Credible Forward Guidance*

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Abstract

We analyze optimal time-consistent forward guidance policies in a sticky-price model with an effective lower bound (ELB) constraint on nominal interest rates. Lower-for-longer strategies, while effective in stimulating the economy at the ELB, are potentially time-inconsistent, as the associated temporary overheating of the economy in the aftermath of a recession is undesirable ex post. However, if reneging on a lower-for-longer promise leads to loss of reputation and prevents the central bank from effectively using lower-for-longer strategies in future recessions, these strategies can be time-consistent. We find that, even without an explicit commitment technology, the central bank can still credibly keep the policy rate at the ELB for an extended period—though not as extended as it would with an explicit commitment technology—and meaningfully mitigate the adverse effects of the ELB constraint on economy activities.

JEL: E32, E52, E61, E62, E63

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1 Introduction

I believe the FOMC should seriously consider pursuing a lower-for-longer or makeup strategy for setting short rates when the zero lower bound binds and should articulate its intension to do so before the next zero lower bound episode.

Janet L. Yellen[1]

Developing effective strategies to manage the adverse consequences of the effective lower bound (ELB) constraint on nominal interest rates is an important task for economists and central bankers. In forward-looking models with ELB, the commitment to keeping the policy rate at the ELB for an extended period—and temporarily overshooting inflation and output targets—is known to be effective in stimulating economic activity during a deep recession, as the anticipation of an overheated economy leads foreward-looking households and firms to increase consumption and set higher prices (Reifschneider and Williams (2000); Eggertsson and Woodford (2003); Jung, Teranishi, and Watanabe (2005); Adam and Billi (2006)).

While the effectiveness of such overheating commitment policy in theory is widely known, central banks that recently faced, or are currently facing, the ELB constraint have not adopted this type of policy, with an exeption of the Bank of Japan. One key argument against the overheating commitment policy is its potential time-inconsistency (or "lack of credibility"). Ex ante, it is desirable to promise to overheat the economy in the future, as the expectations of future overheating stimulate inflation and output when the economy faces headwinds and the ELB is a binding constraint. However, once the headwinds dissipate, the central bank will have an incentive to renege on the promise of overheating the economy by raising the policy rate, as the overheating is ex post undesirable . A number of policymakers have stated that this time-inconsistency problem is one reason for why the commitment policy may be not as effective in reality as in theory.[2]

Nakata (2018) has recently shown that, once the central bank's reputational concern is taken into account, the *optimal* overheating commitment policy can be made time-consistent provided that the policy rate is expected to fall into the ELB in the future with sufficient frequency and the loss of reputation lasts for a sufficiently long duration. In his analysis, if the central bank reneges on the promise of temporarily overheating the economy in the aftermath of a crisis, it loses reputation and private-sector agents will not believe similar promises in future crises. If private-sector agents do not believe the central bank's promise to overheat the economy, future ELB episodes will be associated with large declines in inflation and output. Thus, concern for maintaining reputation gives the central bank an incentive to fulfill the promise of keeping the policy rate "lower-for-longer" in the aftermath of a crisis. According to Nakata (2018), this incentive to maintain reputation often dominates the short-run incentive to eliminate the overheating of the economy, and as a result, the

[1] Yellen (2018), "Comments on Monetary Policy at the Effective Lower Bound" `https://www.brookings.edu/wp-content/uploads/2018/09/ES_20180910_Yellen-ELB.pdf`.

[2] See Nakata (2015) and Appendix H for quotes from various policymakers discussing the time-inconsistency of the commitment policy.

optimal commitment policy is credible. However, Nakata (2018) is silent about what the central bank can achieve when the optimal commitment policy is not credible.

In this paper, we study the optimal time-consistent overheating policy in a sticky-price model with the ELB to understand the best allocations the central bank can credibly achieve when the optimal commitment policy is not credible. Specifically, we formulate and solve an optimal sustainable policy problem in which the central bank at time one chooses state-contingent allocations to maximize welfare subject to not only private-sector equilibrium conditions, but also an incentive compatibility constraint—known as the sustainability constraint. The sustainability constraint requires that the continuation value associated with the chosen state-contingent allocation has to be at least as large as the continuation value associated with the optimal discretionary policy (or a temporary deviation to it) at any time and after any history of shocks. According to Nakata (2018), the optimal commitment policy is often credible. That is, the sustainability constraint does not bind and the optimal sustainable policy coincides with the optimal commitment policy under a wide range of parameter values. However, there are cases in which the optimal commitment policy is not credible, for example, when crises occur very infrequently or when the loss of reputation lasts for only a short amount of time after reneging on the promise of overheating the economy. Our main interest is optimal sustainable policies in those cases in which the sustainability constraint occasionally binds.

Our main result is that, even when the optimal commitment policy is not credible, the central bank can still credibly keep the policy rate at the ELB for an extended period in the aftermath of a crisis—though not as extended as under the optimal commitment policy. As in the optimal commitment policy, such lower-for-longer policy generates a temporary overheating of the economy in the aftermath of a recession and mitigates the declines in economic activities during a recession through expectations. Under reasonable assumptions regarding how long the central bank suffers from loss of reputation after reneging on the promise of lower-for-longer, the welfare cost of the ELB constraint is substantially lower under the optimal sustainable policy than under the optimal discretionary policy.

One key feature of the optimal sustainable policy is that it is less history-dependent than the optimal commitment policy. As discussed in detail by Eggertsson and Woodford (2003), a key feature of the optimal commitment policy is its history dependence. In particular, under the optimal commitment policy, the additional period at which to keep the policy rate at the ELB in the aftermath of a crisis—and, as a result, the magnitude of output and inflation overshoot—increases with the realized duration of the crisis shock. When reputational force is strong, the optimal sustainable policy exhibits a qualitatively similar history-dependence, but the degree of history dependence is weaker. When reputational force is weak and the optimal sustainable policy is closer to the optimal discretionary policy, the optimal sustainable policy does not feature any history dependence. That is, the additional ELB duration as well as the magnitude of output and inflation overshoot do not depend on the realized crisis shock duration.

Our optimal sustainable policy, especially when the sustainability constraint binds sufficiently strongly, is of interest to central banks for two reasons. First, by construction, the optimal sustainable policy is time-consistent, Thus, it is immune to the criticism that the promised overshoot of inflation and output associated with any "lower-for-longer" strategies may not be credible. Second, when the punishment length is sufficiently short, the optimal sustainable policy is not history dependent, or not as history dependent as the optimal commitment policy is. Thus, it overcomes the criticism that, because the policy rate path associated with a "lower-for-longer" strategy is complex, it is difficult for the central bank to clearly explain these strategies to the public.

Our paper builds on the literature on optimal monetary policies in the New Keynesian model with the ELB. This literature has demonstrated the value of an explicit commitment technology in liquidity traps by characterizing the optimal commitment policy and contrasting it with the optimal discretionary poicy (Eggertsson and Woodford (2003); Jung, Teranishi, and Watanabe (2005); Adam and Billi (2006); Adam and Billi (2007); and Nakov (2008)). In particular, these papers have shown that the central bank can effectively mitigate the adverse consequence of the ELB constraint by committing to keeping the policy rate at the ELB for an extended period and temporarily overheating the economy in the aftermath of a crisis. Our paper contributes to this body of work by characterizing the optimal sustainable policy in the model with the ELB and showing that, even without an explicit commitment technology, the central bank can achieve an outcome identical, or similar to, that under the optimal commitment policy.

Our paper is closely related to Nakata (2018) and Walsh (2018). Nakata (2018) has shown that the optimal commitment policy in the New Keynesian model with the ELB can be credible once the reputational concern on the part of the central bank is explicitly modeled. Walsh (2018) examines credibility of non-optimal, simple forward guidance policies—those that promise to keep the policy rate at the ELB for a fixed number of additional periods after the crisis shock is gone, and reaches the conclusion similar to that of Nakata (2018).[3] Our paper is different from Nakata (2018) because we study the optimal sustainable policy and show that, even when the optimal commitment policy is not credible, the central bank can still credibly adopt lower-for-longer strategies. Our paper is different from Walsh (2018) because we characterize the optimal allocation the central bank can credibly achieve, whereas Walsh (2018) studies credibility of simple policy rules with a lower-for-longer feature.[4] It turns out that there is an interesting relationship between our optimal sustainable policy and the forward guidance policy of Walsh (2018), which will be discussed in detail in Section 4.1

Finally, our paper is related to a set of papers that characterize optimal allocations in macroeconomic models with sustainability constraints. Kehoe and Perri (2002) characterize the optimal allocation in an international business cycle model in which a deviation from the promised plan would push the economy to autarky. Fujiwara, Kam, and Sunakawa (2016b) study the optimal

[3]See also Sukeda (2018) which extends the analysis of Walsh (2018) to a model with discounted Euler equation and Phiillips curve.

[4]Nakata (2018) and Walsh (2018) in turn build on earlier work of reputation in macroeconomics, including Barro and Gordon (1983), Rogoff (1987), Chari and Kehoe (1990), Chang (1998), and Phelan and Stacchetti (2001), among many others. Recent contributions include Kurozumi (2008), Loisel (2008), and Dong (2015).

sustainable policy in an two-country model in which the deviation from the promised cooperative plan would push the countries into a non-cooperative regime. Most closely related to our paper is Sunakawa (2015) which characterizes the optimal sustainable policy in a New Keynesian model with cost-push shocks, but without the ELB constraint. Our paper applies the same analytical framework and methodology used in these papers to the model with the ELB.

The rest of the paper is organized as follows. Section 2 describes the model and the central bank's optimization problems. Section 3 presents the results. Section 4 provide additional discussions and results. Section 5 concludes.

2 Model

2.1 Private sector

Our main model is a semi-loglinear New Keynesian model with a static Phillips curve. The private-sector equilibrium conditions of this model are given by:

$$y_t(s^t) = E_t y_{t+1}(s^{t+1}) - \sigma(i_t(s^t) - E_t \pi_{t+1}(s^{t+1}) - r^*) + s_t \tag{1}$$

$$\pi_t(s^t) = \kappa y_t(s^t) \tag{2}$$

$$i_t(s^t) \geq i_{ELB} \tag{3}$$

where y_t is output, π_t is inflation, and i_t is the policy rate. Equations (1) and (2) are the Euler equation and the Phillips curve, respectively. Inequality (3) imposes the ELB constraint (i_{ELB}) on the policy rate. σ is the intertemporal elasticity of substitution, $r^* > 0$ is the natural rate of interest at the deterministic steady state, and κ is the slope of the static Phillips curve. s_t is a natural rate shock. s^t denotes the history of shocks up to time t. That is, $s^t := \{s_k\}_{k=1}^t$. Since there is uncertainty, the allocations are state-contingent and depend on s^t. We refer to the state-contingent sequence of consumption, inflation, and the nominal interest rate, $\{y_t(s^t), \pi_t(s^t), i_t(s^t)\}_{t=1}^{\infty}$, as *an outcome*. Given a process of s_t, an outcome is said to be *competitive* if, for all $t \geq 1$ and $s^t \in \mathbb{S}^t$, (i) $y_t(s^t) \in \mathbb{R}$, $\pi_t(s^t) \in \mathbb{R}$, $i_t(s^t) \in \mathbb{R}_{\geq 0}$ where $\mathbb{R}$ denotes a set of real numbers and $\mathbb{R}_{\geq 0}$ denotes a set of non-negative real number and (ii) the equations above are satisfied.

We assume that s_t follows a two-state Markov process. $s_t = r^* > 0$ in the "high" state, or the "normal" state, and $s_t = r_c < 0$ in the low state, or the "crisis" state. The probability of moving from the high/normal state to the low/crisis state is denoted by p_H ("crisis frequency"), whereas the probability of moving from the low/crisis state to the low/crisis state is denoted by p_L ("crisis persistence"). Following Nakata (2018) and **?**, we allow p_H to be non-zero, which opens up the possibility for a reputational concern to make a lower-for-longer promise credible.

The government's *value* at period t is given by

$$V_t(s^t) := E_t \sum_{j=0}^{\infty} \beta^j u(\pi_{t+j}(s^{t+j}), y_{t+j}(s^{t+j})) \tag{4}$$

where the per-period objective function is given by the following function.

$$u(\pi, y) := -\frac{1}{2}\Big[\pi^2 + \lambda y^2\Big] \tag{5}$$

This objective function can be obtained as the second-order approximation to the household's welfare.[5] For any outcome, there is an associated state-contingent sequence of values, $\{V_t(s^t)\}_{t=1}^{\infty}$, which will be referred to as *the value sequence.*

We use this model with a static Phillips curve as our baseline model for a computational reason. As described in Section 2.4, we use a time-iteration method—a commonly used numerical method to solve nonlinear models—to solve our model. Regardless of the specification of the Phillips curve, this solution method fails to converge if the punishment duration—a key parameter, introduced shortly, governing how long the central bank is prohibited from engaging in state-contingent policies after it reneges on the previsouly annoucend policy—is sufficiently short. We can devise an alternative solution method if the punishment duration is sufficiently short in the model with a static Phillips curve, while we cannot in the model with a forward-looking Phillips curve. Thus, the model can be solved for a wider range of the punishment lengths in the model with a static Phillips curve.[6] In Section 4.3, we present some select results from the model with a forward-looking Phillips curve and confirm that key features of the optimal sustainable policy from the model with the static Phillips curve also hold in the model with a forward-looking Phillips curve.[7]

2.2 Central bank

We will consider three classes of outcomes that differ in how the central bank sets its interest rate policy—the discretionary outcome, the commitment outcome, and the sustainable outcome.

2.2.1 The discretionary outcome

At each time t, the discretionary central bank's optimization problem is to choose $\{y_t,\ \pi_t,\ i_t\}$ to maximize the value today, taking as given the value and allocations in the next period. That is,

$$W_t(s_t) = \max_{\pi_t, y_t, i_t} \quad u(y_t, \pi_t) + \beta E_t W_{t+1}(s_{t+1}), \tag{6}$$

[5] See, for example, Woodford (2003) and Galí (2015).

[6] The instability of the time-iteration method in the presence of a sustainability constraint is not specific to the model with ELB. The second author of this paper experienced a similar instability issue in Fujiwara, Kam, and Sunakawa (2016a). We leave the task of developing a robust algorithm for models with sustainability constraints to future research.

[7] See also Bilbiie (2016) who uses a model with a static Phillips curve.

subject to (1), (2), and (3).

Let $\{W_d(\cdot), \pi_d(\cdot), y_d(\cdot), i_d(\cdot)\}$ be a set of time-invariant value and policy functions that solve the Bellman equation above in which the ZLB binds only in the crisis state.[8] They are functions of today's shock realization, s_t. *The discretionary outcome* is defined as, and denoted by, the state-contingent sequence of output, inflation, and the policy rate, $\{y_{d,t}(s^t),\ \pi_{d,t}(s^t),\ i_{d,t}(s^t)\}_{t=1}^{\infty}$ such that $y_{d,t}(s^t) = y_d(s_t)$, $\pi_{d,t}(s^t) = \pi_d(s_t)$, and $i_{d,t}(s^t) = i_d(s_t)$ and *the discretionary value sequence* is defined as, and denoted by, $\{V_{d,t}(s^t)\}_{t=1}^{\infty}$ such that $V_{d,t}(s^t) = W_d(s_t)$. We will also refer to the discretionary outcome as the outcome under the optimal discretionary policy.

2.2.2 The commitment outcome

At the beginnig of time zero and for each $s_1 \in \mathbb{S}$, the central bank with a commitment technology chooses a state-contingent allocation, $\{y_t(s^t),\ \pi_t(s^t),\ i_t(s^t)\}_{t=1}^{\infty}$, to maximize the expected discounted sum of future utility flows.[9] That is,

$$V_{c,1}(s_1) = \max_{\{y_t(s^t),\pi_t(s^t),i_t(s^t)\}_{t=1}^{\infty}} \quad E_1 \sum_{t=1}^{\infty} \beta^{t-1} u(y_t(s^t), \pi_t(s^t)), \tag{7}$$

subject to (1), (2), (3) for all $t \geq 1$ and for all histories of shocks s^t. *The Ramsey outcome*, or *the commitment outcome*, is defined as the solution to this optimization problem. In other words, the Ramsey outcome is a competitive outcome with the highest time-one value. We will denote the Ramsey outcome by $\{y_{c,t}(s^t),\ \pi_{c,t}(s^t),\ i_{c,t}(s^t)\}_{t=1}^{\infty}$. The value sequence associated with the Ramsey outcome is denoted by $\{V_{c,t}(s^t)\}_{t=1}^{\infty}$, and will be referred to as *the Ramsey value sequence*. We will also refer to the commitment outcome as the outcome under the optimal commitment policy.

2.2.3 The sustainable outcome

At the beginnig of time one, the central bank chooses a state-contingent allocation, $\{y_t(s^t),\ \pi_t(s^t),\ i_t(s^t)\}_{t=1}^{\infty}$, to maximize the time-one value:

$$V_{s,1} = \max_{\{y_t,\pi_t,i_t\}_{t=1}^{\infty}} \quad E_1 \sum_{t=1}^{\infty} \beta^{t-1} u(y_t, \pi_t), \tag{8}$$

subject to (1), (2), (3), and the following sustainability constraint:

$$E_t \sum_{k=0}^{\infty} \beta^n u(y_{t+k}(s^{t+k}), \pi_{t+k}(s^{t+k})) \geq W_d^N(s_t), \tag{9}$$

[8] There also exists a time-invariant solution to this discretionary government's problem in which the ZLB binds in both states. See Armenter (2017), Nakata (2018), and **?** for extensive analyses of such deflationary Markov-perfect equilibrium.

[9] An alternative way to define the Ramsey outcome would be to let the central bank optimize before they observe s_1, provided that the distribution of s_1 is not degenerate.

for all $t \geq 0$ and for all histories of states s^t. *The sustainable outcome* is defined as the solution to this infinite-horizon optimization problem. We will also refer to the sustainable outcome as the outcome under the optimal sustainable policy.

The left hand side of the sustainability constraint is the continuation value of implementing a chosen state-contingent allocation at time t after s^t. The right hand side, $W_d^N(s_t)$, is the continuation value the central bank gets if it deviates from the chosen state-contingent allocation, with N indicating the punishment duration. $W_d^N(s_t)$ is recursively defined as follows. $W_d^0(s) := V_{s,1}(s)$, $\pi_d^0(s) := \pi_{s,1}(s)$, $y_d^0(s) := y_{s,1}(s)$. For any $k \geq 1$,

$$W_d^k(s) = \max_{\pi,y,i} \quad u(\pi, y) + \beta E[W_d^{k-1}(s')|s]$$

where the maximization is subject to the private sector equilibrium conditions and taking as given the allocations for the next period (that is, $\pi_d^{k-1}(\cdot)$ and $y_d^{k-1}(\cdot)$). Note that $W_d^{\infty}(s) = W_d(s)$.

Note that the punishment value, $W_d^N(s_t)$, is determined jointly with the sustainable outcome, except when $N = \infty$. When $N = \infty$, the punishment lasts forever and its value, $W_d(s)$, is independent on the sustainable outcome. For any finite N, the punishment eventually ends and the economy returns to the allocations consistent with the sustainable outcome. Thus, The punishment value and the sustainable outcome are not independent of each other. An increase (decrease) in the value associated with the sustainable outcome implies an increase (decrease) in the punishment value.

As described in detail in Appendix **??**, once the sustainable outcome is computed from the optimization problem above, we can construct a plan—a pair of central bank and private-sector strategies—which induces the sustainable outcome and which has a trigger-type structure. In particular, we can construct a revert-to-discretion plan in which (i) the economy follows the sustainable outcome as long as the central bank has never deviated from the policy rate path consistent with the sustainable outcome in the past and (ii) the economy follows the discretionary outcome—or a temporary deviation to a discreionary regime—otherwise. By construction, such a revert-to-discretion plan is credible; because the sustainability constraint is imposed on the central bank's optimization problem, the continuation value under the sustainable outcome is at least as large as the punishment continuation value. Thus, the central bank has no incentive to deviate from the policy rate consistent with the sustainable outcome. Even though the deviation does not occur in equilibrium, the specification of what would happen if the central bank were to deviate from the sustainable outcome does affect what happens under the sustainable outcome.

Note that, if the sustainability constraint does not bind at any time t and after any history of shocks, the sustainable outcome coincides with the commitment outcome. Also, if the sustainability constraint always binds—which happens, for example, when the punishment length (N) is zero or when the crisis frequency (p_H), is zero—the sustainable outcomes coincides with the discretionary outcome. Our main interest is those cases in which the sustainability constraint occasionally binds.

2.3 Parameter values

Table 1 shows the baseline parameter values. The quarterly frequency of crises is set to 0.5/100 (2/400), and is motivated by the fact that, in the United States, there have been two large crises that pushed the short-term nominal interest rate to the ELB over roughly the last hundred years (400 quarters) since the creation of the Federal Reserve System. The crisis shock persistence is set to 3/4, which implies the expected duration of the crisis shock 4 quarters. σ is set to 1. κ and s_L are chosen so that output and inflation declines 7 percent and 1 percentage point (annualized), respectively, in the crisis state under the optimal discretionary policy. This severity of the crisis is consistent with that considered in Boneva, Braun, and Waki (2016) and Nakata (2018), and is intended to capture the severity of the Great Recession of 2007-2009 in the United States.

Table 1: Baseline Parameter Values

Parameter	Description	Values
β	Discount factor	0.9925
σ	Intertemporal elasticity of substitution	1
κ	Slope of the Phillips Curve	0.25/7
i_{ELB}	Effective lower bound on the policy rate	0
N	Duration of reputation loss	[20, 60, ∞]
p_H	Frequency of the crisis state	0.5/100
p_L	Persistence of the crisis state	3/4
r_c	Natural-rate shock	**Chosen so that $y_{MP}(s_t = r_c) = -0.07$

We mainly consider three values for the duration of lost reputation (20, 60, and ∞), which are chosen to cover various qualitatively distinct cases that can arise in analyzing the optimal sustainable policy. To put these valuse into perspective, note that the assumption that the central bank can restore its reputation after a finite number of periods can be motivated by the fact that the tenure of governorship at central banks is finite and the reputation is specific to the leader of the central bank, as opposed to the institution. As shown in Appendix G, the average tenure of the governorship in the central banks in economies currently grappling with the ELB ranges from about 5 years (20 quarters) in the Bank of Japan to about 10 years (40 quarters) for the Bank of Canada. The maximum tenure duration exceeds 15 years (60 quarters) at several central banks (the Federal Reserve, Bank of Canada, Bank of England, and Sveriges Riksbank).

2.4 Solution method

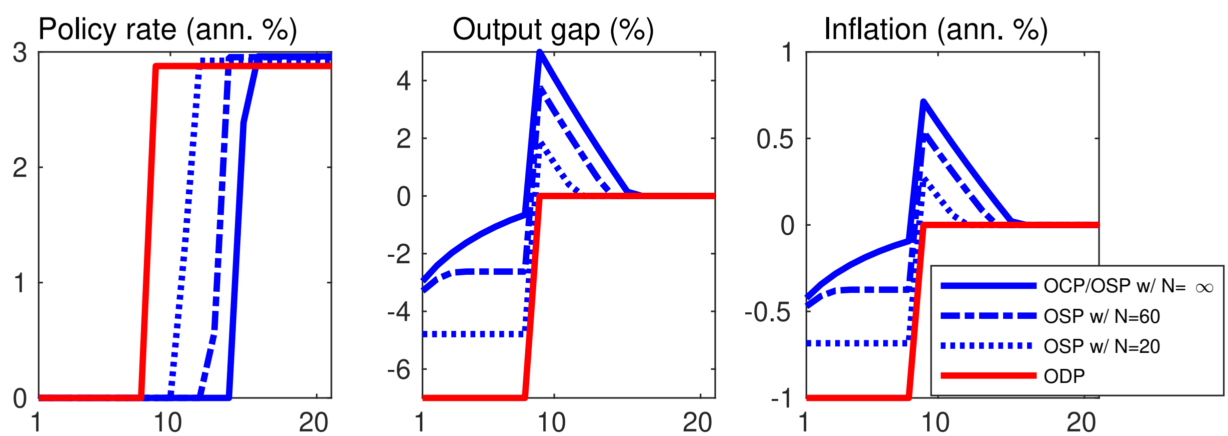
The model is highly nonlinear, featuring two inequality constraints—the ELB and constraint and the sustainability constraint—and cannot be solved analytically. Following Kehoe and Perri (2002) and Sunakawa (2015), we recursify the infinite-horizon optimization problem of the central bank into a saddle-point functional equation using the Lagrange multiplier on the Euler equation as the pseudo-state variable. We then apply a time-iteration method to find the set of time-invariant policy functions that solve the saddle-point functional equation. Appendix D describes the details of the solution method as well as its accuracy.[10]

3 Results

3.1 Dynamics

Figure 1 shows the dynamics of the economy under the optimal discretionary policy, the optimal commitment policy, and the optimal sustainable policy with $N = [20, 60, \infty]$. In this figure, the crisis shock hits the economy at time 1 and stays there until time 8. The crisis shock disappears at time 9 and the economy is in the normal state from that point on.

Figure 1: Dynamics



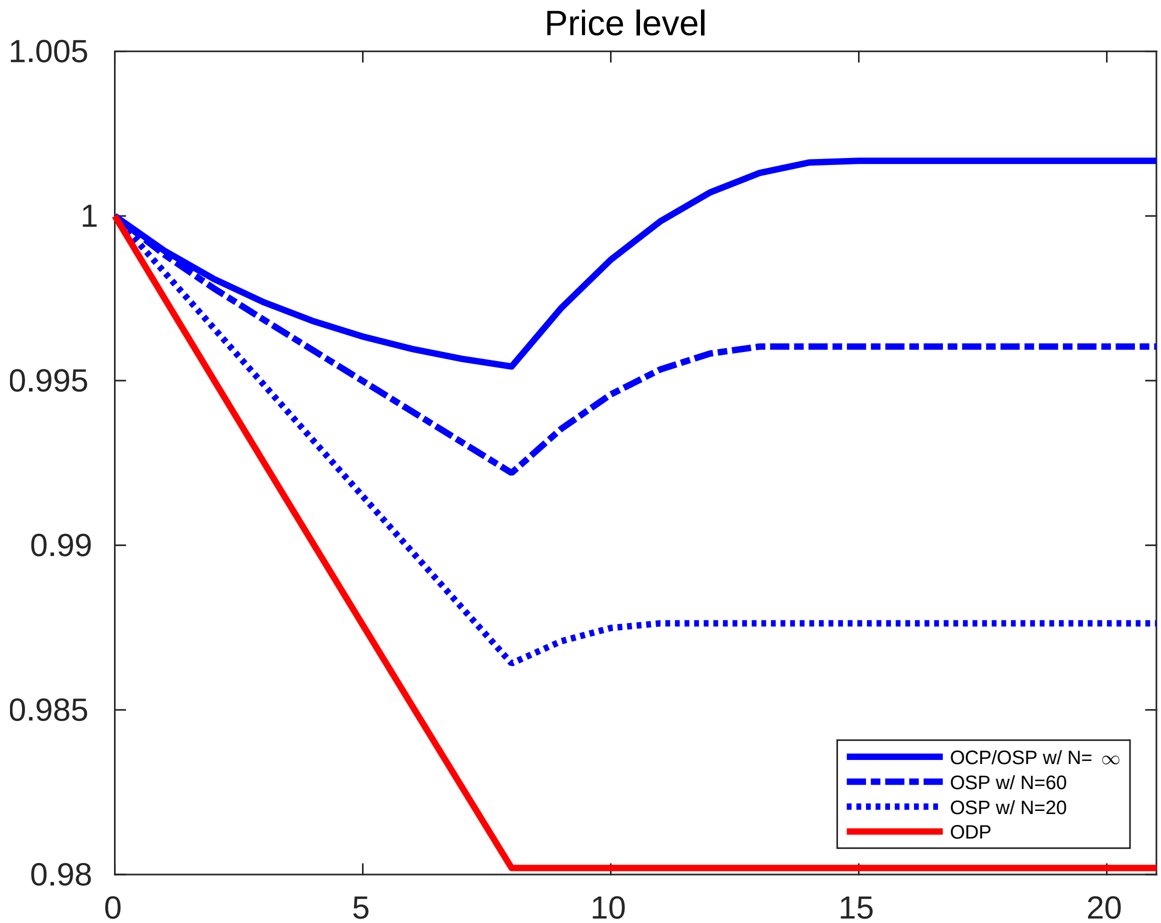
Note: The rate of inflation is expressed in annualized percent. The output gap is expressed in percent.

Under the optimal discretionary policy (ODP)—shown by the solid red lines—the central bank keeps the policy rate at the ELB as long as the crisis shock continues and raises the policy rate immediately after the crisis shock disappears. Under the optimal commitment policy (OCP)—shown by the solid blue lines—the central bank keeps the policy rate at the ELB for a while even after the crisis shock disappears and engineers the overshooting of inflation and output above their targets. Since households are forward looking, the anticipation of high inflation and high output stimulates economy activity during the crisis. The declines in inflation and output are substantially smaller under the OCP than under the ODP.

The allocations under the optimal sustainable policy (OSP) with $N = \infty$ are identical to those under the OCP. In our calibration, the crisis shock is sufficiently frequent so that the cost of being unable to effectively manage expectations in the future forever outweighs the benefit of eliminating the temporary overshooting of inflation and output targets. Thus, the sustainability constraint does not bind at any states of the world, as shown in the left panel of Figure **??**. When the

[10] As discussed earlier, our solution method fails to converge for some parameter values, in particular for some values of N. We examined many alternative variations of our time-iteration methods, including the use of alternative grid points, such as Smolyak grids and simulation-based grids, and the use of alternative basis functions.

Figure 2: Value of fulfilling versus reneging on the promised allocations



Note: The rate of inflation is expressed in annualized percent. The output gap is expressed in percent.

punishment length is shorter, the cost of reneging on the lower-for-longer promise after the crisis shock disappears is smaller. With $N = 20$ and $N = 60$, the sustainability constraint binds right after the crisis shock disappears—as shown in the middle and right panels of Figure **??**, limiting the magnitude of the inflation and output overshoot. With smaller overshooting of inflation and output, inflation and output decline by more during the crisis under the OSP than under the OCP. However, these declines remain substantially smaller than under the ODP.

Reflecting the less severe crisis, welfare cost of the ELB—shown in Table 2—is substantially lower under the OSPs than under the ODP. With $N = 60$, welfare cost of the ELB is about 20 percent of that under the ODP and is only slightly larger than under the OCP. Even with $N = 20$, welfare cost of the ELB is only about half of the ODP.

3.2 History Dependence

One key feature of the OCP is its history dependence. In particular, the additional periods to keep the policy rate at the ELB after the crisis shock disappears—as well as the magnitude of

Table 2: Welfare Cost of the ELB

	Abs($E[V]$)
Optimal commitment policy	29.5 (0.23)
Optimal sustainable policy	
with $N = \infty$	29.5 (0.23)
with $N = 100$	31.0 (0.24)
with $N = 60$	34.6 (0.27)
with $N = 20$	63.3 (0.49)
Optimal discretionary policy	128.1 (1.00)

inflation and output overshoot—are larger when the crisis shock has lasted longer. For example, when the realized crisis shock duration is 1, 4, and 8 quarters, the additional ELB duration is 2, 4, and 6 quarters, respectively, as can be seen in the bottom panels of Figure 3 and the left panel of Figure 4. When the realized crisis shock duration is 1, 4, and 8 quarters, the magnitude of the output overshoot is 2 percentage points, 3.8 percentage points, and 5 percentage points, respectively, as can be seen in the top panels of Figure 3 and the right panel of Figure 4.

The OSPs with small Ns are less history-dependent than the OCP. With $N = 60$ and $N = 100$, there is history-dependence in the additional ELB duration and the magnitude of the output overshoot, but the degree of history dependence is less than that under the OCP, as can be seen in Figures 2 and 3. With $N = 20$, there is no history-dependence at all: the additional ELB duration is 2 quarters and the size of the output overshoot is 2 percent regardless of the realized crisis shock duration.

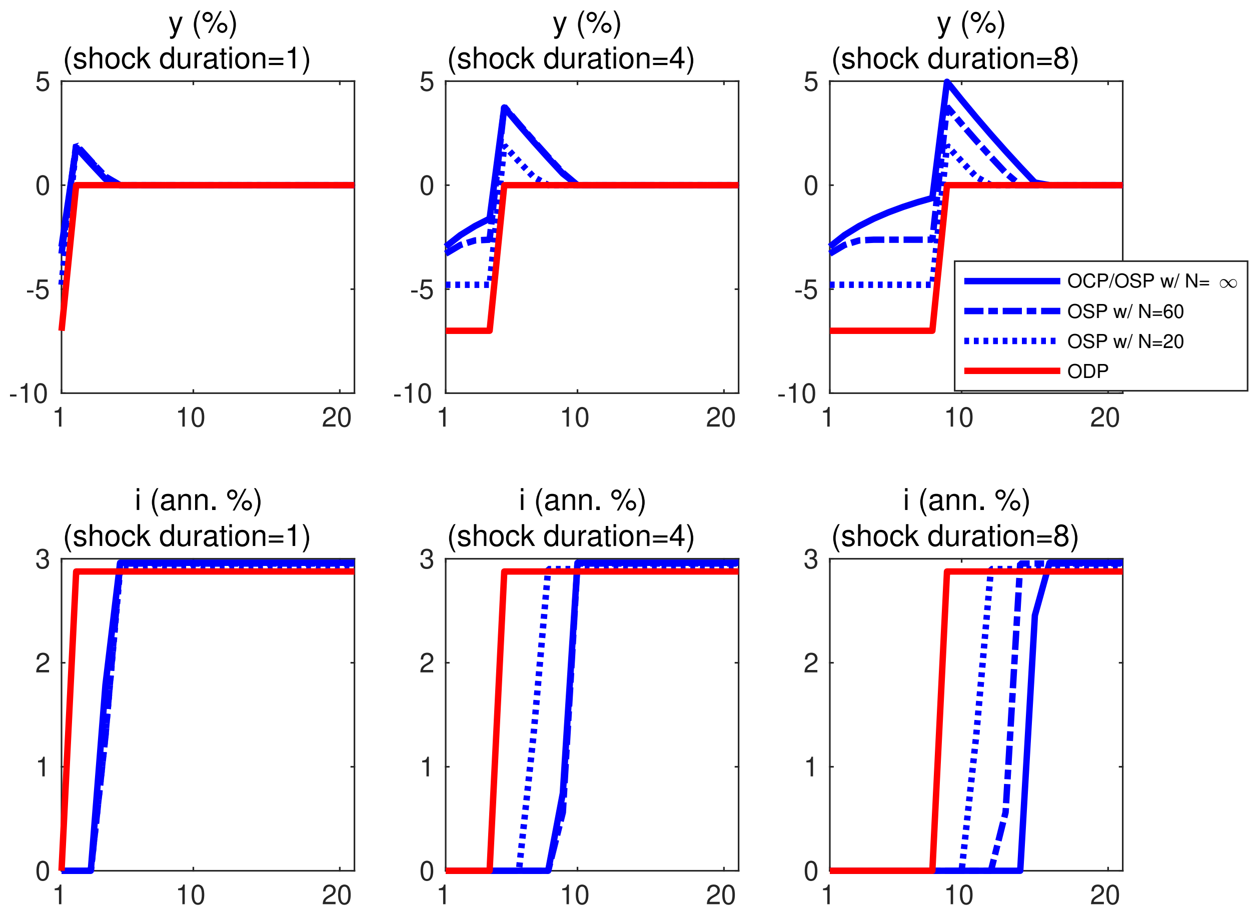
4 Additional results and discussion

4.1 Relation to the simple forward guidance policy of Walsh (2018)

We find that, when N is sufficiently small, the optimal sustainable policy is not history dependent; the policy rate path after the crisis shock disappears does not depend on the realized duration of the crisis shock. Thus, the optimal sustainable policy bears some resemblance to the simple forward guidance policy considered by Walsh (2018). Under the simple forward guidance policy of Walsh (2018), the central bank keeps the policy rate at the ELB for a fixed number of periods after the crisis shock disappears and letting it return to the steady state level immediately thereafter, regardless of the realized duration of the crisis shock. The only difference is that, under the optimal sustainable policy, the policy rate does not return to the steady state level immediately after liftoff.

The similarity between the simple forward guidance policy and the optimal sustainable policy with when N is small points to a benefit of the optimal sustainable policy over the optimal commitment policy; it is easier to explain to the public than the optimal commitment policy. One key criticism against the optimal commitment policy is that it is complex because of its history

Figure 3: History Dependence (I)
—Dynamics with Alternative Realized Crisis Shock Duration—



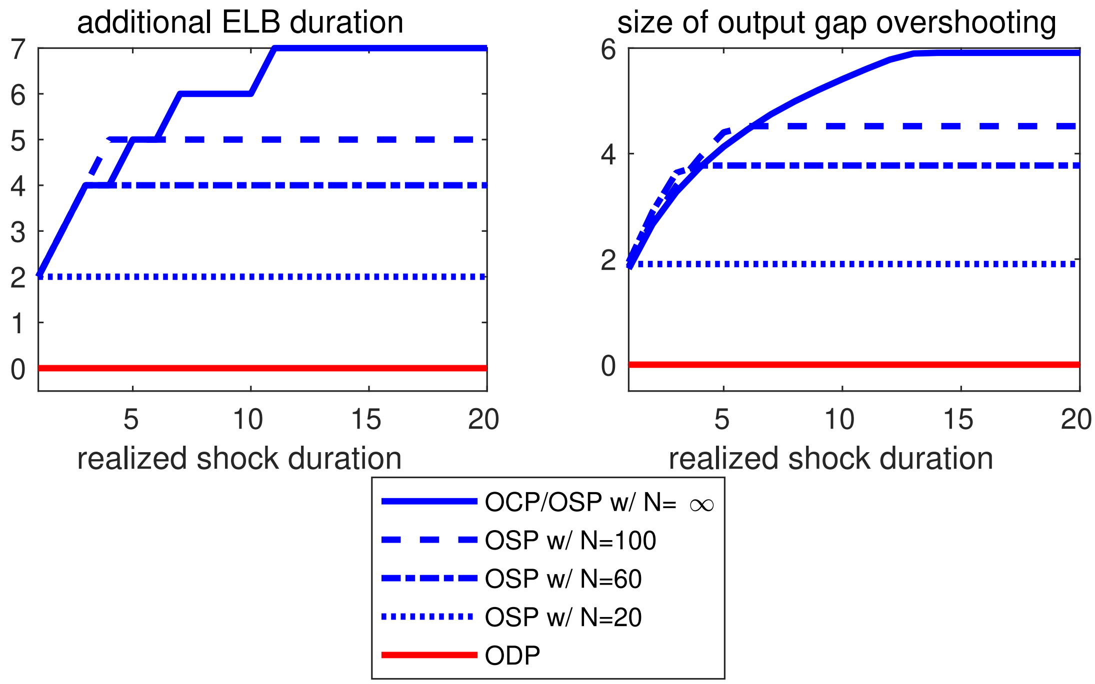
Note: The rate of inflation is expressed in annualized percent. The output gap is expressed in percent.

dependence. In particular, the additional periods at which to keep the policy rate at the ELB as well as the magnitude of inflation and output overshoots depend importantly on how long the crisis shock has lasted. Under the optimal sustainable policy, the additional ELB duration and the magnitude of inflation and output overshoots are independent of how long the crisis shock lasted.

4.2 Relation to the loose commitment approach of Bodenstein, Hebden, and Nunes (2012)

Under the optimal sustainable policy, the central bank achieves an allocation that is "in between" that under the optimal discretionary policy and that under the optimal commitment policy. This feature of the optimal sustainable policy is reminiscent of the optimal policy obtained in the loose commitment approach in which the central bank reoptimizes with a constant probability

Figure 4: History Dependence (II)



Note:

every period regardless of the incentive to renege on the prior commitment.[11]. While these two approaches differ from each other in many ways, both approaches share the same spirit that they are intended to shed light on what one can expect the central bank to achieve when there is no explicit commitment technology. A recent work by Fujiwara, Kam, and Sunakawa (2016a) shows that, using a model without ELB, the allocations under the loose commitment approach with an appropriately chosen re-optimization probability can approximate the allocation under the optimal sustainable policy with N period punishment reasonably well. While we believe their result is likely to extend to the model with ELB, it would be useful to verify the validity of their claim in the model with ELB. We leave such investigation to future research.

4.3 Results from the model with a forward-looking Phillips curve

As discussed earlier, we have focused on the model with a static Phillips curve, instead of the model with a forward-looking Phillips curve, because the range of N under which we can solve the model is wider in the model with static Phillips cuve. A natural question is whether our key results thus far would survive in the model with a forward-looking Phillips curve. In this section, we provide some select results from the model with a forward-looking Phillips curve.

The parameter values used are shown in Table 3. The values for β, σ, p_H, and s_H are the same

[11]See Bodenstein, Hebden, and Nunes (2012) for an analysis of optimal monetary policy under loose commitment in the model with ELB

as in the previous section. κ is set to 0.005. p_L is set to 0.5, implying the expected duration of the crisis state of 2 quarters. λ is set to 1/16, a value consistent with equal weights on the volatility of the output gap and the volatility of the annualized rate of inflation. A high value of λ and a low value of p_L increase a range of the punishment duration values under which we can solve the model. With these parameter values, we could solve the model for $75 \leq N$. We show the dynamics of the model under the optimal sustainable policy with three values of $N = \{80, 160, \infty\}$.

Table 3: Parameter Values
—Model with the Forward-Looking Phillips Curve

Parameter	Description	Parameter Value
β	Discount rate	$\frac{1}{1+0.0075} \approx 0.9925$
σ	Inverse intertemporal elasticity of substitution	1
κ	Slope of the Phillips curve	0.005
λ	Relative weight on output volatility	[1/16]
s_H	Natural rate of interest in the high (normal) state	$\frac{1}{\beta} - 1$ (=0.0075)
s_L	Natural rate of interest in the low (crisis) state	−0.0125
p_H	Crisis shock frequency	0.5/100
p_L	Crisis shock persistence	0.5
N	Punishment length	[80, 160, ∞]

Figure 5 shows the dynamics of the economy under the optimal discretionary policy, the optimal commitment policy, and optimal sustainable policies with three values of N.

The dynamics of the economy under the optimal discretionary policy and the optimal commitment policyshown by the solid red and blue lines, respectivelyare consistent with those from the model with a static Phillips curve as well as the results in the existing studies. Under the ODP, the central bank keeps the policy rate at the ELB as long as the crisis shock continues, but raises the policy rate immediately once the crisis shock disappears. Under the OCP, the central bank keeps the policy rate at the ELB for a while even after the crisis shock disappears and engineers the overshooting of inflation and output above their targets.

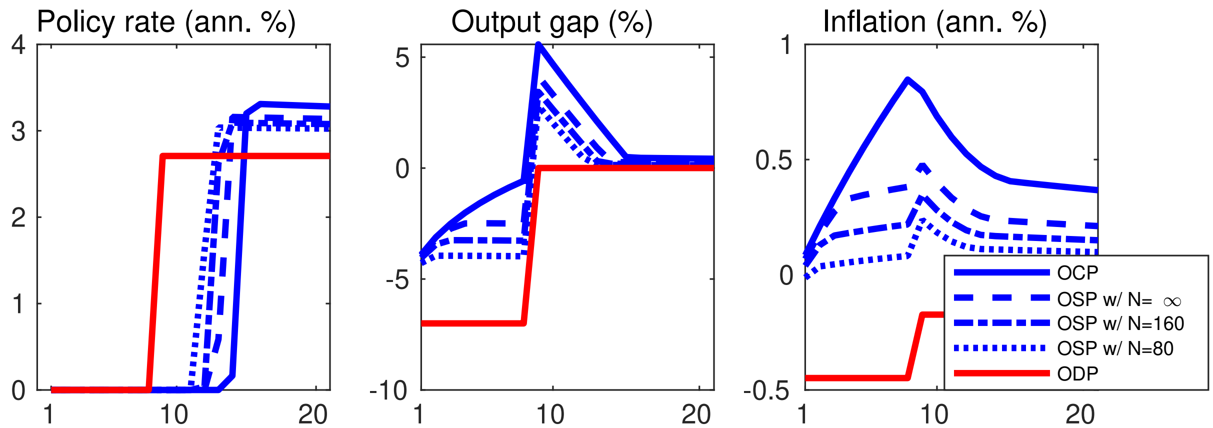
Table 4 shows the welfare cost of the ELB under various optimal sustainable policies.

Table 4: Welfare Cost of ELB
—Model with Forward-Looking Phillips Curve—

	Abs($E[V]$)
Optimal commitment policy	26.8 (0.39)
Optimal sustainable policy	
with $N = \infty$	27.1 (0.39)
with $N = 160$	28.0 (0.40)
with $N = 80$	29.9 (0.43)
Optimal discretionary policy	68.9 (1.00)

Under the OSPs, the central bank keeps the policy rate at the ELB even after the crisis shock disappears, but not as long as it would do under the OCP. The magnitudes of the inflation and

Figure 5: Dynamics
—Model with Forward-Looking Phillips Curve—

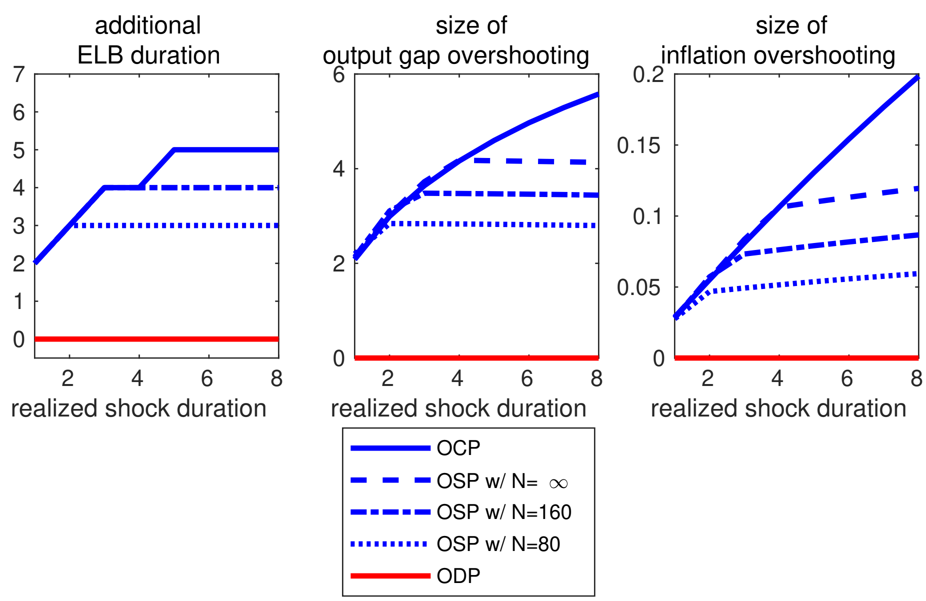


Note: The rate of inflation is expressed in annualized percent. The output gap is expressed in percent.

output overshoots are smaller under the optimal sustainable policies than under the optimal commitment policy, with the overshoot smaller when N is smaller. These lower-for-longer policy rate paths nontrivially reduces the welfare cost of the ELB constraint, as shown in Table 4.

History dependence: One key feature of the OSP in the model with the static Phillips curve was that it is less history dependent than the optimal commitment policy. We can examine whether this feature is present also in the model with the forward-looking Phillips curve by examining how the additional ELB duration varies with the realized crisis shock duration, which is shown in Figure 6.

Figure 6: History Dependence
—Model with Forward-Looking Phillips Curve—



According to the figure, Like in the model with a static Phillips curve, the OSPs are less history dependent than the OCP. In particular, the additional duration to keep the policy rate at the ELB in the aftermath of a crisis as well as the magnitude of the inflation and output overshoot are less sensitive to the realized duration of the crisis shock.

All told, qualitatively, the key insights from the model with a static Phillips curve carry over to the model with a forward-looking Phillips curve.

5 Conclusion

In this paper, we have characterized optimal sustainable policy in a model with ELB constraint and recurring crises. We find that, even when the optimal commitment policy is not credible, the central bank can still credibly commit to keeping the policy rate at the ELB in the aftermath of a crisis—though not as long as the Ramsey planner would—and substantially mitigate the adverse consequences of the ELB constraint.

By construction, the optimal sustainable policy is time-consistent and thus overcomes the criticism that the temporary overheating of the economy associated with lower-for-longer strategies is not credible. When the punishment length is sufficiently short, we find that the optimal sustainable policy is not history dependent, or not as history dependent as the optimal commitment policy. Thus, it overcomes the criticism that the implied policy rate path is too complex for the central bank to be able to clearly explain to the public.

Finally, while we focused on the time-consistency aspect of lower-for-longer strategies in this paper, there are other aspects of these strategies that could make them less attractive in reality than in theory. For example, the public may not understand the temporary nature of the inflation overshooting, leading to the unanchoring of the long-run inflation expectations (Kohn (2012) and Yellen (2018)). The overheating of the economy may be more undesirable for policymakers in reality than what's implied by the New Keynesian model, if the overheating leads to financial instability (Yellen (2018)). It would be useful to formally analyze how these factors affect the effectivenss and desirability of lower-for-longer strategies. We leave such analysis to future research.

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Technical Appendix for Online Publication

This technical appendix is organized as follows:

- Appendix C describes the model with a forward-looking Phillips curve.
- Appendix D describes the numerical solution method and report the solution accuracy.
- Appendix G documents the average tenure of chairpersons/governors in selected central banks.

A Definition of a plan and credibility

This section defines a plan, credibility, and the revert-to-discretion plan. The definitions closely follow Chang (1998).

A.1 Plan

A government strategy, denoted by $\sigma_g := \{\sigma_{g,t}\}_{t=1}^{\infty}$, is a sequence of functions that maps a history of the nominal interest rates up to the previous period and a history of states up to today into today's nominal interest rate. Formally, $\sigma_{g,t}$ is given by $\sigma_{g,1} : \mathbb{S} \rightarrow \mathbb{R}_{\geq 0}$ and $\sigma_{g,t} : \mathbb{R}_{\geq 0}^{t-1} \times \mathbb{S}^t \rightarrow \mathbb{R}_{\geq 0}$ for all $t \geq 2$.[12] Given a particular realization of $\{s_t\}_{t=1}^{\infty}$, a sequence of nominal interest rates will be determined recursively by $i_1 = \sigma_{g,1}(s_1)$ and $i_t = \sigma_{g,t}(i^{t-1}, s^t)$ for all $t > 1$ and for all $s^t \in \mathbb{S}^t$. A government strategy is said to *induce* a sequence of the nominal interest rates. *A private-sector strategy*, denoted by $\sigma_p := \{\sigma_{p,t}\}_{t=1}^{\infty}$, is a sequence of functions mapping a history of nominal interest rates up to today and a history of states up to today into today's consumption and inflation. Formally, $\sigma_{p,t}$ is given by $\sigma_{p,t} : \mathbb{R}^t \times \mathbb{S}^t \rightarrow (\mathbb{R}, \mathbb{R})$ for all t.

Given a government and private-sector strategy, a sequence of consumption and inflation will be determined recursively by $(y_t, \pi_t) = \sigma_{p,t}(i^t, s^t)$ for all $t \geq 1$ and for all $s^t \in \mathbb{S}^t$. A private sector strategy, together with a government strategy, is said to *induce* a sequence of consumption and inflation.[13] *A plan* is defined as a pair of government and private sector strategies, (σ_g,σ_p). Notice that a plan induces an outcome—a state-contingent sequence of consumption, inflation, and the nominal interest rate. As discussed earlier, there is a value sequence $\{w_t(s^t)\}_{t=1}^{\infty}$, associated with any outcome.

A.2 Credibility

Let us use $CE_t^R(s)$ to denote a set of state-contingent sequences of the nominal interest rate consistent with the existence of a competitive equilibrium when $s_t = s$. Formally, for each $s \in \mathbb{S}$, $CE_t^R(s) := \{\boldsymbol{i}_t(s) \in \mathbb{R}^{\infty} |\ \exists\ (\boldsymbol{y}_t(s), \boldsymbol{\pi}_t(s))$ s.t. $(\boldsymbol{y}_t(s), \boldsymbol{\pi}_t(s), \boldsymbol{i}_t(s)) \in CE_t(s)\}$. σ_g is said to be *admissible* if, after any history of policy actions, i^{t-1}, and any history of states, s^t, $\boldsymbol{i}_t(s)$ induced by the continuation of σ_g belongs to $CE_t^R(s_t)$.

A plan, (σ_g, σ_p), is *credible* if (i) σ_g is admissible, (ii) after any history of policy actions, i^t, and any history of states, s^t, the continuation of σ_p and σ_g induce a $(\boldsymbol{y}_t(s_t), \boldsymbol{\pi}_t(s_t), \boldsymbol{r}_t(s_t)) \in CE_t(s_t)$,

[12] The first period is a special case, as there is no previous policy action.

[13] Note that, while the nominal interest rate today depends on the history of nominal interest rates up to the previous period, consumption and inflation today depend on the history of nominal interest rates up to today. The implicit within-period-timing protocol behind this setup is that the government moves before the private sector does.

and (iii) after any history i^{t-1} and s^t, $\boldsymbol{i}_t(s_t)$ induced by σ_g maximizes the government's objective over $CE_t^R(s_t)$ given σ_p. In plain languages, a plan is said to be credible if neither the private sector nor the government has incentive to deviate from the strategies associated with it.

An outcome is said to be credible if there is a credible plan that induces it. When a certain plan A is credible and the plan A induces a certain outcome α, we say that the outcome α can be made credible, or time-consistent, by the plan A.

A.3 The revert-to-discretion plan

I now define a key object of this paper, the revert-to-discretion plan, and discuss the condition under which this plan is credible.

The revert-to-discretion plan, $(\sigma_g^{rtd}, \sigma_p^{rtd})$, consists of (i) the following government strategy: $\sigma_{g,1}^{rtd} = i_{os,1}(s_1)$ for any $s_1 \in \mathbb{S}$, $\sigma_{g,t}^{rtd}(i^{t-1}, s^t) = i_{os,t}(s^t)$ if $i_j = i_{os,j}(s^j)$ for all $j \leq t-1$, and $\sigma_{g,t}^{rtd}(i^{t-1}, s^t) = i_{d,t}(s^t)$ otherwise, and (ii) the following private-sector strategy: $\sigma_{p,t}^{rtd}(i^t, s^t) = (y_{os,t}(s^t), \pi_{ram,t}(s^t))$ if $r_j = i_{ram,j}(s^j)$ for all $j \leq t$, $\sigma_{p,t}^{rtd}(i^t, s^t) = (y_{br}(s_t, i_t), \pi_{br}(s_t, i_t))$ otherwise,[14] where

$$y_{br}(s_t, r_t) = E_t y_{d,t+1}(s^{t+1}) - \sigma\Big[\big[i_t - E_t\pi_{d,t+1}(s^{t+1}) - r^*\big] + s_t\Big] \tag{10}$$

$$\pi_{br}(s_t, r_t) = \kappa y_{br}(s_t, r_t) + \beta E_t \pi_{d,t+1}(s^{t+1}) \tag{11}$$

The government strategy instructs the government to choose the nominal interest rate consistent with the Ramsey outcome, but chooses the interest rate consistent with the discretionary outcome if it has deviated from the Ramsey outcome at some point in the past. The private sector strategy instructs the household and firms to choose consumption and inflation consistent with the Ramsey outcome as long as the government has never deviated from the Ramsey outcome. If the government has ever deviated from the nominal interest rate consistent with the Ramsey outcome, the private sector strategy instructs the household and firms to choose consumption and inflation today based on the belief that the government in the future will choose the nominal interest rate consistent with the discretionary outcome. By construction, the revert-to-discretion plan induces the Ramsey outcome, and the implied value sequence is identical to the Ramsey value sequence.

It is relatively straightforward to show that the revert-to-discretion plan is credible. See online appendix B for proof. By construction, $w_{os,t}(s^t) \geq w_{d,t}(s^t)$ for all $t \geq 1$ and all $s^t \in \mathbb{S}^t$, making sure that the government does not have an incentive to deviate from the instruction given by the government strategy after any history i^{t-1} and s^t in which the optimal sustainable policy has been followed.

The revert-to-discretion plan that induces the optimal sustainable outcome with a finite period punishment is defined in a similar way. See Nakata (2018). It is also straighforward to show the plan's credibility.

[14] Subscript *br* stands for *b*est *r*esponse.

B Model with a static Phillips Curve

The policymaker maximizes

$$
\begin{aligned}
& v_0 = -E_0 \sum_{t=0}^{\infty} \beta^t y_t^2, \\
& \text{subject to} \\
& y_t = (1+\sigma^{-1}\kappa) E_t y_{t+1} - \sigma^{-1}(r_t - s_t), && (12) \\
& r_t \geq -r^*, && (13) \\
& v_t = -E_t \sum_{n=0}^{\infty} \beta^n y_{t+n}^2 \geq v^d(s_t), && (14)
\end{aligned}
$$

for all $t \geq 0$. We have used a relationship based on the static Phillips curve $E_t\pi_{t+1} = \kappa E_t y_{t+1}$. The shock s_t follows two-state Markov chain, $s_t \in \{s_h, s_l\}$ where $s_h > s_l$. Transition probability matrix is given as $P = \begin{bmatrix} 1-p_h & p_h \\ 1-p_l & p_l \end{bmatrix}$, where p_h is the frequency of the crisis and p_l is the persistence of the crisis. $v^d(s_t)$ is the value under the optimal discretionary policy, i.e., the Markov perfect equilibrium (MPE).

The analytical solution for the Markov perfect equilibrium We focus on the following equilibrium. The ZLB is slack in the good state, $\phi_h = 0$, and the ZLB is binding in the bad state, $\phi_l = \phi > 0$. Then the equilibrium confitions become

$$
\begin{aligned}
& y_h = 0, \\
& y_l = -\phi, \\
& y_h + \sigma^{-1}(r_h - s_h) - (1+\sigma^{-1}\kappa)(p_h y_l + (1-p_h) y_h) = 0, \\
& y_l + \sigma^{-1}(-r^* - s_l) - (1+\sigma^{-1}\kappa)(p_l y_l + (1-p_l) y_h) = 0, \\
& v_h = -y_h^2 + \beta(p_h v_l + (1-p_h) v_h), \\
& v_l = -y_l^2 + \beta(p_l v_l + (1-p_l) v_h),
\end{aligned}
$$

Then we have

$$
\begin{aligned}
y_h = 0, & \qquad y_l = -\phi = \frac{r^* + s_l}{\sigma(1-(1+\kappa\sigma^{-1})p_l)}, \\
r_h = s_h + \sigma(1+\kappa\sigma^{-1}) p_h y_l, & \qquad r_l = -r^*, \\
v_h = \frac{-\beta p_h y_l^2}{(1-\beta)(1-\beta(p_l - p_h))}, & \qquad v_l = \frac{-y_l^2 + \beta(1-p_l) v_h}{(1-\beta p_l)}.
\end{aligned}
$$

Solving for the sustainable equilibrium Lagrangean is set up as

$$
\begin{aligned}
\mathcal{L} & \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -y_t^2 + 2\phi_t \left(y_t + \sigma^{-1}(r_t - s_t) - (1+\sigma^{-1}\kappa) E_t y_{t+1} \right) + \psi_t \left(-E_t \sum_{n=0}^{\infty} \beta^n y_{t+n}^2 - v^d(s_t) \right) \right\}, \\
& = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\Psi_t y_t^2 + 2\phi_t \left(y_t + \sigma^{-1}(r_t - s_t) \right) - \frac{1}{\beta} 2\phi_{t-1}(1+\sigma^{-1}\kappa) y_t - \psi_t v^d(s_t) \right\},
\end{aligned}
$$

where $\Psi_t = 1 + \psi_0 + ... + \psi_t < \infty$. The FOCs are

$$\begin{aligned}
\partial y_t: &\quad -\Psi_t y_t - \phi_t + \beta^{-1}(1+\sigma^{-1}\kappa)\phi_{t-1} = 0, \\
\partial \phi_t: &\quad y_t + \sigma^{-1}(r_t - s_t) - (1+\sigma^{-1}\kappa)E_t y_{t+1} = 0.
\end{aligned}$$

Normalizing the first equation by Ψ_t, we have

$$-y_t - \tilde{\phi}_t + \beta^{-1}(1+\sigma^{-1}\kappa)z_t\tilde{\phi}_{t-1} = 0,$$

where $\tilde{\phi}_t = \phi_t/\Psi_t$ and $z_t = \Psi_{t-1}/\Psi_t \in (0,1]$. The complementary slackness conditions are

$$\begin{aligned}
&\phi_t(r_t + r^*) = 0, \\
&\phi_t \geq 0, \\
&\psi_t\left(v_t - v^d(s_t)\right) = 0, \\
&\psi_t \geq 0.
\end{aligned}$$

We solve for the policy functions by the time iteration method. There are four patterns of binding constraints.

(i) $r_t > -r^*$ and $v_t > v^d(s_t)$:

$\phi_t = 0$ and $z_t = 1$. Note that, given the policy functions in the previous iteration, $f_y(\phi_{-1}, s)$ and $f_V(\phi_{-1}, s)$, $E_t y_{t+1} = \int f_y(0, s_{t+1})\mu(s_{t+1}|s_t)$ and $E_t V_{t+1} = \int f_V(0, s_{t+1})\mu(s_{t+1}|s_t)$ are obtained. Then we have

$$\begin{aligned}
y_t &= \beta^{-1}(1+\sigma^{-1}\kappa)\tilde{\phi}_{t-1}, \\
r_t &= \sigma\left((1+\sigma^{-1}\kappa)E_t y_{t+1} - y_t\right) + s_t, \\
V_t &= -y_t^2 + \beta E_t V_{t+1}.
\end{aligned}$$

(ii) $r_t > -r^*$ and $v_t \leq v^d(s_t)$:

$\phi_t = 0$ and $z_t \in (0,1)$. The sustainnability constraint is binding, $V_t = V^d(s_t)$. Then we have

$$\begin{aligned}
y_t &= \left(-V^d(s_t) + \beta E_t V_{t+1}\right)^{.5}, \\
r_t &= \sigma\left((1+\sigma^{-1}\kappa)E_t y_{t+1} - y_t\right) + s_t, \\
z_t &= \beta(1+\sigma^{-1}\kappa)^{-1} y_t/\tilde{\phi}_{t-1}.
\end{aligned}$$

(iii) $r_t \leq -r^*$ and $V_t > V^d(s_t)$:

$\phi_t > 0$ and $z_t = 1$. The ZLB is binding, $r_t = -r_t^*$. Note that (ϕ_t, y_t) can be obtained by solving the following equations:

$$\begin{aligned}
&y_t = -\phi_t + \beta^{-1}(1+\sigma^{-1}\kappa)\tilde{\phi}_{t-1}, \\
&y_t + \sigma^{-1}(-r_t^* - s_t) - (1+\sigma^{-1}\kappa)\int f_y(\phi_t, s_{t+1})\mu(s_{t+1}|s_t) = 0.
\end{aligned}$$

(iv) $r_t \leq -r^*$ and $V_t \leq V^d(s_t)$:

$\phi_t > 0$ and $z_t \in (0,1)$. Both constraints are binding, $r_t = -r_t^*$ and $V_t = V^d(s_t)$. (ϕ_t, y_t) can be obtained by solving the following equations:

$$V^d(s_t) = -y_t^2 + \beta \int f_V(\phi_t, s_{t+1})\mu(s_{t+1}|s_t),$$
$$y_t + \sigma^{-1}(-r_t^* - s_t) - (1 + \sigma^{-1}\kappa) \int f_y(\phi_t, s_{t+1})\mu(s_{t+1}|s_t) = 0.$$

Note that these equations do not depend on $\tilde{\phi}_{t-1}$.

C Model with a forward-looking Phillips Curve

$$\sigma y_t(s^t) = \sigma E_t y_{t+1}(s^{t+1}) + E_t \pi_{t+1}(s^{t+1}) - r_t(s^t) + r^* + s_t \quad (15)$$
$$\pi_t(s^t) = \kappa y_t(s^t) + \beta E_t \pi_{t+1}(s^{t+1}) \quad (16)$$
$$r_t \geq r_{ELB} \quad (17)$$

The optimal discretionary policy, the optimal commitment policy, and the optimal sustainable policy are defined in a way that is similar to how they are defined for the model with the static Phillips curve in the main text.

The first-order necessary conditions of the central bank's optimization problem under the sustainable policy are given by:

[To be completed by Takeki]

D Time-iteration method

[To be completed by Takeki: Only for the model with static Phillips curve.]

E A detailed account of the model with a static Phillips curve

. In this section, we provide a detailed account of the dynamics of the model with a static Phillips curve from the vantage of the policy functions. Our goals are two-fold. First, seeing the dynamics of the economy in this way helps us better understand what's happening in the model. Second, it helps us to understand why the dynamics of the economy are history independent when N is sufficiently small and why it is possible to solve the model in this case in an alternative way that we later describe in Appendix F.

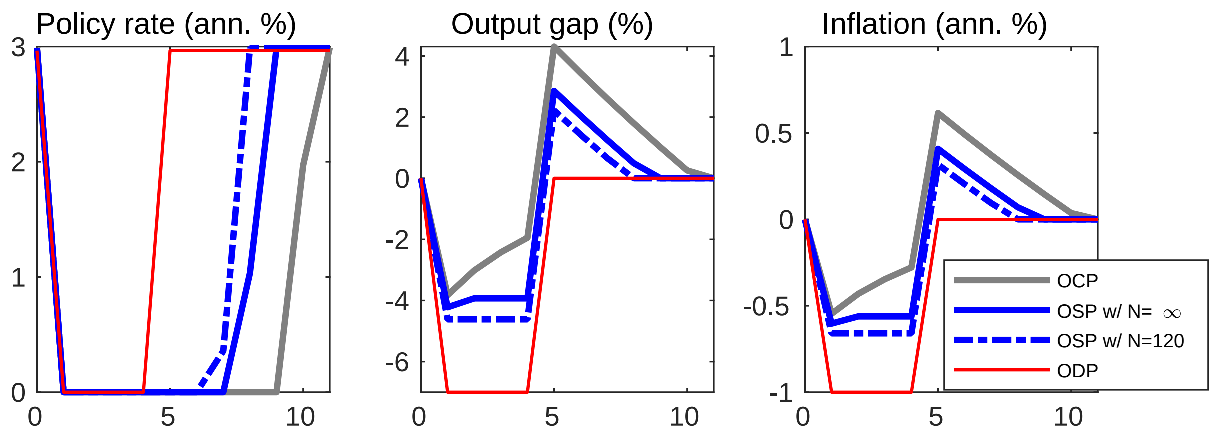
We can accomplish the first goal with the baseline calibration of the main text, but cannot accomplish the second goal because the time-iteration does not converge for any small values of N consistent with history-independent dynamics. Accordingly, we will use an alternative calibration in which the time-iteration method converges for some values of N consistent with history independent dynamics. In this alternative calibration, $p_H = 0.2/100$ and $p_L = 0.5$. The values for other parameters are the same as the baseline values from the main text.

We first describe the dynamics of the economy under the Ramsey and Markov perfect policy. We then describe the dynamics of the economy under the optimal sustainable policy.

E.1 Ramsey Policy

Suppose that the economy is initially at its risky steady state at time 0 where the policy rate is positive and the Lagrange multiplier is zero. The economy falls into the crisis state at time 1 and stays there until time 4. The economy is back in the normal state from time 5 on. The economy's dynamics in this case are shown by the solid black lines in Figure 7.

Figure 7: Dynamics with an alternative calibration



The solid black lines in Figure 8 are the policy functions associated with the Ramsey equilibrium. The black dots in Figure **??** traces the dynamics of the economy in this scenario along the policy functions. The dynamics of the economy from time 1 to time 4 are governed by the policy function in the right panel. As the economy stays in the crisis state, the Lagrange multiplier—the sole endogenous state variable of the mode—increases. The pace of the increase decelerates the longer the economy is in the crisis state. If the economy were to stay in the crisis state forever (which is a zero probability event), the Lagrange multiplier converges to a finite value.

Once the economy is back in the normal state, the dynamics of the economy are governed by the policy functions in the left panel. In the first period back in the normal state, the lagged Lagrange multiplier is positive, which implies positive output at time one. As the time rolls on, the Lagrange multiplier gradually declines, eventually returns to zero.

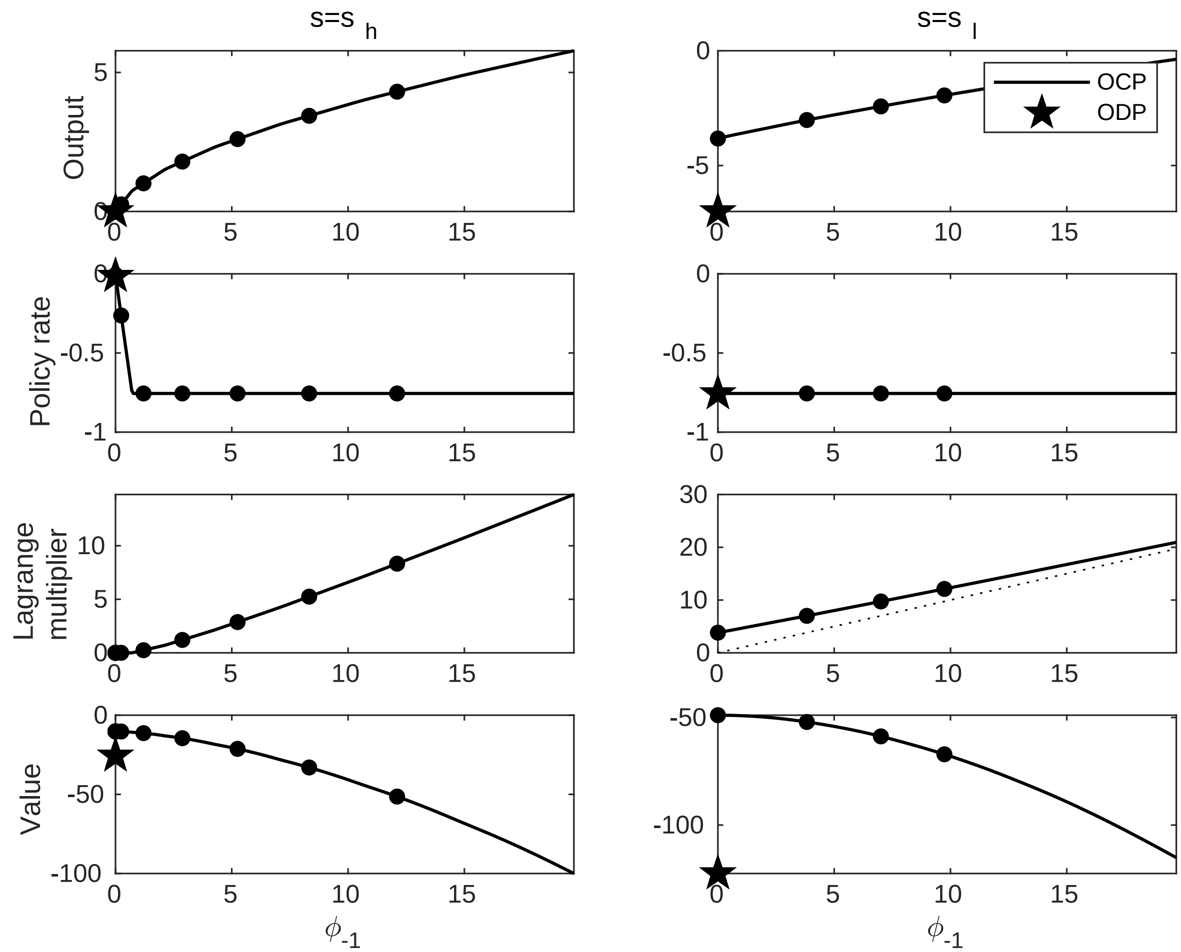
E.2 Markov Perfect Policy

It is useful to understand the dynamics of the economy under the Markov perfect policy in a similar manner. Although the value and policy functions from the Markov perfect equilibrium are functions of the crisis shock only, one can also see them as a flat function of the Lagrange multiplier, which are shown by the solid blue lines in Figure 8. When the economy moves to the crisis state, the allocations are given by the black pentagram in the right panels of the figure. When the economy is back in the normal state, the allocations are given by the black pentagram in the left panel of the figure.

E.3 Optimal Sustainable Policy

Value/policy functions associated with the optimal sustainable policies with $N = 60$ and $N = 100$ are shown by the solid black and blue lines in Figure **??** , respectively. The dynamics of the

Figure 8: Value/policy functions
—OCP and ODP—



economy under the scenario considered above are traced by black dots and pentagrams for $N = \infty$ and $N = 120$.

$N = \infty$:

In the crisis state, the dynamics of the economy are given by [One or two paragraphs carefully describing the dynamics]

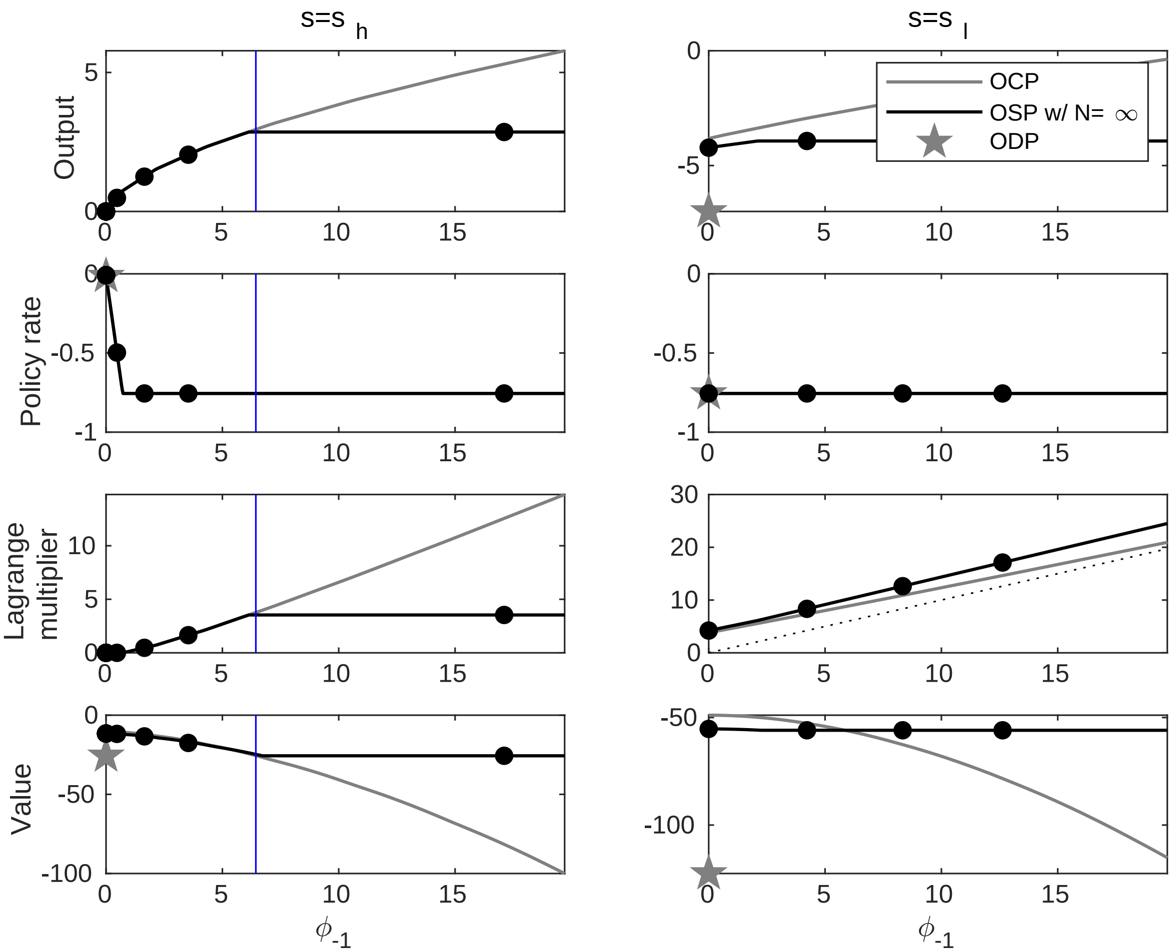
$N = 120$:

[two or three paragraphs carefully describing the dynamics; explain what makes the dynamics history independent]

F Alternative solution method when N is low

The previous section described why the dynamics are history-independent when N is sufficiently small and suggested that, in that case, the dynamics of the economy are fully characterized by a vector of scalars solving a system of nonlinear equations and satisfying certain inequality constraints.

Figure 9: Value/policy functions
—OSP with $N = \infty$—



As a result, it is not necessary to rely on the time-iteration method to find the dynamics of the model. In this section, we provide the details of the solution algorithm we use when the economy's dynamics are history dependent.

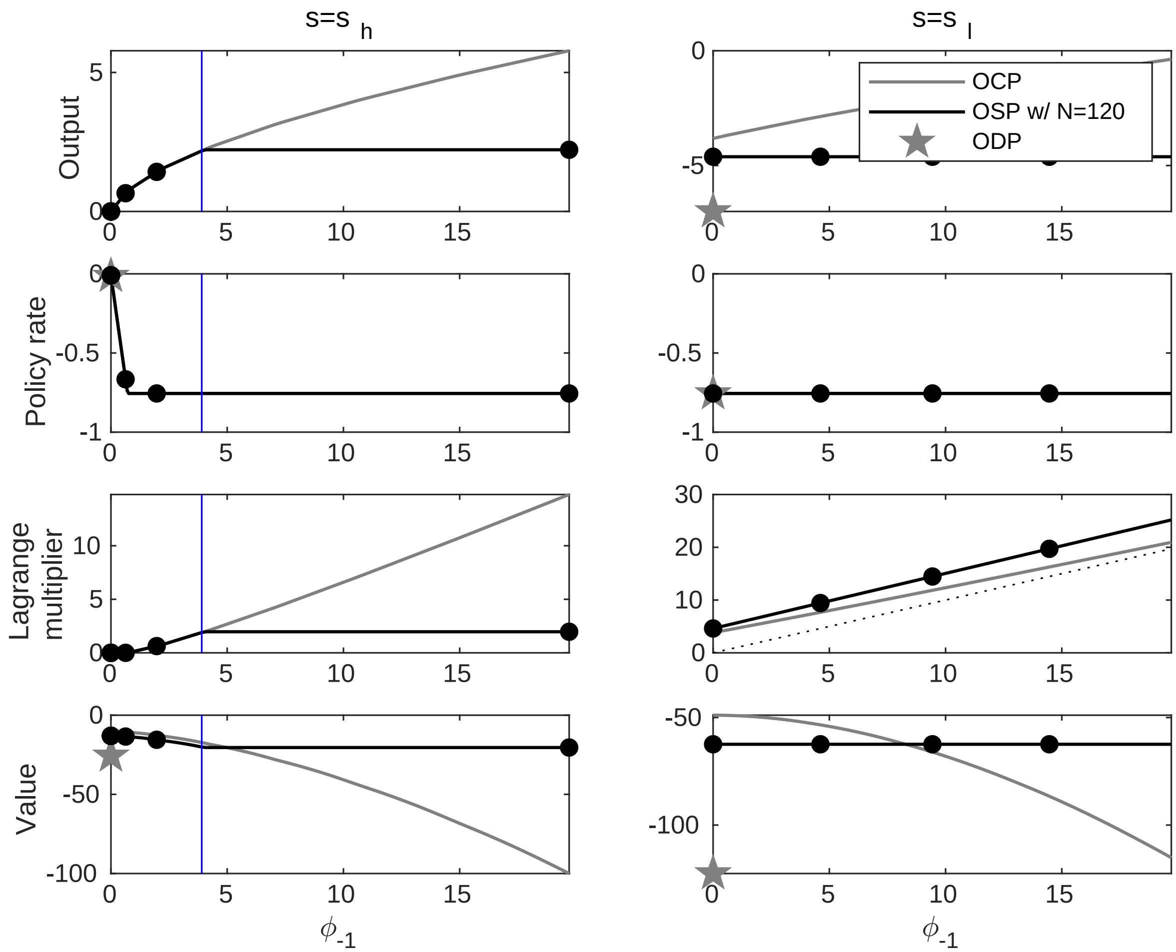
The big picture of the solution algorithm is as follows. Assuming that the additional ELB duration, denoted by τ, is t. Compute the allocation satisifying the model's equilibrium conditions that are captured by equality constraints, . Check (i) whether the model's inequality constraints (ELB and sustainability constraints) are satisfied and (ii) whether the history-independence assumption is indeed valid. If these two requirements are met, then we have found the solution. If not, continue to search for the value of τ that satisfies all the equilibrium conditions, both those represented by equality and inequality constraints.

Below, we describe the set of equality and inequality constraints the solution has to satisfy given τ. We also describe how to verify that the history-independence assumption is satisfied.

Case 1: $\tau = 0$ (liftoff occurs immediately after the crisis shock disappears)

We describe how to find the solution to the model if the economy's dynamics are history

Figure 10: Value/policy functions
—OSP with $N = 120$—



independent and the policy rate is above the ELB right after the crisis shock disappears. In this case, the dynamics of the economy are fully characterized by $\{i_H, i_M, i_L, y_H, y_M, y_L, v_H, v_M, v_L\}$ where the subscript denote one of the three states of this economy:

- In L state, the crisis shock is present. Only the ELB constraint is binding.
- In M state, the crisis shock is absent. Only the sustainability constraint is binding. This state follows L state.
- In H state, the crisis shock is absent. No constraint is binding.

By definition of this case, the following equality and inequality constraints must be satisfied:

$$i_H > -r^*$$
$$i_M > -r^*$$
$$i_L = -r^*$$

and

$$
\begin{aligned}
v_H &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\
v_M &= v_{Punish,H}(v_H, v_L, y_H, y_L) \\
v_L &> v_{Punish,L}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

where $v_{Punish,H}(v_H, v_L, y_H, y_L)$ and $v_{Punish,L}(v_H, v_L, y_H, y_L)$ are given by:

$$
\begin{aligned}
v_{D,H}^{i-1} &= \beta\left[(1-p_H)v_{D,H}^{i} + p_H v_{D,L}^{i}\right], \\
v_{D,L}^{i-1} &= -y_{D,L}^2 + \beta\left[(1-p_L)v_{D,H}^{i} + p_L v_{D,L}^{i}\right], \\
y_{D,L}^{i-1} &= -\sigma^{-1}\left(-r^* - s_L\right) + (1+\sigma^{-1}\kappa)\left[(1-p_L)y_{D,H}^{i} + p_L y_{D,L}^{i}\right],
\end{aligned}
$$

for $i = 1, ..., K$, where $(v_{D,H}^K, v_{D,L}^K, y_{D,H}^K, y_{D,L}^K) = (v_H, v_L, y_H, y_L)$ and $v_{Punish,H} = v_{D,H}^0$. K is the length of punishment. In this case, the equilibrium conditions are given by:

$$
\begin{aligned}
y_H &= (1+\sigma^{-1}\kappa)\left[(1-p_H)y_H + p_H y_L\right] - \sigma^{-1}\left(i_H - s_H\right), \\
y_M &= (1+\sigma^{-1}\kappa)\left[(1-p_H)y_H + p_H y_L\right] - \sigma^{-1}\left(i_M - s_H\right), \\
y_L &= (1+\sigma^{-1}\kappa)\left[(1-p_L)y_M + p_L y_L\right] - \sigma^{-1}\left(-r^* - s_L\right), \\
v_H &= -y_H^2 + \beta\left[(1-p_H)v_H + p_H v_L\right], \\
v_M &= -y_M^2 + \beta\left[(1-p_H)v_H + p_H v_L\right] = v_{Punish,H}(v_H, v_L, y_H, y_L), \\
v_L &= -y_L^2 + \beta\left[(1-p_L)v_M + p_L v_L\right],
\end{aligned}
$$

where we used $i_L = -r^*$. Because the central bank only cares about output, $y_H = 0$. Thus,

$$
\begin{aligned}
i_H &= \sigma(1+\sigma^{-1}\kappa)p_H y_L + s_H, \\
i_M &= \sigma(1+\sigma^{-1}\kappa)p_H y_L + s_H - \sigma y_M, \\
y_L &= (1+\sigma^{-1}\kappa)\left[(1-p_L)y_M + p_L y_L\right] - \sigma^{-1}\left(-r^* - s_L\right), \\
v_H &= \beta\left[(1-p_H)v_H + p_H v_L\right], \\
v_M &= -y_M^2 + \beta\left[(1-p_H)v_H + p_H v_L\right], \\
v_L &= -y_L^2 + \beta\left[(1-p_L)v_M + p_L v_L\right], \\
v_M &= v_{Punish,H}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

There are 7 equations and 7 unknowns: $(i_H, i_M, y_M, y_L, v_H, v_M, v_L)$.
Guess (y_L, v_L). Then,

- From the first three equations, we can determine (i_H, i_M, y_M).
- From the fifth and sixth equations, we can determine (v_H, v_M).

We need to check whether the fourth and seventh equations hold:

$$
\begin{aligned}
v_H &= \beta\left[(1-p_H)v_H + p_H v_L\right], \\
v_M &= v_{Punish,H}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

That is, we can basically reduce the system to two-unknowns in two equations. Once you solve the system of equations, we need to verify the following four inequalities:

$$\begin{aligned} i_H &> -r^* \\ i_M &> -r^* \\ v_H &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\ v_L &> v_{Punish,L}(v_H, v_L, y_H, y_L) \end{aligned}$$

When $\tau = 0$, the Lagrange multiplier is zero in the first period after the crisis shock disappears. Thus, the dynamics of the economy cannot depend on the realized duration of the shock. That is, the economy's dynamics are history independent by construction.

Case 2: $\tau > 0$ (liftoff occurs at least two periods after the crisis shock disappears)

In this section, we discuss how to solve for cases in which the policy rate stays at the ZLB for at least one period after the crisis shock disappears:

- In L state, the crisis shock hits the economy. Only the ZLB constraint is binding.
- In M_1 state, the crisis shock is absent. Both the ZLB and sustainability constraints are binding. This state follows L state.
- In M_i state from $i = 2$ to $i = \tau - 1$, the crisis shock is absent. Only the ZLB constraint is binding. This state follows M_{i-1} state.
- In M_τ state, the crisis shock is absent. No constraint is binding. This state follows $M_{\tau-1}$ state.
- In H state, the crisis shock is absent. No constraint is binding. And, output gap is zero.

By construction, the following equality and inequality constraints must be satisfied:

$$\begin{aligned} i_H &> -r^* \\ i_{M,\tau} &> -r^* \\ i_{M,\tau-1} &= -r^* \\ &\dots \\ i_{M,1} &= -r^* \\ i_L &= -r^* \end{aligned}$$

and

$$\begin{aligned} v_H &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\ v_{M,\tau} &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\ &\dots \\ v_{M,2} &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\ v_{M,1} &= v_{Punish,H}(v_H, v_L, y_H, y_L) \\ v_L &> v_{Punish,L}(v_H, v_L, y_H, y_L) \end{aligned}$$

where $v_{Punish,H}(v_H, v_L, y_H, y_L)$ and $v_{Punish,L}(v_H, v_L, y_H, y_L)$ are given by:

$$\begin{aligned}
v_{D,H}^{i-1} &= \beta\left[(1-p_H)v_{D,H}^{i} + p_H v_{D,L}^{i}\right],\\
v_{D,L}^{i-1} &= -y_{D,L}^2 + \beta\left[(1-p_L)v_{D,H}^{i} + p_L v_{D,L}^{i}\right],\\
y_{D,L}^{i-1} &= -\,\sigma^{-1}\left(-r^* - s_L\right) + (1+\sigma^{-1}\kappa)\left[(1-p_L)y_{D,H}^{i} + p_L y_{D,L}^{i}\right],
\end{aligned}$$

for $i = 1, ..., K$, where $(v_{D,H}^K, v_{D,L}^K, y_{D,H}^K, y_{D,L}^K) = (v_H, v_L, y_H, y_L)$ and $v_{Punish,H} = v_{D,H}^0$. K is the length of punishment. In this case, the equilibrium conditions are given by:

$$\begin{aligned}
&y_H = (1+\sigma^{-1}\kappa)\left[(1-p_H)y_H + p_H y_L\right] - \sigma^{-1}\left(i_H - s_H\right),\\
&y_{M,\tau} = (1+\sigma^{-1}\kappa)\left[(1-p_H)y_H + p_H y_L\right] - \sigma^{-1}\left(i_{M,\tau} - s_H\right),\\
&y_{M,\tau-1} = (1+\sigma^{-1}\kappa)\left[(1-p_H)y_{M,\tau} + p_H y_L\right] - \sigma^{-1}\left(-r^* - s_H\right),\\
&...\\
&y_{M,k} = (1+\sigma^{-1}\kappa)\left[(1-p_H)y_{M,k+1} + p_H y_L\right] - \sigma^{-1}\left(-r^* - s_H\right),\\
&...\\
&y_{M,1} = (1+\sigma^{-1}\kappa)\left[(1-p_H)y_{M,2} + p_H y_L\right] - \sigma^{-1}\left(-r^* - s_H\right),\\
&y_L = (1+\sigma^{-1}\kappa)\left[(1-p_L)y_{M,1} + p_L y_L\right] - \sigma^{-1}\left(-r^* - s_L\right),
\end{aligned}$$

and

$$\begin{aligned}
&v_H = -y_H^2 + \beta\left[(1-p_H)v_H + p_H v_L\right],\\
&v_{M,\tau} = -y_{M,N}^2 + \beta\left[(1-p_H)v_H + p_H v_L\right],\\
&v_{M,\tau-1} = -y_{M,\tau-1}^2 + \beta\left[(1-p_H)v_{M,\tau} + p_H v_L\right],\\
&...\\
&v_{M,k} = -y_{M,k}^2 + \beta\left[(1-p_H)v_{M,k+1} + p_H v_L\right],\\
&...\\
&v_{M,2} = -y_{M,2}^2 + \beta\left[(1-p_H)v_{M,3} + p_H v_L\right],\\
&v_{M,1} = -y_{M,1}^2 + \beta\left[(1-p_H)v_{M,2} + p_H v_L\right],\\
&v_L = -y_L^2 + \beta\left[(1-p_L)v_{M,1} + p_L v_L\right],\\
&v_{M,1} = v_{Punish,H}(v_H, v_L, y_H, y_L)
\end{aligned}$$

Because the central bank only cares about output stabilization, $y_H = 0$. Using $y_H = 0$, we obtain

$$
\begin{aligned}
&i_H = \sigma(1+\sigma^{-1}\kappa)p_H y_L + s_H, \\
&i_{M,\tau} = \sigma(1+\sigma^{-1}\kappa)p_H y_L + s_H - \sigma y_{M,\tau}, \\
&y_{M,\tau-1} = (1+\sigma^{-1}\kappa)\left[(1-p_L)y_{M,\tau} + p_L y_L\right] - \sigma^{-1}\left(-r^* - s_H\right), \\
&... \\
&y_{M,k} = (1+\sigma^{-1}\kappa)\left[(1-p_L)y_{M,k+1} + p_L y_L\right] - \sigma^{-1}\left(-r^* - s_H\right), \\
&... \\
&y_{M,2} = (1+\sigma^{-1}\kappa)\left[(1-p_L)y_{M,3} + p_L y_L\right] - \sigma^{-1}\left(-r^* - s_H\right), \\
&y_{M,1} = (1+\sigma^{-1}\kappa)\left[(1-p_L)y_{M,2} + p_L y_L\right] - \sigma^{-1}\left(-r^* - s_H\right), \\
&y_L = (1+\sigma^{-1}\kappa)\left[(1-p_L)y_{M,1} + p_L y_L\right] - \sigma^{-1}\left(-r^* - s_L\right),
\end{aligned}
$$

and

$$
\begin{aligned}
&v_H = \beta\left[(1-p_H)v_H + p_H v_L\right], \\
&v_{M,\tau} = -y_{M,\tau}^2 + \beta\left[(1-p_H)v_H + p_H v_L\right], \\
&v_{M,\tau-1} = -y_{M,\tau-1}^2 + \beta\left[(1-p_H)v_{M,\tau} + p_H v_L\right], \\
&... \\
&v_{M,k} = -y_{M,k}^2 + \beta\left[(1-p_H)v_{M,k+1} + p_H v_L\right], \\
&... \\
&v_{M,2} = -y_{M,2}^2 + \beta\left[(1-p_H)v_{M,3} + p_H v_L\right], \\
&v_{M,1} = -y_{M,1}^2 + \beta\left[(1-p_H)v_{M,2} + p_H v_L\right], \\
&v_L = -y_L^2 + \beta\left[(1-p_L)v_{M,1} + p_L v_L\right], \\
&v_{M,1} = v_{Punish,H}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

We are solving for $(y_L, i_H, i_{M,\tau})$, $\{y_{M,i}\}_{i=1}^{\tau}$, (v_H, v_L) and $\{v_{M,i}\}_{i=1}^{\tau}$. $2\tau+5$ unknowns in $2\tau+5$ equations.

Guess (y_L, v_L). Then,

- From the first $\tau+2$ equations, we can determine $(i_H, i_{M,\tau})$ and $\{y_{M,i}\}_{i=1}^{\tau}$.
- From the equations for $v_{M,i}$ and v_L ($\tau+1$ equations), we can determine v_H and $\{y_{M,i}\}_{i=1}^{\tau}$.

Then, we need to check whether the following two equations hold:

$$
\begin{aligned}
&v_H = \beta\left[(1-p_H)v_H + p_H v_L\right], \\
&v_M = v_{Punish,H}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

That is, we can basically reduce the system to two unknowns in two equations. Once you solve the system of equations, we need to verify the following $N+3$ inequalities:

$$\begin{aligned} i_H &> -r^* \\ i_{M,N} &> -r^* \\ v_H &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\ v_{M,\tau} &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\ &\dots \\ v_{M,2} &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\ v_L &> v_{Punish,L}(v_H, v_L, y_H, y_L) \end{aligned}$$

Finally, we need to check whether the dynamics of the economy are indeed history independent. As we saw in the previous section, whether the history independence assumption is valid depends on whether the value of the Lagrange multiplier in the first period of the crisis state is larger than a certain cutoff value. While we need to know policy functions to know the value of this cutoff value, we can find the approximate cutoff value consistent with the solution we just computed.

G Tenure duration of leadeship at central banks

Table 5: Average Tenure Duration of Chairpersons in Select Central Banks

Central Bank	Year of foundation	No. of leaders since foundation	No. of leaders since 1946	Average tenure since foundation	Average tenure since 1946
Federal Reserve System	1914	16	10	6.9	8.1
European Central Bank	1998	3	3	6.5	6.5
Bank of Canada	1934	9	8	9.9	9.7
Bank of Japan	1882	31	15	4.4	4.9
Bank of England	1694	120	9	2.7	8.5
Sveriges Riksbank	1901	14	11	8.1	6.1
Swiss National Bank	1907	14	10	8.1	7.4

Table 6: Maximum Tenure Duration of Chairpersons in Select Central Banks

Central Bank	Max duration since foundation	Max duration since 1946
Federal Reserve System	18 yrs and 10 months (Martin)	18 yrs and 10 months (Martin)*
European Central Bank	8 (Trichet)	8 (Trichet)
Bank of Canada	20 yrs and 4 months (Towers)	14 (Boey)
Bank of Japan	8 yrs and 6 months (Ichimada)	8 yrs and 6 months (Ichimada)
Bank of England	24 (Norman)	12 (Cobbold)
Sveriges Riksbank	19 (Rooth)	18 (Asbrink)
Swiss National Bank	14 (Bachmann)	11 (Leutwiler)

Note: The tenure of Alan Greenspan lasted for 18 years and 6 months.

H Time-inconsistency of the commitment policy in the words of policymakers

The time-inconsistency of the commitment policy at the ELB is not a mere theoretical curiosity. Policymakers in many central banks have pointed out the potential time-inconsistency of the commitment-type forward guidance policy. Some have argued that the time-inconsistency is one key reason for why most central banks refrained from making the overheating commitment. Below are some examples:

H.1 Bean (2013)

"In particular, we signalled our intention not to countenance tightening policy until unemployment has fallen to at least 7 percent."

"**This guidance is intended primarily to clarify our reaction function and thus make policy more effective, rather than to inject additional stimulus by pre-committing to a time-inconsistent lower for longer' policy path in the manner of Woodford (2012). While such a time-inconsistent policy may be desirable in theory, in an individualistic committee like ours, with a regular turnover of members, it is not possible to implement a mechanism that would credibly bind future members in the manner required.**"

H.2 Bullard (2013)

"The New Keynesian, sticky price literature has been influential in U.S. monetary policymaking. The literature has been led by Michael Woodford. This line of research argues that policy accommodation can be provided even when the policy rate is near zero. The extra accommodation comes from a promise to maintain the near zero policy rate into the future, beyond the point when ordinary policymaker behavior would call for an increase in the policy rate. This promise must be credible to have an impact.

The "Woodford period" approach to forward guidance relies on a credible announcement made today that future monetary policy will deviate from normal. The central bank does not actually behave differently today. One might argue that such an announcement is unlikely to be believed. Why should future monetary policy deviate from normal once the economy is growing and inflation is rising? But if the announcement is not credible, then the private sector will not react with more consumption and investment today. That is, any effects would be minimal."

H.3 Carney (2012)

"Today, to achieve a better path for the economy over time, a central bank may need to commit credibly to maintaining highly accommodative policy even after the economy and, potentially, inflation picks up. **Market participants may doubt the willingness of an inflation-targeting central bank to respect this commitment if inflation goes temporarily above target. These doubts reduce the effective stimulus of the commitment and delay the recovery.**"

H.4 Cœuré (2013)

"Most notably, the central bank may try to convince markets that it would keep interest rates low, even if this would imply inflation well above its previous objective, at least temporarily. The promise of higher future inflation, if credible, induces private agents to substitute future for current consumption, hence providing additional stimulus today. This type of forward guidance is closer to the academic concept of forward guidance in the strict sense—as discussed, for example, in Woodford (2012).

The main challenge of such guidance is its inherent inconsistency over time and thus lack of credibility. When the time comes, the central bank may be tempted to deviate from its prior commitment: once the benefits of higher inflation expectations in terms of front-loaded spending have been reaped, the central bank may not be willing to pay the bill in terms of higher inflation afterwards. If the public foresees this temptation, expectations might remain unaffected in the first instance and the desired inter-temporal substitution of spending might not materialise. This is a possible explanation why, in practice, central banks have refrained from using forward guidance in a way that implies a major change in strategy. Therefore, central banks' forward guidance has rather aimed at providing greater clarity on the reaction function and the assessment of future economic conditions."

H.5 Dudley (2013)

"With respect to forward guidance, it is important to distinguish between two specific forms that this guidance may take. In the first form the central bank provides its forecast for the future path of the policy rate and, possibly, some sense of the degree of uncertainty around this path. In the second, the central bank pre-commits to a specific future path for its policy rate.

Providing a forecast for the policy rate by itself does not create any budget or reputational risk for the Federal Reserve, so I generally do not see the first form of forward guidance as posing much risk to central bank independence.

The second form of forward guidance—pre-commitment to a policy rate path—could create more risk for the central bank. In particular, consider a scenario in which the central bank decided to increase monetary accommodation by committing to maintain a low short-term interest rate for a long time even if this commitment resulted in inflation overshooting the central bank's objective in the future. I could see how this could create a potential threat to the central bank's independence. That is because the commitment could force the central bank in the future to conduct monetary policy in a way that was inconsistent with the inflation portion of its mandate. Although this second form of forward guidance could create greater risk for the central bank with respect to its future independence, this is not a policy that has been adopted by the Federal Reserve. There are implementation challenges with this approach. **In particular, it is difficult for a monetary policy committee today to institutionally bind future monetary policy committees to follow actions that could conflict with their objectives in the future. Without such a credible forward commitment, such policies would likely be ineffective in affecting expectations in the manner needed to provide additional monetary policy accommodation.**"

H.6 Lacker (2013)

"Designing such conditional guidance involves trade-offs, however. Credibility requires consistency, over time, between a central bank's statements and its actual subsequent actions. A central bank's statements will have greater immediate effect on the public's expectations the more they are seen as limiting the central bank's future choices. Yet there are likely to be circumstances, ex post, in which the central bank feels constrained by past statements. Yielding to the temptation to implicitly renege by reworking decision criteria or citing unforeseen economic developments may have short-term appeal, but widely perceived discrepancies between actual and foreshadowed behavior will inevitably erode the faith people place in future central bank statements. So central banks face an ex ante trade-off, as well, between the short-run value of exercising discretion and the ability to communicate effectively and credibly in the future."

H.7 Plosser (2013)

"Note, however, that the central bank's ability to influence the public's belief about the future path of policy and the economy depends critically on the bank's commitment to that policy path and the credibility of that commitment in the eyes of the public. **The public must believe that even after the economy begins to strengthen, the central bank will hold rates lower than it otherwise might have found desirable to do had it not been at the zero bound in the past.**"

H.8 Ueda (2013)

"If Max (the Taylor rule rate, zero) describes the usual central bank's reaction function to the macroeconomic environment, the central bank can generate easing effects by offering a new reaction function to the market with a promise of a longer period at the zero rate than the above rule suggests. **To the extent that the Taylor rule represents an optimal response of the central bank to macroeconomic environment, however, this forward guidance strategy amounts to "irresponsible" central bank behaviour. In other words, the strategy is time-inconsistent. This means that when the economy no longer requires a zero rate, it is better to raise the interest rate, reneging on the promise made. If people foresaw this ex ante, however, the strategy would become ineffective.** Thus, the central bank would be sending a confusing signal if it was using forward guidance in this sense and insisted that it was still behaving in a "responsible" way. Also, the central bank does not seem to get much mileage out of a vague promise, such as the maintenance of a low policy rate "for an extended period," unless there is much confusion in the market as to where the policy rate would go in the short term.

The BOJ seems to have faced the time-inconsistency problem in 2000."

H.9 Williams (2012)

"Although forward policy guidance has proven to be a very useful policy tool, it's not a perfect substitute for the kind of monetary stimulus that comes from lower interest rates. One issue is that, for the forward guidance policy to work as desired, the public has to believe that the FOMC will really carry out the policy as it says it will. But, the Fed doesn't have the ability to tie its hands that way. This point was made by Finn Kydland and Edward Prescott in the late 1970s. Let me explain. For forward policy guidance to have its maximum effect, the Fed must commit to keeping the short-term policy rate lower than it otherwise would to compensate for the fact that the short-term interest rate cannot be lowered today. **But when the time comes to carry out the commitment made in its forward guidance, it may no longer want to do so. For instance, it might be hard to resist raising rates earlier than promised to head off an increase in inflation. So, even when central bankers say they will keep rates unusually low for a set time, the public may worry that the central bank will raise rates earlier to fight budding inflation pressures.**"