

New opinion dynamics theory including both trust and distrust between human relation for social simulation and social analysis

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Opinion Dynamics

⇒ Social Simulation

Consensus formation



Social division



Collaborator

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Gunma University



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Tottori University



Previous theories of opinion dynamics

- Voter Model (Clifford-Sudbury(1973) 、 Holly-Leggett(1975))

- Galam model (Galam 1999)

$$S_j = +1 \text{ or } -1$$

Ising model in solid state theory on magnetism

- Galam local majority model (Galam 1997)

Application of renormalization group theory

- Bounded Confidence Model(Hegselmann-Krause 2002)

$$x_i(t + 1) = \sum_{j=1}^N D_{ij} x_j(t) \quad 0 \leq x_i \leq 1$$

- Deffuant-Weisbuch Model(G Deffuant et al 2000)

$$x_i(t + 1) = x_i(t) + \mu [x_j(t) - x_i(t)] \\ 0 \leq x_i \leq 1$$

Previous theories of opinion dynamics

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 $S_j = +1$ or -1
Ising model in solid state theory on magnetism
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Application of renormalization group theory

**Opinion is
binary value**

- Bounded Confidence Model(Hegselmann-Krause 2002)
 $x_i(t + 1) = \sum_{j=1}^N D_{ij} x_j(t)$ $0 \leq x_i \leq 1$

**Opinion is
Continuous value**

- Deffuant-Weisbuch Model(G Deffuant et al 2000)

$$x_i(t + 1) = x_i(t) + \mu[x_j(t) - x_i(t)]$$
$$0 \leq x_i \leq 1$$

Bounded Confidence Model

Deffuant et al 2000

$$x_i(t + 1) = x_i + \mu(x_j(t) - x_i(t))$$
$$\mu > 0$$

G Deffuant, D Neau, F Amblard, G Weisbuch, Advances in Complex Systems 3 no.4, 87 (2000)

Hegselmann-Krause 2002

$$x_i(t + 1) = \sum_{j=1}^N D_{ij} x_j(t)$$
$$D_{ij} > 0$$

Hegselmann R and U Krause,

"Opinion Dynamics and Bounded Confidence Models, Analysis, and Simulation"

Journal of Artificial Society and Social Simulation 5 no.3, (2002) 33 page

Deffuant et al 2000の計算

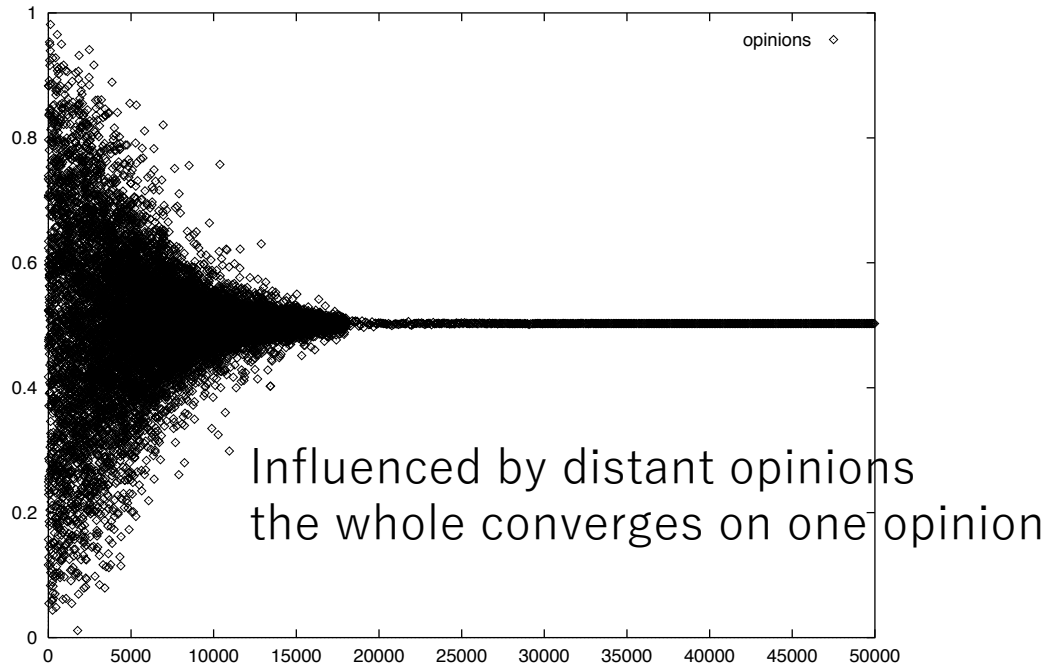


Figure 1. Time chart of opinions ($d = 0.5$ $\mu = 0.5$ $N = 2000$). One time unit corresponds to sampling 1000 pairs of agents.

In this case, it converges to two opinions if it is not influenced by the nearby opinion only

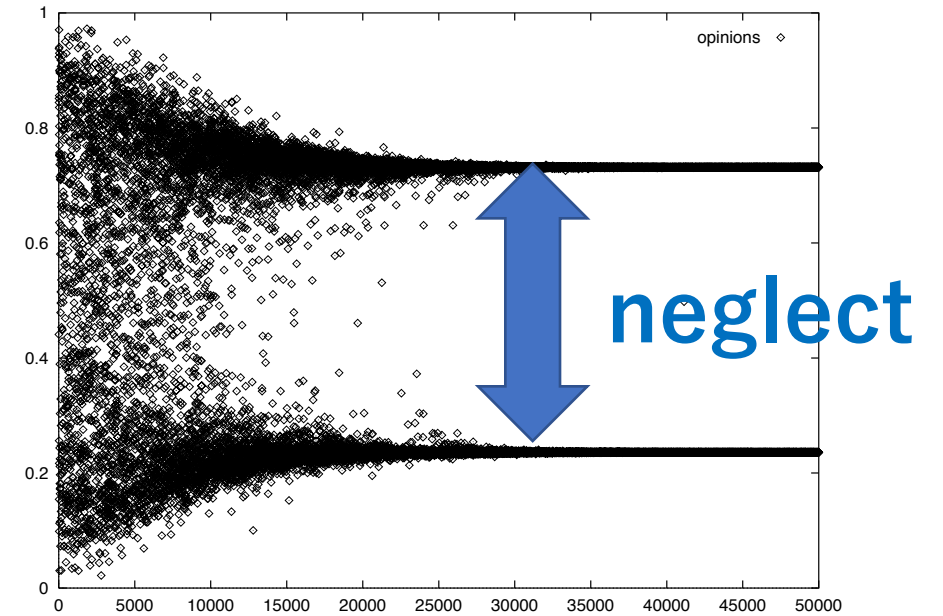
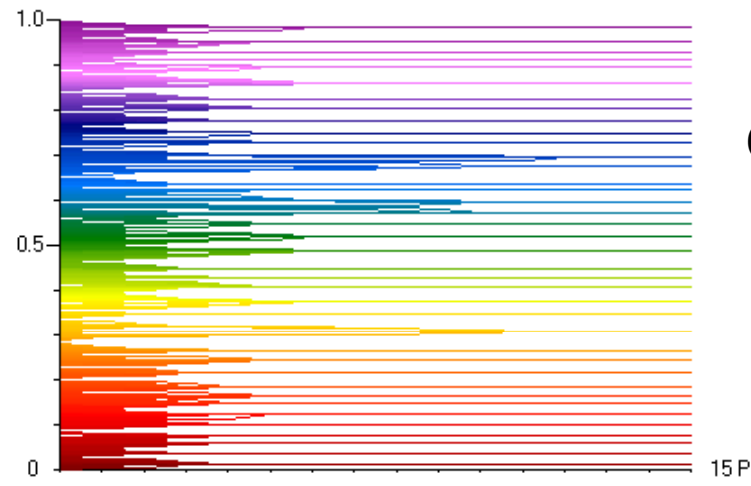


Figure 2. Time chart of opinions ($d = 0.2$ $\mu = 0.5$ $N = 1000$). One time unit corresponds to sampling 1000 pairs of agents.

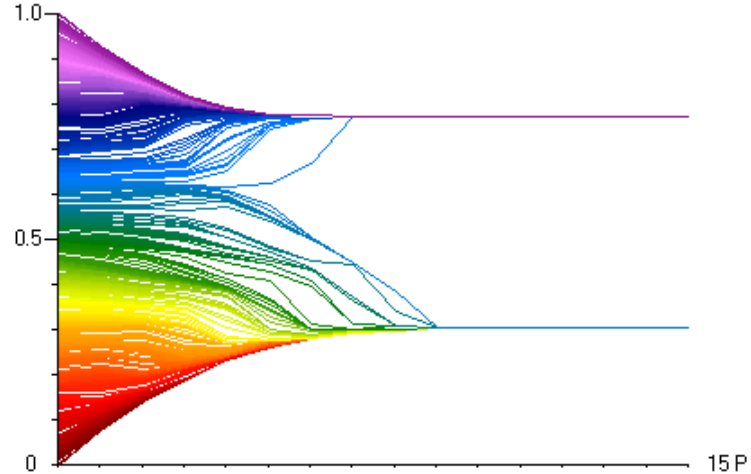
Opinions are aggregated into a small number under the assumption that everyone is a trusting relationship

Converge to many

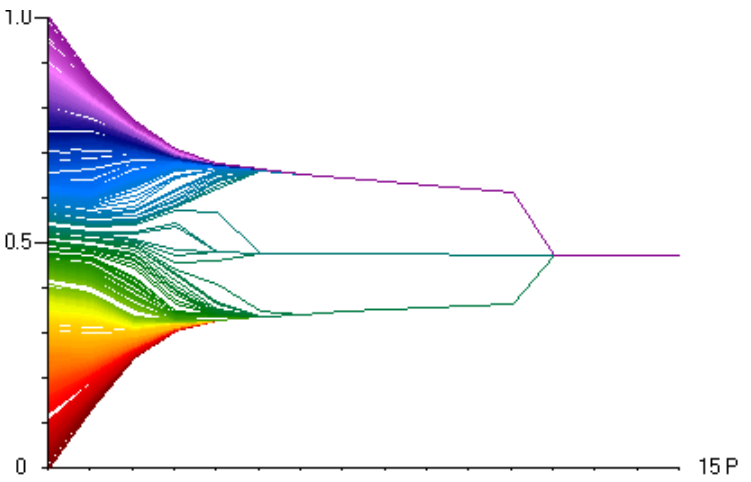
(a) $\varepsilon_l = \varepsilon_r = 0.01$



(b) $\varepsilon_l = \varepsilon_r = 0.15$
Converge to two



(c) $\varepsilon_l = \varepsilon_r = 0.25$
Converge to one



Opinions are aggregated into a small number under the assumption that everyone is a trusting relationship

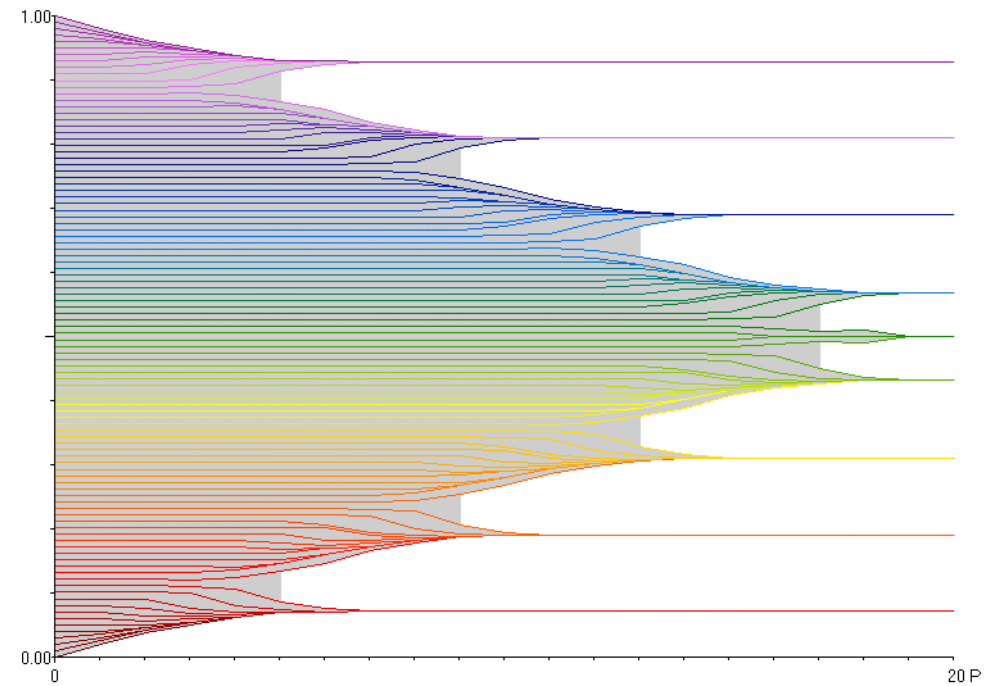


Figure 7: 100 opinions, $\varepsilon_l = \varepsilon_r = 0.05$, 8 splits.

Converge to eight

Our new opinion dynamics theory

- Based on the Bounded Confidence Model
- Including both **trust and distrust** into human relationship
- **Mass media effects** are included

A Ishii, Lecture Notes in Business Information Processing series. Vol. 351
(Proceedings book of GDN2019)

Our new opinion dynamics theory

$$\frac{dI_i(t)}{dt} = C_i A(t) + \sum_{j=1}^N D_{ij} \Phi(I_j(t), I_i(t)) (I_j(t) - I_i(t))$$

$$\Phi(I_i, I_j) = \frac{1}{1 + \exp(\beta |I_i - I_j| - b)}$$

Hegselmann-Krause 2002

$$D_{ij} > 0 \quad i \text{ trust } j$$

Our new opinion dynamics theory

$$\frac{dI_i(t)}{dt} = C_i A(t) + \sum_{j=1}^N D_{ij} \Phi(I_j(t), I_i(t)) (I_j(t) - I_i(t))$$

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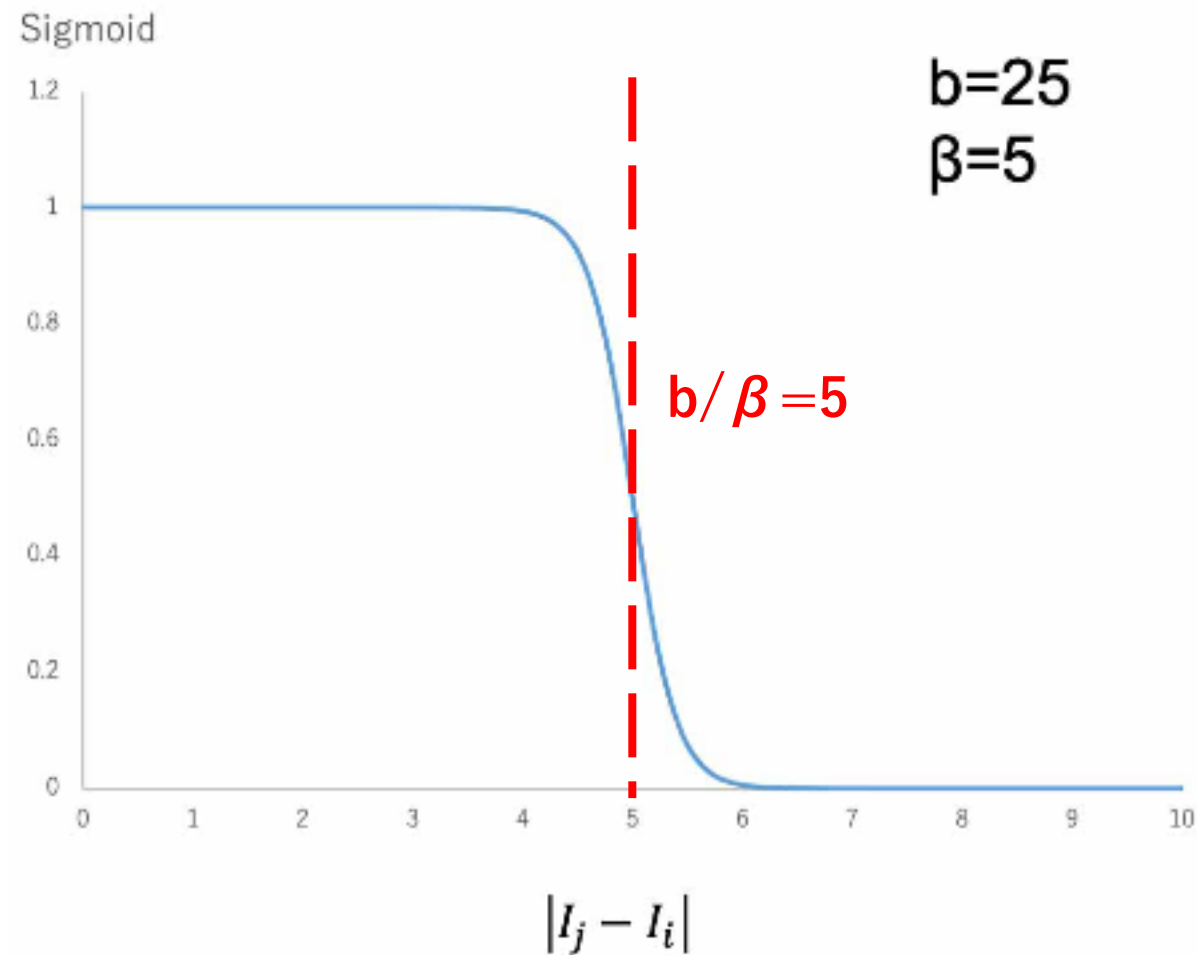
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value range of $I_i(t)$ is $-\infty \leq I_i(t) \leq +\infty$.

$D_{ij} > 0$ i trust j
 $D_{ij} < 0$ i distrust j

$D_{ij} \neq D_{ji}$

$$\Phi(I_i, I_j) = \frac{1}{1 + \exp(\beta |I_i - I_j| - b)}$$



Our new opinion dynamics theory

$$\frac{dI_i(t)}{dt} = \underline{C_i A(t)} + \sum_{j=1}^N D_{ij} \Phi(I_j(t), I_i(t)) (I_j(t) - I_i(t))$$

External Media Effects

$$\Phi(I_i, I_j) = \frac{1}{1 + \exp(\beta |I_i - I_j| - b)}$$

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Our new opinion dynamics theory

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Interaction with other persons

$$\Phi(I_i, I_j) = \frac{1}{1 + \exp(\beta |I_i - I_j| - b)}$$

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value range of $I_i(t)$ is $-\infty \leq I_i(t) \leq +\infty$.

$D_{ij} > 0$ i trust j
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Our new opinion dynamics theory

$$\underline{m} \frac{dI_i(t)}{dt} = C_i A(t) + \sum_{j=1}^N D_{ij} \Phi(I_j(t), I_i(t)) (I_j(t) - I_i(t))$$

Strength of the will

$$\Phi(I_i, I_j) = \frac{1}{1 + \exp(\beta |I_i - I_j| - b)}$$

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Our new opinion dynamics theory

$$m \frac{dI_i(t)}{dt} = C_i A(t) + \sum_{j=1}^N D_{ij} \Phi(I_j(t), I_i(t)) (I_j(t) - I_i(t))$$

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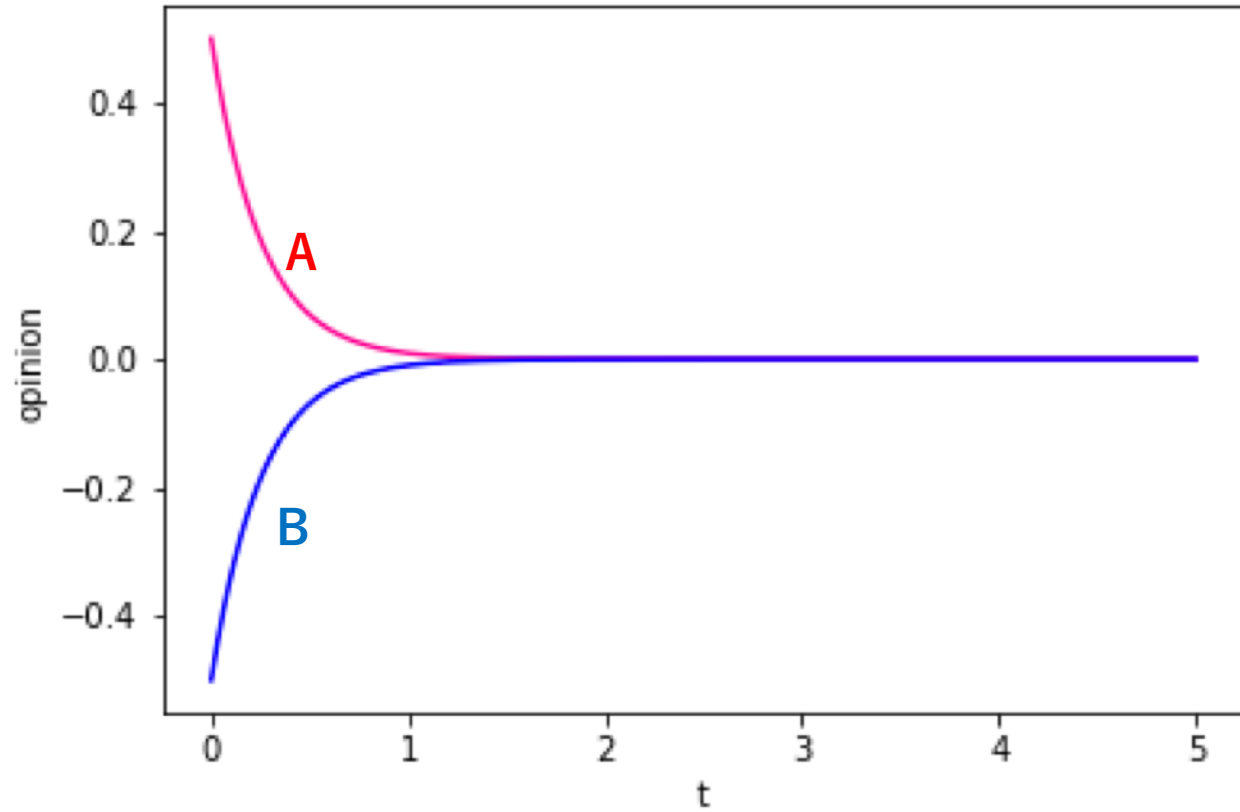
Network structure can be included here.
Random Network is tested in this presentation.

value range of $I_i(t)$ is $-\infty \leq I_i(t) \leq +\infty$.

$D_{ij} > 0$ i trust j
 $D_{ij} < 0$ i distrust j

$$D_{ij} \neq D_{ji}$$

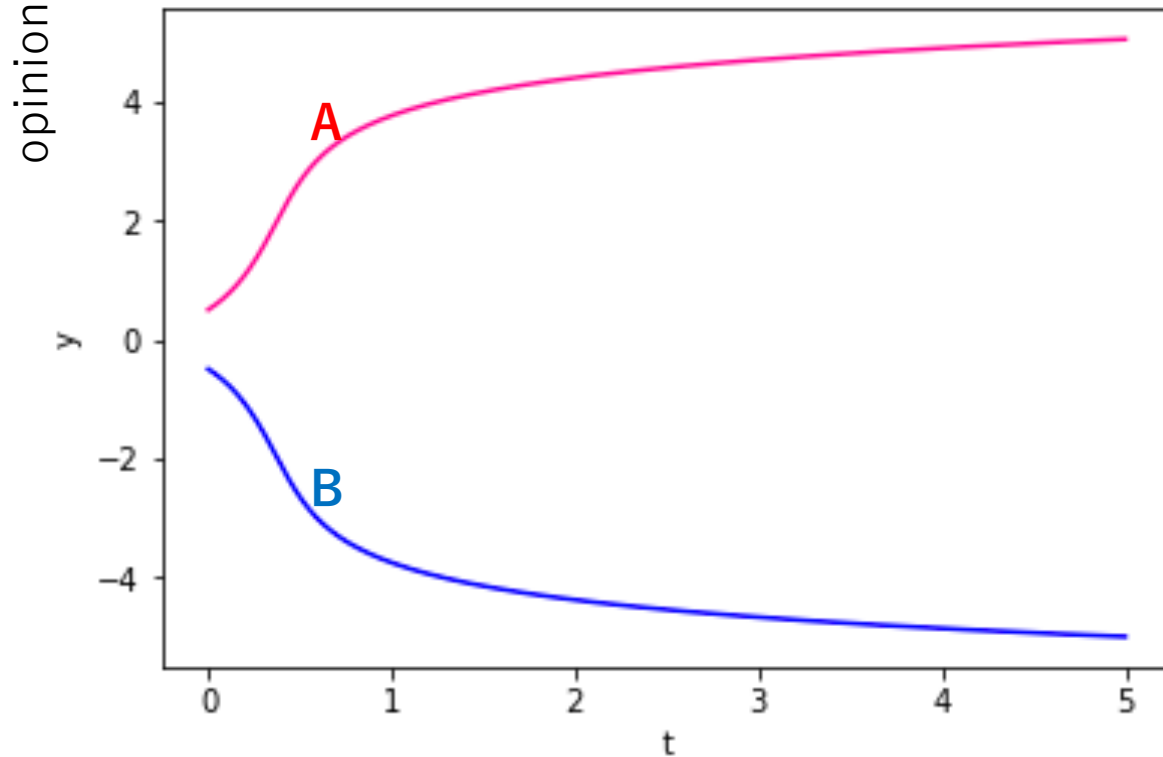
Agreement of opinions



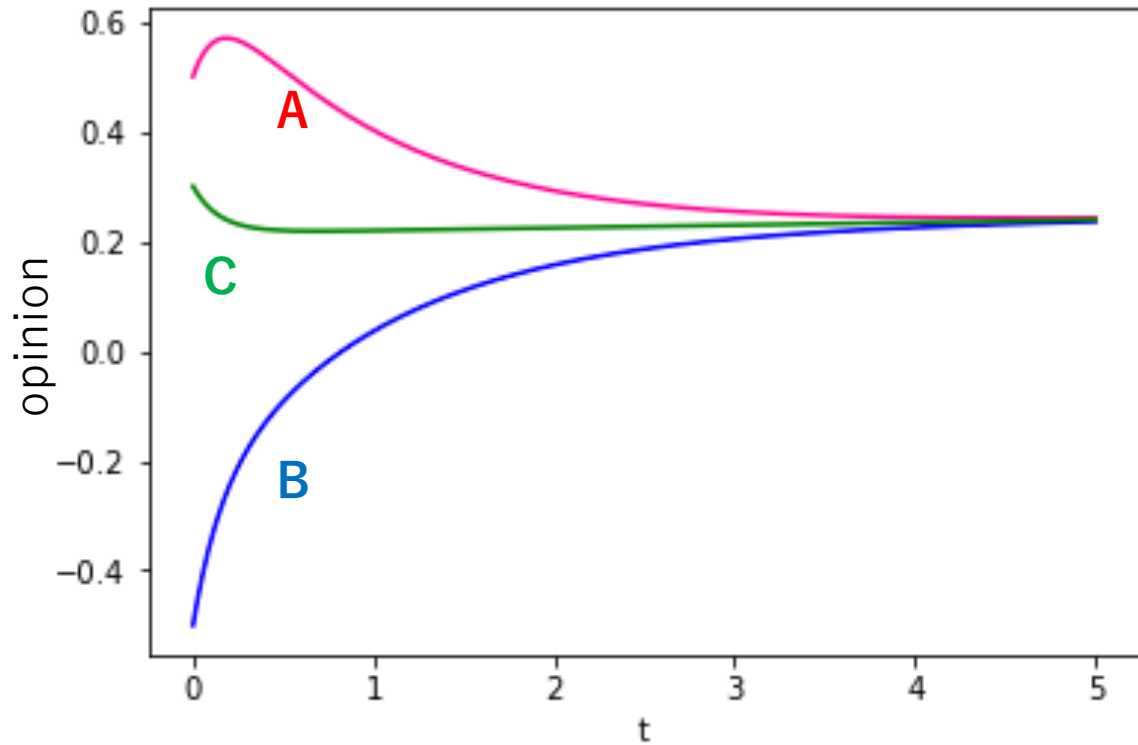
$$D_{AB} > 0$$

$$D_{BA} > 0$$

Disagreement



A and B with different opinions repel each other.
When opinions are far apart, they ignore each other.



A:Red B : Blue C:Green

A and B have no trust in each other and opinions are also different. Both A and B have strong trust in C,

With the trust of Mr. C, the opinions of A and B changed, and it is likely that an agreement will be obtained.

C would be Nelson R Mandela?



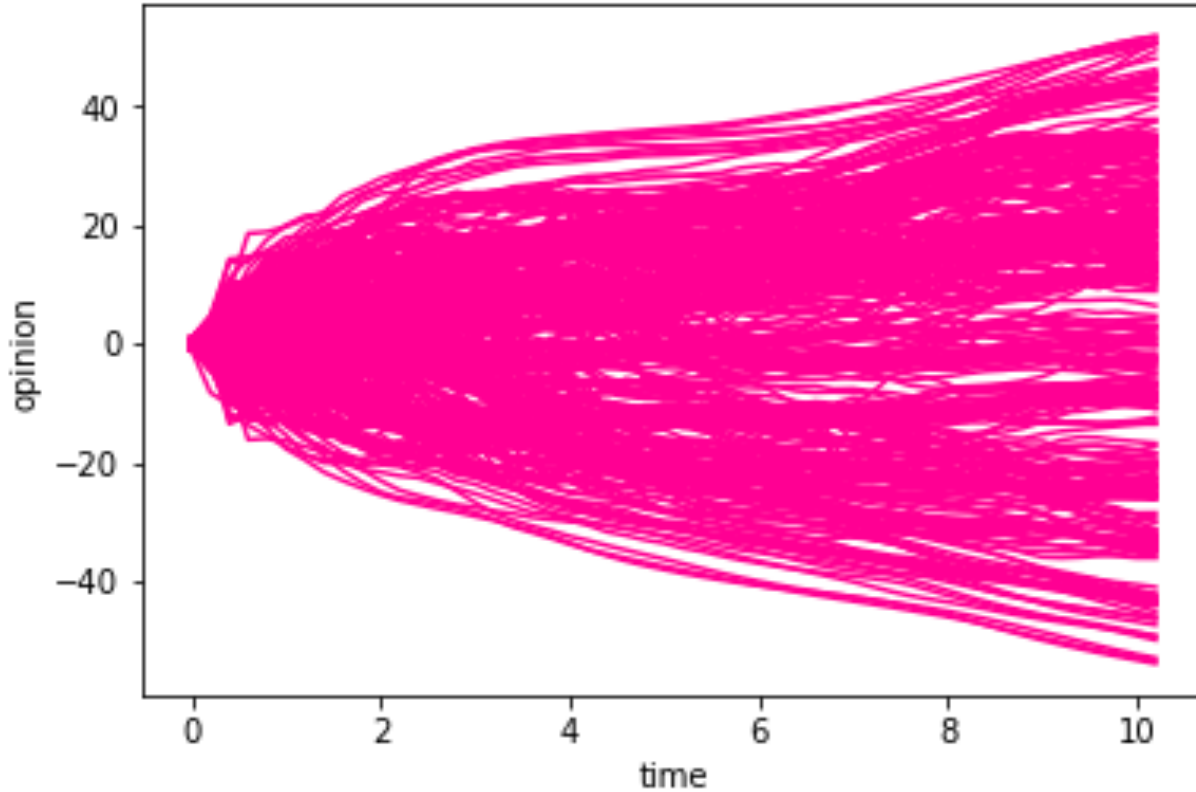
Calculation for 300 persons

300 persons connect to everybody as Complete Graph.

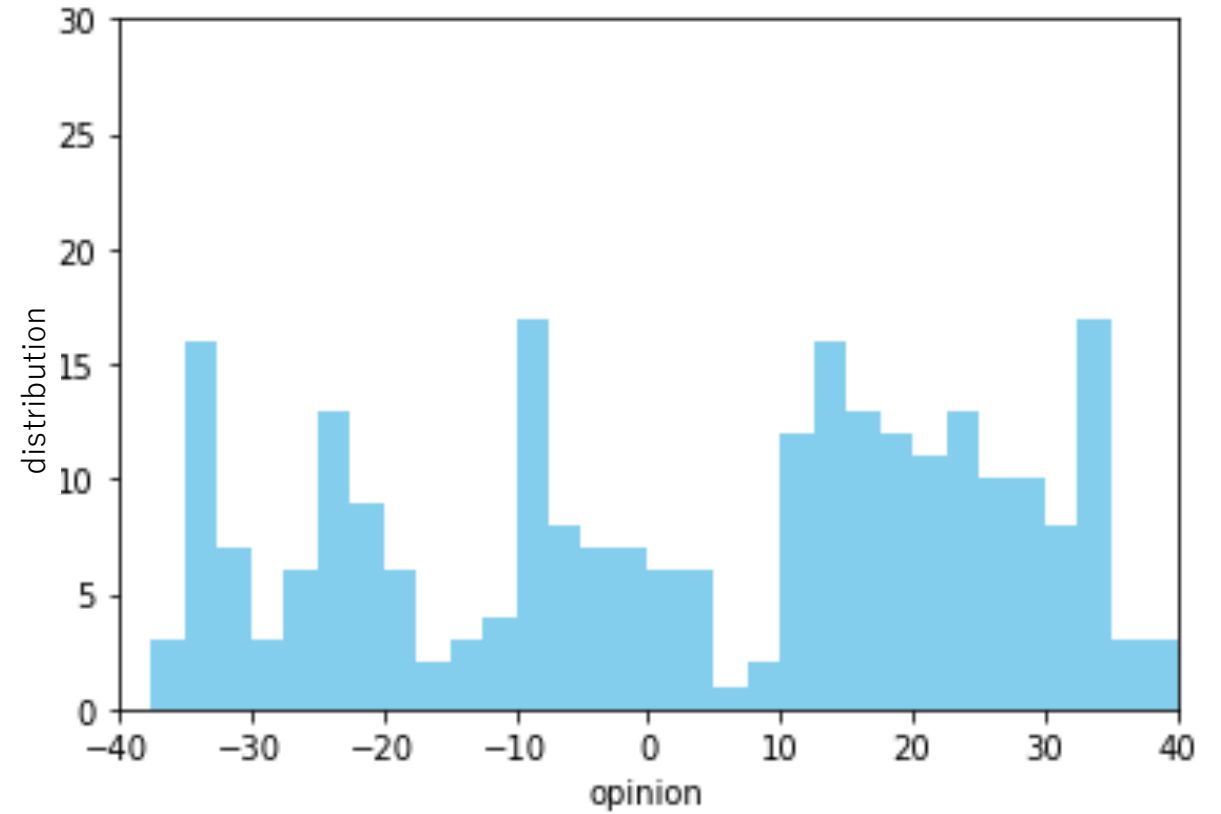
D_{ij} is decided in range $[-1,+1]$ using random number

Initial opinions are decided in range $[-1,+1]$ using random number

$N=300$



Time development of opinions for 300 persons.

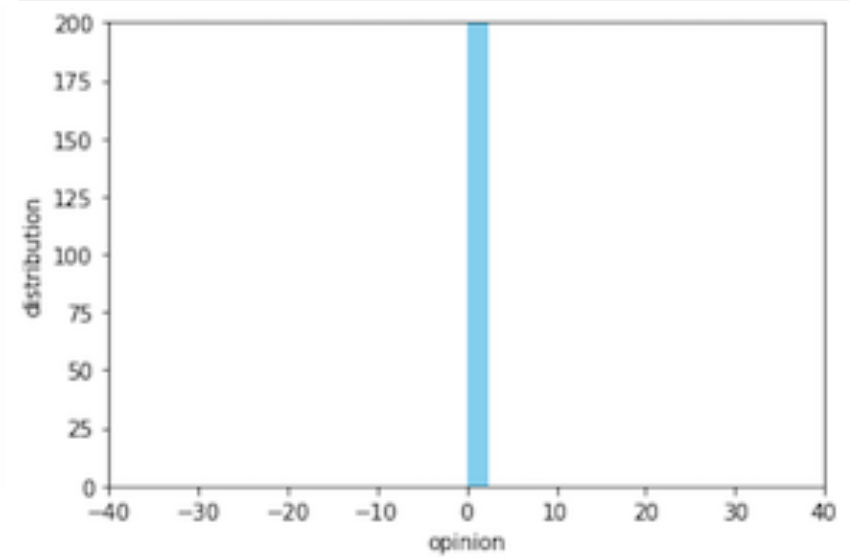
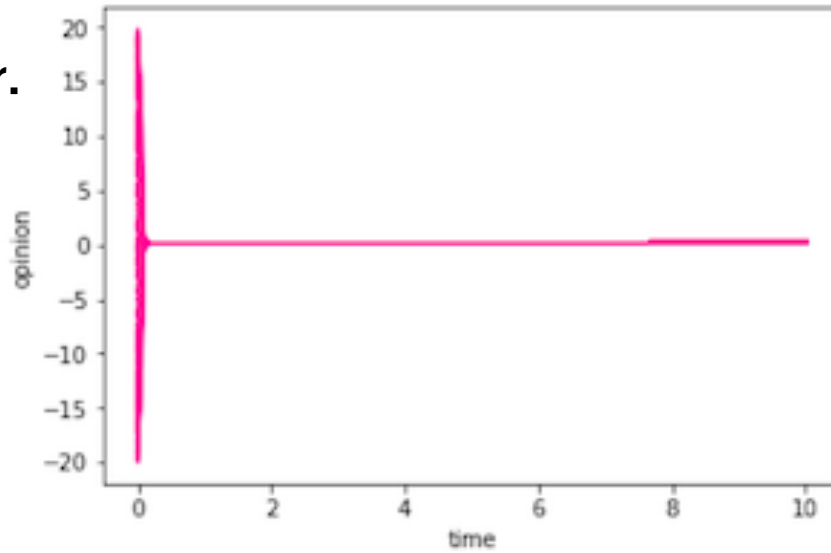


Opinion distribution at Time=10

Everyone has trust each other.

Just like bounded confidence model, we obtain consensus building.

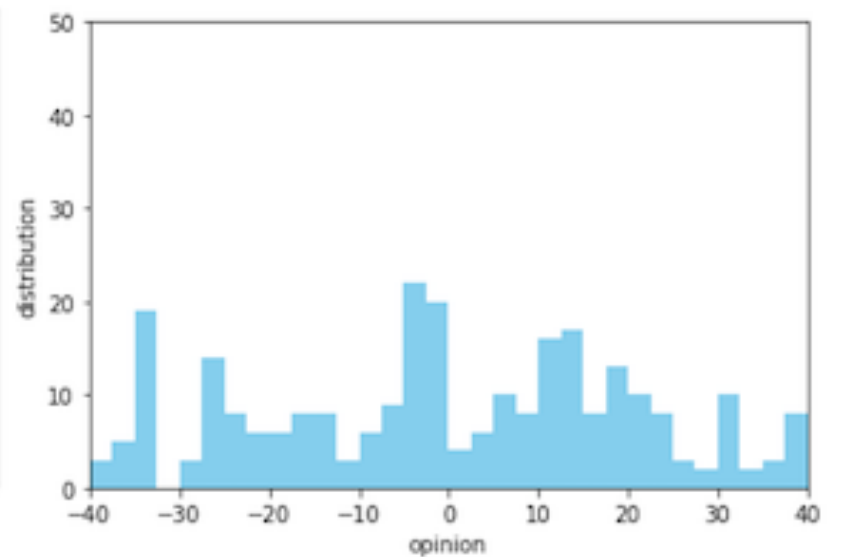
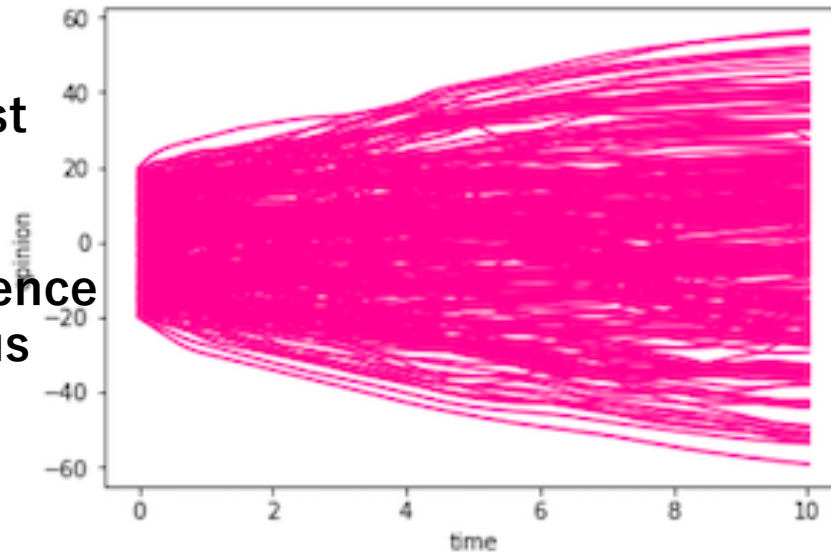
Trust 100%

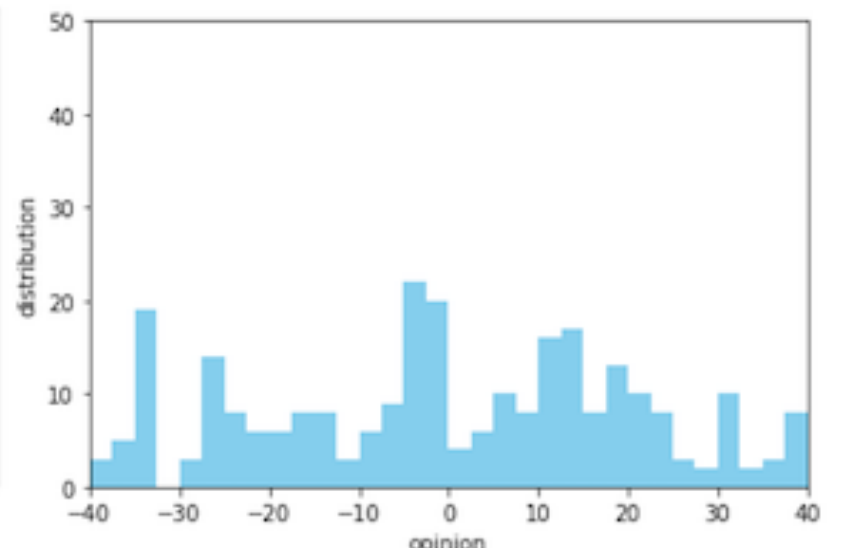
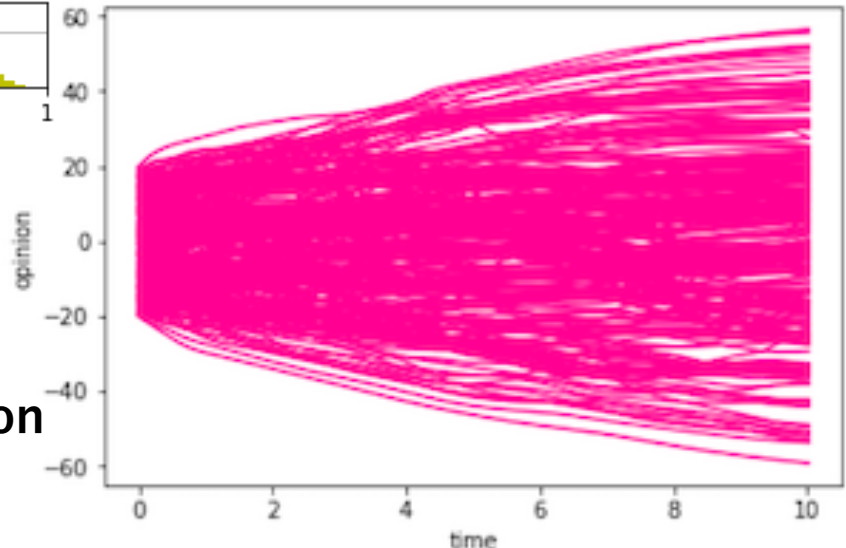
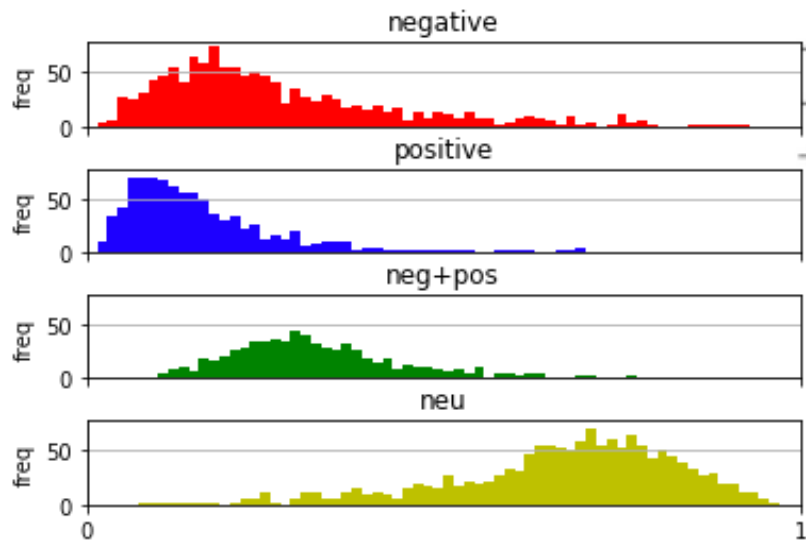
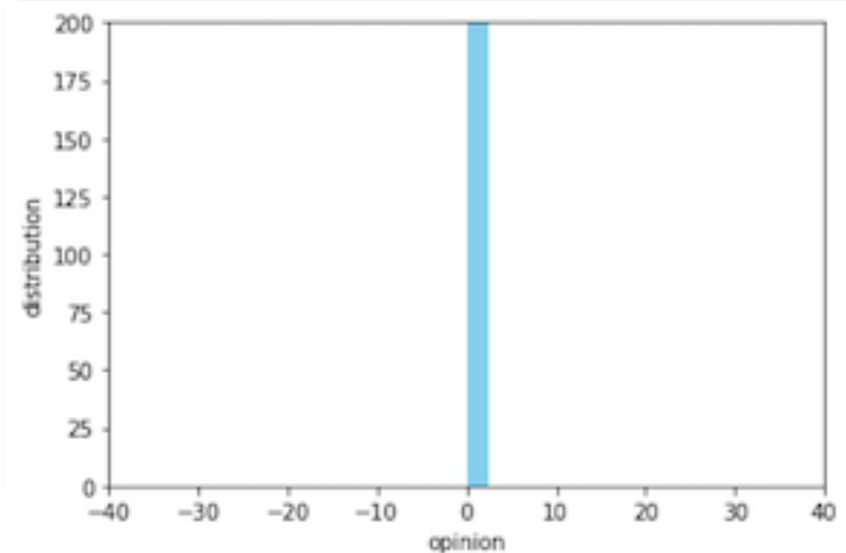
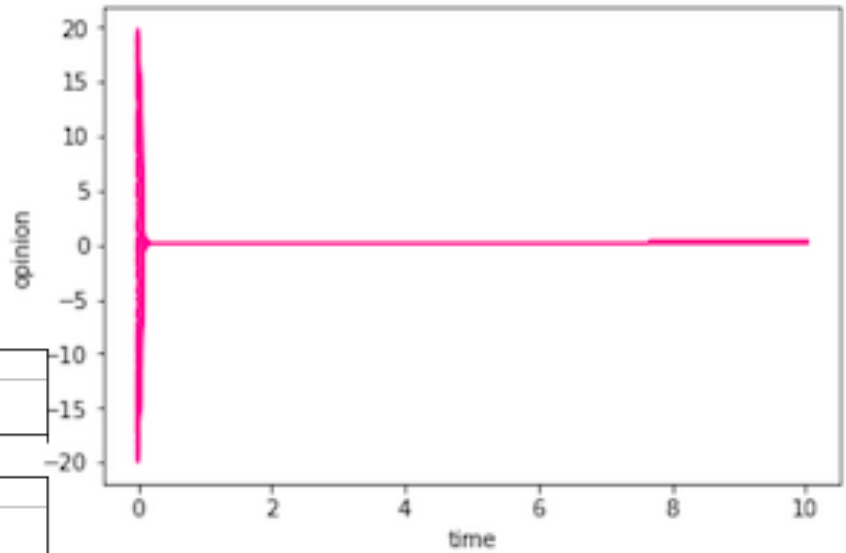
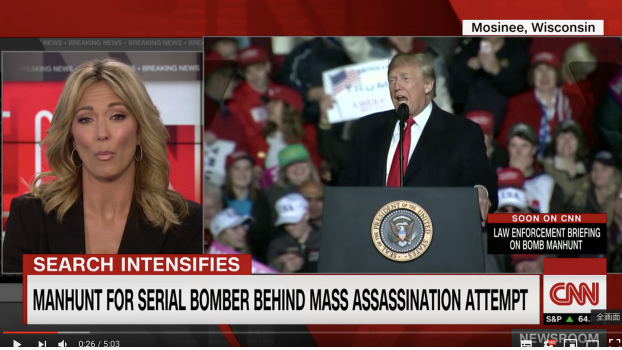


Everyone has trust and distrust about half half.

In contrast to bounded confidence model, we obtain no consensus building.

Trust 50%
Distrust 50%





Compare with observation of comments on news of YouTube, observed distribution is close to non consensus building case.

Suppose that there are two groups of positive opinion and negative opinion.
Everyone in a group is in a trusting relationship and distrusts people in other groups

- Suppose that there are two groups of positive opinion and negative opinion. Everyone in a group is in a trusting relationship and distrusts people in other groups

Suppose that there are **two groups** of positive opinion and negative opinion. Everyone in a group is in a trusting relationship and distrusts people in other groups

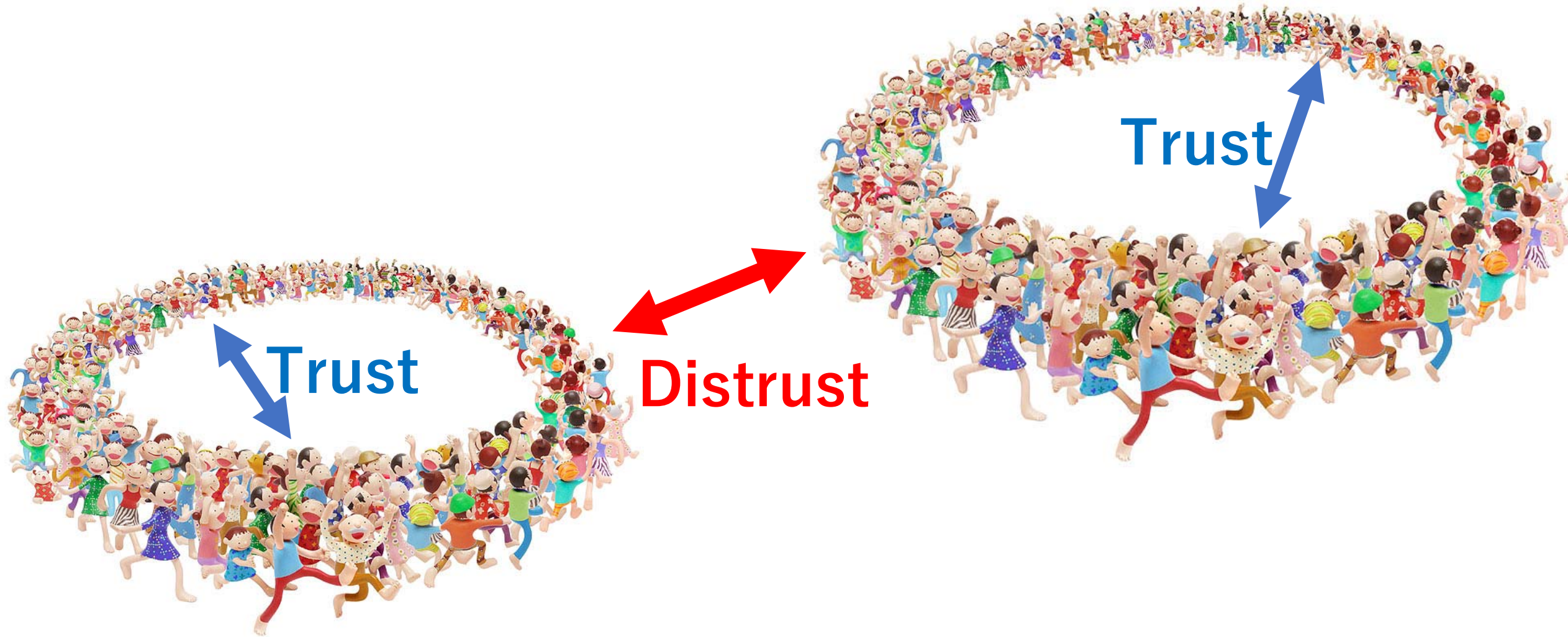


Positive opinion people



Negative opinion people

Conflict of Two Groups



Positive opinion group (pink) 150 persons
Negative opinion group (light blue) 150 persons
 $b=2$

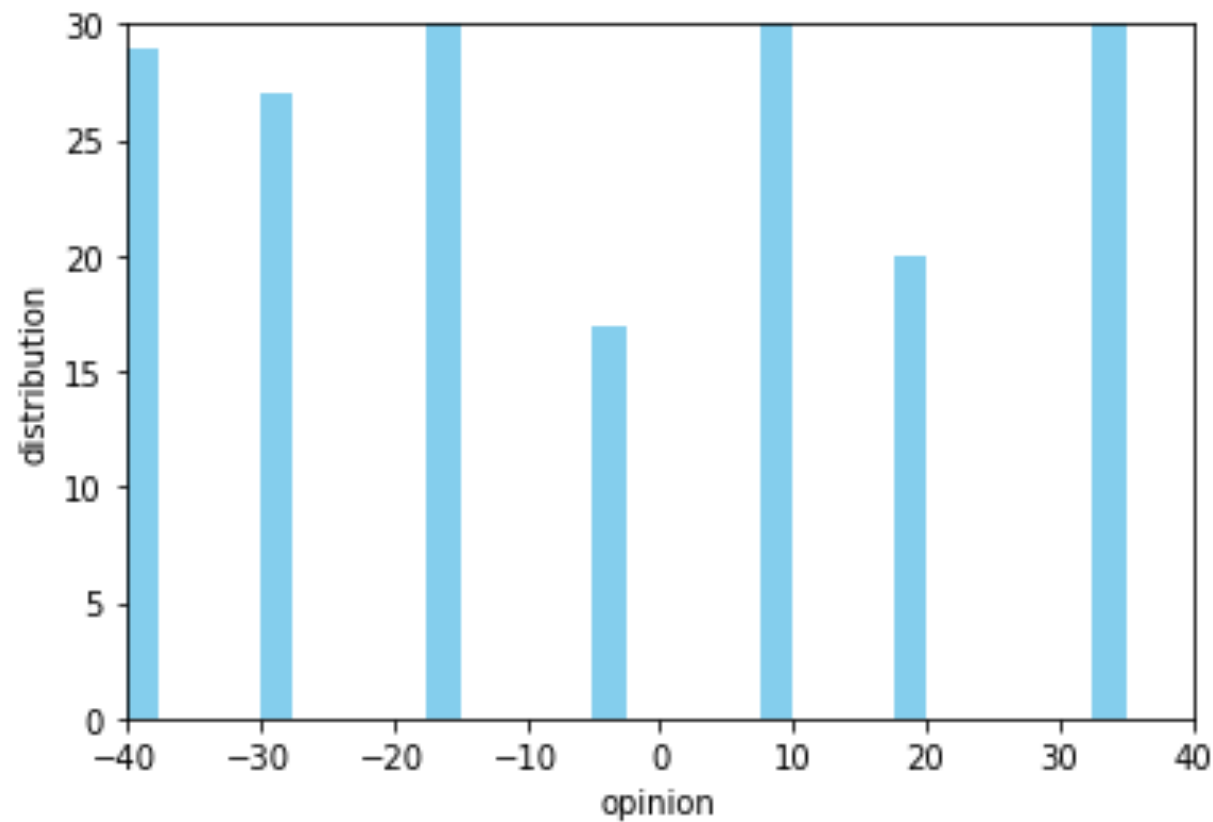
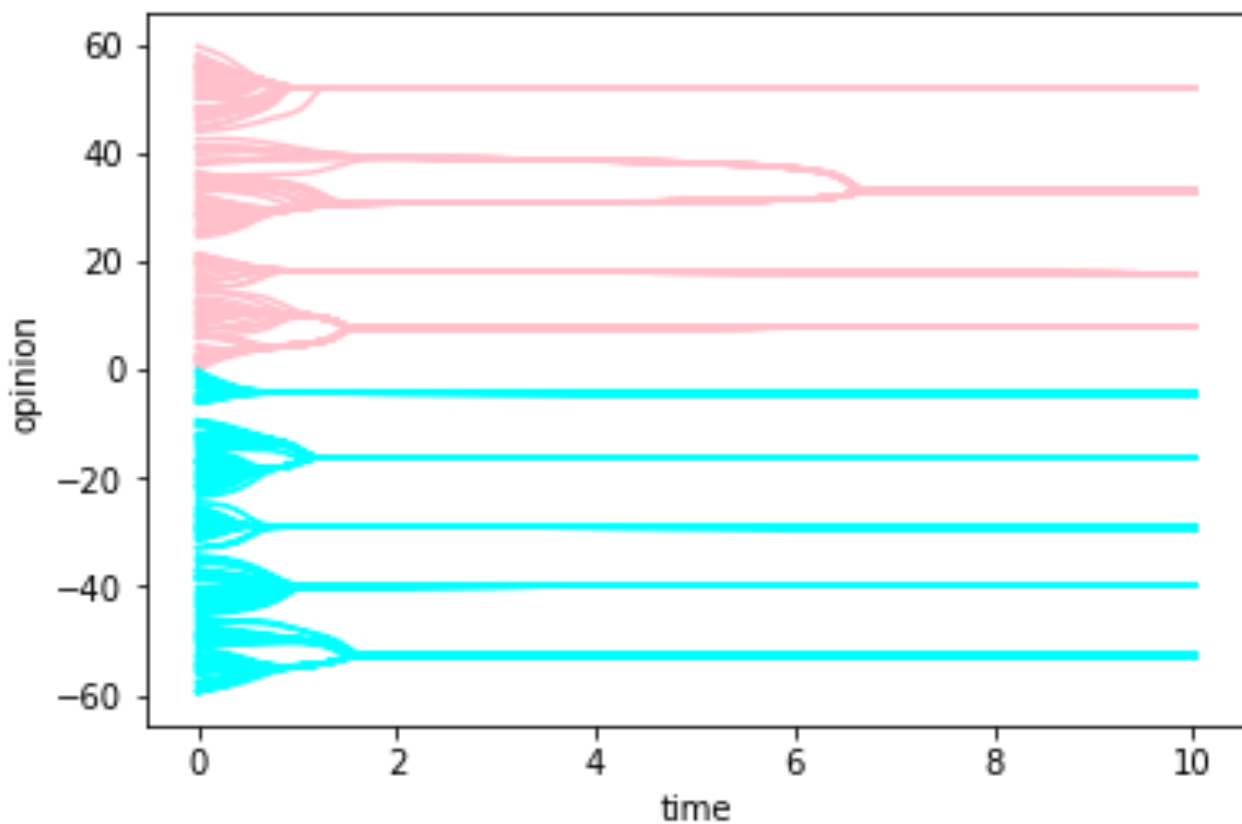
Pink-Pink $D_{ij} > 0$

Light blue-Light blue $D_{ij} > 0$

Pink-Light Blue $D_{ij} < 0$

Pink Initial opinion $+30 \pm 30$

Light blue initial opinion -30 ± 30



Positive opinion group (pink) 150 persons
Negative opinion group (light blue) 150 persons
 $b=5$

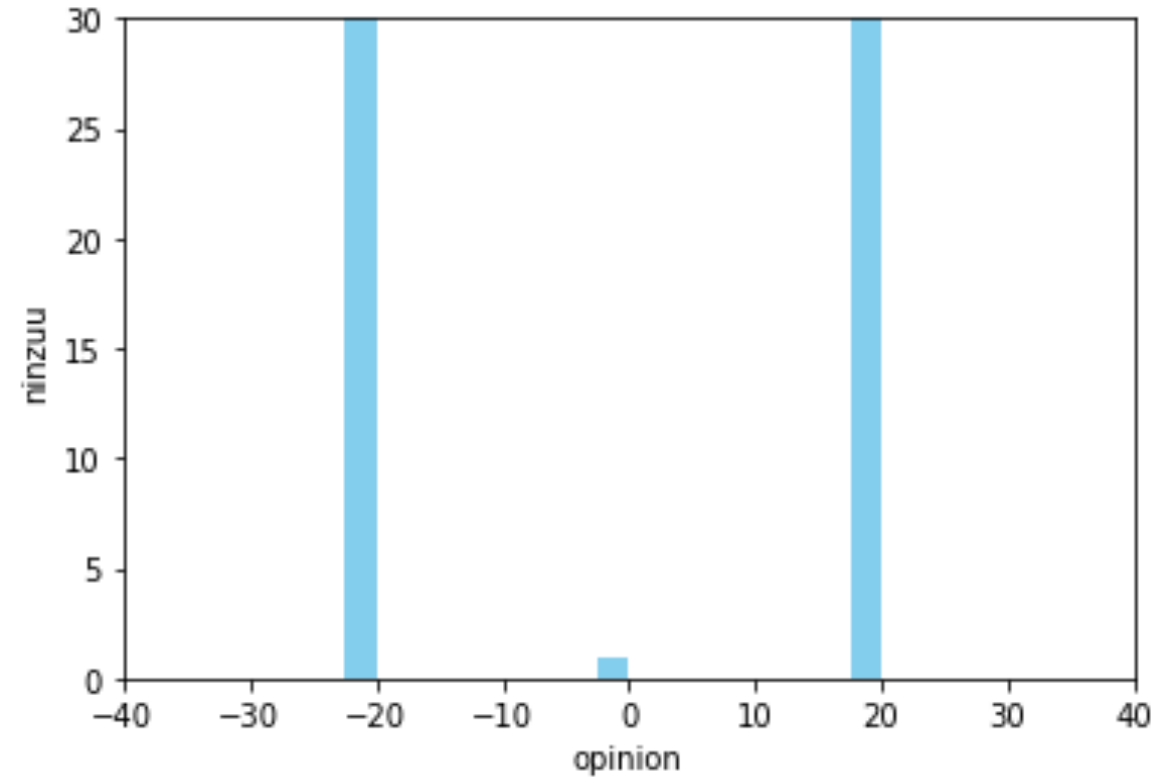
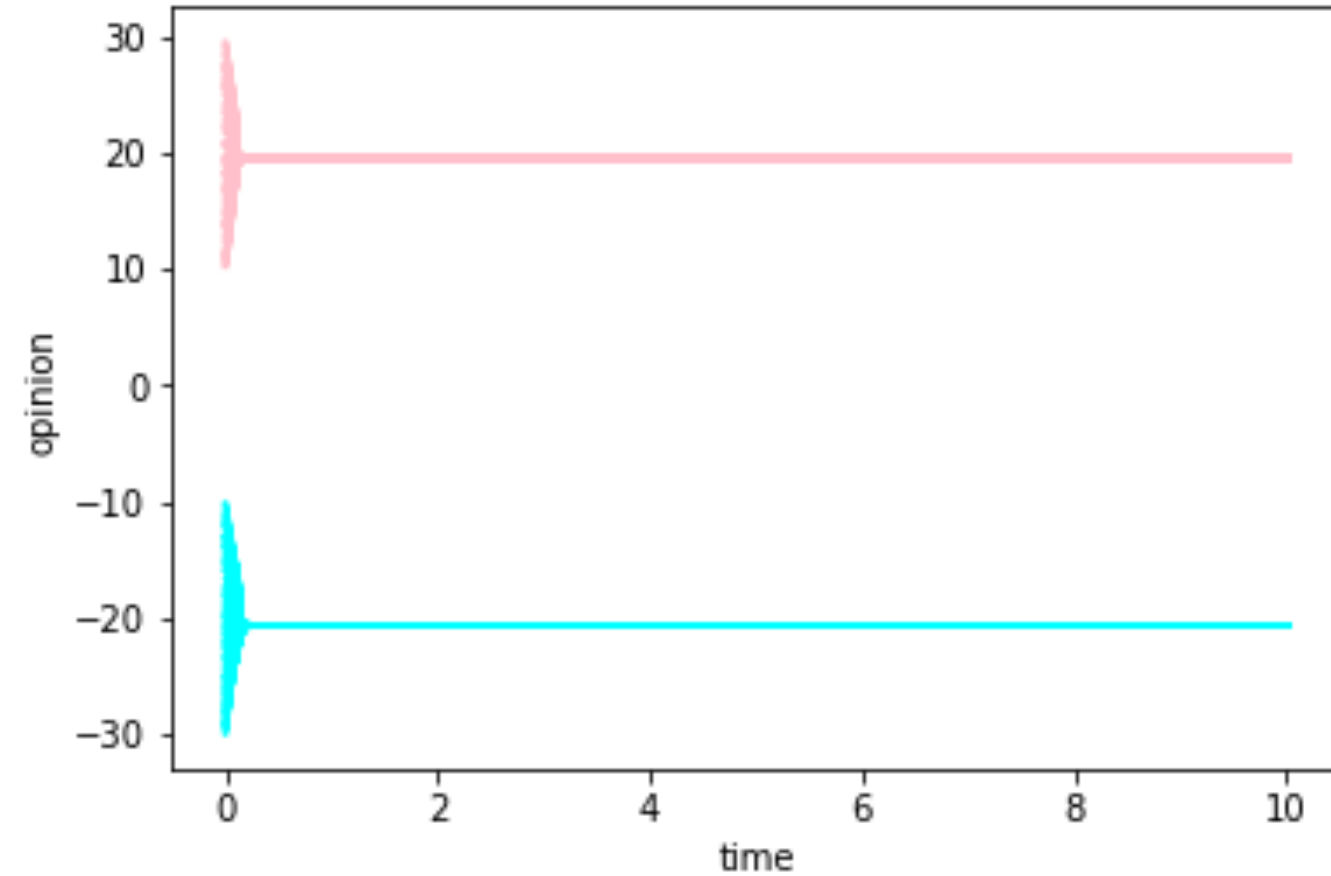
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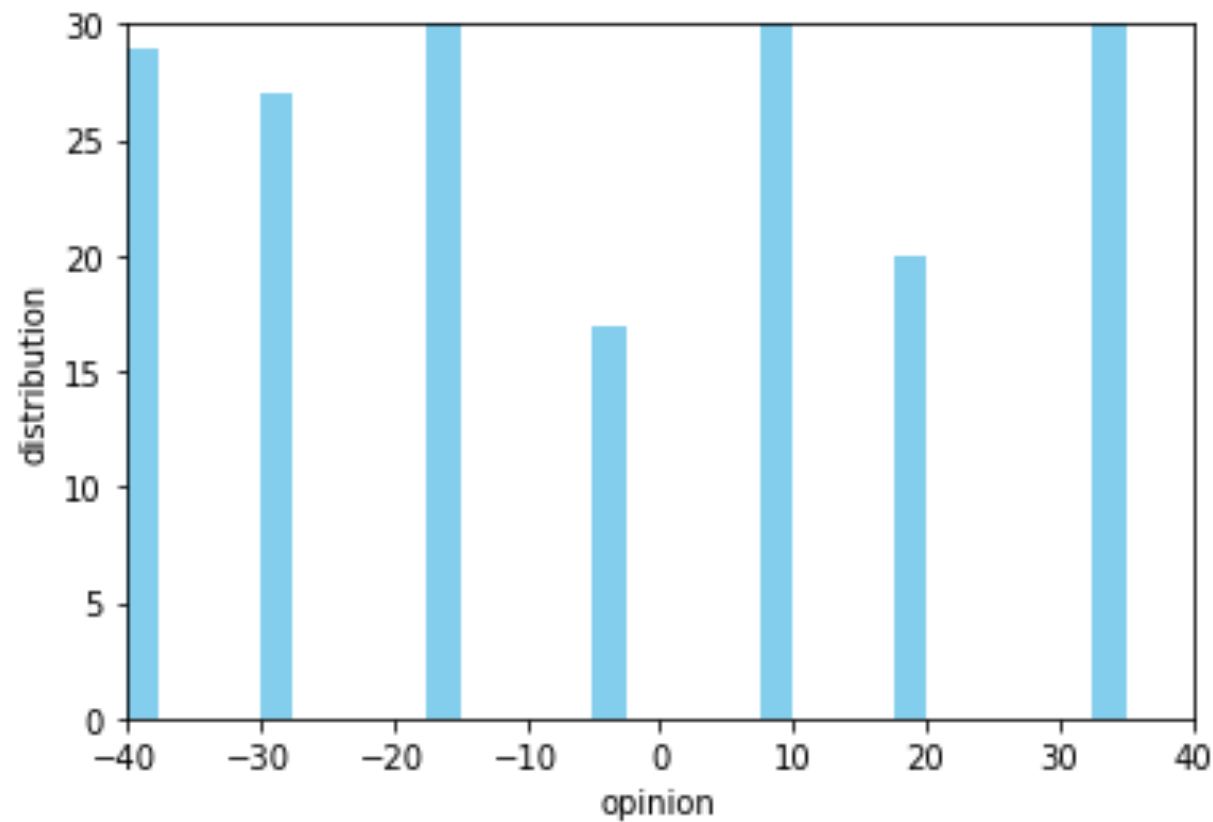
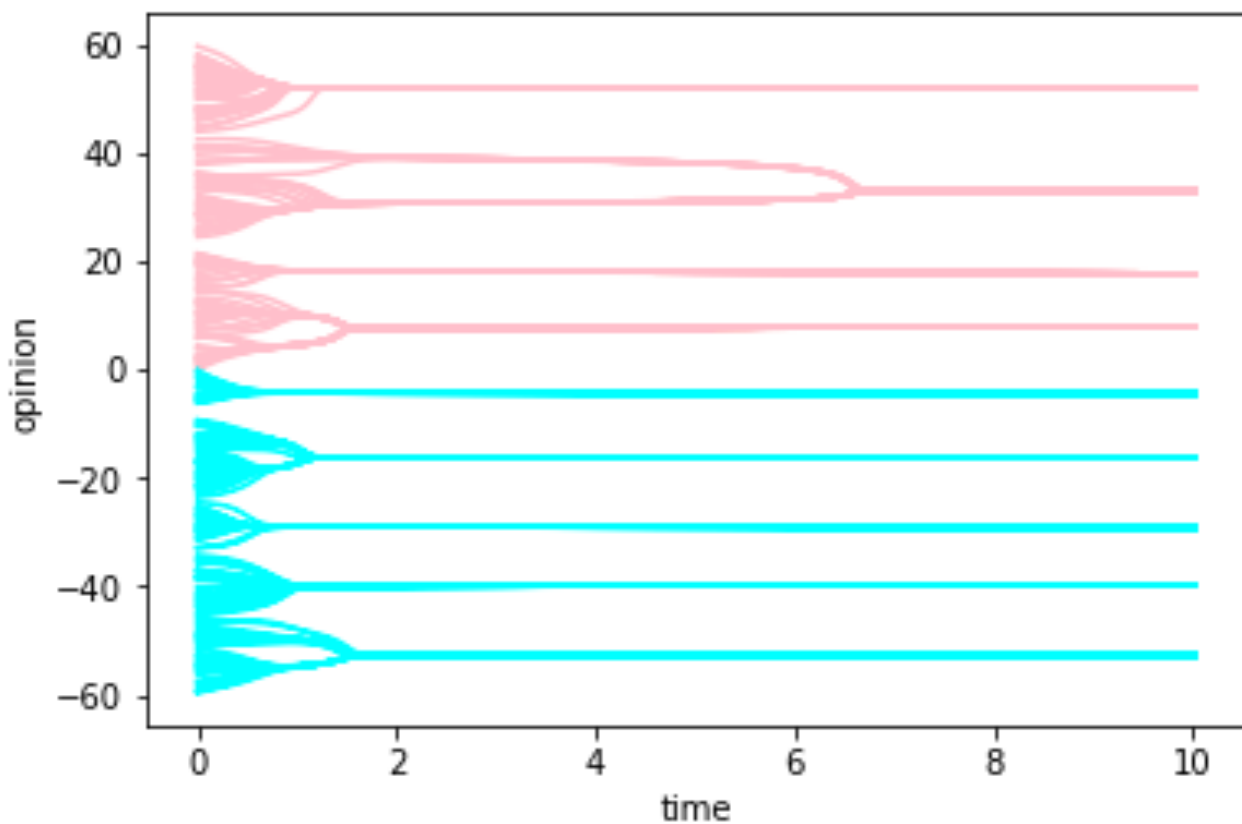


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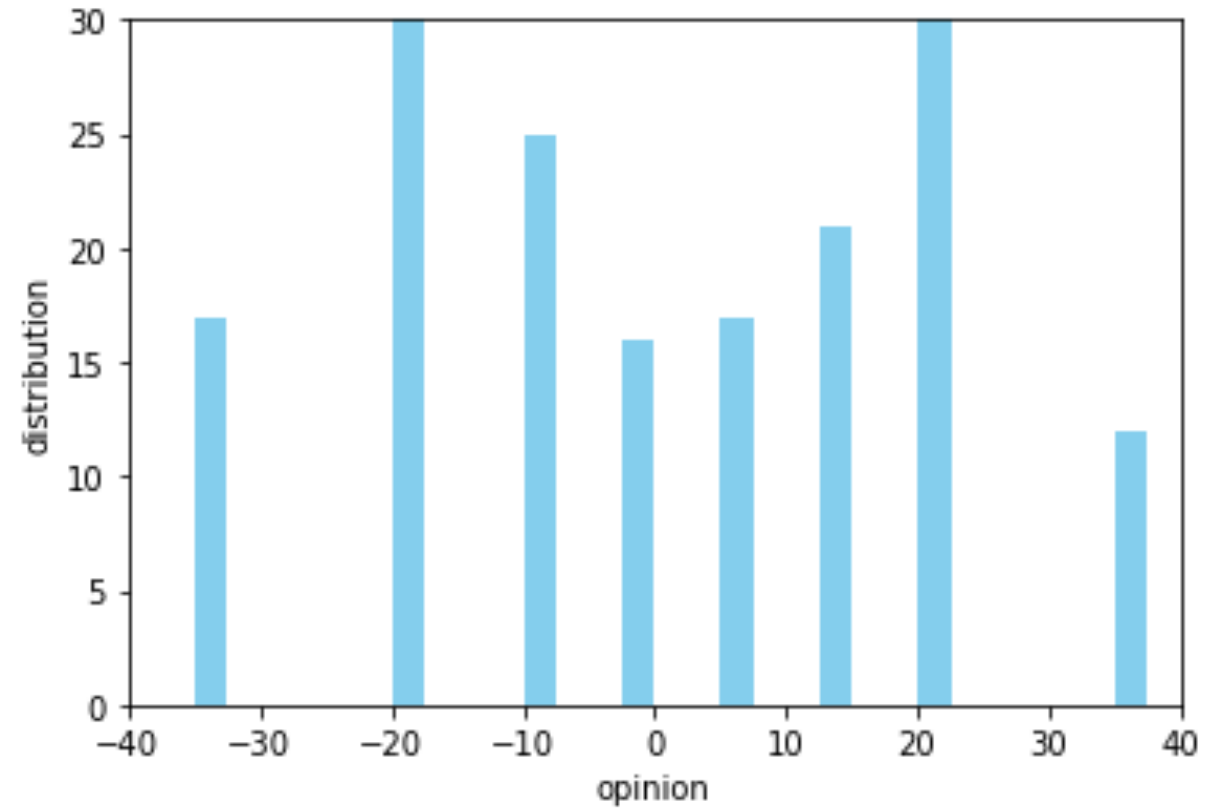
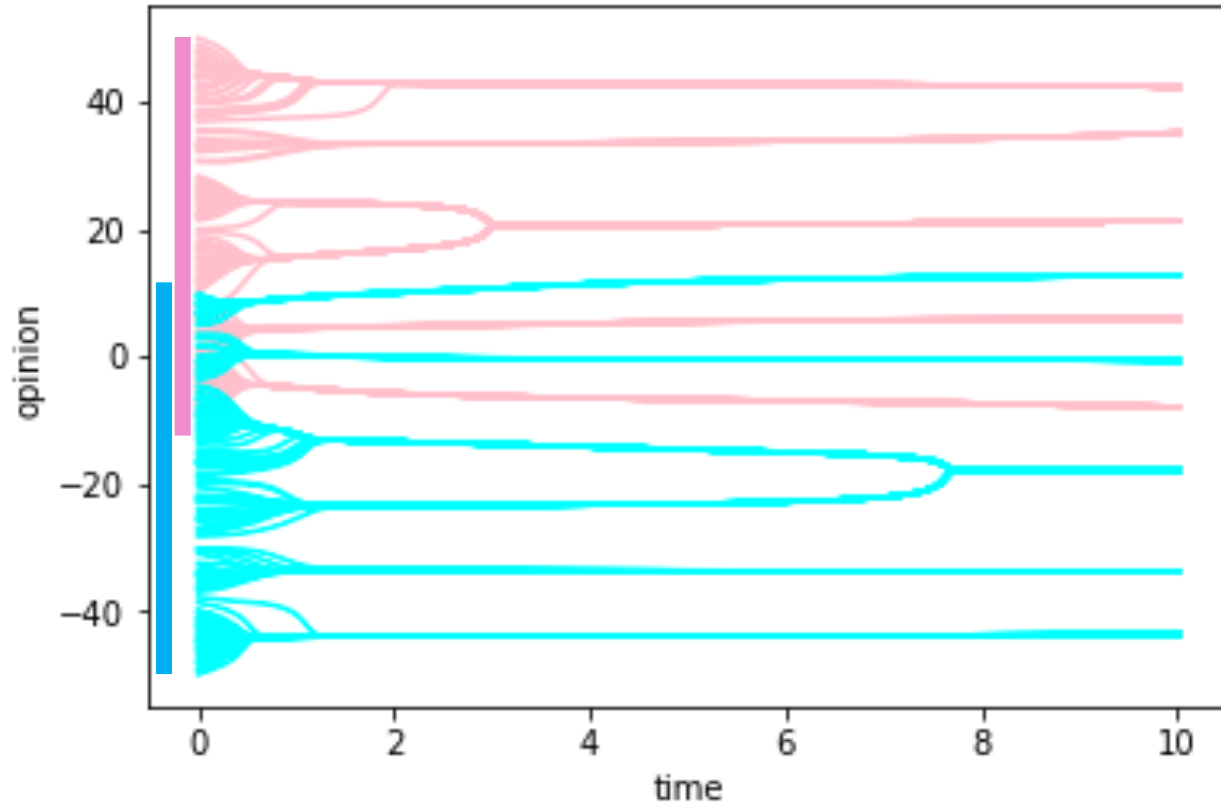
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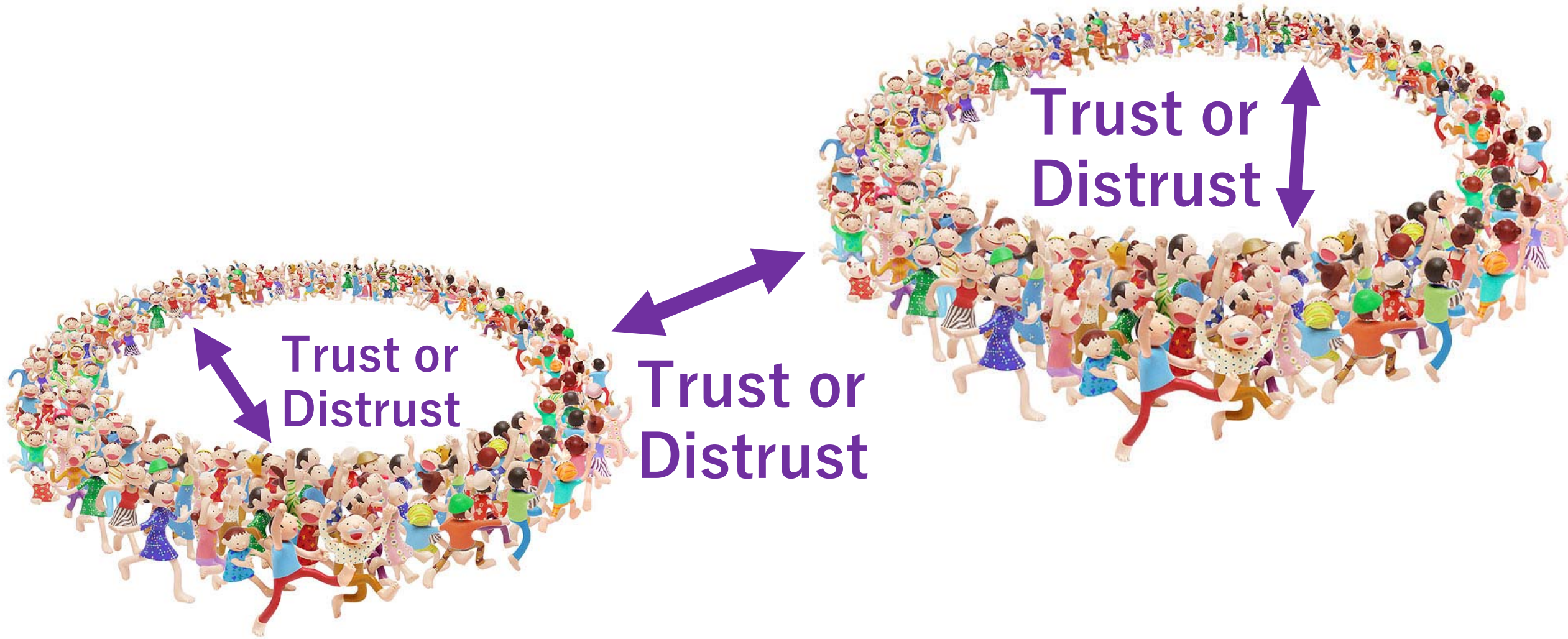
Pink-Light Blue $D_{ij} < 0$

Pink Initial opinion $+30 \pm 30$

Light blue initial opinion -30 ± 30

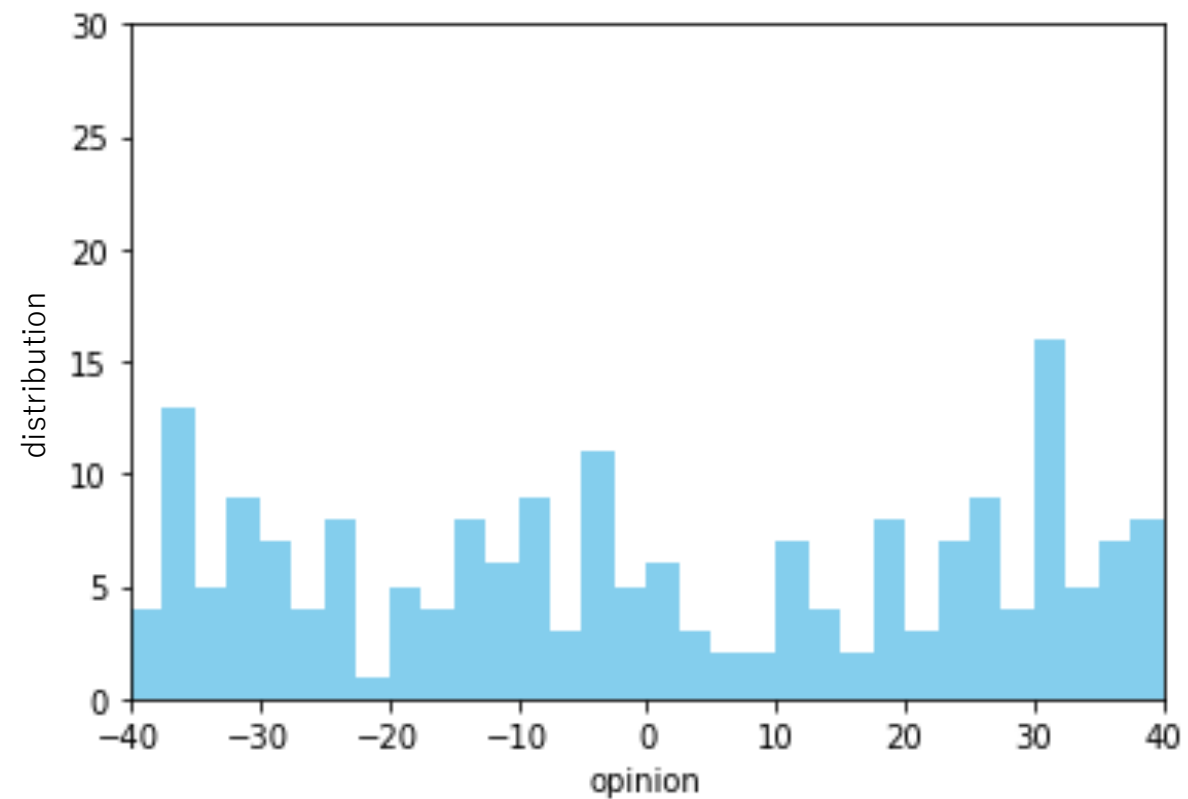
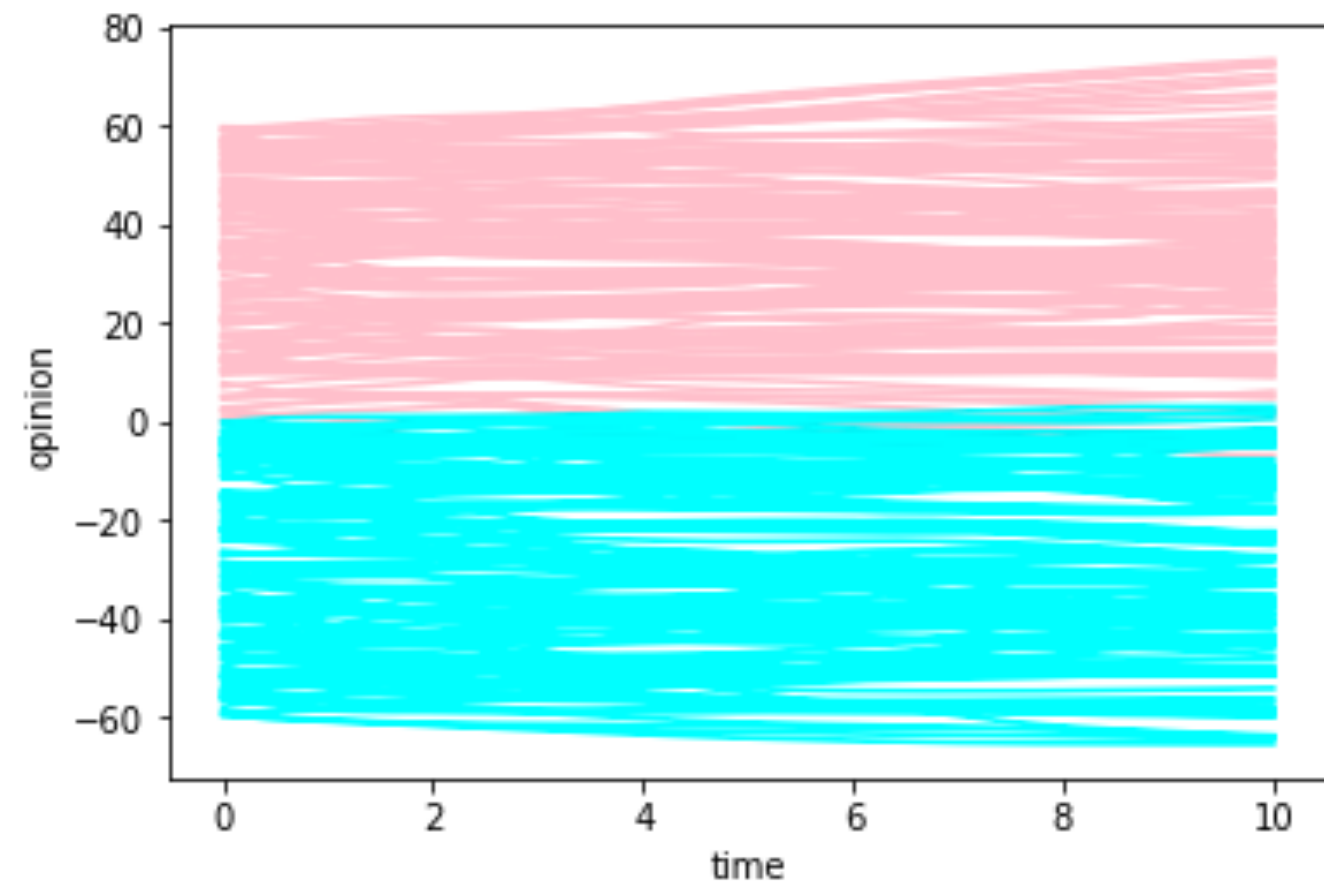


Conflict of Two Groups



Positive opinion group (pink) 150 persons
Negative opinion group (light blue) 150 persons
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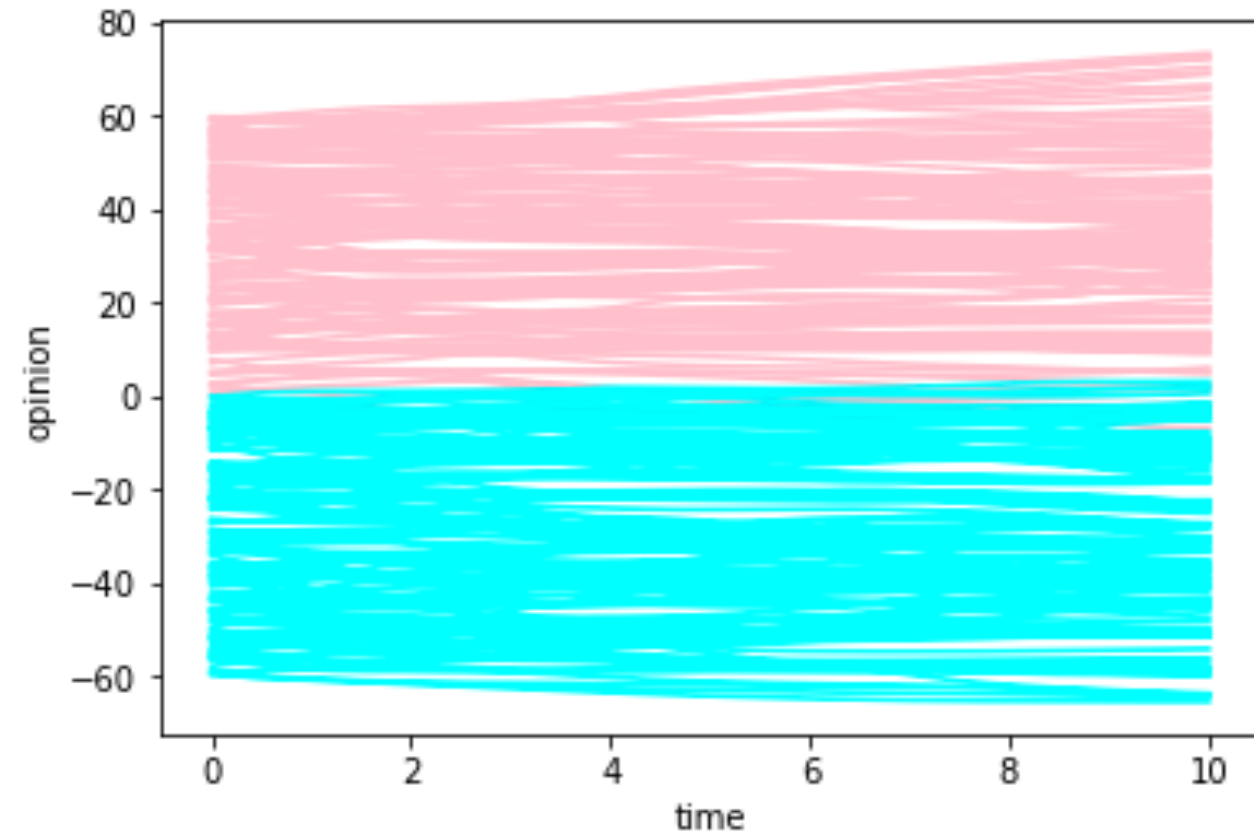
$D_{ij} = -1 \sim +1$ for all persons



Which is the distribution of real society's opinion?

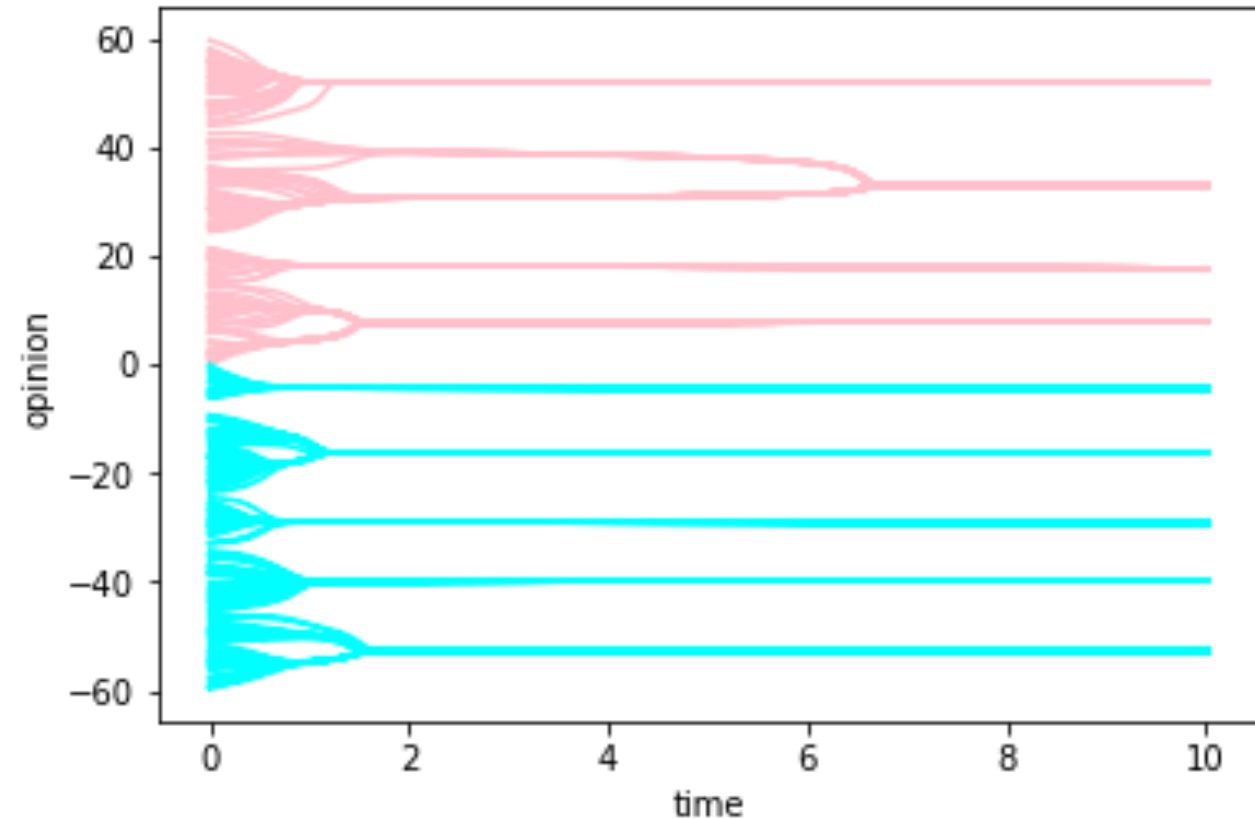
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Trust and distrust for all persons

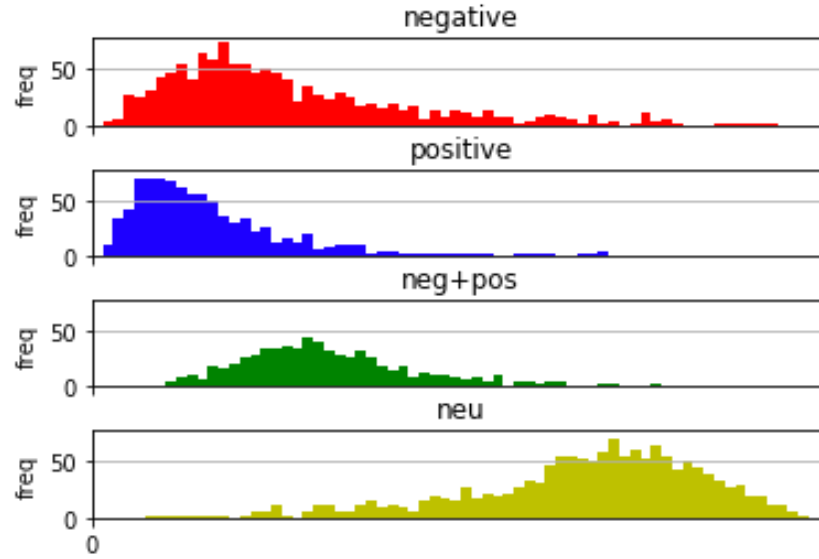


Everyone is in a trusting relationship with each other.

It has the same settings as Deffuant and Hegselmann-Krause.

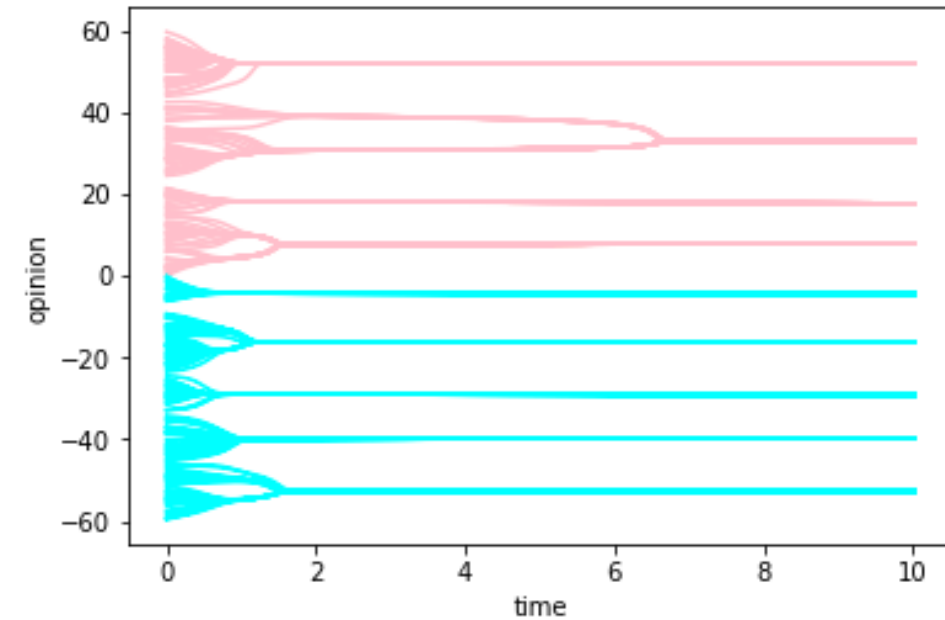
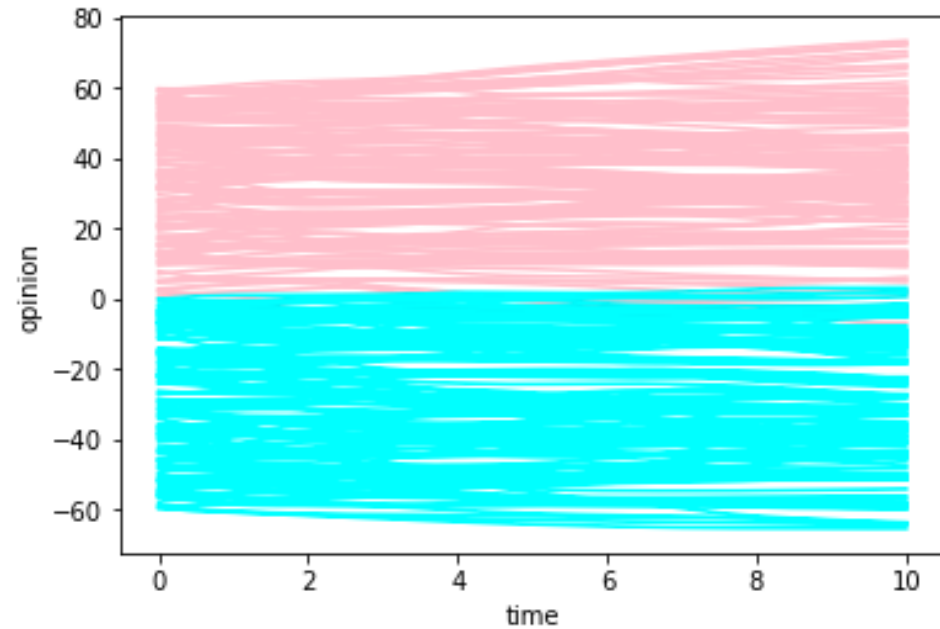


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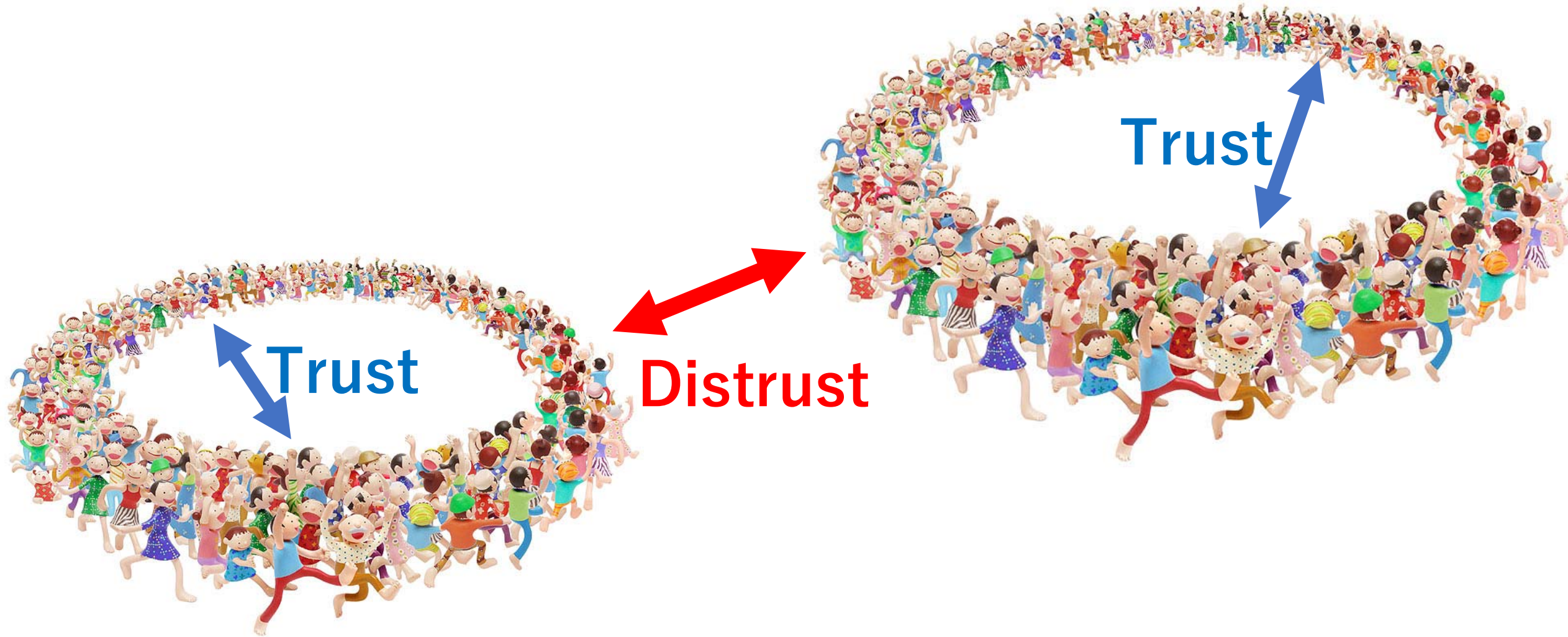


Ishii2019
Trust and distrust for all persons

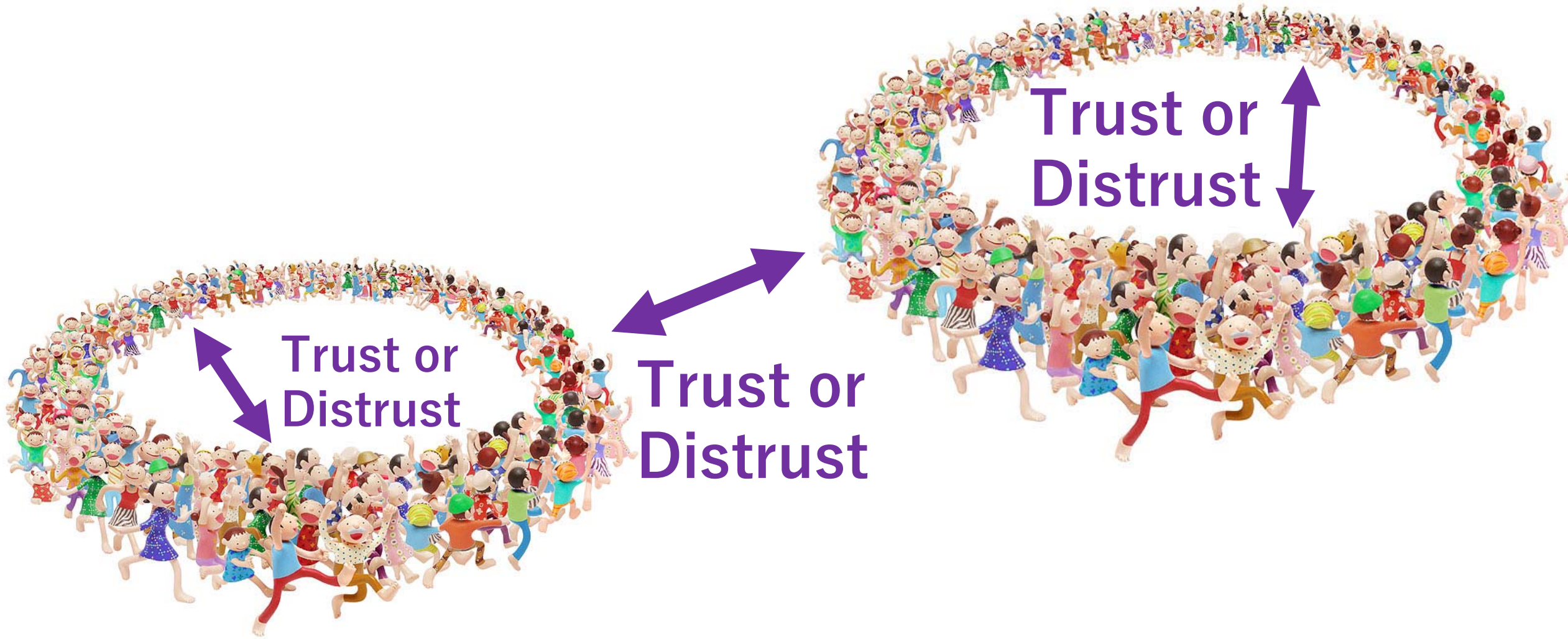
Everyone is in a trusting relationship with each other. It has the same settings as Deffuant and Hegselmann-Krause.



Conflict of Two Groups



Conflict of Two Groups



Coefficient of trust D_{ij}

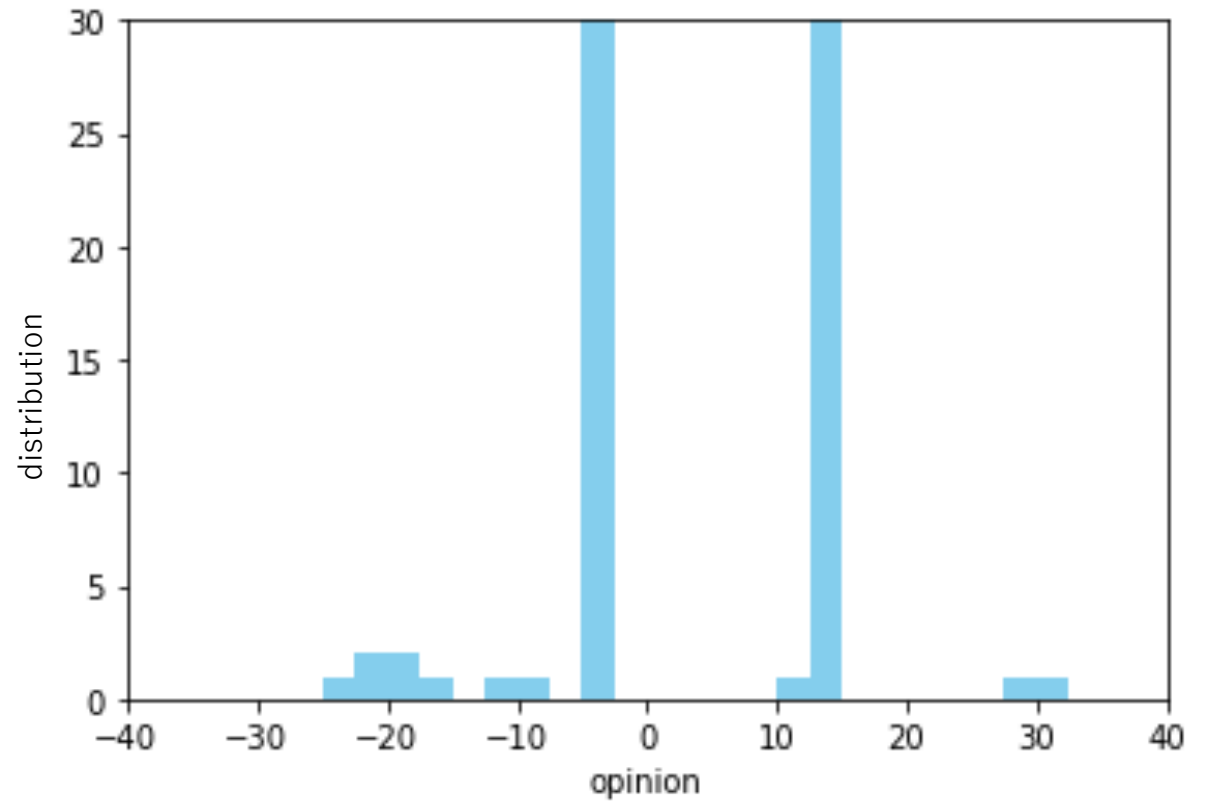
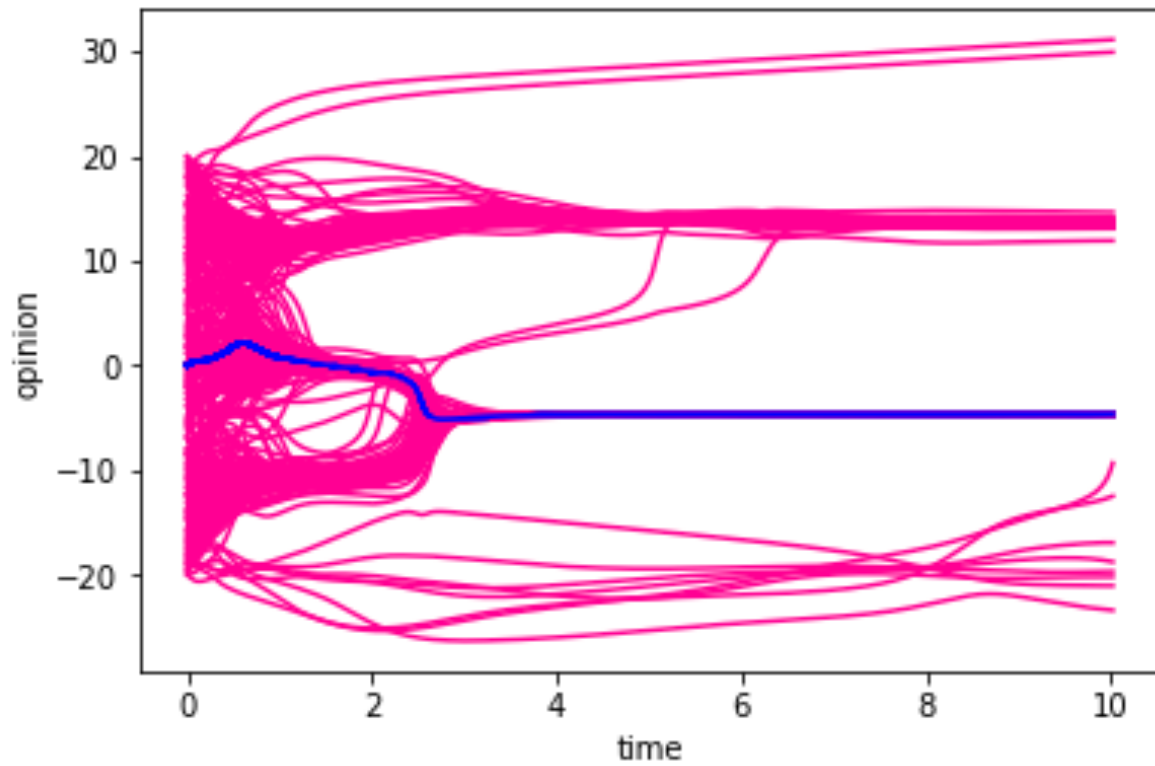
Change in the rate of trust
and distrust

δ = number of positive D_{ij} / number of all D_{ij}

N=300

Initial opinion range -20~+20

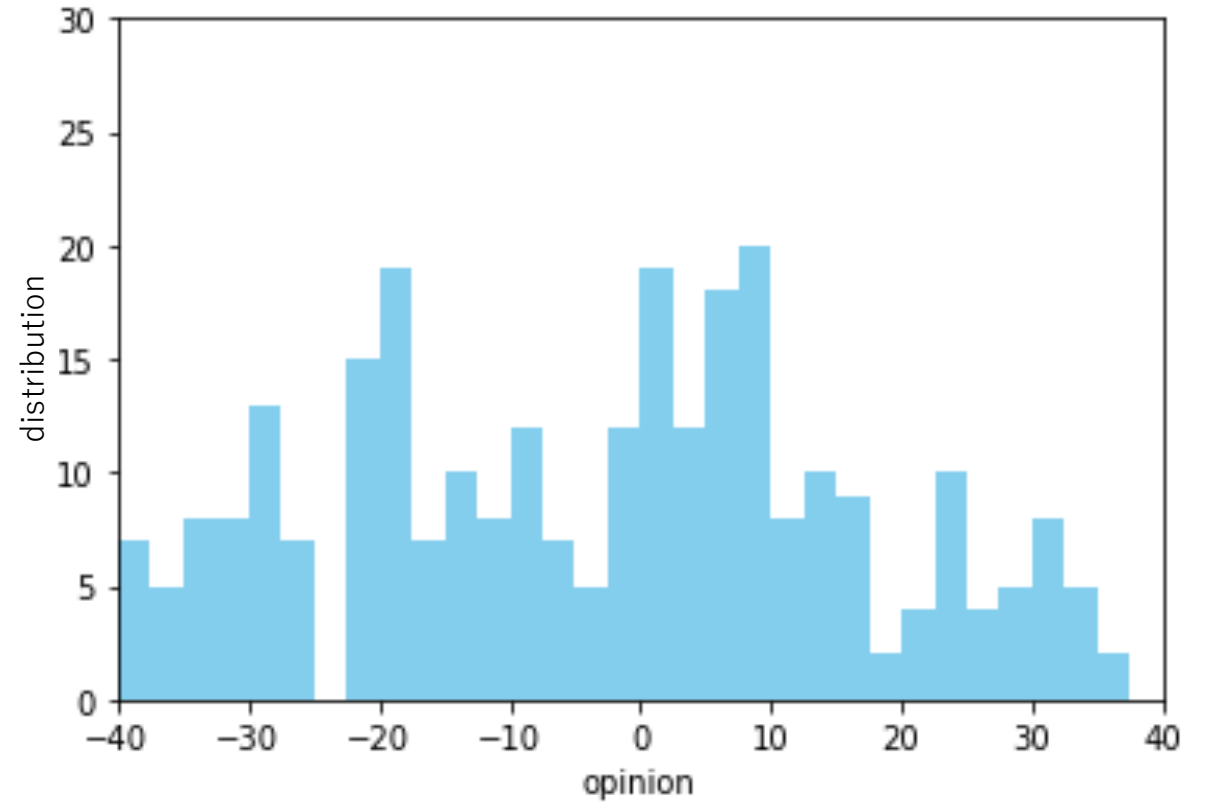
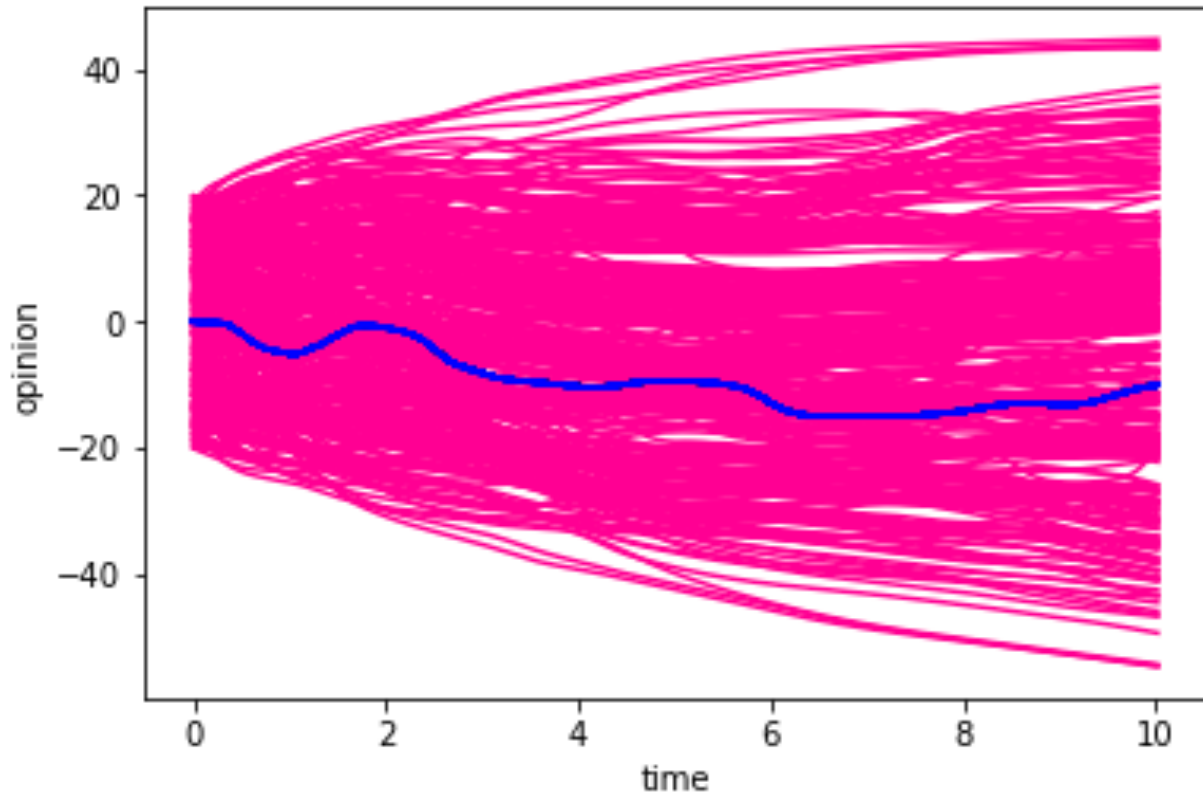
Dij is determined by using random number
(positive 60%)



N=300

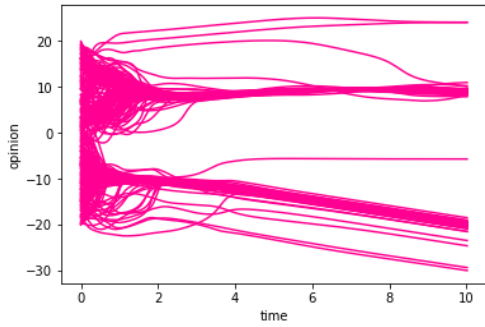
Initial opinion range -20~+20

Dij is determined by using random number
(positive 50%)

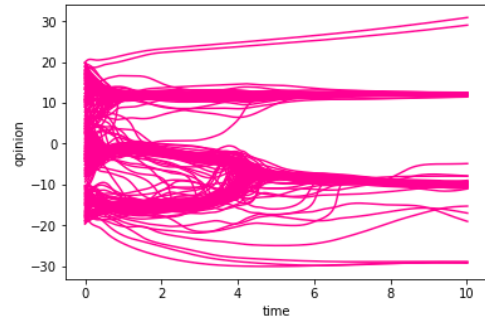


55% would be a critical point
percent can be changed by network structure

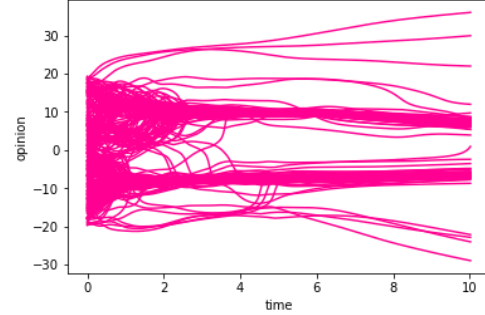
0.60



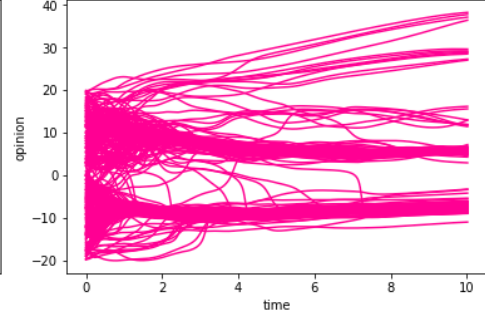
0.59



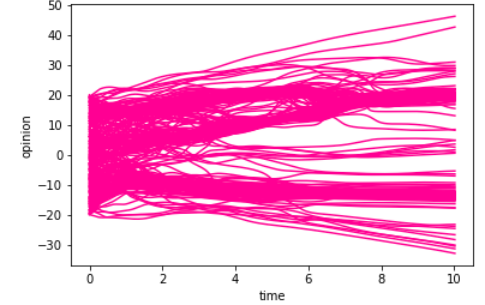
0.58



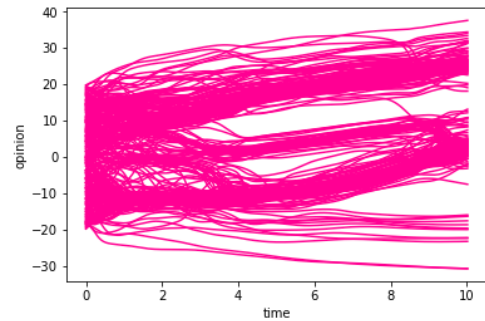
0.57



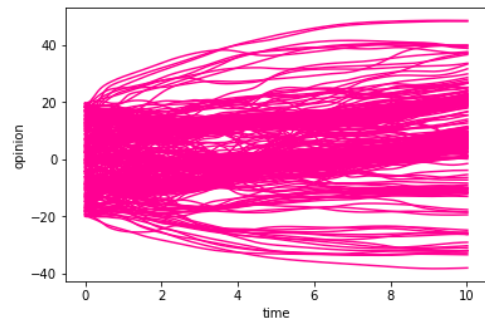
0.56



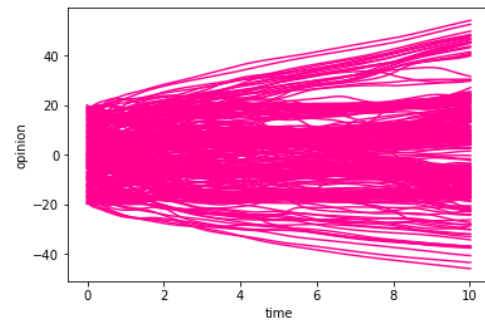
0.55



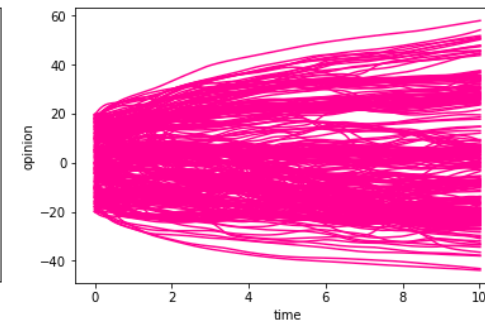
0.54



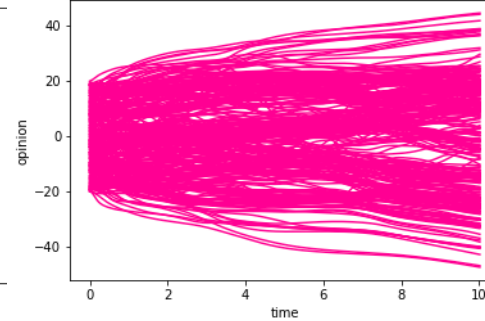
0.53



0.52

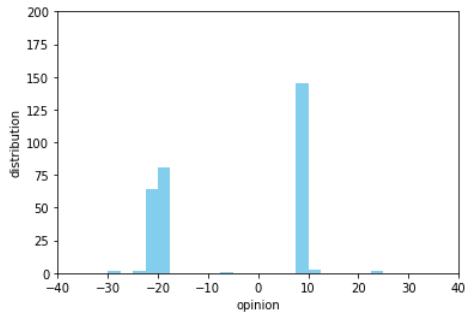


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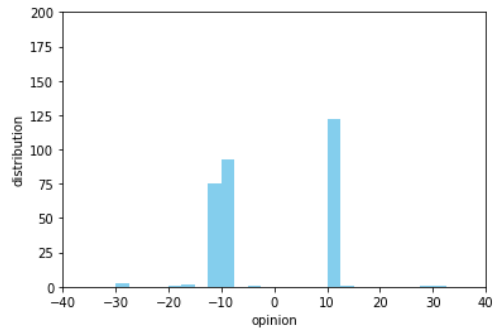


55% would be a critical point
percent can be changed by network structure

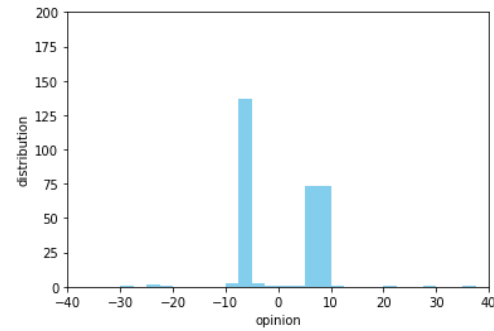
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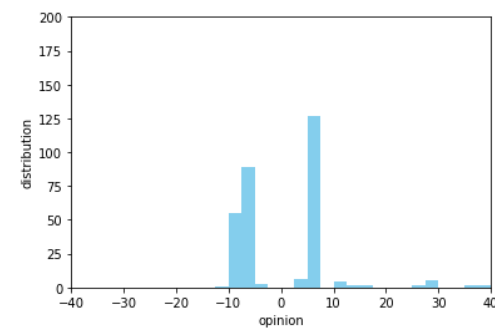
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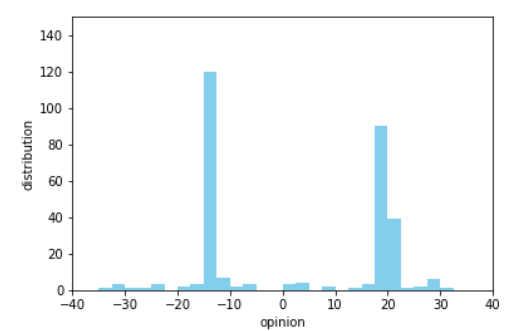
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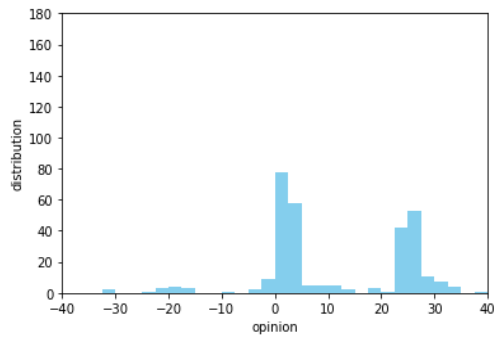
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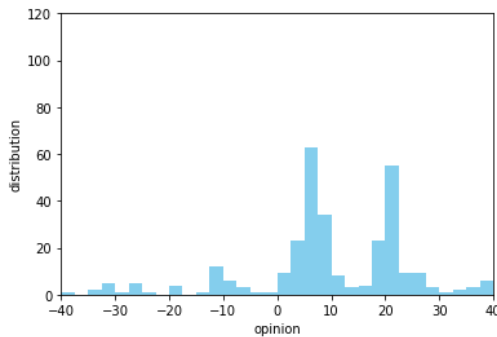
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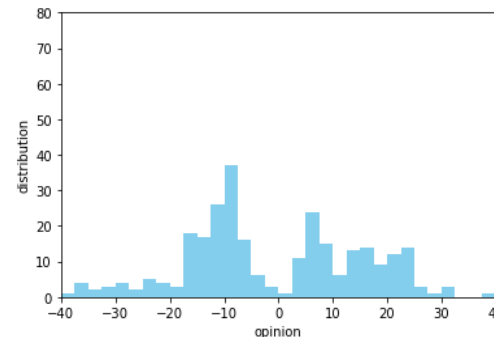
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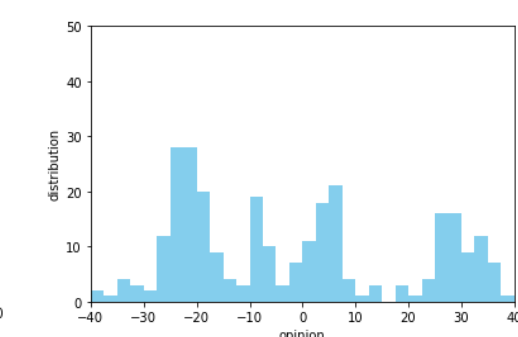
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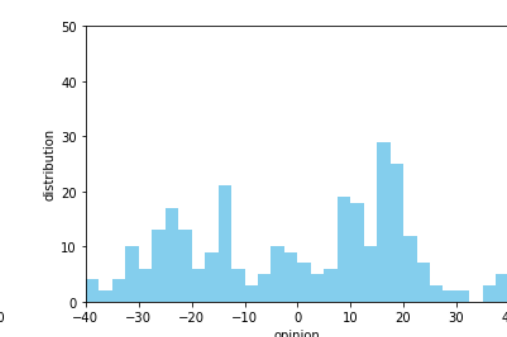
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0.52



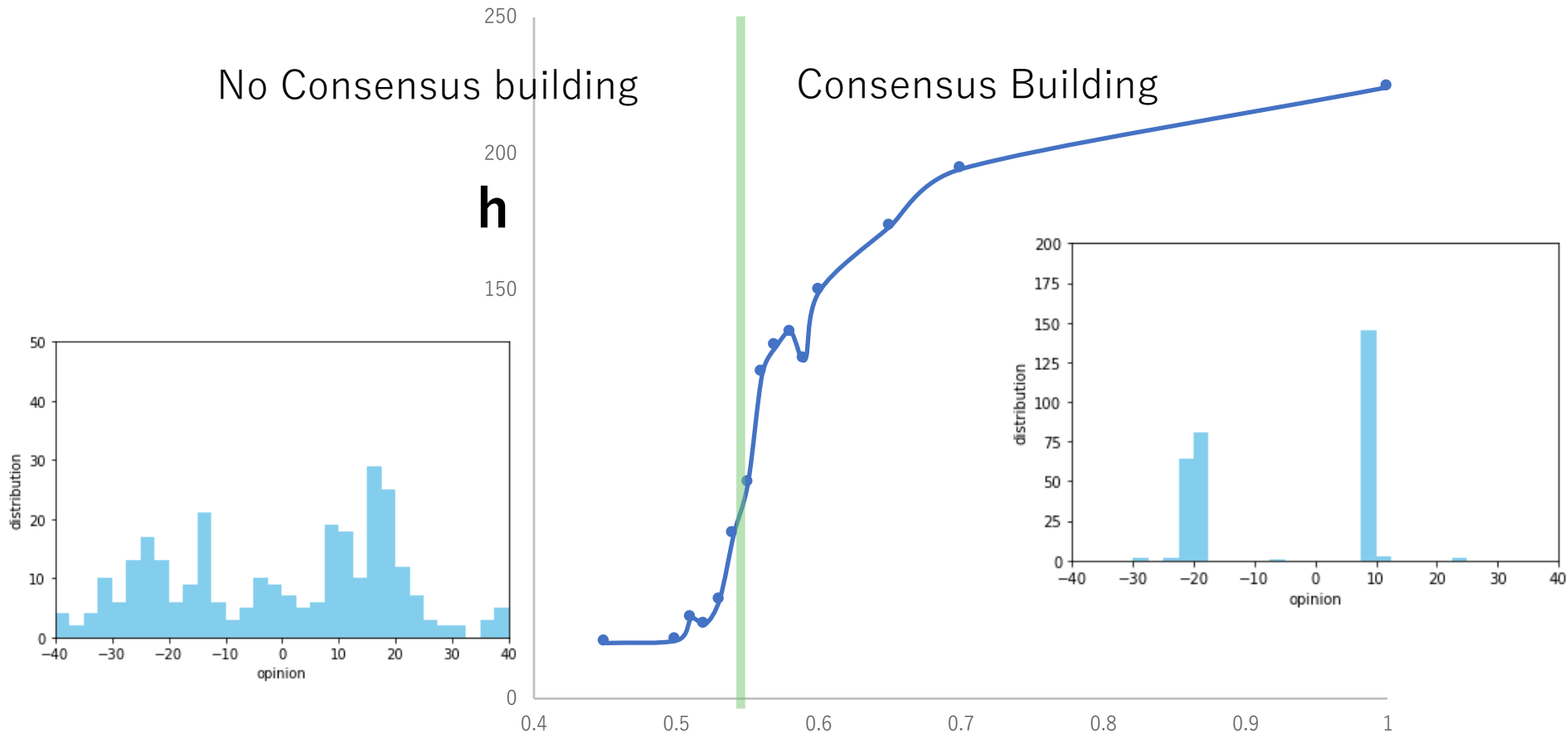
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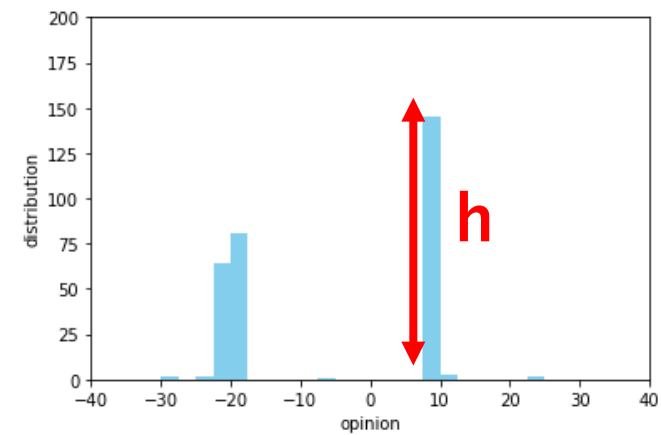
No Consensus building

Consensus Building

h

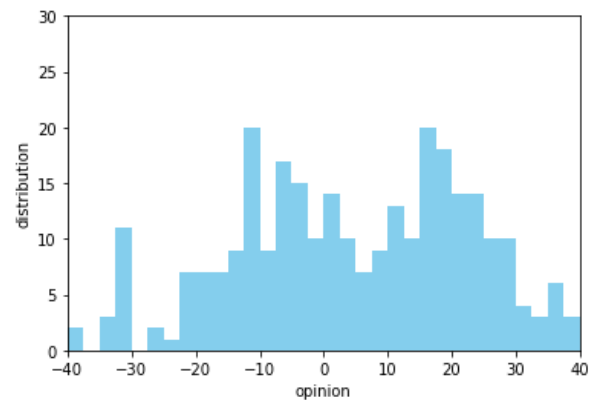
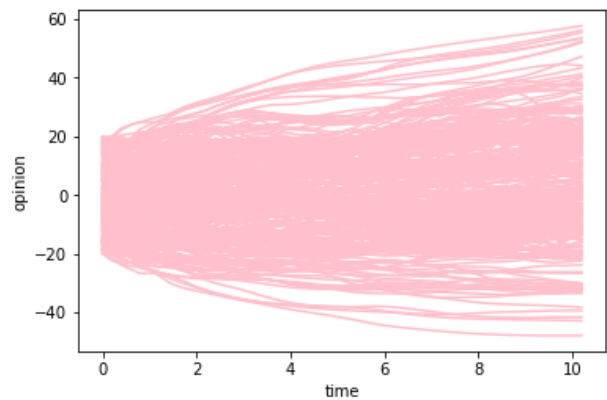


Positive ratio of D_{ij}

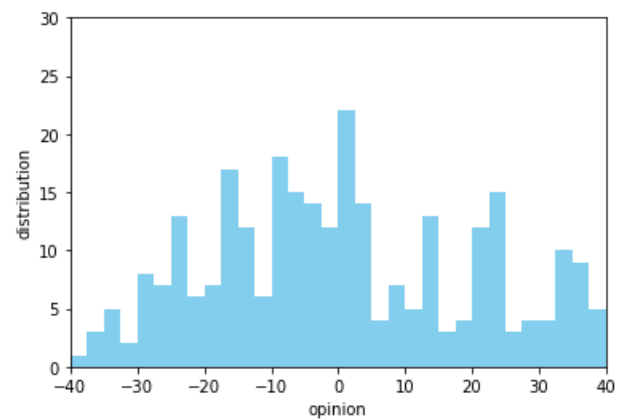
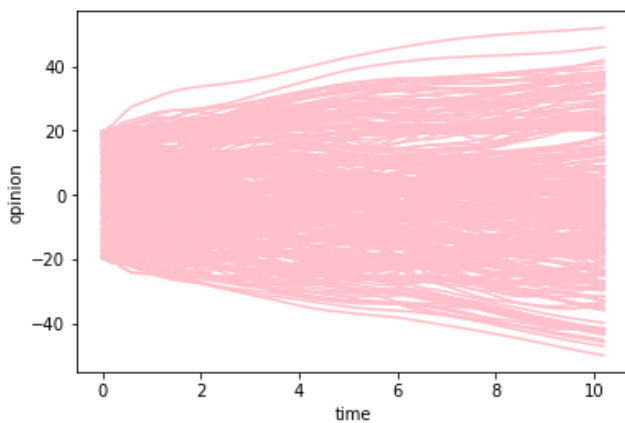


Random Network

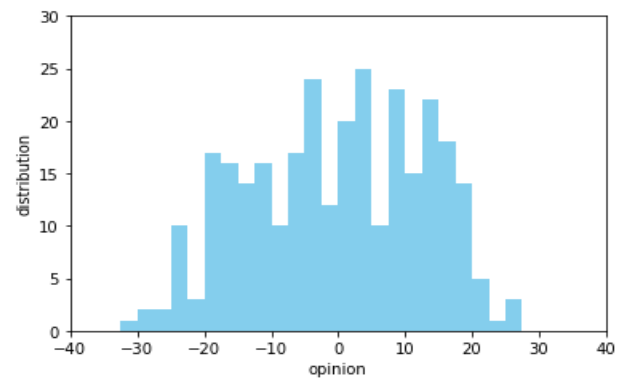
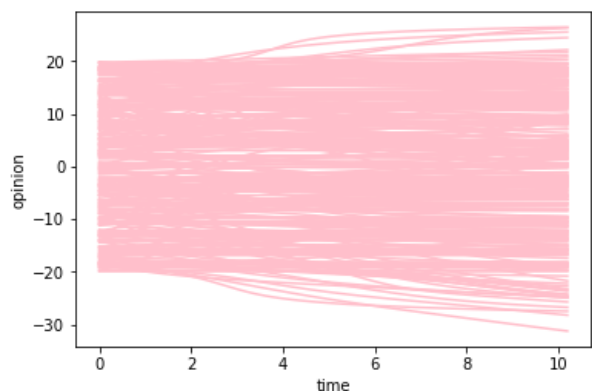
Connection 90%

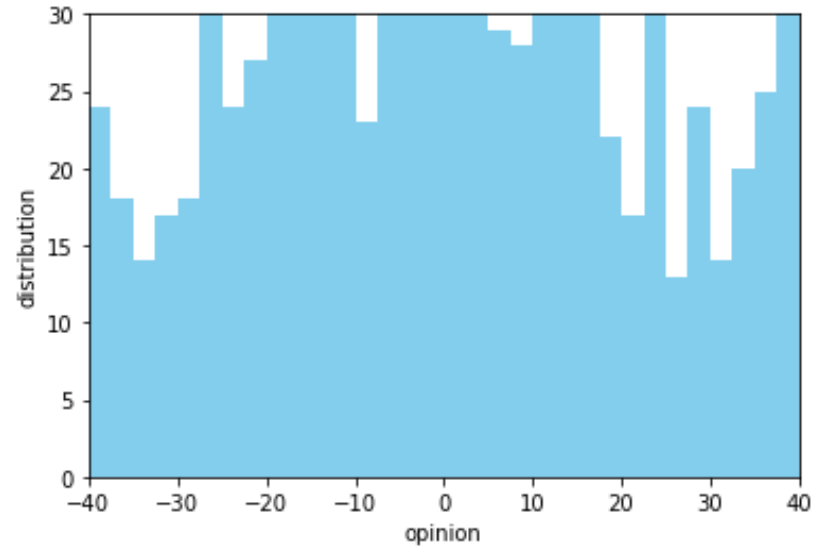
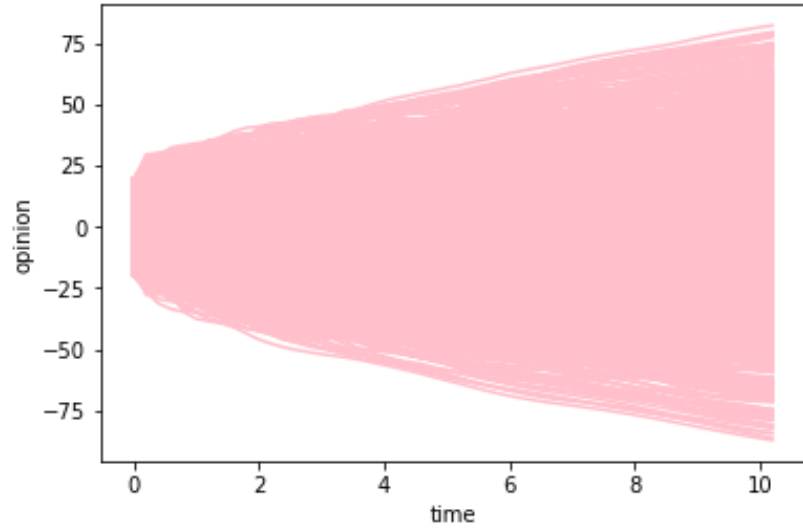


Connection 50%

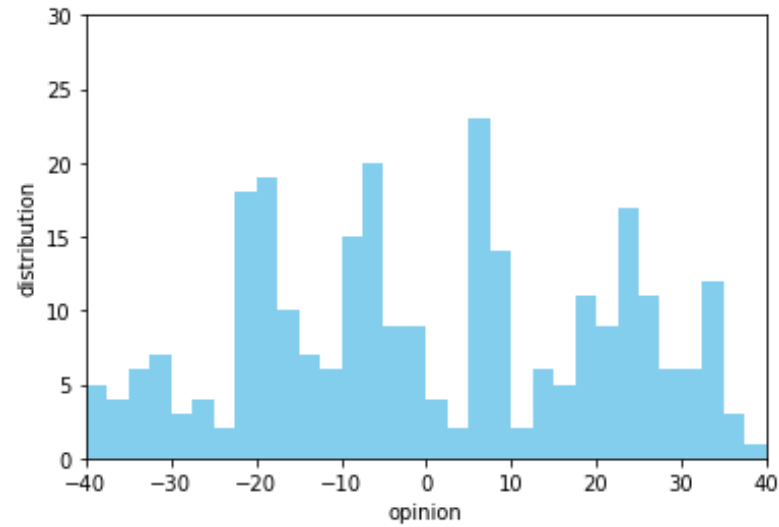
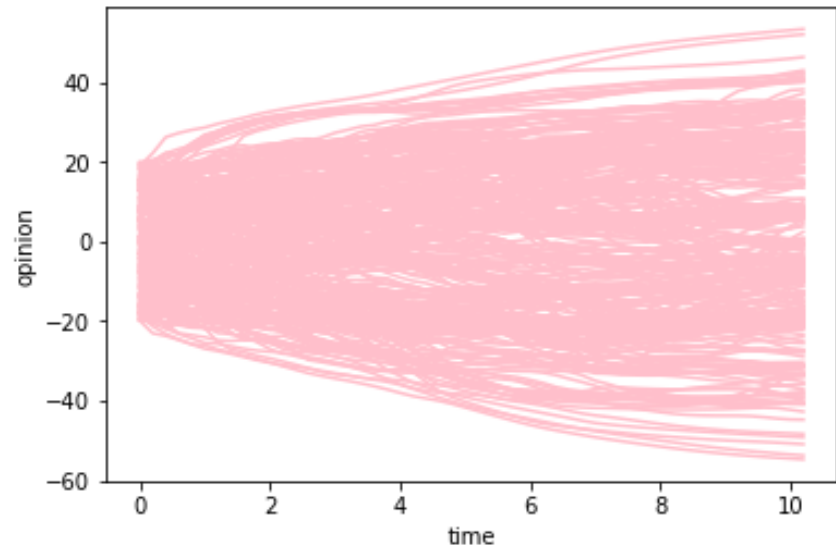


Connection 1 %





Random Network
N=1200 link 25%



Random Network
N=300 link 100%

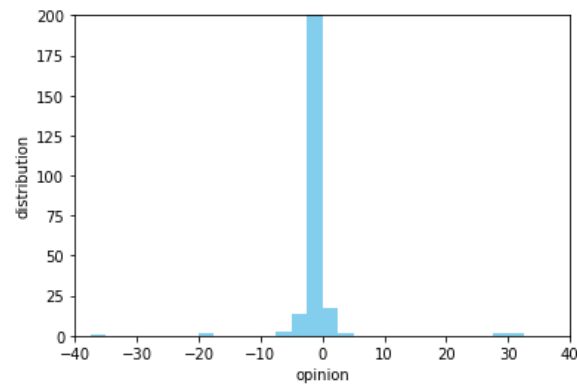
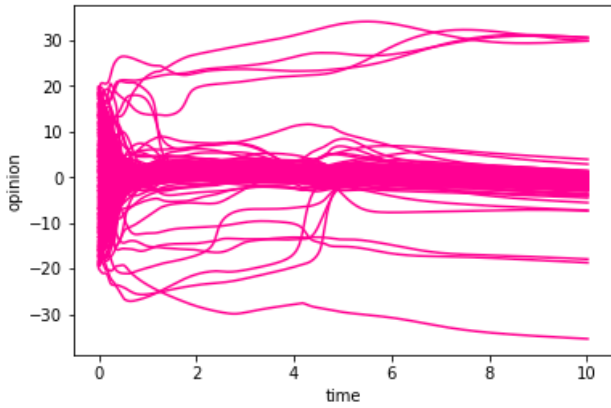
N=300

Initial opinion range -20~+20

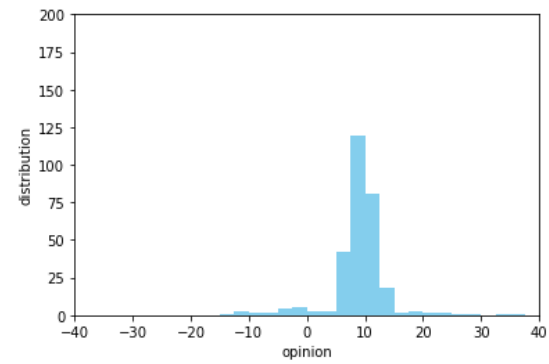
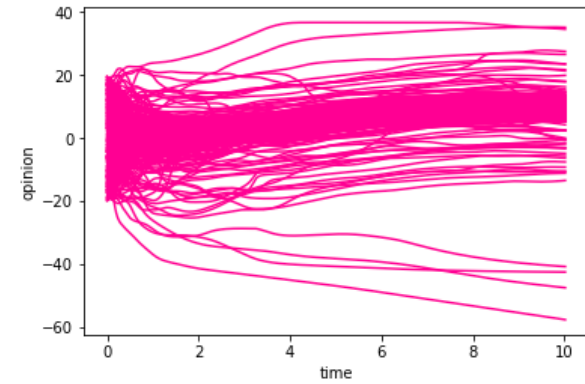
Dij is determined by using random number
(positive 60%)

Difference for link ratio of random network

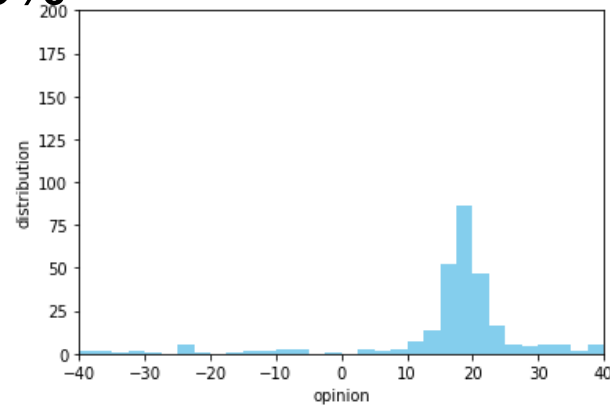
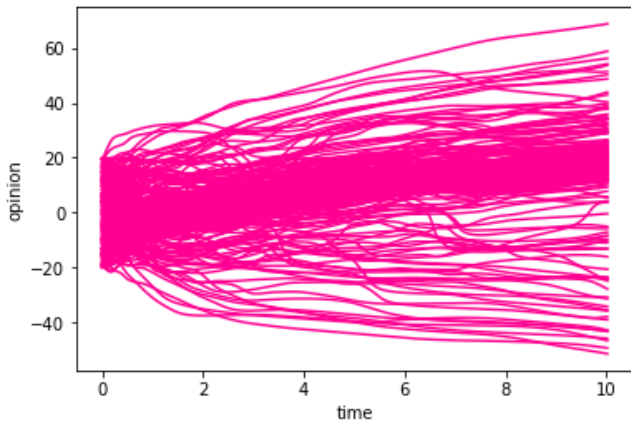
50%



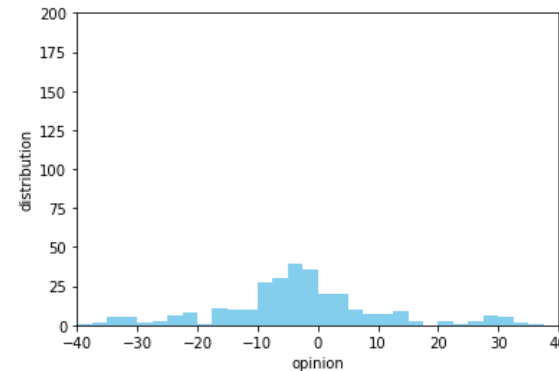
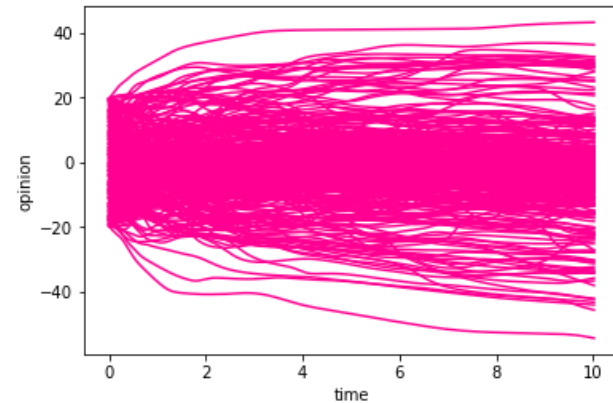
30%



20%



10%

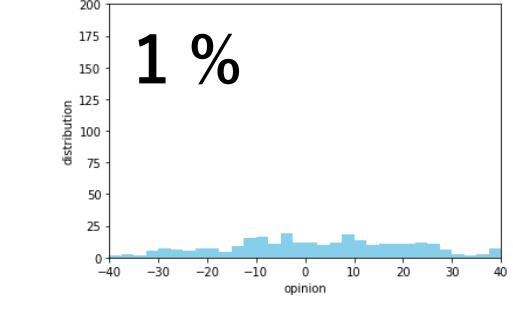
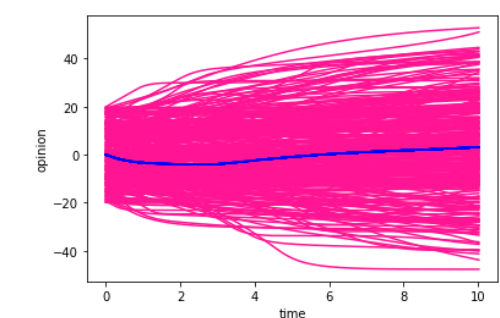
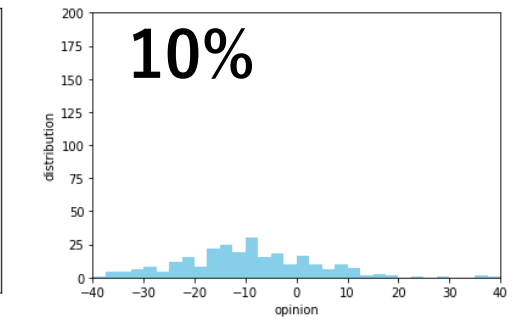
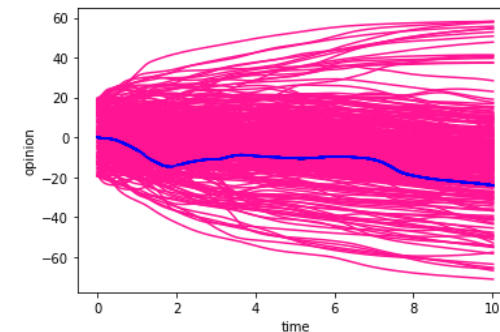
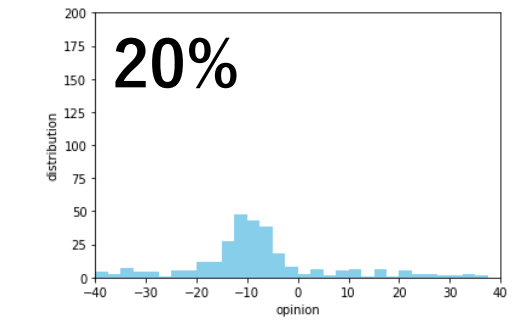
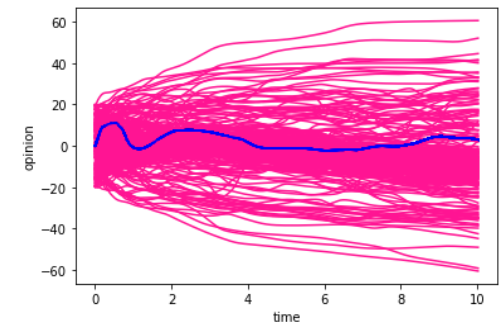
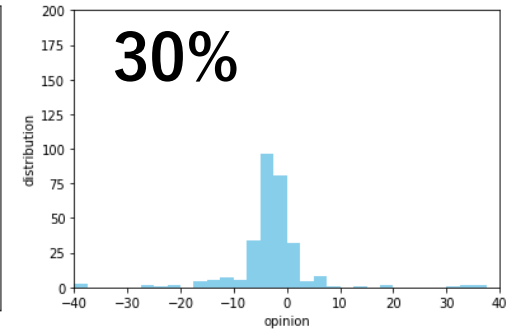
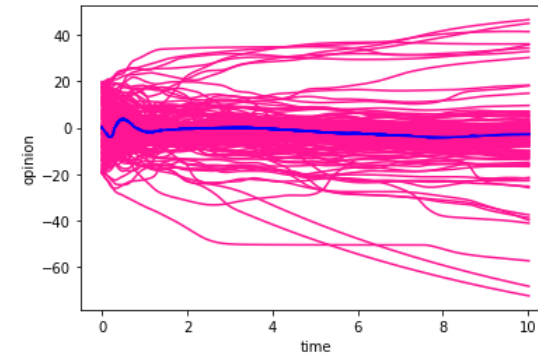
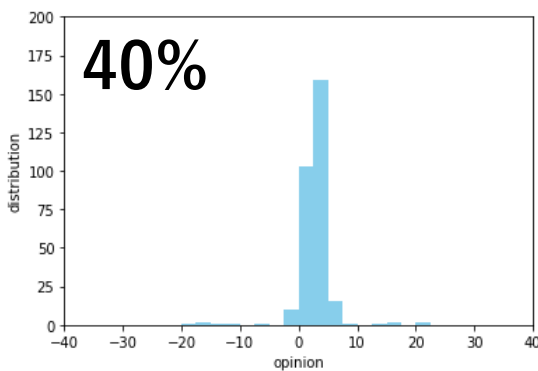
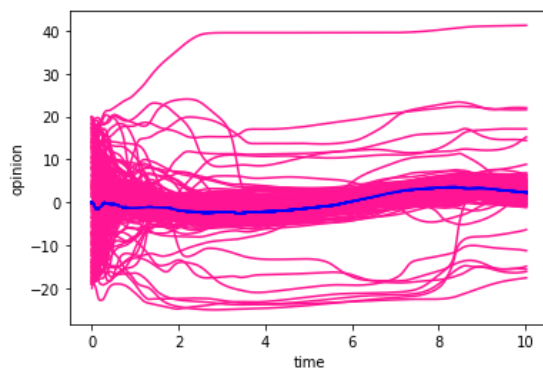
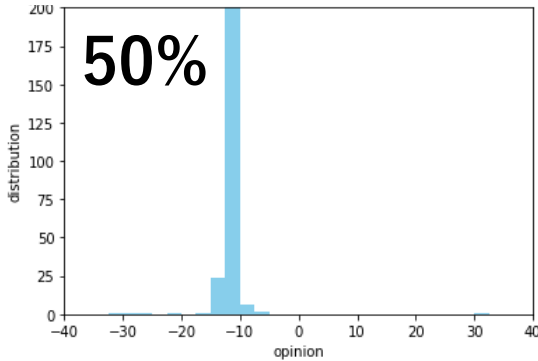
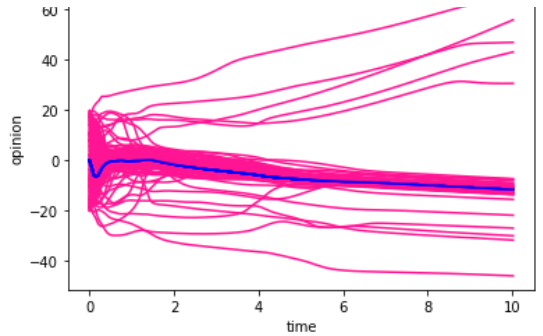
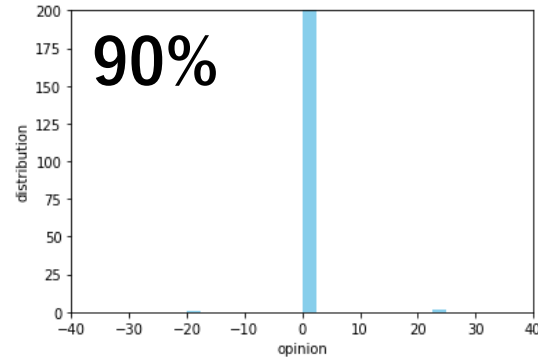
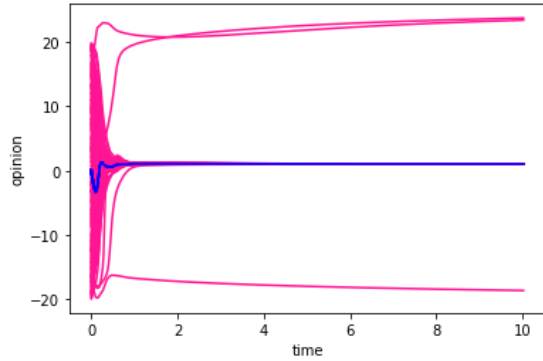


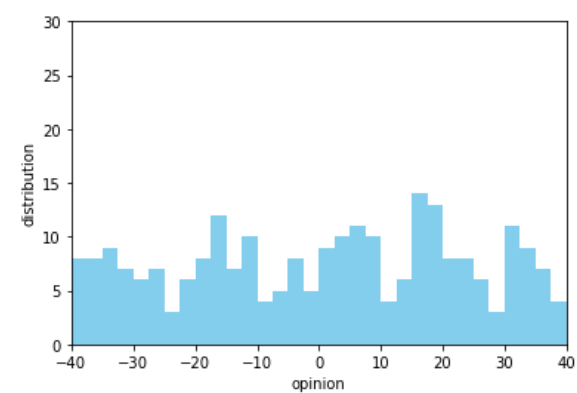
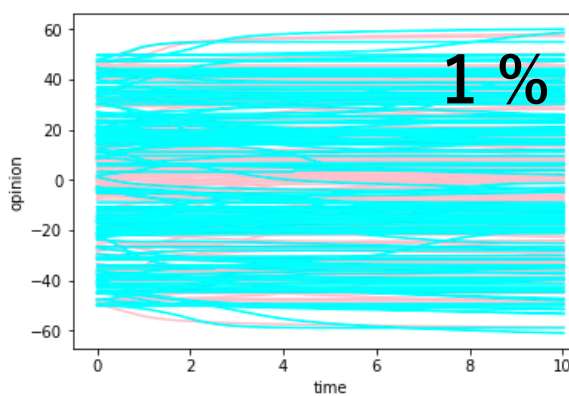
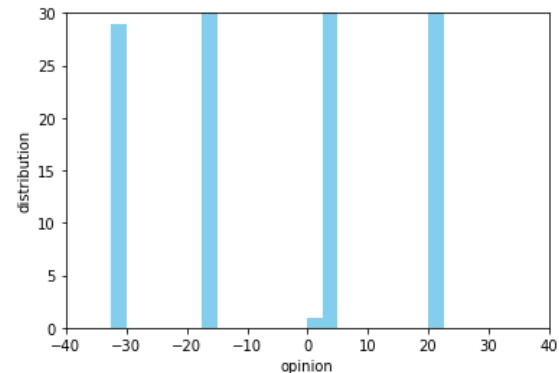
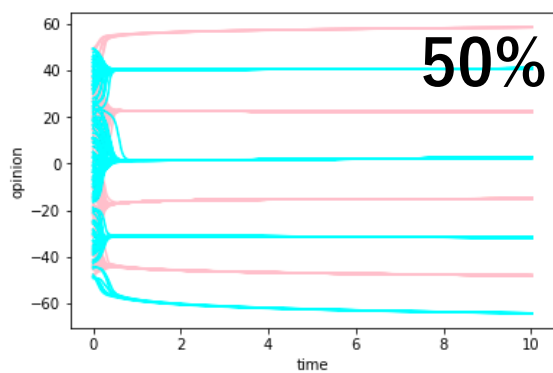
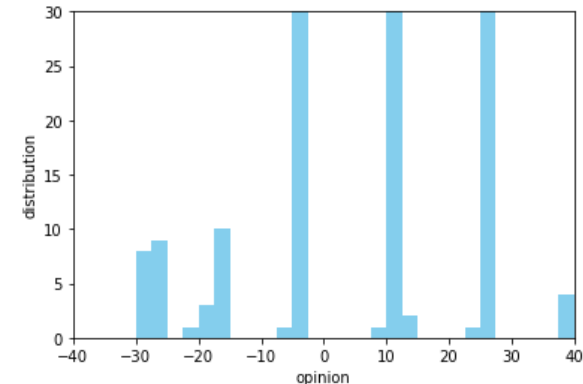
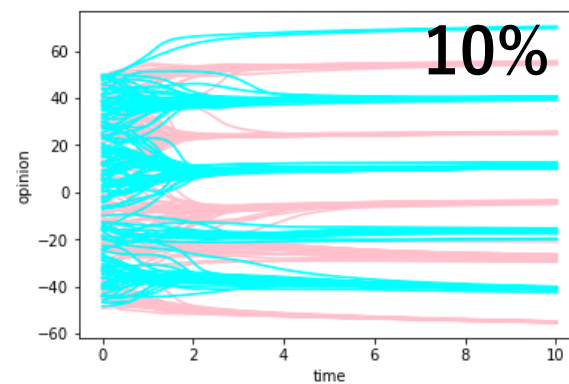
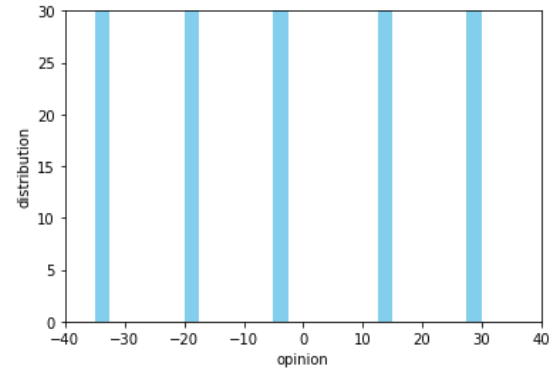
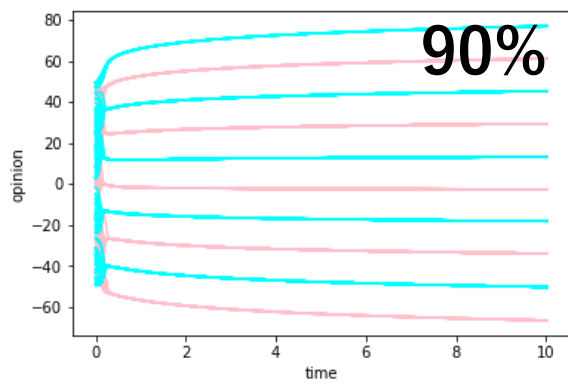
N=300

Initial opinion range -20~+20

Link ratio
1% - 90%

Dij is determined by using random number
(positive 60%)





T Tanaka, 2019, “Does the Internet cause Polarization? Panel survey in Japan”

Keio-IES Discussion Paper Series, No. 12129



Japanese version
Book, 2019

ネットは社会を
分断しない、

田中辰雄 浜屋 敏

集英社

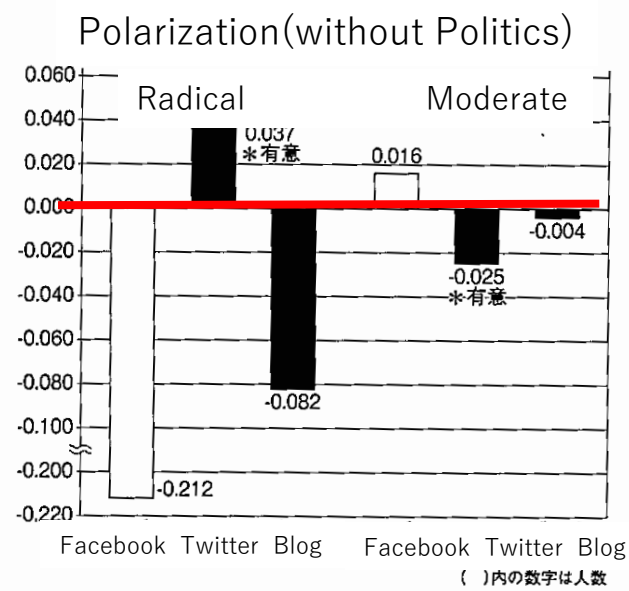
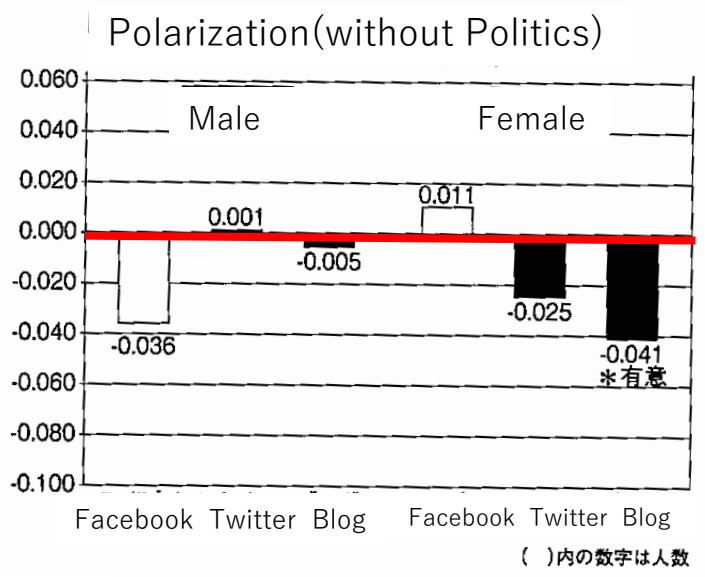
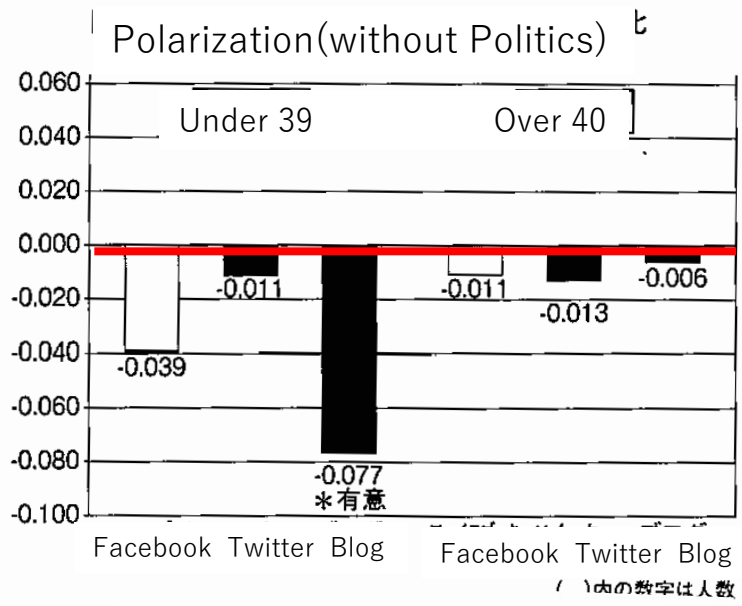
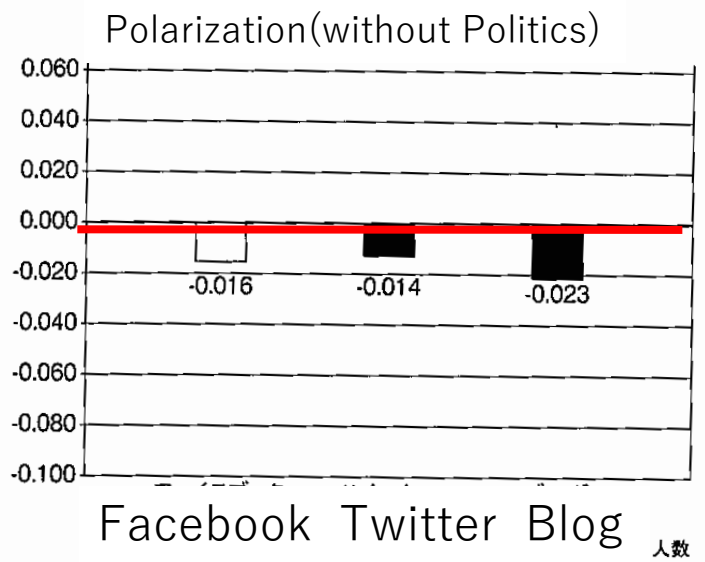
There is concern that the Internet causes ideological polarization through selective exposure and the echo chamber effect. **This paper examines the effect of social media on polarization by applying a difference-in-difference approach to panel data of 50 thousand respondents in Japan.** Japan is good case for this research because other factors affecting polarization like huge wealth gap and massive immigration are not serious issue, thus it offers quasi natural experimental situation to test the effect of the Internet. The results show that people who started using social media during the research period (targets) were no more polarized than people who did not (controls). There was a tendency for younger and politically moderate people to be less polarized. The only case in which the Internet increased polarization was for already radical people who started using Twitter. However, since radical people represent only 20% of the population and there was no effect for Facebook or blogs, the overall effect of the Internet was moderation, not polarization

T Tanaka, 2019, "Does the Internet cause Polarization? Panal survey in Japan"

Keio-IES Discussion Paper Series, No. 12129



Japanese version
Book, 2019



In general, people using internet is more moderate than people of non-internet.

This report agrees with our calculation that more link make society to be consensus formation.

Introducing the strength of the will

Introduce the strength of the will for each person.
If the strength of intention m is large, the trajectory of the opinion does not shake much

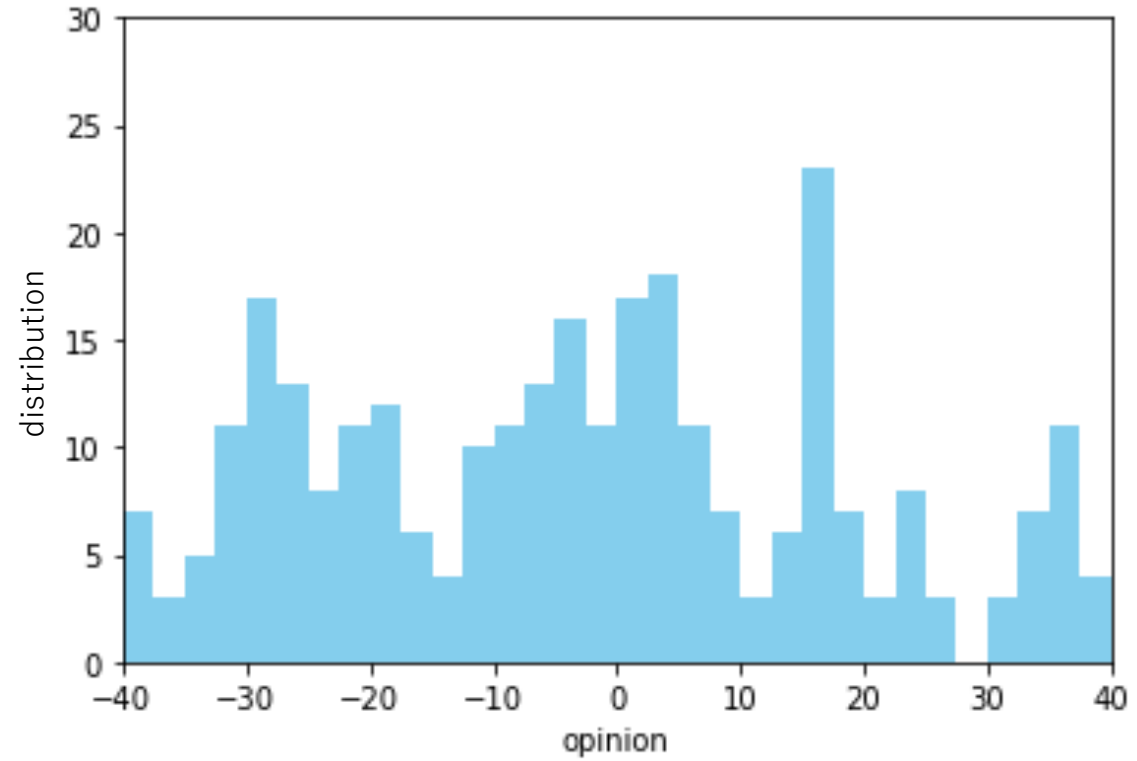
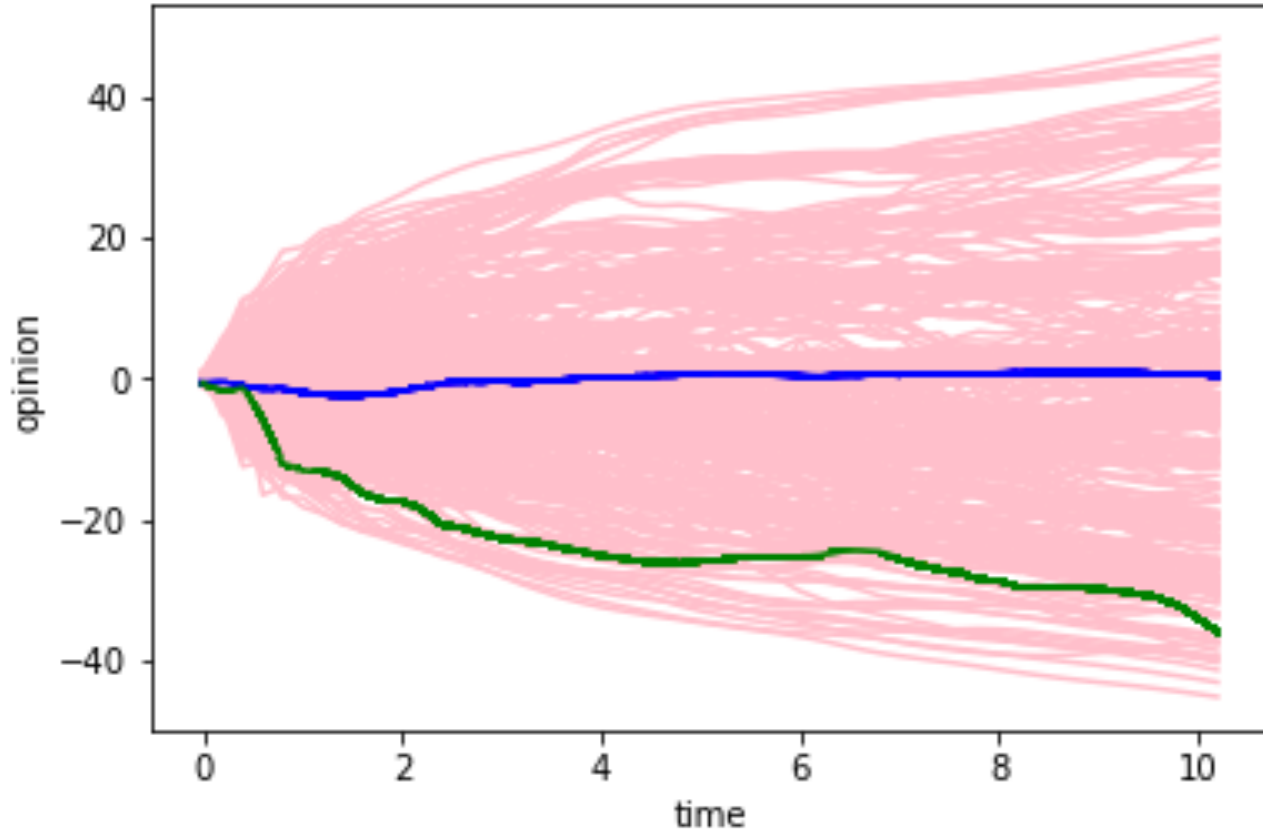
$$\underline{m} \frac{dI_i(t)}{dt} = C_i A(t) + \sum_{j=1}^N D_{ij} \Phi(I_j(t), I_i(t)) (I_j(t) - I_i(t))$$

Strength of the will

$$\Phi(I_i, I_j) = \frac{1}{1 + \exp(\beta |I_i - I_j| - b)}$$

The strength of the will is 10

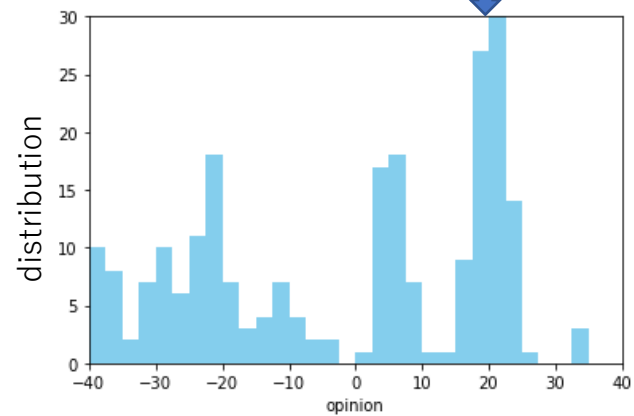
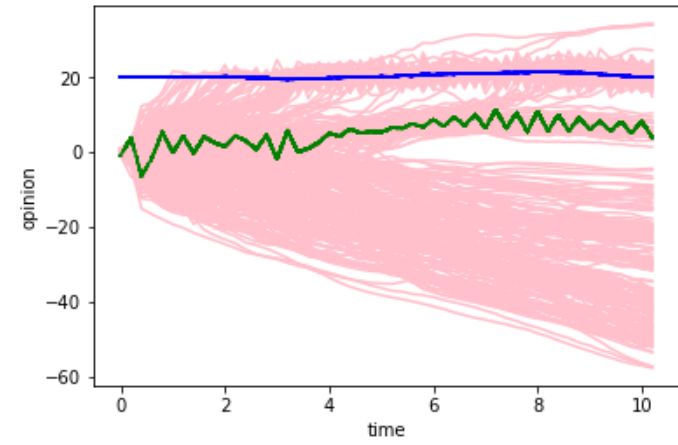
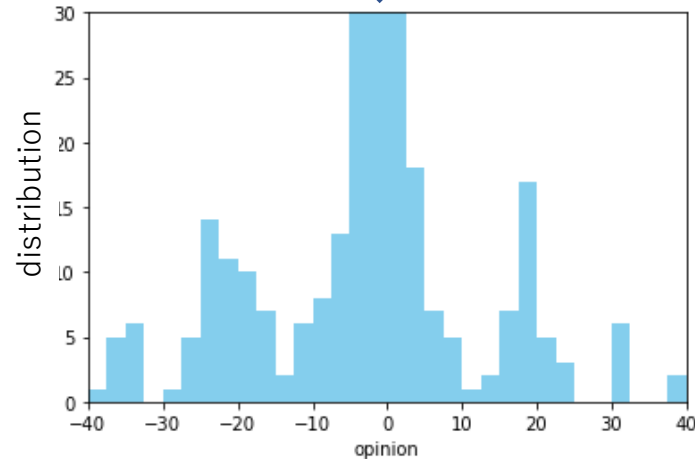
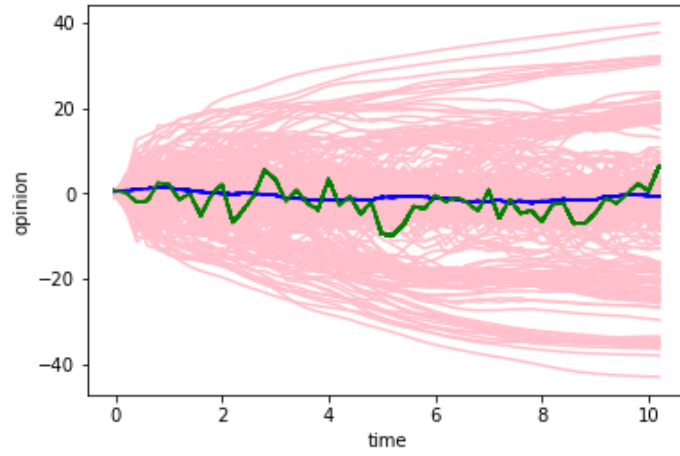
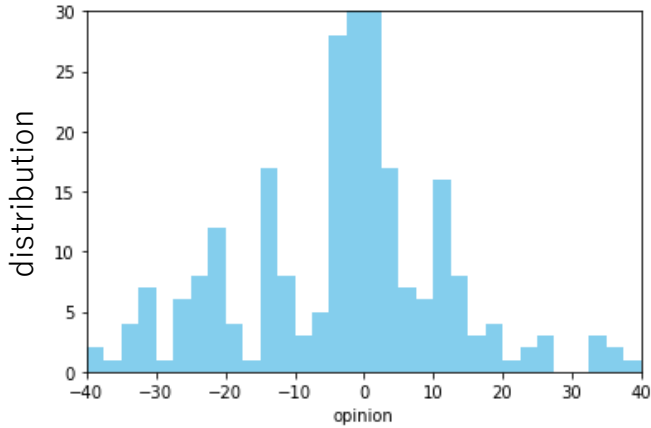
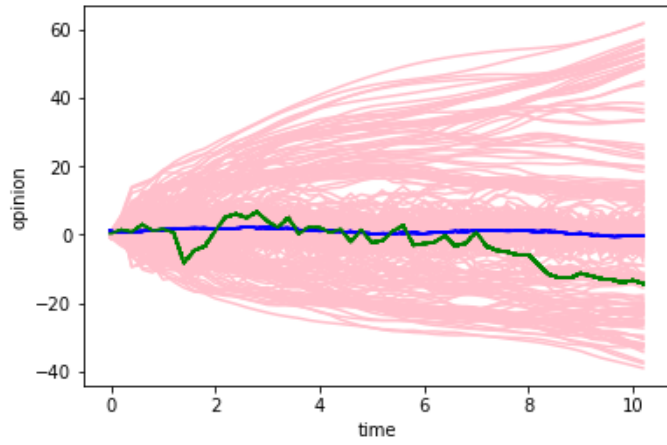
general	$m = 1$
green	$m = 1$
Blue	$m = 10$



Calculation

A person who has received
charismatic trust

N=300 One charismatic person whose will is 5.
Trust to charismatic person $D_{i_0}=5$



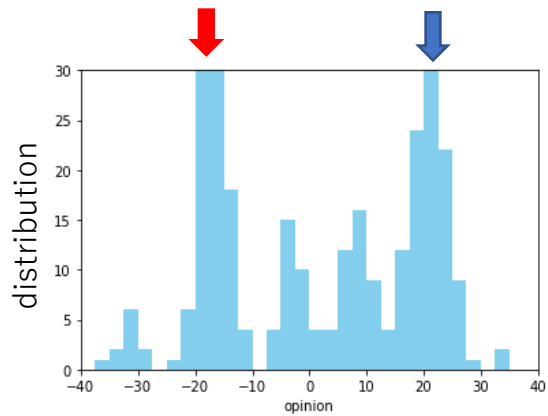
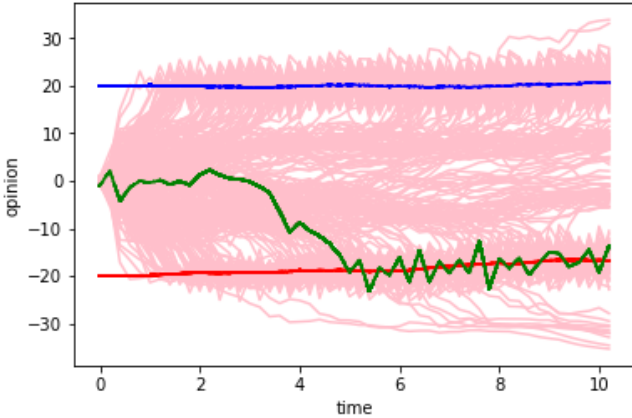
2 charismatic persons

The strength of the will for the 2 charismatic is 1 0

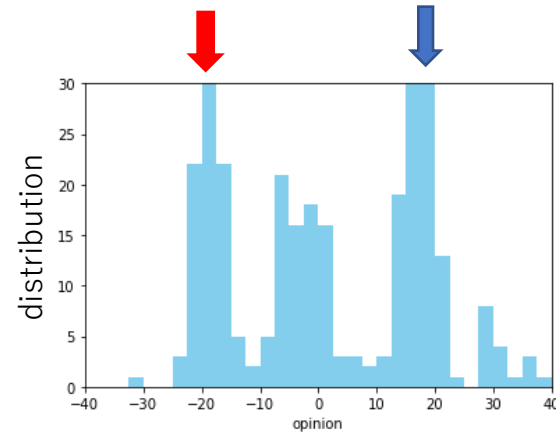
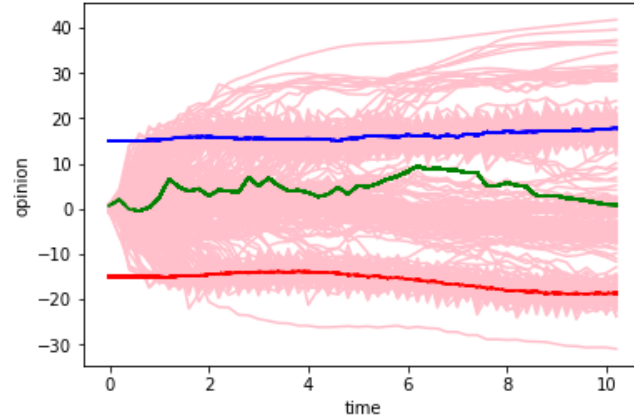
Trust received from ordinary people is 10

$$-1 < D_{ij} < 1$$

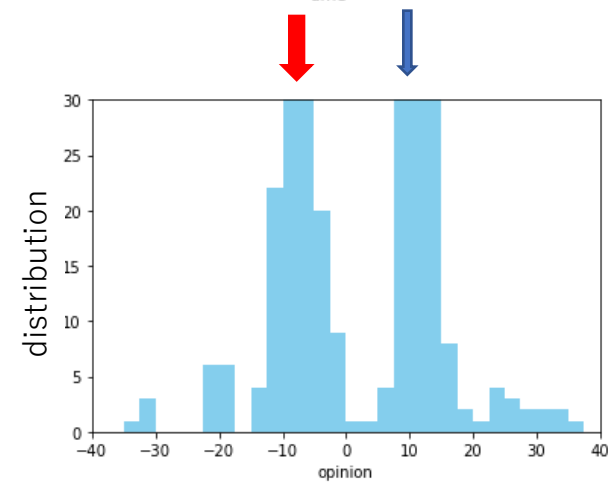
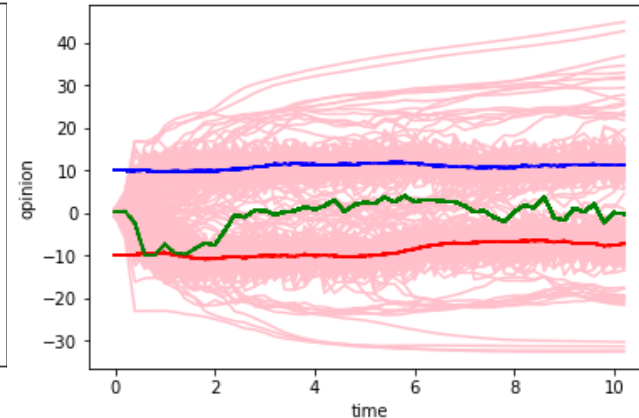
Initial opinion of
2 charismatic
+ 2 0 and - 2 0



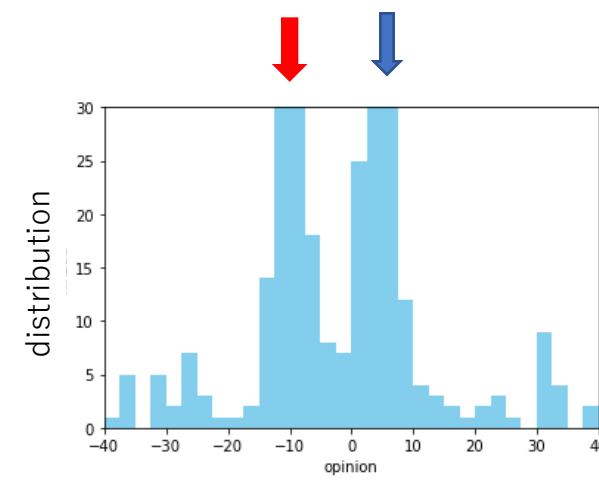
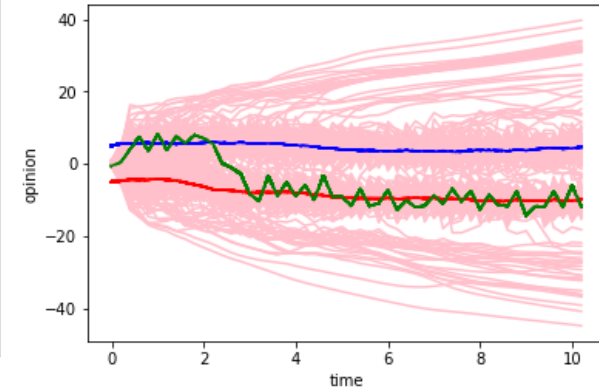
Initial opinion of
2 charismatic
+ 1 5 and - 1 5



Initial opinion of
2 charismatic
+ 1 0 and - 1 0

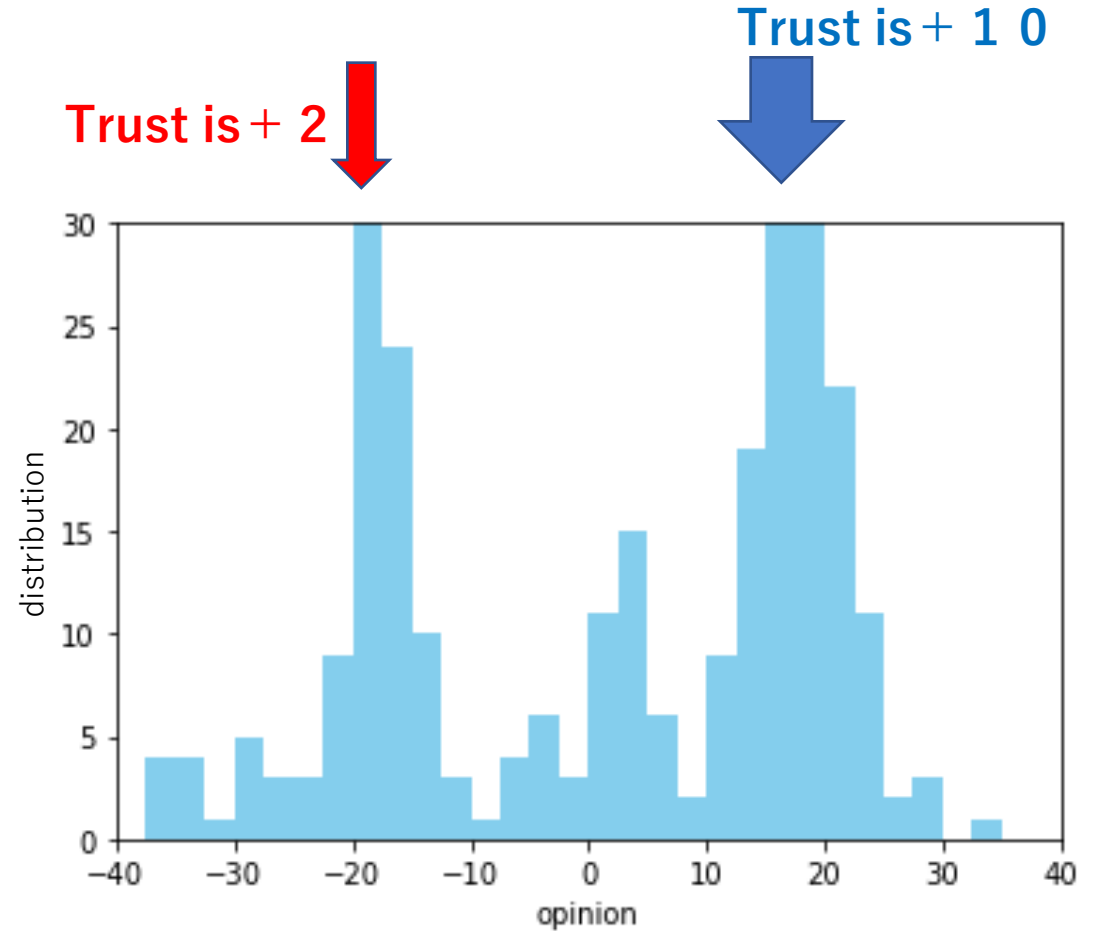
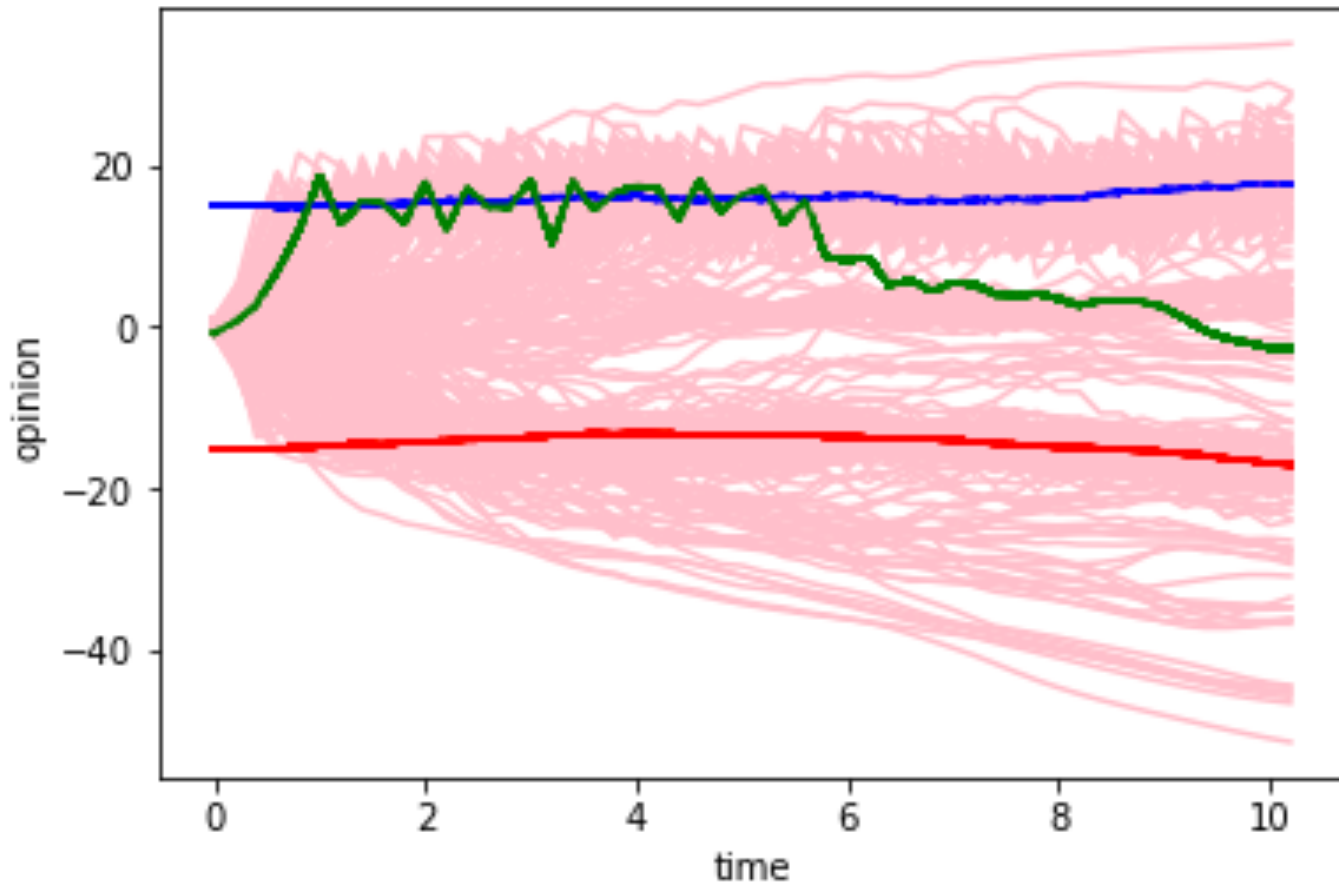


Initial opinion of
2 charismatic
+ 5 and - 5



Difference due to difference in degree of charisma

Charismatic person, blue receives trust + 1 0
red receives trust + 2

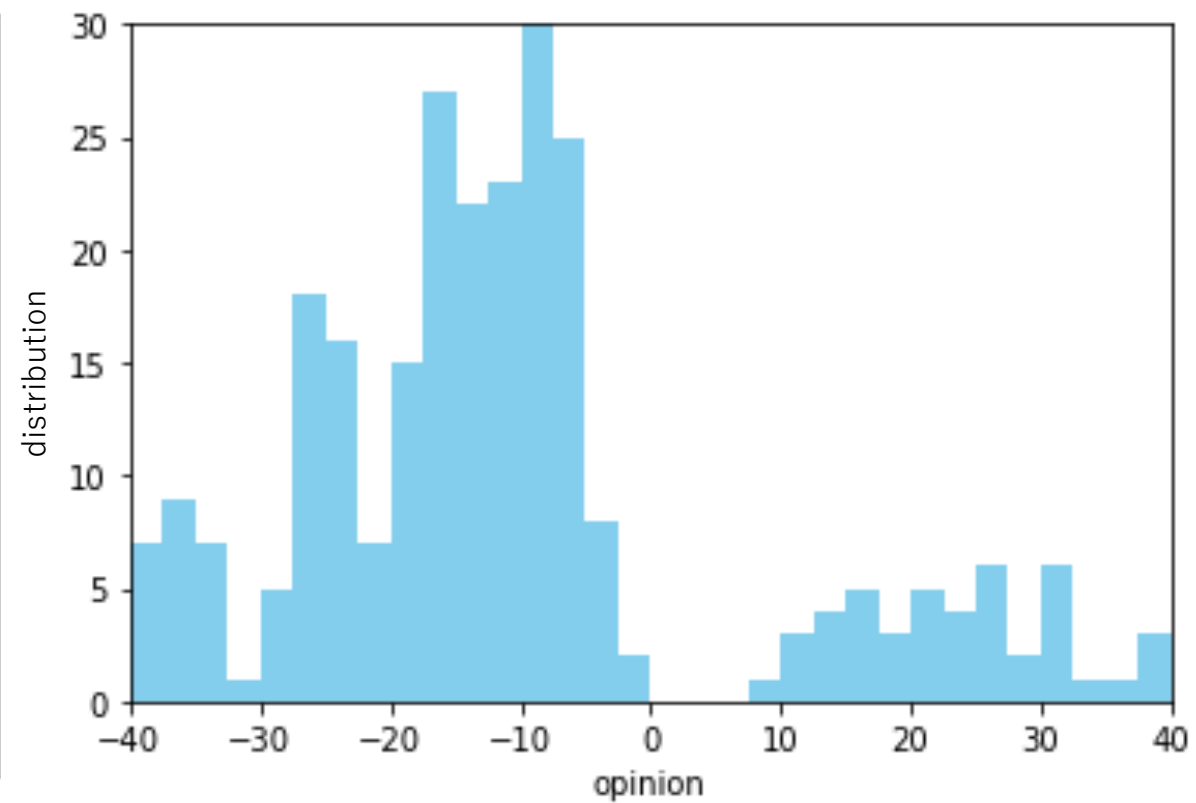
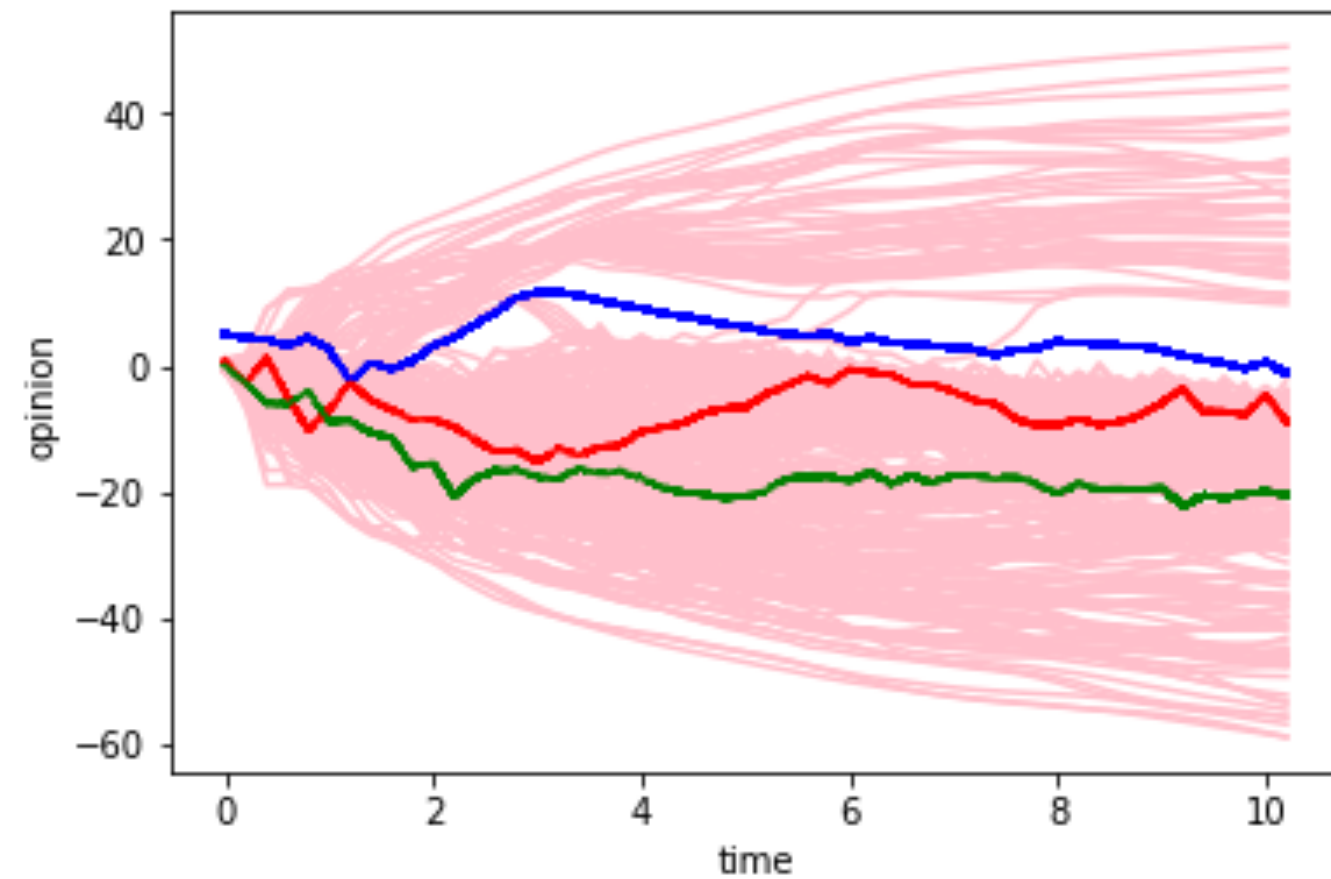


A case of
person who is not trusted
by all

Person who is not trusted by all.

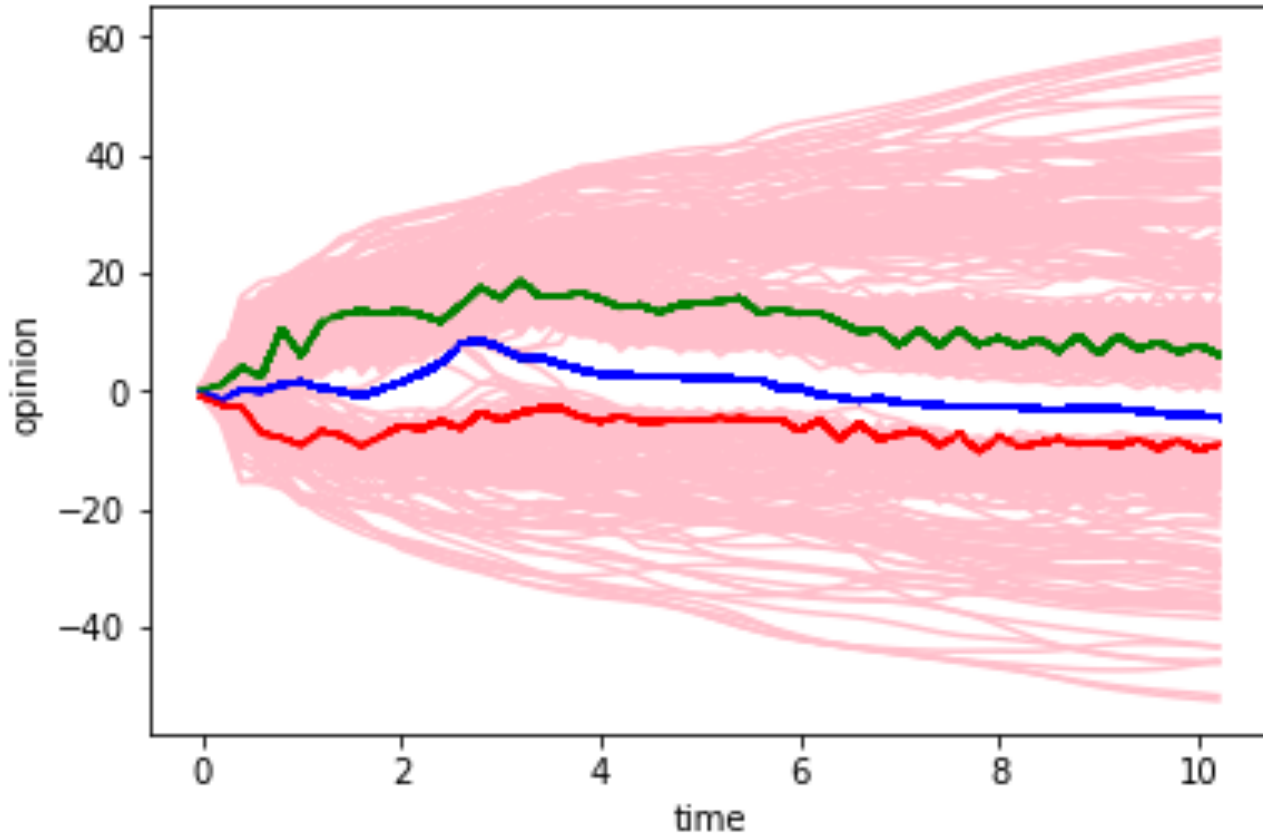
Distrust value by all - 5

Person who is not trusted by all : Blue line

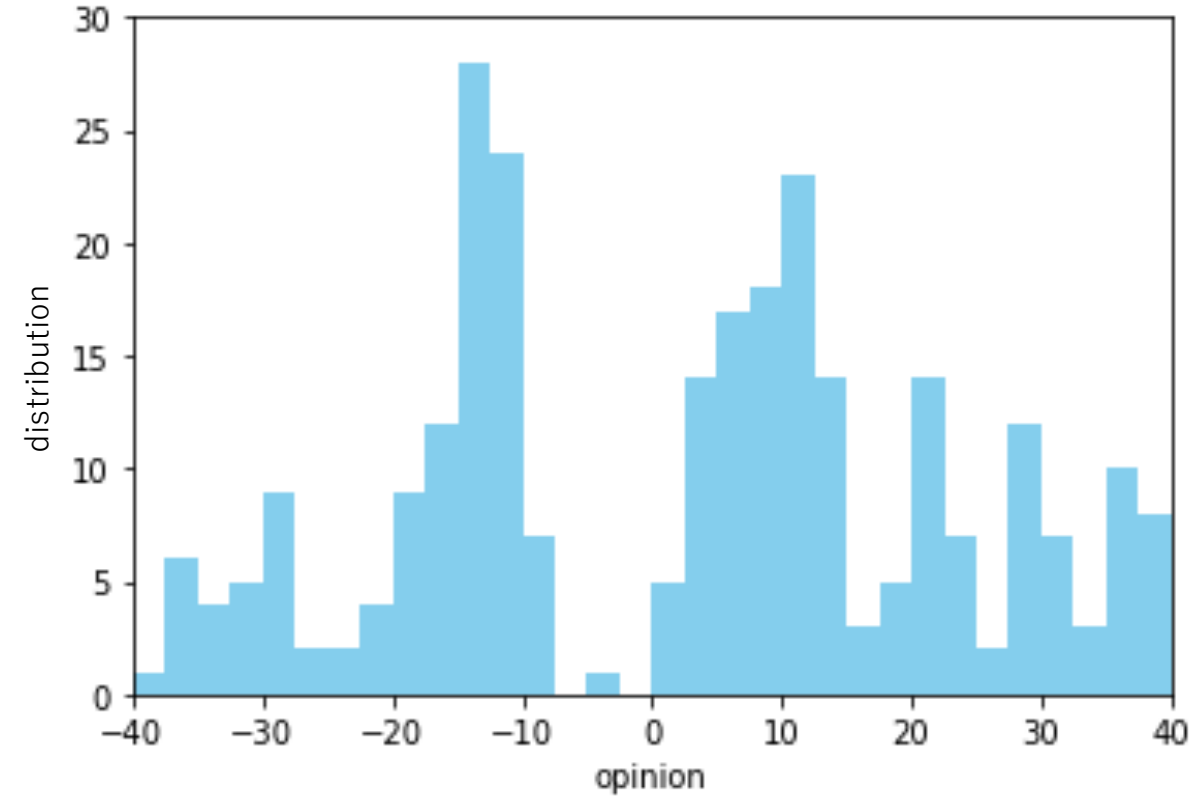


Person who is not trusted by all.

Distrust value by all - 5



Person who is not trusted by all : Blue line



When charisma person trust people of distrust from all

Distrust by all -5

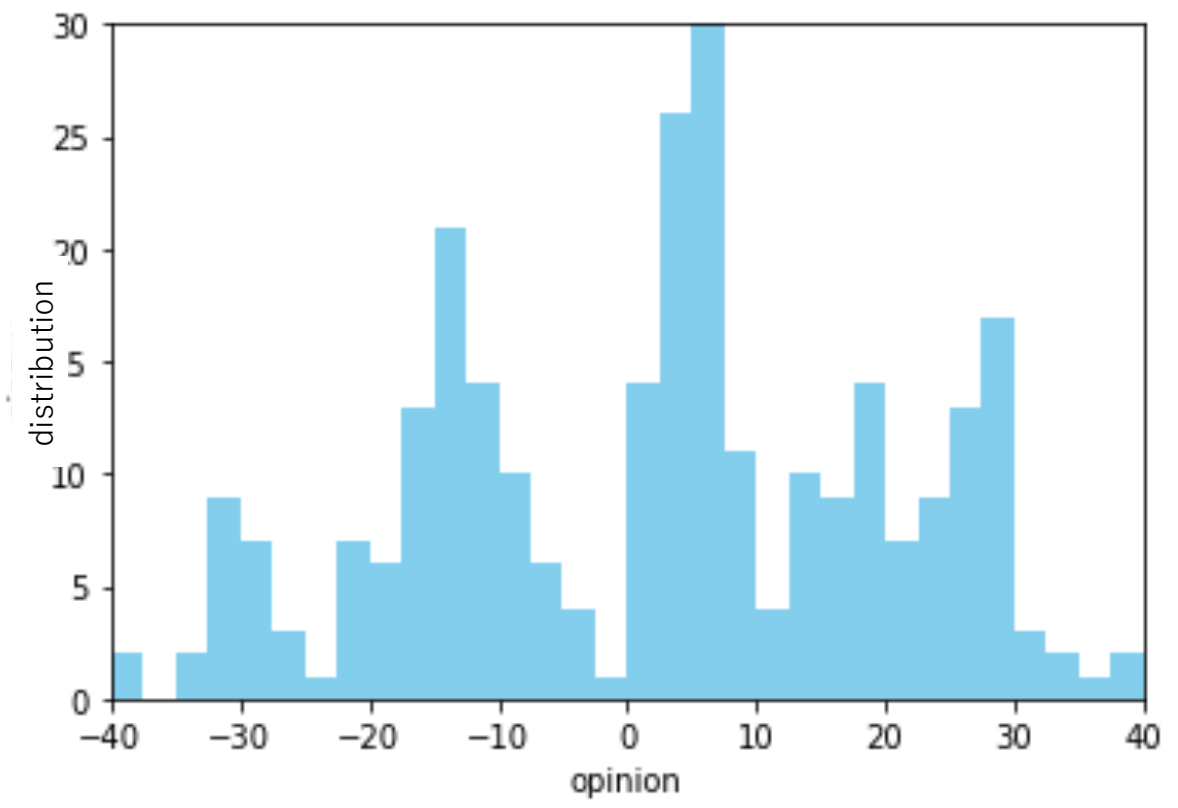
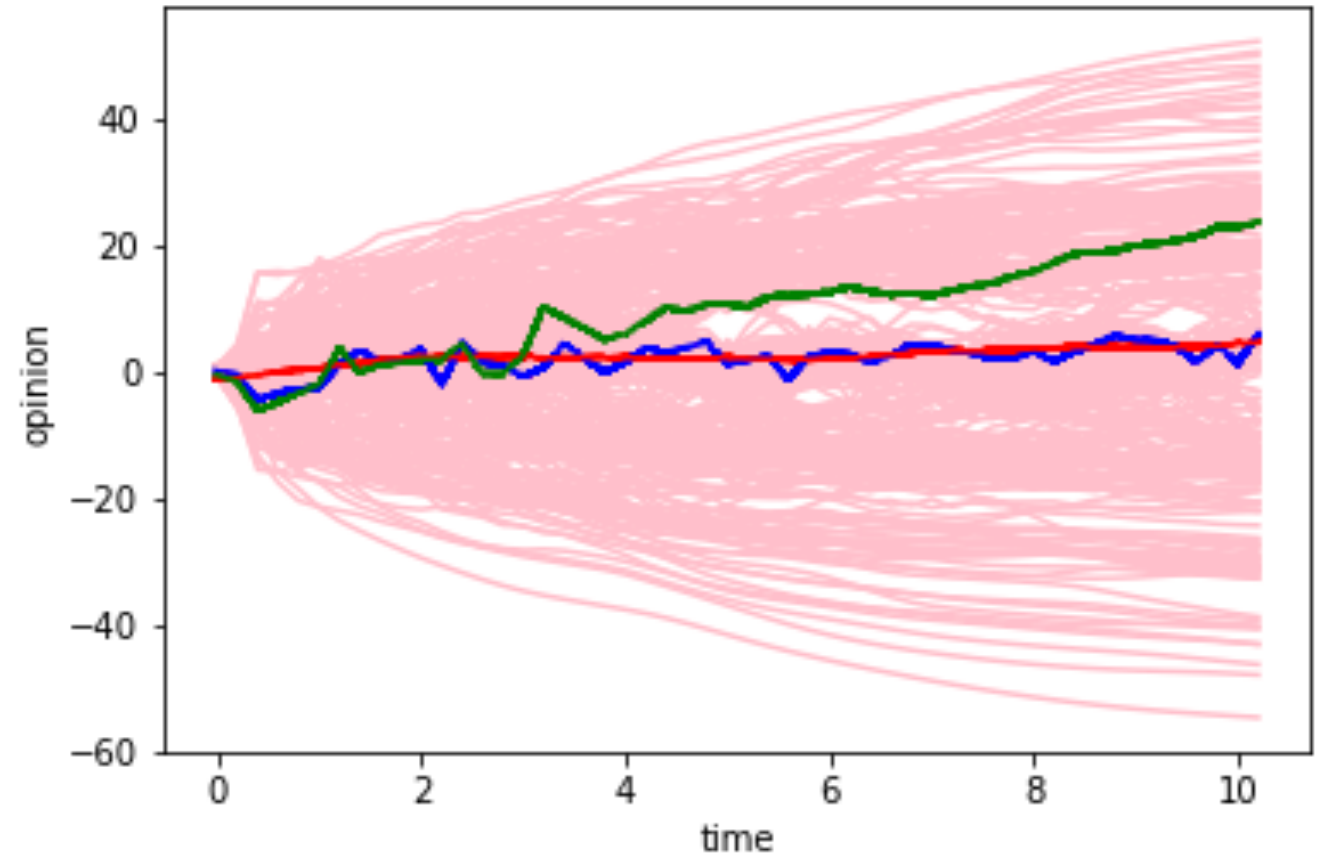
Person of distrust by all : blue line

Charismatic person (red line)

Trust by all +5

Strength of the will 1 0

Charismatic person trust the person of distrust by all +1



It is important for charismatic people to reach out to socially isolated people



Conclusion

- Constructed an opinion dynamics theory that **incorporates both interpersonal relationships of trust and distrust**
- **Mass media effect** is included
- Calculation results of two people and three people seems plausible
- While trust is the only thing in society, the opinion of the society reaches consensus building, but when there is a distrust relationship, it **does not reach consensus building**
- **Many link** between people make people to be consensus formation under random network.
- We discuss **charismatic person**
- **Social isolation** can be avoided by charismatic person