CHARACTERIZING THE DYNAMICS OF FINANCIAL NETWORKS

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Introduction of myself

**Background:** Macroeconomics, Monetary policy
**Recent research interest:** Complex networks, financial systemic risk, social networks

**Collaborators:**

**Physics:**
N. Masuda (NYS Buffalo)
T. Takaguchi (LINE Corp.)
A. Barrat (CNRS)

**Applied Math:**
C.D. Brummitt
T. Yaguchi (Kobe U, PRESTO)

**CS:**
E. Ferrara (USC)
A. Sapienza (USC)

**Econ:**
Muto (BOJ)
T. Sugo (BOJ) etc.
1. Data analysis of interbank networks
Temporality of interbank networks

• Overnight transactions form daily networks
• Network structure changes day to day
Conventional approach of interbank network analysis

- Overnight lending-borrowing, but aggregate networks (weekly, monthly, etc.)

- Why aggregated?
Are networks random at the daily scale?

“We show that the networks appear to be random at the daily level, but contain significant non-random structure for longer aggregation periods.”


“For the e-mid, we initially looked at daily snapshot of loans among banks. However, we found that the high volatility of the links at this time scale prevented a robust estimation of the network properties.”

Objective of this work

• Characterize dynamical patterns at the daily scale!
## Data: e-MID (Italian interbank market)

### Sample

<table>
<thead>
<tr>
<th>Duration</th>
<th>Time</th>
<th>Rate</th>
<th>Amount</th>
<th>Quoter</th>
<th>Agressor</th>
<th>Verb</th>
</tr>
</thead>
<tbody>
<tr>
<td>ON</td>
<td>2000-09-04 09:06:00</td>
<td>4.62</td>
<td>5</td>
<td>IT0159</td>
<td>IT0094</td>
<td>Buy</td>
</tr>
</tbody>
</table>

### Details

- **Duration**: ON (overnight), ONL (overnight large)
- **Data period**: Sep, 2000 - Dec, 2015 (3922 business days)
- **# banks**: 308 (in total)
- **# transactions**: 1,187,415
Size of daily networks

- Network size varies daily
- Non-stationary (downward trends)

Max: $N = 162$
Min: $N = 13$
Daily dynamical patterns
1. Size and the # of edges

\[ M \propto N^{1.5} \]
\[ \langle k \rangle \propto \sqrt{N} \]

"Superlinear scaling"

\[ \log M = -1.27 + 1.49 \log N \]
\[ R^2 = 0.973 \]
2. Duration and interval time (days)
Duration and interval time (for pairs)

Duration, 2000–2006

Data

\[ \gamma = 2.56 \]

Duration, 2007–2009

Data

\[ \gamma = 2.63 \]

Duration, 2010–2015

Data

\[ \gamma = 2.90 \]

Interval, 2000–2006

\[ \tau, \Delta \tau \]

days

Interval, 2007–2009

\[ \tau, \Delta \tau \]

days

Interval, 2010–2015

\[ \tau, \Delta \tau \]

days
Social systems
Similarity to social networks:

Superlinear scaling

1. # of mobile phone users vs. # of pairs
2. Population vs. time of calls


Pan et al, Nature Communications, 2013
Similarity to social networks:

Duration and interval time

1. Face-to-face interaction
2. Cattle trade movements between livestock premises
Model
Model: A dynamic Fitness Model

Step 0. There are $N_p$ many isolated banks

Bank $i$ has activity $a_i \in [0, 1]$

Step 1. Edge creation with prob. $p_{ij} = (a_i a_j)^\alpha$

Step 2. Update activity

with prob. $h \rightarrow$ redraw $a_i$ from $\mathcal{U}[0,1]$

with prob. $1-h \rightarrow$ update as $a_i = |\cos \theta_i |$

$\theta_i :$ random walk

Go to Step 0.
Synthetic networks

- $N_p$ controls the average size of networks

Visualized by graph-tool
Result: Emergence of superlinear scaling

\[ \log M = -1.27 + 1.49 \log N \]

empirical regression line
Result: duration and interval (pairwise)

For a given $N_p$, a sequence of “daily” networks is generated
Estimating the potential network size $N_p$

Simulated histogram: $f(N, M | N_p)$

ML estimator:

$$N_{p,ML}(t) = \arg\max_{N_p} f(N_t, M_t | N_p)$$
Result: Estimation of market size

Daily estimates of $N_p$

![Graph showing daily estimates of $N_p$ from 2000 to 2015 with data and model comparisons.](image)

**Data**

$log(M) = -1.27 + 1.49 \log(N)$

$R^2 = 0.973$

**Model**

$log(M) = -1.27 + 1.49 \log(N)$
Conclusion

Daily interbank networks have explicit patterns
- superlinear relation, power-law duration distribution

Banks are social creatures
- Banks trade in the same way that people find conversation partners

Fitness model as a generative model of financial networks
- can explain many properties simultaneously
- contribute to systemic risk studies (Battiston et al. 2016, Science)
2. Extracting significant ties in temporal networks
Research question

- Long duration for trades with particular pairs...
- Cannot happen if there are no preferences.
Research question

- Long duration for trades with particular pairs...
- Cannot happen if there are no preferences.

- How do banks choose trading partners?
- Could it be explained by random chance?
Research question

- Long duration for trades with particular pairs...
- Cannot happen if there are no preferences.

• How do banks choose trading partners?

• Could it be explained by random chance?

If not, “relationship lending!”
Relationship lending?

Commonly used measures

- # transactions between two banks
- Share of lending to a particular bank

...may be disturbed by

bank size, # total transactions, and market activity.
The aim of this work

- Identify relationship lending in a statistically rigorous manner
Backboning

Extracting essential edges, i.e., “significant ties.”
— The backbone of networks
Methods
Null model

“Fitness model”

- Daily matching probability

\[ u(a_i, a_j) = a_i a_j \]

\[ u_{i\rightarrow j}(a_i^{\text{out}}, a_j^{\text{in}}) = a_i^{\text{out}} a_j^{\text{in}} \]

\[ u(a_i, a_j, t) = a_i(t) a_j(t) \]

Undirected

Directed

Time-varying

“activity” \( \propto \) # trades
Random matching (= Null hypothesis): 

If there is a strong partnership:
Identification of significant ties

- **Edge-based test**

Under the null, $m_{ij}$ follows a binomial distribution:

$$m_{ij} \sim B(\tau, u(a^*_i, a^*_j))$$

Banks $i$ and $j$ are connected by a *significant tie*. 
Estimation of activity

Under random matching, # bilateral transactions should follow a binomial distribution:

\[ p(\{m_{ij}\} | \tilde{a}) = \prod_{i,j: i \neq j} \left( \begin{array}{c} \tau \\ m_{ij} \end{array} \right) u(a_i, a_j)^{m_{ij}} (1 - u(a_i, a_j))^{\tau - m_{ij}}, \]
Estimation of activity

Under random matching, # bilateral transactions should follow a binomial distribution:

\[
p(\{m_{ij}\}|\tilde{a}) = \prod_{i,j: i\neq j} \left( \frac{\tau}{m_{ij}} \right) u(a_i, a_j)^{m_{ij}} (1 - u(a_i, a_j))^{\tau - m_{ij}},
\]

ML estimator of activity:

\[
F_i(\tilde{a}^*) \equiv \sum_{j: j \neq i} \frac{m_{ij} - \tau(a_i^* a_j^*)}{1 - (a_i^* a_j^*)} = 0, \ \forall \ i = 1, \ldots, N,
\]
Node-based test

Under the null, aggregate degree $K_i$ follows a Poisson binomial distribution, approximated as:

$$f(K_i | a^*) \approx \frac{\lambda_i^{K_i} e^{\lambda_i}}{K_i!}$$

where $\lambda_i \equiv \sum_{j:j \neq i} [1 - (1 - u(a_i, a_j))^T]$}

$K_i < K_i^C$ indicates bank $i$ is *relationship-dependent*. 
Results
Tests on synthetic temporal networks

- Introduce “relationship lending”

1. Create random temporal networks
2. Assign a fraction of pairs as relationship pairs
3. Decreasing hazard prob for terminating a relationship:

\[ p_{ij}^{\text{norel}}(t) = \frac{b_0}{b_1 + b_2 D_{ij}(t - 1)}, \]
Tests on synthetic temporal networks

True fraction = 0.2

(a) $\tau = 10$, $b_1 = 1$, $b_2 = 0$

- Bonferroni, $\alpha = 0.01$
- $\alpha = 0.001$
- $\alpha = 0.01$

(b) $\tau = 10$, $b_1 = 1$, $b_2 = 5$

(c) $\tau = 10$, $b_1 = 5$, $b_2 = 5$

(d) $\tau = 20$, $b_1 = 5$, $b_2 = 5$
Model fit

# trades: real $\approx$ model
# unique partners: real $<$ model

$\rightarrow$ Relationship lending?
Identification of significant ties

- Undirected edge

a. Edge-based test

b. Node-based test

c. Fraction of significant ties

d. Fraction of rel.-dependent banks

years

Bonferroni, $\alpha = 0.01$

$\alpha = 0.001$

$\alpha = 0.01$
Impacts on trade conditions

Difference in interest rates

Difference in trade amount

(relationship - non-relationship)
Application to face-to-face networks

Nodes: High school students
Edges: Contacts
Application to face-to-face networks

Nodes: High school students
Edges: Contacts
Application to face-to-face networks

Nodes: High school students
Edges: Contacts

School class
Conclusion

1. Significant ties and relationship-dependent banks are identified in a statistically rigorous manner.

2. Fraction of significant ties increased during the GFC.

3. The filtering method is also applicable to social networks.
Call for papers for the *Japanese Economic Review* special issue: Economics and Complex Networks

Guest editors: Teruyoshi Kobayashi (Kobe University) and Naoki Masuda (University of Bristol)

Since the late 1990s, network analysis has been playing an increasingly important role in various fields of social sciences, natural sciences, engineering, and industry among others. This new research field, collectively called network science, has been benefiting from interdisciplinary research efforts and a growing quantity and variety of network data.

Submission deadline: May 31, 2020
Papers

“Social dynamics of financial networks”,

“Identifying relationship lending in the interbank market: A network approach”,

“The structured backbone of temporal social ties”,
T. Kobayashi, Taro Takaguchi, A. Barrat,
*Nature Communications*, 2019.

Review article

“Network models of financial systemic risk: A review”,
*Journal of Computational Social Science* 1, 2018,
Fabio Caccioli, Paolo Barucca, T. Kobayashi,
Duration and interval time (for nodes)

Any direction

Incoming

Outgoing

Duration, node

Duration, incoming edge

Duration, outgoing edge

Interval, node

Interval, incoming edge

Interval, outgoing edge

CCDF vs. Duration

CCDF vs. Interval

Any direction

Incoming

Outgoing

Duration, node

Duration, incoming edge

Duration, outgoing edge

Interval, node

Interval, incoming edge

Interval, outgoing edge

CCDF vs. Duration

CCDF vs. Interval

Any direction

Incoming

Outgoing
Identification of significant ties

- Directed edge

(a) Directed edge test
- Bonferroni, $\alpha = 0.01$
- $\alpha = 0.001$
- $\alpha = 0.01$

(b) Lending dependency

(c) Borrowing dependency

(d) Directed edge test, variable activity

(e) Lending dependency, variable activity

(f) Borrowing dependency, variable activity
Previous research (excl. Econ)

- **Theoretical analysis of default cascades in financial networks** *(EPJB 2013, PRE 2015)*
- **Detection of important nodes in networks with community structure** *(Sci. Rep. 2016)*
- **Data analysis of interbank markets** *(EPJ Data Science 2018)*
- **Identification of “relationship lending” in the interbank market** *(JBF 2018)*
- **Backbone of temporal networks** *(Nat. Commun. 2019)*