\% = 0.28\%$, and $\frac{\Delta m}{1 - u} = \frac{0.28\% \times 0.03}{0.95} \approx 0.01\%$
- Lower $\eta$ relieves the problem (see the next page).
Lower $\eta$ and Truncated 5% shock: AQ Economy

Unemployment Rate Path

- Low eta: $\eta = 0.5$
- Benchmark: $\eta = 0.72$