Bubbly Recessions

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Motivation: Japanese post-bubble recession
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Motivation: U.S. post-bubble recession
This paper

- Asks: can collapse of bubbles precipitate long recessions?
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  - bursting of asset bubbles
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  - frictional labor market (sticky wages)
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- Methodology:
  - Expansionary rational bubble: à-la Hirano Yanagawa (RES 2016)
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  - Sticky wage: à-la Schmitt-Grohe Uribe (JPE 2016)
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  - bursting of asset bubbles
  - frictional labor market (sticky wages)
  - zero lower bound
- Methodology:
  - Expansionary rational bubble: à-la Hirano Yanagawa (RES 2016)
  - Sticky wage: à-la Schmitt-Grohe Uribe (JPE 2016)
  - Simple model → analytical solution
Main findings/contributions

Figure: K & L before, during & after a bubble episode

1. Collapse of bubbles → “overshooting” & protracted recessions
Main findings/contributions

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   - “Leaning against bubbles” policies can help
Main findings/contributions

Figure: K & L before, during & after a bubble episode

1. Collapse of bubbles $\rightarrow$ “overshooting” & protracted recessions
   - “Leaning against bubbles” policies can help

2. Collapse of large bubbles $\rightarrow$ liquidity trap, which exacerbates unemployment & recession
Related literature

- Rational bubbles (esp. with infinite-lived agents)
  - Miao Wang (2011), Hirano Yanagawa (2017), Hirano et al. (2016), ...
  - Samuelson (1958), Diamond (1965), Tirole (1985), ...
- Rational bubbles & unemployment:
- Rational bubbles & sticky prices
- New Keynesian models
  - Krugman (1998), Eggertsson Krugman (2012), Schmitt-Grohe Uribe (2016), ...
Outline

1. Model
2. Equilibrium dynamics
3. Policy discussion
4. Liquidity trap (preliminary)
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**Model**
Firms

- Single perishable good
Firms

- Single perishable good
- Firms, entrepreneurs, workers
Firms

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- Firms, entrepreneurs, workers
- Competitive firms:

\[ y_t = k_t^\alpha l_t^{1-\alpha} \]
\[ w_t = (1 - \alpha) \left( \frac{K_t}{L_t} \right)^\alpha \]
\[ q_t = \alpha \left( \frac{K_t}{L_t} \right)^{\alpha-1} \]
Workers & Entrepreneurs

- Identical preferences:

$$E_0 \left( \sum_{t=0}^{\infty} \beta^t \ln c^t \right)$$
Workers & Entrepreneurs

- Identical preferences:
  \[ E_0 \left( \sum_{t=0}^{\infty} \beta^t \ln c_t^i \right) \]

- Workers: unit measure, are “hand-to-mouth”:
  \[ c_t^w = w_t l_t + T_t \]
Workers & Entrepreneurs

- Identical preferences:
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- Workers: unit measure, are “hand-to-mouth”:
  \[ c_t^w = w_t l_t + T_t \]

- Entrepreneurs: unit measure, provide capital
Entrepreneurs

- Idiosyncratic productivity \( a^j_t \in \{ a^H, a^L \} \), w. prob. \( h, 1 - h \)
Entrepreneurs

- Idiosyncratic productivity $a_t^j \in \{a^H, a^L\}$, w. prob. $h, 1 - h$
- Entrepreneurs accumulate capital (after knowing their type):

$$k_{t+1}^j = a_t^j i_t^j \geq 0$$
Entrepreneurs

- Idiosyncratic productivity $a^j_t \in \{a^H, a^L\}$, w. prob. $h, 1-h$
- Entrepreneurs accumulate capital (after knowing their type):
  \[
  k^j_{t+1} = a^j_t i^j_t \geq 0
  \]
- H-type want to borrow from L-type, but face credit constraint (à-la Kiyotaki Moore):
  \[
  R_{t+1} d^j_t \leq \theta q_{t+1} k^j_{t+1} \tag{CC}
  \]
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  \] (CC)
  - $R_{t+1}$: interest rate between $t$ & $t+1$
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  - $R_{t+1}$: interest rate between $t$ & $t+1$
  - $q_{t+1} k_{t+1}^j$: collateral value at $t+1$
Entrepreneurs

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  - $\theta \in [0,1]$: pledgeability
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  ▶ $R_{t+1}$: interest rate between $t$ & $t+1$
  ▶ $q_{t+1}k^j_{t+1}$: collateral value at $t+1$
  ▶ $\theta \in [0, 1]$: pledgeability
  ▶ Throughout, assume $\theta$ small so (CC) binds for H-type
Bubble asset

- Besides trading in credit market, entrepreneurs can trade bubble asset (after knowing their type)
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- Bubble asset:
  - Tradable durable asset in fixed unit supply
  - Pays no dividend

\[ p_{bt} = \begin{cases} p_{bt}w. \text{prob.} \rho & \text{W-type} \\ 1 - \rho & \text{H-type} \end{cases} \]

Once collapsed, bubble will not re-emerge

In equilibrium, bubble serves as savings instrument: L-type buy bubble and sell it when become H-type
Bubble asset

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- Bubble asset:
  - Tradable durable asset in fixed unit supply
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  - Risky (Weil, 1987):

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\bar{p}_t^b = \begin{cases} 
p_t^b & \text{w. prob. } \rho \\
0 & \text{w. prob. } 1 - \rho
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    \tilde{p}_t^b = \begin{cases} 
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    0 & \text{w. prob. } 1 - \rho 
    \end{cases}
    \]
    Once collapsed, bubble will not re-emerge
- In equilibrium, bubble serves as savings instrument: L-type buy bubble and sell it when become H-type
Tax & Entrepreneur’s Problem

- Taking prices, productivity shock, tax $\tau$ as given:

$$\max_{\{c^j_t, i^j_t, b^j_t, d^j_t\}_{t=0}^\infty} E_0 \left( \sum_{t=0}^\infty \beta^t \ln c^j_t \right)$$

s.t.

$$c^j_t + i^j_t + R_t d^j_{t-1} + (1 + \tau) \tilde{p}^b_t b^j_t = q_t a^j_t i^j_{t-1} + d^j_t + \tilde{p}^b_t b^j_{t-1} \quad \text{(BC)}$$

$$R_{t+1} d^j_t \leq \theta q_{t+1} a^j_t i^j_t \quad \text{(CC)}$$

$$i^j_t, b^j_t \geq 0$$

- Macropurudential policy: speculation tax $\tau$. Budget balance: $\tau \tilde{p}^b_t = T_t$
Labor market

- (Exogenous) downward nominal wage rigidity (SGU 2016)

\[ P_{t+1}w_{t+1} \geq \gamma_n P_t w_t, \forall t \geq 0 \]
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- \( \gamma_n \in [0, 1] \): degree of nominal wage rigidity
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- \( P_t \) price level of consumption good
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  - \( P_t \): price level of consumption good

- For now, assume fixed inflation \( \frac{P_{t+1}}{P_t} \equiv \bar{\Pi} \geq 1 \). Then:

  \[ w_{t+1} \geq \gamma w_t \]

  (DWR)

  \[ \gamma \equiv \frac{\gamma_n}{\bar{\Pi}} \]
Labor market

- (Exogenous) downward nominal wage rigidity (SGU 2016)
  \[ P_{t+1}w_{t+1} \geq \gamma_n P_tw_t, \quad \forall t \geq 0 \]
  - \( \gamma_n \in [0, 1] \): degree of nominal wage rigidity
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  \[ w_{t+1} \geq \gamma w_t \] (DWR)
  \[ \gamma \equiv \frac{\gamma_n}{\bar{\Pi}} \]

- Labor market may not clear. Workers take employment from firm as given:
  \[ l_t = L_t \leq 1 \]
  \[ (1 - L_t)(w_t - \gamma w_{t-1}) = 0 \]
Equilibrium

- Given $\tau$, $k_0^j = K_0$, $d_0^j = 0$, $b_0^j = 1$, $p_0^b$, a competitive equilibrium consists of prices $\{w_t, q_t, R_{t+1}, p_t^b\}$, quantities $\{i_t^j, k_{t+1}^j, c_t^j\}, \{l_t, c_t^w\}$, $\{K_{t+1}, L_t\}$ s.t.
Equilibrium

- Given $\tau, k^j_0 = K_0, d^j_0 = 0, b^j_0 = 1, p^b_0$, a competitive equilibrium consists of prices $\{w_t, q_t, R_{t+1}, p^b_t\}$, quantities $\{i^j_t, k^j_{t+1}, c^j_t\}, \{l_t, c^w_t\}, \{K_{t+1}, L_t\}$ s.t.:
  - Entrepreneurs & firms optimize
Equilibrium

Given $\tau$, $k^j_0 = K_0$, $d^j_0 = 0$, $b^j_0 = 1$, $p^b_0$, a competitive equilibrium consists of prices $\{w_t, q_t, R_{t+1}, p^b_t\}$, quantities $\{i^j_t, k^j_{t+1}, c^j_t\}, \{l_t, c^w_t\}, \{K_{t+1}, L_t\}$ s.t.:

- Entrepreneurs & firms optimize
- Credit market clears: $hd^H_t + (1 - h)d^L_t = 0$
Equilibrium

Given \( \tau, k^j_0 = K_0, d^j_0 = 0, b^j_0 = 1, p^b_0 \), a competitive equilibrium consists of prices \( \{ w_t, q_t, R_{t+1}, p^b_t \} \), quantities \( \{ i^j_t, k^j_{t+1}, c^j_t \}, \{ l_t, c^w_t \}, \{ K_{t+1}, L_t \} \) s.t.:

- Entrepreneurs & firms optimize
- Credit market clears: \( hd^H_t + (1 - h)d^L_t = 0 \)
- Bubble market clears: \( hb^H_t + (1 - h)b^L_t = 1 \) if \( \tilde{p}^b_t > 0 \)
Equilibrium

- Given $\tau$, $k^j_0 = K_0$, $d^j_0 = 0$, $b^j_0 = 1$, $p^b_0$, a competitive equilibrium consists of prices $\{w_t, q_t, R_{t+1}, p^b_t\}$, quantities $\{i^j_t, k^j_{t+1}, c^j_t\}$, $\{l_t, c^w_t\}$, $\{K_{t+1}, L_t\}$ s.t.:
  - Entrepreneurs & firms optimize
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  - Bubble market clears: $hb^H_t + (1 - h)b^L_t = 1$ if $\tilde{p}^b_t > 0$
  - Goods market clears: $h(c^H_t + i^H_t) + (1 - h)(c^L_t + i^L_t) + c^w_t = K^\alpha_t L_t^{1-\alpha}$
Equilibrium

Given $\tau$, $k^j_0 = K_0$, $d^j_0 = 0$, $b^j_0 = 1$, $p^b_0$, a competitive equilibrium consists of prices $\{w_t, q_t, R_{t+1}, p^b_t\}$, quantities $\{i^j_t, k^{j}_{t+1}, c_t\}, \{l_t, c^w_t\}, \{K_{t+1}, L_t\}$ s.t.:

- Entrepreneurs & firms optimize
- Credit market clears: $hd^H_t + (1 - h)d^L_t = 0$
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- Goods market clears: $h(c^H_t + i^H_t) + (1 - h)(c^L_t + i^L_t) + c^w_t = K^\alpha_t L^{1-\alpha}_t$
- Labor market conditions: DWR and

$$l_t = L_t \leq 1$$

$$(1 - L_t)(w_t - \gamma w_{t-1}) = 0$$
Equilibrium dynamics

1. Bubble-less dynamics
2. Bubble dynamics
3. Post-bubble dynamics
Bubble-less equilibrium \((p^b_t = 0, \forall t)\)

- Assume \(K_0\) small, so DWR does not bind \((L_t = 1\forall t)\)
Bubble-less equilibrium \( (p^b_t = 0, \ \forall t) \)

- Assume \( K_0 \) small, so DWR does not bind \( (L_t = 1 \ \forall t) \)
- From binding CC & credit market clearing:

\[
K_{t+1} = \left( \frac{h(a^H - a^L)}{1 - \frac{\theta a^H}{a^L}} + a^L \right) \beta q_t K_t
\]

\[
R_{t+1} = a^L q_{t+1}
\]
Bubble-less equilibrium \((p_t^b = 0, \forall t)\)

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\]

\[
R_{t+1} = a^L q_{t+1}
\]

- Bubble-less steady state:

\[
K_{nb} = (\alpha \Omega)^{\frac{1}{1-\alpha}}, \quad \Omega \equiv \left( \frac{h(a^H - a^L)}{1 - \frac{\theta a^H}{a^L}} + a^L \right) \beta
\]

\[
R_{nb} = a^L \alpha K_{nb}^{\alpha-1}
\]
Equilibrium dynamics

1. Bubble-less dynamics
2. Bubble dynamics
3. Post-bubble dynamics
Bubble equilibrium \( (p^b_t > 0, \forall t) \)

- Focus: DWR doesn’t bind when bubble persists \( (L_t = 1 \text{ if } \tilde{p}^b_t > 0) \)
Bubble equilibrium ($p_t^b > 0$, $\forall t$)

- Focus: DWR doesn’t bind when bubble persists ($L_t = 1$ if $\tilde{p}_t^b > 0$)
- Bubble has two effects on capital
Bubble equilibrium \( (p_t^b > 0, \forall t) \)

- Focus: DWR doesn’t bind when bubble persists \( (L_t = 1 \text{ if } \tilde{p}_t^b > 0) \)
- Bubble has two effects on capital
  - Crowd-in: bubble sales raise entrepreneurs’ net worth

\[
K_{t+1} = \begin{cases} 
\alpha(H - \alpha L)^{1-\theta} \alpha H - \alpha L(1+\tau) & \text{if } R_t = \alpha H q_{t+1} + 1 \\
\alpha(H - \alpha L)^{1-\theta} \alpha H - \alpha L(1+\tau) & \text{if } R_t > \alpha H q_{t+1} + 1 
\end{cases}
\]

Where \( \phi_t \equiv \beta p_t^b q_t K_t + p_t^b \phi_t - \alpha L (1+\tau) \) denotes bubble size (relative to agg. savings)
Bubble equilibrium \((p^b_t > 0, \forall t)\)

- Focus: DWR doesn’t bind when bubble persists \((L_t = 1\text{ if } \tilde{p}^b_t > 0)\)
- Bubble has two effects on capital
  - Crowd-in: bubble sales raise entrepreneurs’ net worth
  - Crowd-out: bubble speculation reduces investment
Bubble equilibrium \((p^b_t > 0, \forall t)\)

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- Bubble has two effects on capital
  - Crowd-in: bubble sales raise entrepreneurs’ net worth
  - Crowd-out: bubble speculation reduces investment
  - Bubble is “large” if it completely crowds out L-type’s k investment

\[
K_{t+1} = \begin{cases} 
\left( \frac{h(a^H-a^L)}{1-\theta a^H a^L} + a^L \right) \beta (q_t K_t + p^b_t) - a^L (1+\tau)p^b_t & \text{if } R_t = a^L q_{t+1} \\
 a^H \beta (q_t K_t + p^b_t) - a^H (1+\tau)p^b_t & \text{if } R_t > a^L q_{t+1} 
\end{cases}
\]
Bubble equilibrium \((p^b_t > 0, \forall t)\)

- **Focus**: DWR doesn’t bind when bubble persists \((L_t = 1 \text{ if } \tilde{p}_t^b > 0)\)
- **Bubble** has two effects on capital
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K_{t+1} = \begin{cases} 
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 a^H \beta(q_t K_t + p^b_t) - a^H(1+\tau)p^b_t & \text{if } R_t > a^L q_{t+1} 
\end{cases}
\]

- **Bubble may raise or lower interest rate**

\[
R_{t+1} = \max \left\{ a^L, \frac{\theta a^H(1-(1+\tau)\phi_t)}{1-h-(1+\tau)\phi_t} \right\} q_{t+1}
\]

where \(\phi_t \equiv \frac{p^b_t}{\beta(q_t K_t + p^b_t)}\) denotes bubble size (relative to agg. savings)
Proposition (Bubble existence)

A bubble steady state exists iff sufficient financial friction:

\[ \theta < \frac{\beta \rho (1 - h)}{1 + \tau} \]

and bubble not too risky:

\[ \rho > \frac{a^L - \theta a^H}{\beta (a^L - \theta a^H) + \beta h (a^H - a^L)} \]
Equilibrium dynamics

1. Bubble-less dynamics
2. Bubble dynamics
3. Post-bubble dynamics
What happens when bubble bursts?

- Assume expansionary bubble \((K_b > K_{nb})\)
What happens when bubble bursts?

- Assume expansionary bubble ($K_b > K_{nb}$)
- Bubble collapses in $T$ (i.e., $\bar{p}_t^b = 0, \forall t \geq T$)
What happens when bubble bursts?

- Assume expansionary bubble ($K_b > K_{nb}$)
- Bubble collapses in $T$ (i.e., $\tilde{p}_b^t = 0, \forall t \geq T$)
- If $\gamma = 0$, then $K_t$ and $w_t$ will ↓ towards the bubble-less SS levels

Figure: $K$ before, during & after bubble: $\gamma = 0$
Binding wage rigidity

- If $\gamma > 0$, wage may not flexibly fall to clear labor market, causing:
  
  ▶ involuntary unemployment ($L < 1$)
  
  ▶ which reduces capital return $q_t = \alpha K^{\alpha} - 1^t L^{1^t - \alpha}$
  
  ▶ which reduces entrepreneur's capital income & thus net worth
  
  ▶ and reduces capital accumulation

  Involuntary unemployment as long as rigid wage floor > market-clearing wage
Binding wage rigidity

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  - which reduces entrepreneur’s capital income & thus net worth
  - and reduces capital accumulation

- Involuntary unemployment as long as rigid wage floor $> \text{market-clearing wage}$
Figure: Equilibrium wage vs. market-clearing wage
Characterizing a slump

- Let $T + s^*$ be the first post-bubble period with full employment:

$$s^* \equiv \min\{s \geq 0 | L_{T+s} = 1\}$$
Characterizing a slump

- Let $T + s^*$ be first post-bubble period with full employment:

$$s^* \equiv \min\{s \geq 0 | L_{T+s} = 1\}$$

- Economy is in a *slump* between $T$ and $T + s^* - 1$. 
Characterizing a slump

- Let $T + s^*$ be first post-bubble period with full employment:
  \[
  s^* \equiv \min\{s \geq 0 | L_{T+s} = 1\}
  \]

- Economy is in a slump between $T$ and $T + s^* - 1$.
- How long and deep is the slump?
Proposition (Post-bubble slump)

1. **Slump duration:**

\[
s^* = \begin{cases} 
0 & \text{if } \gamma = 0 \\
\left\lceil \omega(\gamma) - 2\alpha \log_{\gamma} K_T \right\rceil & \text{if } \gamma \in (0, 1) \\
\infty & \text{if } \gamma = 1 
\end{cases}
\]

where \( \omega(\gamma) \equiv \frac{2\alpha}{1-\alpha} \log_{\gamma}(\alpha \Omega) - \frac{3-\alpha}{1-\alpha} \)
Proposition (Post-bubble slump)

1. **Slump duration:**

\[ s^* = \begin{cases} 
0 & \text{if } \gamma = 0 \\
\lceil \omega(\gamma) - 2\alpha \log_\gamma K_T \rceil & \text{if } \gamma \in (0, 1) \\
\infty & \text{if } \gamma = 1 
\end{cases} \]

where \( \omega(\gamma) \equiv \frac{2\alpha}{1-\alpha} \log_\gamma(\alpha \Omega) - \frac{3-\alpha}{1-\alpha} \)

2. **During the slump:**

\[ \begin{align*}
   w_{T+s} &= \gamma^s w_T \\
   L_{T+s} &< 1 \\
   K_{T+s+1} &= \alpha \Omega \left( \frac{w_{T+s}}{1-\alpha} \right)^{\frac{\alpha-1}{\alpha}} K_{T+s}
\end{align*} \]
Simulation

(a) Bubble/savings ratio
(b) Real wage
(c) Employment
(d) Capital
(e) Output
(f) Real interest rate

Figure: Bubbly boom-bust (relative to bubble-less SS)
“Proof”: backward & forward induction

- After $T + s^*$: economy follows full employment bubble-less dynamics
  
  $$w_{T+s} = w_{T+s}^{\text{full}} \equiv (1 - \alpha)K_{T+s}^\alpha, \quad \forall s \geq s^*$$

  $$K_{T+s+1} = \alpha \Omega \left( \frac{w_{T+s}}{1 - \alpha} \right)^{\frac{\alpha - 1}{\alpha}} K_{T+s}$$
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- Between \( T \) and \( T + s^* - 1 \): DWR binds
  \[
  w_{T+s} = \gamma^s w_b, \quad \forall 0 \leq s < s^*
  \]
  \[
  K_{T+s+1} = \alpha\Omega \left( \frac{\gamma^s w_b}{1 - \alpha} \right)^{\frac{\alpha-1}{\alpha}} K_{T+s}
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"Proof": backward & forward induction

- After $T + s^*$: economy follows full employment bubble-less dynamics

  \[ w_{T+s} = w_{T+s}^{\text{full}} = (1 - \alpha)K_T^{\alpha s}, \quad \forall s \geq s^* \]

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- By definition:

  \[ s^* \equiv \min \left\{ s \geq 0 \mid w_{T+s}^{\text{full}} \geq \gamma^s w_b \right\} \]

  \[ = \min \left\{ s \geq 0 \mid K_T^{\alpha s} \geq \gamma^s K_b^\alpha \right\} \]

  \[ = \ldots = \left\lceil \omega - 2\alpha \log_{\gamma} K_b \right\rceil \]
Policy discussion
### Proposition (Worker’s expected utility in SS)

1. **Bubble-less SS:**
   \[ W_{nb}(K) \equiv \Gamma_2 + \frac{\alpha}{1 - \beta \alpha} \log K \]

2. **Bubble SS:**
   \[
   W_b = \frac{\log c_w + \beta (1 - \rho) W_{burst}(K_b)}{1 - \beta \rho}
   \]
   \[
   W_{burst}(K_b) \equiv \log[(1 - \alpha)(K_b)^{\alpha}]
   \]
   \[
   s^* - 1 + \sum_{s=1}^{s^*} \beta^s (\Gamma_1(s) - ((1 - \alpha)s - \alpha \log K_b))
   \]
   \[
   + \beta^{s^*} W_{nb} \left( \gamma^{-\left(\frac{1-\sigma}{\sigma}\right)\frac{s^*+1}{2}} \left[ \alpha \Omega \cdot (K_b)^{\alpha-1} \right]^{s^*} K_b \right)
   \]
Proposition (Welfare-reducing bubble)

Assume bubble sufficiently risky:

\[ \beta (\beta - \alpha) (1 - \rho) > \alpha (1 - \beta)^2 \]

Then:

\[ W_{nb} > \lim_{\gamma \to 1^-} W_b \]
Effect of macroprudential policy

- Changing bubble tax can change how bubble affects capital accumulation
Effect of macroprudential policy

- Changing bubble tax can change how bubble affects capital accumulation
- Tradeoff: smaller economic boom vs. less severe bust
Simulation: Effects of bubble tax

(a) Bubble/savings ratio
(b) Real wage
(c) Employment
(d) Capital
(e) Output
(f) Real interest rate
Effect of changing inflation target

- Higher $\bar{\Pi}$ lowers real wage rigidity $\gamma = \frac{\gamma_n}{\bar{\Pi}}$
Effect of changing inflation target

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  - Model lacks endogenous cost of inflation (e.g., via sticky prices)
Effect of changing inflation target

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  - A sufficiently high $\bar{\Pi}$ would restore full employment $\forall t$
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  - Model lacks endogenous cost of inflation (e.g., via sticky prices)
  - So far model is also silent about ZLB
Simulation: Effects of changing inflation target

(g) Bubble/savings ratio
(h) Real wage
(i) Employment
(j) Capital
(k) Output
(l) Real interest rate
Summary

- Embed DWR in rational bubble model
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- Embed DWR in rational bubble model
- Find: Collapse of bubble can $\rightarrow$ persistent & inefficient slump. Warrants policy interventions
Liquidity trap (preliminary)
Collapse of large bubble & overshooting R

Proposition (Post-bubble interest rate)

Suppose economy reaches steady state with large expansionary bubble; then bubble collapses in $T$.

If $K_{lb} >$ some threshold $\bar{K}$, then post-bubble nominal interest rate is negative:

$$R_{T+1}\bar{\Pi} < 1.$$
Intuition

- $R$ depends on productivity of marginal investor & on MPK
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- Bubble bursts $\rightarrow$ marginal $k$ investor switches from H-type to L-type

Note: Slower depreciation rate (e.g., housing) $\rightarrow$ higher persistence of low MPK

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Intro Model Equilibrium Policy ZLB

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- Bubble bursts $\rightarrow$ marginal $k$ investor switches from H-type to L-type
- Bubble causes “overinvestment” in capital relative to bubble-less SS $\rightarrow$ low $MPK$
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Introducing money holding

To microfound ZLB, assume entrepreneurs choose cash holding:

\[
c^j_t + i^j_t + (1 + \tau)\tilde{\rho}_t^b b_t^j + \frac{M_t^j - M_{t-1}^j}{P_t} = q_t k^j_t + d^j_t - R_{t-1,t} d^j_{t-1} + \tilde{\rho}_t^b b_{t-1}^j
\]

(BC)

\[
\frac{M_t^j}{P_t} \geq \varepsilon
\]

(CIA)
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- Cash-less limit: \( \varepsilon \to 0^+ \)
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    \frac{M^j_t}{P_t} &\geq \varepsilon
\end{align*} \]  

(BC)  

(CIA)

- Cash-less limit: \( \varepsilon \to 0^+ \)

- Money supply grows at exogenous rate \( \bar{\Pi} \geq 1 \)
  - All seignorage transferred to workers
Inflation in equilibrium

In equilibrium:

\[ E_t \left[ u'(c^j_{t+1}) R_{t+1} \right] \geq E_t \left[ u'(c^j_{t+1}) \frac{P_t}{P_{t+1}} \right], \forall t \geq 0 \quad \text{(ZLB)} \]
Inflation in equilibrium

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- Price level determination (via money market clearing)
Inflation in equilibrium

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- Price level determination (via money market clearing)
  - If CIA binds: \( P_t = \frac{M_t}{\varepsilon} \)
  - If CIA does not bind (liquidity trap): Price determined by binding ZLB
Bursting bubble $\rightarrow$ liquidity trap

- Assume no liquidity trap in steady states:

$$\bar{\Pi} \geq \max \left\{ \frac{1}{R_{nb}}, \frac{1}{R_{b}} \right\}$$
Bursting bubble $\rightarrow$ liquidity trap

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  - large bubble ($\tau < \bar{\tau}$)
  - and investment boom ($K_b$) sufficiently large that $a^L \alpha K_{ib}^{\alpha-1} \bar{\Pi} < 1$,
  Then
Bursting bubble $\rightarrow$ liquidity trap

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  \[ \bar{\Pi} \geq \max \left\{ \frac{1}{R_{nb}}, \frac{1}{R_b} \right\} \]

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  \begin{itemize}
  \item large bubble ($\tau < \bar{\tau}$)
  \item and investment boom ($K_b$) sufficiently large that $a^L \alpha K_{lb}^{\alpha-1} \bar{\Pi} < 1$
  \end{itemize}

  Then
  
  \begin{itemize}
  \item Liquidity trap in $T+1$,
  \end{itemize}

  \[ R_{T+1} \frac{P_{T+1}}{P_T} = 1 \]  \quad (ZLB binds)
Liquidity trap → deflated price level

- Focus on parameters s.t. liquidity trap lasts for only one period (as in Krugman 1998, Eggertsson Krugman 2012)
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- Then $P_{T+1}$ fixed at target $P^*_T = \frac{M_{T+1}}{\epsilon}$
Liquidity trap $\rightarrow$ deflated price level

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- Then $P_{T+1}$ fixed at target $P^*_T$ $\equiv \frac{M_{T+1}}{\varepsilon}$
- Binding ZLB becomes:

$$R_{T+1} \frac{P^*_T}{P_T} = 1$$
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- So $\downarrow R_{T+1}$ (due to bubble bursting) must be associated with $\downarrow P_T$ (deflated price level)
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- So $\downarrow R_{T+1}$ (due to bubble bursting) must be associated with $\downarrow P_T$ (deflated price level)

- $\downarrow P_T$ exacerbates DWR:

$$ w_T \geq \frac{\gamma}{P_T/P^*_T} \frac{w_b}{\Pi} \frac{\gamma}{w_b} $$
Simulation: ZLB

(m) Bubble/savings ratio
(n) Real wage
(o) Employment
(p) Capital
(q) Output
(r) Real interest rate

Figure: Post-bubble liquidity trap.

U.S. data
Taking stock: Bubble $\rightarrow$ ZLB $\rightarrow$ slump

- Bursting bubble in $T$
Taking stock: Bubble $\rightarrow$ ZLB $\rightarrow$ slump

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Taking stock: Bubble $\rightarrow$ ZLB $\rightarrow$ slump

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Taking stock: Bubble $\rightarrow$ ZLB $\rightarrow$ slump

- Bursting bubble in $T$
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- Exacerbated wage rigidity: $\Pi_T \downarrow \Rightarrow \frac{\gamma}{\Pi_T} \uparrow$
Taking stock: Bubble $\rightarrow$ ZLB $\rightarrow$ slump

- Bursting bubble in $T$
  $\rightarrow$ Overshooting $R_{T+1}$
  $\rightarrow$ Liquidity trap
  $\rightarrow$ Deflated price level
  $\rightarrow$ Exacerbated wage rigidity: $\Pi_T \downarrow \Rightarrow \frac{\gamma}{\Pi_T} \uparrow$
  $\rightarrow$ Sufficient deflation ($\frac{\gamma}{\Pi_T} > 1$) causes unemployment ($L_T < 1$)
Conclusion

Collapse of large bubbles can trigger persistent slump and liquidity trap

Figure: K & L before, during & after a bubble episode
U.S. post-bubble ZLB & deflation

**US: Real and Nominal Wages**
- **Average Real Wages**
- **Average Nominal Wages**

**US: Nominal Interest Rate and Inflation**
- **Nominal Interest Rate**
- **Inflation**