Credible Forward Guidance

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\(^1\)The views expressed in this presentation are not necessarily those of the Federal Reserve Board of Governors or the Federal Reserve System.
Question: What is the optimal, time-consistent, forward guidance (FG) policy?

Context:

One well-known form of FG policy at the ELB:

“lower-for-longer” / overheating commitment policy

- CB promises to keep the policy rate at the ELB for an extended period so as to generate a temporary overheating.

- Very effective (Eggertsson and Woodford (2003)).

- But, CB has an incentive to renege on this promise, ex post: Time-inconsistency problem.
Central banks have not adopted this overheating commitment policy.$^2$

- The time-inconsistency problem was one key reason.

$^2$...except for the Bank of Japan.
“The main challenge of such [commitment-type] guidance is its inherent inconsistency over time and thus lack of credibility. When the time comes, the central bank may be tempted to deviate from its prior commitment: once the benefits of higher inflation expectations in terms of front-loaded spending have been reaped, the central bank may not be willing to pay the bill in terms of higher inflation afterwards. [...]”

This is a possible explanation why, in practice, central banks have refrained from using forward guidance in a way that implies a major change in strategy.” (September 2013)
However, there is a growing interest in adopting this type of policy in future crises.


“I believe the FOMC should seriously consider pursuing a lower-for-longer or makeup strategy for setting short rates when the zero lower bound binds and should articulate its intension to do so before the next zero lower bound episode.”

Janet Yellen (2018), “Comments on Monetary Policy at the Effective Lower Bound”
A way to overcome the criticism that the commitment policy is time-inconsistent: **Reputation**

Nakata (2018) ("Reputation and Liquidity Traps," *RED*) shows that the *optimal* commitment policy can be made time-consistent if

- (i) reneging on the promise leads to loss of reputation, making “lower-for-longer” policy ineffective in future crises.
- (ii) crises occur with sufficient frequency.
- (iii) loss of reputation lasts for a sufficiently long period.

However, Nakata (2018) is silent about what CB can credibly achieve if the *optimal* commitment policy is not credible.
What We Do

We characterize the best allocations the central bank can **credibly achieve** when the optimal commitment policy is not credible.

- by analyzing the *optimal sustainable policy problem* in a sticky-price model with ELB.
  - CB’s optimization problem is subject to a sustainability constraint.
  - ...assuming that, if deviation from OSP occurs, the economy falls into a discretionary regime (as in Nakata (2018)).
Main Results

Even when the optimal commitment policy (OCP) is not credible, CB can still credibly adopt “lower-for-longer” policies.

- Shorter ELB duration and smaller overheating of the economy under optimal sustainable policy (OSP) than under OCP.
- Welfare cost of ELB is substantially smaller under OSP than under optimal discretionary policy.

OSP is less history-dependent than OCP.

- Easier for CB to communicate to the public.
Related Literature

▶ Reputation in macroeconomics:

▶ Monetary policy at the ELB:
  ▶ Reifschneider and Williams (2000), Eggertsson and Woodford (2003), Jung et al. (2005), Adam and Billi (2006)
Outline of the Talk

- Model with a static Phillips curve
  - Setup
  - Results

- Model with a forward-looking Phillips curve
Model

Private-sector equilibrium conditions:

\[ y_t(s^t) = \mathbb{E}_t y_{t+1}(s^{t+1}) - \sigma(i_t(s^t)) - \mathbb{E}_t \pi_{t+1}(s^{t+1}) - s_t \]
\[ \pi_t(s^t) = \kappa y_t(s^t) \]
\[ i_t(s^t) \geq i_{ELB} \]

Notation: \( s^t := \{s_j\}_{j=1}^t \).

A state-contingent sequence of output, inflation, and the policy rate, \( \{y_t(s^t), \pi_t(s^t), i_t(s^t)\} \), is called an outcome.

- an outcome that satisfies EE, PC, and ELB constraint is called a competitive outcome.
An exogenous shock \((s_t)\)

Governed by a two-state Markov shock:

- In the normal/high state, \(s_t = r^* > 0\).
- In the crisis/low state \(s_t = r_c < 0\).
- Crisis frequency: \(\text{Prob}(s_{t+1} = r_c | s_t = r^*) = p_H\).
- Crisis persistence: \(\text{Prob}(s_{t+1} = r_c | s_t = r_c) = p_L\).

Notation: \(\mathbb{S} := \{r^*, r_c\}\).
Value

\[ V_t(s^t) = u(y_t(s^t), \pi_t(s^t)) + \beta \mathbb{E}_t V_t(s^{t+1}) \]

with

\[ u(y, \pi) = -\frac{1}{2} \left[ \pi^2 + \lambda y^2 \right] \]
Three outcomes

- Discretionary outcome [“optimal discretionary policy (ODP)”].
- Commitment outcome [“optimal commitment policy (OCP)”].
- Sustainable outcome [“optimal sustainable policy (OSP)”].
Discretionary outcome

At each period $t$, the discretionary CB’s problem is given by

$$W_t(s_t) = \max_{i_t, y_t, \pi_t} u(y_t) + \beta E_t W_{t+1}(s_{t+1})$$

subject to the private-sector equilibrium conditions at time $t$ and taking as given $\{W_{t+1}(\cdot), y_{t+1}(\cdot), \pi_{t+1}(\cdot)\}$.

Let $\{W_d(\cdot), i_d(\cdot), y_d(\cdot), \pi_d(\cdot)\}$ be the time-invariant value and policy functions that solve this problem and in which the normal-state policy rate is positive.

The discretionary outcome is defined as, and denoted by $\{i_{d,t}(s^t), y_{d,t}(s^t), \pi_{d,t}(s^t)\}_{t=1}^{\infty}$ such that $y_{d,t}(s^t) = y_d(s_t)$, $\pi_{d,t}(s^t) = \pi_d(s_t)$, and $i_{d,t}(s^t) = i_d(s_t)$. 
Commitment outcome

At the beginning of $t = 1$, the central bank chooses a state-contingent sequence, $\{i_t(s^t), y_t(s^t), \pi_t(s^t)\}_{t=1}^{\infty}$, to maximize

$$V_1(s_1)$$

subject to the private sector equilibrium conditions for all $t$.

The commitment/Ramsey outcome is the solution to this problem and is denoted by $\{i_{c,t}(s^t), y_{c,t}(s^t), \pi_{c,t}(s^t)\}_{t=1}^{\infty}$. The sequence of values associated with the commitment outcome is denoted by $\{V_{c,t}(s^t)\}_{t=1}^{\infty}$. 
Sustainable outcome

At the beginning of $t = 1$, the central bank chooses a state-contingent sequence, $\{i_t(s^t), y_t(s^t), \pi_t(s^t)\}_{t=1}^{\infty}$, to maximize

$$V_1(s_1)$$

subject to the private sector equilibrium conditions and a sustainability constraint for all $t$ and after all $s^t$.

$$V_t(s^t) \geq W_d(s_t).$$

The sustainable outcome is the solution to this problem and is denoted by $\{i_{s,t}(s^t), y_{s,t}(s^t), \pi_{s,t}(s^t)\}_{t=1}^{\infty}$. The sequence of values associated with the sustainable outcome is denoted by $\{V_{s,t}(s^t)\}_{t=1}^{\infty}$. 
Once we compute the sustainable outcome, we can construct a plan—a pair of central bank and private-sector strategies—that induces it and that is sustainable.

CB strategy \((\sigma_g, t)\): A sequence of functions mapping a history of states and a history of the policy rates (up to the previous period) into today’s policy rate.

\[
\sigma_{g,1} : S \rightarrow \mathbb{R}, \quad \text{and} \quad \sigma_{g,t} : \mathbb{R}^{t-1} \times S^t \rightarrow \mathbb{R} \quad \text{for all} \quad t \geq 2.
\]

PS strategy \((\sigma_p, t)\): A sequence of functions mapping a history of states and a history of the policy rates into today’s inflation and output.

\[
\sigma_{p,t} : \mathbb{R}^t \times S^t \rightarrow \mathbb{R} \times \mathbb{R} \quad \text{for all} \quad t.
\]

***Note that a plan induces an outcome.

A plan is said to be sustainable (credible/time-consistent) if “neither CB/private-sector agents have incentives to deviate from the instruction given by the plan.”
Construct the **revert-to-discretion plan** in which

- the economy follows the sustainable outcome as long as the central bank chooses a policy rate consistent with the sustainable outcome.
- if the central bank has ever deviated from the policy rate consistent with the sustainable outcome, the economy follows the discretionary outcome. [“the central bank loses reputation”; “punishment”]

By construction,

- the **revert-to-discretion plan** induces the **sustainable outcome**.
- the **revert-to-discretion plan** is sustainable (because \( V_{s,t}(s^t) \geq W_d(s_t) \))
Sustainable outcome w/ finite punishment

Sustainability constraint with $N$-period punishment:

$$V_t(s^t) \geq W_d^N(s_t)$$

where, for $k = 1$,

$$W_d^1(s) = \max u(y, \pi) + \beta \mathbb{E}[V_1(s')|s]$$

and, for $k \geq 2$,

$$W_d^k(s) = \max u(y, \pi) + \beta \mathbb{E}[W_d^{k-1}(s')|s]$$
Outline of the Talk

- Model with a static Phillips curve
  - Setup
  - Results
- Model with a forward-looking Phillips curve
Solution Method

We recurisify the infinite-horizon optimization problem by the method of Kehoe and Perri (2002) and Sunakawa (2015) (a modification of Marcet and Marimon (2017)).

We then use a time-iteration method.
**Table: Parameter Values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9925</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>IES</td>
<td>1</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Slope of the Phillips Curve</td>
<td>0.25/7</td>
</tr>
<tr>
<td>$i_{ELB}$</td>
<td>Effective lower bound</td>
<td>0</td>
</tr>
<tr>
<td>$N$</td>
<td>Punishment duration</td>
<td>[20, 60, $\infty$]</td>
</tr>
<tr>
<td>$p_H$</td>
<td>Frequency of the crisis state</td>
<td>0.5/100</td>
</tr>
<tr>
<td>$p_L$</td>
<td>Persistence of the crisis state</td>
<td>3/4</td>
</tr>
<tr>
<td>$r^*$</td>
<td>Natural-rate in the normal state</td>
<td>3/400</td>
</tr>
<tr>
<td>$r_c$</td>
<td>Natural-rate in the crisis state</td>
<td><strong>Chosen so that $y_d(s_t = r_c) = -0.07$</strong></td>
</tr>
</tbody>
</table>

**The recession severity consistent with Boneva, Braun, and Waki (2016).**

*Note that the value of $\lambda$ does not matter for allocations in the model with a static Phillips curve.*
Discretionary and commitment outcomes

Policy rate (ann. %)
Output gap (%)
Inflation (ann. %)

-1
-0.5
0
0.5
1

-6
-4
-2
0
2
4

0
1
2
3

OCP
ODP
Sustainable outcomes
Sustainable outcomes
Benefit and cost of reneging on the lower-for-longer promise:

- **Benefit:** Eliminate the temporary overheating of the economy.
- **Cost:** cannot promise any overheating in future crises.

For the promise to be credible, the cost has to be larger than, or equal, to the benefit.

- when the punishment lasts for shorter, the cost is smaller.
- so, for the promise to be credible, the promised overheating has to be smaller.
Welfare

Table: Welfare Cost of ELB

<table>
<thead>
<tr>
<th>Optimal Commitment</th>
<th>$29.5 \ (0.23)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal Sustainable ($N = \infty$)</td>
<td>$29.5 \ (0.23)$</td>
</tr>
<tr>
<td>Optimal Sustainable ($N = 100$)</td>
<td>$31.0 \ (0.24)$</td>
</tr>
<tr>
<td>Optimal Sustainable ($N = 60$)</td>
<td>$34.6 \ (0.27)$</td>
</tr>
<tr>
<td>Optimal Sustainable ($N = 20$)</td>
<td>$63.3 \ (0.49)$</td>
</tr>
<tr>
<td>Optimal Discretion</td>
<td>$128.1 \ (1)$</td>
</tr>
</tbody>
</table>

- Even with $N = 20$, the welfare cost of ELB is about half as large as under the optimal discretion.
<table>
<thead>
<tr>
<th>Institution</th>
<th>Average in yrs (since 1946)</th>
<th>Max in yrs (since 1946)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRB</td>
<td>8.1</td>
<td>18.8 (Martin)</td>
</tr>
<tr>
<td>ECB</td>
<td>6.5</td>
<td>8 (Trichet)</td>
</tr>
<tr>
<td>Bank of Canada</td>
<td>9.7</td>
<td>14 (Boey)</td>
</tr>
<tr>
<td>Bank of Japan</td>
<td>4.9</td>
<td>8.5 (Ichimada)</td>
</tr>
<tr>
<td>Bank of England</td>
<td>8.5</td>
<td>12 (Cobbold)</td>
</tr>
<tr>
<td>Riksbank</td>
<td>6.1</td>
<td>18 (Asbrink)</td>
</tr>
<tr>
<td>Swiss National Bank</td>
<td>7.4</td>
<td>11 (Leutwiler)</td>
</tr>
</tbody>
</table>

Note: The tenure of Alan Greenspan lasted for 18 years and 6 months.
History-dependence

- **y (%)**
  - (shock duration=1)
  - (shock duration=4)
  - (shock duration=8)

- **i (ann. %)**
  - (shock duration=1)
  - (shock duration=4)
  - (shock duration=8)

Graphs showing the response of y (%) and i (ann. %) for different shock durations.
History-dependence

additional ELB duration

size of output gap overshooting

realized shock duration

realized shock duration

OCP

ODP
History-dependence

- **y (%) (shock duration=1)**
- **y (%) (shock duration=4)**
- **y (%) (shock duration=8)**

- **i (ann. %) (shock duration=1)**
- **i (ann. %) (shock duration=4)**
- **i (ann. %) (shock duration=8)**

Legend:
- OCP/OSP w/ N=∞
- OSP w/ N=60
- OSP w/ N=20
- ODP
History-dependence

additional ELB duration

realized shock duration

size of output gap overshooting

realized shock duration

- OCP/OSP w/ $N = \infty$
- OSP w/ $N = 60$
- OSP w/ $N = 20$
- ODP
History-dependence

When $N$ is sufficiently small, there is no history-dependence under OSP:

- OSP becomes (almost) identical to “simple FG” policies of Walsh (2018).

- Under the simple FG policies of Walsh (2018), the policy rate is kept for a fixed duration, $k \geq 1$, after the shock disappears regardless of the realized duration of the shock.

- after $k$ periods, the policy rate is set according to a Taylor rule.
History-dependence

- One key criticism against OCP is that it is too “complex” and difficult for the public to understand.

- Our OSP is “less complex” than the OCP, and thus overcomes this criticism.
Outline of the Talk

- Model with a static Phillips curve
  - Setup
  - Results

- Model with a forward-looking Phillips curve
Private-sector equilibrium conditions:

\[
y_t(s^t) = \mathbb{E}_t y_{t+1}(s^{t+1}) - \sigma(i_t(s^t) - \mathbb{E}_t \pi_{t+1}(s^{t+1}) - s_t)
\]

\[
\pi_t(s^t) = \kappa y_t(s^t) + \beta \mathbb{E}_t \pi_{t+1}(s^{t+1})
\]

\[
i_t(s^t) \geq i_{ELB}
\]

An exogenous shock \((s_t)\):

- The same two-state Markov shock as before.

Value:

- \(V_t(s^t) = u(\pi_t(s^t), y_t(s^t)) + \beta \mathbb{E}_t V_t(s^{t+1})\)
- \(u(\pi, y) = -\frac{1}{2} [\pi^2 + \lambda y^2]\)
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<tr>
<td>$\kappa$</td>
<td>Slope of the Phillips Curve</td>
<td>0.005</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Weight on $y_t^2$</td>
<td>0.0625**</td>
</tr>
<tr>
<td>$i_{ELB}$</td>
<td>Effective lower bound</td>
<td>0</td>
</tr>
<tr>
<td>$N$</td>
<td>Punishment duration</td>
<td>[80, 160, $\infty$]</td>
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<td>$p_H$</td>
<td>Frequency of the crisis state</td>
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**This $\lambda$ implies equal weights on (annualized) inflation and output volatility.**
Discretionary and commitment outcomes

Policy rate (ann. %)

Output gap (%)

Inflation (ann. %)

OCP

ODP
Sustainable outcomes

Policy rate (ann. %)

Output gap (%)

Inflation (ann. %)
## Welfare

### Table: Welfare Cost of ELB

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<tr>
<td>Optimal Sustainable ($N = \infty$)</td>
<td>27.1 (0.39)</td>
</tr>
<tr>
<td>Optimal Sustainable ($N = 160$)</td>
<td>28.0 (0.40)</td>
</tr>
<tr>
<td>Optimal Sustainable ($N = 80$)</td>
<td>29.9 (0.43)</td>
</tr>
<tr>
<td>Optimal Discretion</td>
<td>68.9 (1)</td>
</tr>
</tbody>
</table>
History-dependence

- **additional ELB duration**
- **size of output gap overshooting**
- **size of inflation overshooting**

Graphs showing the relationship between realized shock duration and various economic indicators, with different lines representing different scenarios such as OCP, OSP with N=∞, OSP with N=160, OSP with N=80, and ODP.
Summary

▶ Even when the commitment policy is not credible, CB can still credibly promise to temporarily overheat the economy.
  
  ▶ Shorter ELB duration (and smaller overshooting of inflation and output) under optimal sustainable policy (OSP) than under optimal commitment policy (OCP).
  
  ▶ Welfare cost of ELB is substantially smaller under OSP than under optimal discretionary policy.

▶ OSP is less history-dependent than OCP.
  
  ▶ Easier for CB to communicate with public.
Extra slides
Digression

There are two time-invariant solutions to the discretionary CB’s problem (see Armenter (2017), Nakata (2018), and Nakata and Schmidt (2018)).

- one in which the ELB binds only in the crisis state.
- the other in which the ELB binds in both states.
- Possible to construct sunspot equilibria “fluctuating” between them. See Nakata and Schmidt (2019): “Simple Analytics of Expectations-Driven Liquidity Traps”.

We use the solution in which the ELB binds only in the crisis state in constructing the discretionary outcome.
1. **Outcome**: A state-contingent sequence of output, inflation, and the policy rate, \( \{y_t(s^t), \pi_t(s^t), \pi_t(s^t)\} \), is called an outcome.

   - an outcome that satisfies EE, PC, and ZLB constraint is called a *competitive outcome*.

Formally, an outcome is a sequence of functions mapping a history of states into today’s inflation, output, and the policy rate.

   - \( \sigma_t : \mathbb{S}^t \rightarrow \mathbb{R} \times \mathbb{R} \times \mathbb{R} \) for all \( t \).

**Notation**: \( \mathbb{S} := \{ r^*, r_c \} \)
Two objects (II)

2. Plan: A plan is a pair of government and private-sector strategies, \( \sigma_g = \{\sigma_{g,t}\}_{t=1}^\infty \) and \( \sigma_p := \{\sigma_{p,t}\}_{t=1}^\infty \).

\( \sigma_{g,t} \): A sequence of functions mapping a history of states and a history of the policy rates (up to the previous period) into today’s policy rate.

- \( \sigma_{g,1} : \mathbb{S} \to \mathbb{R} \), and
- \( \sigma_{g,t} : \mathbb{R}^{t-1} \times \mathbb{S}^t \to \mathbb{R} \) for all \( t \geq 2 \).

\( \sigma_{p,t} \): A sequence of functions mapping a history of states and a history of the policy rates into today’s inflation and output.

- \( \sigma_{p,t} : \mathbb{R}^t \times \mathbb{S}^t \to \mathbb{R} \times \mathbb{R} \) for all \( t \).

***Note that a plan induces an outcome.
Definition of sustainability

A plan, \((\sigma_g, \sigma_p)\), is sustainable/credible if

1. after any history \(i^t\) and \(s^t\), the continuation of \(\sigma_p\) and \(\sigma_g\) induce a competitive outcome, and
2. after any history \(i^{t-1}\) and \(s^t\), the sequence of the policy rates induced by \(\sigma_g\) maximizes the government’s objective given \(\sigma_p\).

An outcome is said to be sustainable if there is a sustainable plan that induces it.

When a certain plan A is sustainable and the plan A induces a certain outcome \(\alpha\), we say that the outcome \(\alpha\) can be made sustainable by the plan A.
Revert-to-discretion plan

Government strategy, $\sigma_g^{rtd}$:

- $\sigma_{g,1}^{rtd} = i_{c,1}(s_1)$ for any $s_1 \in \mathcal{S}$
- $\sigma_{g,t}^{rtd}(i^{t-1}, s^t) = i_{c,t}(s^t)$ if $i_k = i_{c,k}(s^k)$ for all $k \leq t - 1$,
- $\sigma_{g,t}^{rtd}(i^{t-1}, s^t) = i_{d,t}(s^t)$ otherwise.

Private-sector strategy, $\sigma_g^{rtd}$:

- $\sigma_{p,t}^{rtd}(i^t, s^t) = (y_{c,t}(s^t), \pi_{c,t}(s^t))$ if $i_k = i_{c,k}(s^k)$ for all $k \leq t$
- $\sigma_{p,t}^{rtd}(i^t, s^t) = (y_{br}(s_t, i_t), \pi_{br}(s_t, i_t))$ otherwise.\(^\text{3}\)

where

$$y_{br}(s_t, i_t) = E_t y_{d,t+1}(s^{t+1}) - \sigma\left[i_t - E_t \pi_{d,t+1}(s^{t+1})\right] - s_t$$

$$\pi_{br}(s_t, i_t) = \kappa y_{br}(s_t, r_t) + \beta E_t \pi_{d,t+1}(s^{t+1})$$

\(^3\)Subscript $br$ stands for best response.