Reallocation Effects of Monetary Policy*

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Abstract

Japanese firm-level data show a relation between firm dynamics and inflation: large firms tend to grow faster than small firms under higher inflation. Motivated by this fact, we construct a model that introduces nominal rigidity into endogenous growth with heterogeneous firms. The model shows that, under a high nominal growth rate, firms inferior in quality bear heavier burdens of menu cost payments than firms superior in quality. This causes superior firms to obtain larger market shares, while some of inferior firms exit from the market. This reallocation effect, if strong, yields a positive effect of monetary expansion on both real growth and welfare. The optimal nominal growth can be strictly positive even under nominal rigidity, whereas standard New Keynesian models often conclude that zero nominal growth is optimal. Moreover, the presence of menu costs improves welfare when the reallocation effect is sufficiently large.

Keywords: Reallocation; firm dynamics; creative destruction; menu cost; optimal inflation rate

JEL classification: E5, O3, O4

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1 Introduction

This study investigates the reallocation effects of monetary policy in a framework that introduces nominal rigidity into endogenous growth with heterogeneous firms. The literature on reallocation or firm dynamics has emphasized that there exists sizable heterogeneity among firms, plants, and establishments even within narrowly defined industries (e.g., Baily et al. (1992), Bartelsman and Doms (2000), and Bartelsman et al. (2004)). Firm dynamics has a significant implication on macroeconomic performance (e.g., Caballero and Hammour (1994), Hsieh and Klenow (2009), Restuccia and Rogerson (2013), and Hsieh and Klenow (2017)). However, these previous studies do not pay attention to the nominal side of the economy, although it is extremely important to consider how monetary policy influences firm dynamics when we discuss optimal monetary policy or the optimal inflation target.

In this study, we construct a model incorporating the following three features: Schumpeterian creative destruction, multi-product firms, and menu costs. The Schumpeterian creative-destruction model developed by Aghion and Howitt (1992) and Grossman and Helpman (1991) enables us to study firm entry and exit as well as endogenous growth. In the basic model, however, each firm is assumed to produce only one product. For all firms, the amount of sales is the same and, thus, a firm-size distribution is degenerate. The second feature, multi-product firms, enables us to analyze firm dynamics. Klette and Kortum (2004) build a model in which incumbent firms can produce more than one product by continuing R&D investment and taking over products supplied by other firms. This leads to resource reallocation from inferior to superior firms and ex post heterogeneity in firm size. Lentz and Mortensen (2005, 2008) introduce ex ante heterogeneity in innovativeness of firms as well as selection after entry to investigate the effect of resource reallocation. The third feature, menu costs, is our theoretical innovation. We shed new light on reallocation stemming from the nominal side of the economy, such as monetary policy, by extending the Lentz and Mortensen (2005) model to introduce menu costs as nominal rigidity (e.g., Sheshinski and Weiss (1977), Golosov and Lucas (2007), and Midrigan (2011)). We adopt menu costs for nominal rigidity because they are a fundamental setup in the New Keynesian literature and we believe they are more realistic than Calvo- or Rotemberg-type pricing models. One of the contributions of this study is that it provides a tractable framework to consider the reallocation effects of monetary policy.

Our study has an empirical relevance. Using Japanese firm-level data, we find that firm dynamics is associated with, and also likely to be influenced by, inflation rates. Specifically, the real growth of large firms in terms of both sales and employment tends to be higher than that of small firms under a higher inflation rate. This result is consistent with our model prediction.

The main results from our model are as follows. First, nominal growth can enhance real growth depending on circumstances. Second, the optimal nominal growth (and inflation) rate can be positive. These results are in a sharp contrast to the results of standard New Keynesian models in which the economy is the most efficient without nominal growth (i.e., the optimal inflation target is zero if the real growth rate is zero, see Goodfriend and King (1997), Khan et al. (2003), Burstein and Hellwig (2008), Schmitt-Grohé and Uribe (2010), and Coibion et al. (2012)). Although nominal rigidity incurs a cost for all firms, the burden is relatively larger for lower-quality firms and, thus, higher-quality firms increase market shares as well as relative R&D expenditures. As a result, the average quality of firms increases under positive nominal growth. If this reallocation effect dominates inefficiency stemming from nominal rigidity, the optimal nominal growth (and inflation) rate can be positive. The key to the welfare implication is heterogeneity in negative externality of R&D, or the business stealing effect, in Schumpeterian growth models. A heavier menu cost burden crowds out low-quality firms whose innovations only provide small social benefits relative to private benefits. Put differently, menu costs serve as a quality-dependent entry barrier. Therefore, nominal rigidity is not necessarily a source of inefficiency but a benefit to the economy. Our numerical simulation suggests that higher menu costs are better in terms of welfare.

The model calibrated to Denmark based on Lentz and Mortensen (2008) reveals that the optimal nominal growth is indeed positive. Under higher nominal growth, the ratio of the sales of large firms to those of small firms increases, which is consistent with our empirical finding. Further, both the average firm size and the average quality of firms increase. However, this is not always the case. The optimal nominal growth rate becomes zero when we assume a uniform distribution for ex ante quality instead of a Pareto distribution. The optimal rate is zero also when there are a large mass of potential entrants, because it decreases incumbents' growth possibility and weakens reallocation effects.

Some existing studies argue that nominal factors such as inflation and nominal interest rates affect both real growth and welfare in endogenous growth models (e.g, Bilbiie et al. (2014), Chu and Cozzi (2014), Oikawa and Ueda (2015), Arawatari et al. (2016), and Chu et al. (2017)). For example, Chu and Cozzi (2014) show that the optimal nominal interest rate is strictly positive if R&D is overinvested in a Schumpeterian model with cash-in-advance (CIA) in R&D. Compared with these studies, our study is the first work to combine a New Keynesian model with an endogenous growth model embedding reallocation and firm dynamics. Thus, we can connect our model to the vast literature on monetary policy based on New Keynesian models where money is absent. Moreover, our model does not need a case for overinvestment in R&D, which rarely occurs under standard parameter values, to have positive optimal nominal growth if there exist sufficient reallocation effects.[1]

To study reallocation effects of monetary policy, other types of models may be useful (e.g., Melitz (2003), Bernard et al. (2003), Luttmer (2007), Restuccia and Rogerson (2013), and Lucas and Moll (2014)). Compared with these models, our model explicitly incorporates firms' R&D investment decision. As documented in Malerba and Orsenigo (1999), the majority of newly granted patents are held by the existing patent holders. Argente et al. (2018) report that firms grow by continuously adding products and that incumbents are the main agents of product reallocation. In our model, R&D plays the key role in firm growth as well as endogenous economic growth.

Our paper also complements the growing literature on the effects of monetary policy on heterogenous agents. For example, Iacoviello (2005) and Gornemann et al. (2016) investigate monetary policy effects on different types of households. A notable difference from these studies is that the focus of our study is not on household heterogeneity but on firm heterogeneity.

Acemoglu et al. (2017) is also related to our study although they do not consider the nominal aspect. They analyze several industry policies, e.g., R&D subsidy, by extending the model of Lentz and Mortensen (2008). One of the main results in their paper is that tax should be imposed only on inferior incumbents to achieve the social optimum. However, as they mention, governments hardly conduct such a selective policy in reality. By contrast, our model suggests that monetary policy that is not

[1] Oikawa and Ueda (2015) is a homogeneous-firm version of the current model (thus, no reallocation). They calibrate the model to the US economy and conclude that there is underinvestment in R&D and that the optimal nominal growth rate is zero.

selective has different impacts across firms: inferior firms are incurred more burdens of menu cost payments under high nominal growth than superior firms. In other words, monetary policy automatically works as an entry barrier only for low quality firms.

The rest of the paper is organized as follows. After Section 2 provides empirical findings, Section 3 builds a theoretical model and Section 4 defines stationary equilibrium. Section 5 investigates the reallocation impacts of nominal growth. Section 6 shows the results of numerical simulations. Section 7 concludes.

2 Empirical Evidence on the Relationship between Inflation and Firm-size Distributions

Before going into the theoretical model, we provide some empirical evidence on the relationship between inflation and firm-size distributions. We first briefly review existing literature and then present observations from Japanese firm-level data.

Existing Literature

Although empirical studies using firm- or plant-level data are growing rapidly, studies on the relation between the nominal side of the economy and firm-size distributions are scarce, as there is almost no theoretical study on this issue. However, some indirect evidence seems to suggests a relation between a firm distribution and inflation. For example, Alfaro et al. (2008) report that less-developed countries, notably India, tend to have a greater mean establishment size than advanced countries. Similar findings are also reported by Bartelsman et al. (2004) and Poschke (2014). Considering that less-developed countries tend to have relatively high inflation rates, this fact seems to suggest that there exists some association between inflation and the growth of large firms relative to small ones in a cross-country dimension.

It should be noted, however, that there are considerable rooms for concerns as to how accurate the measurement of firm distributions or what drives different firm distributions.[2] To alleviate these concerns, we next analyze the relation between in-

[2]There are considerable measurement errors for firm- or plant-distributions due to the large cost of collecting data and the presence of informal small establishments. The latter problem is serious particularly in less-developed countries, because there are a large mass of informal small establishments. Thus, the more we include informal small establishments as *firms*, the less the share

flation and firm-size distributions using Japanese firm-level data. Policies and institutions are considered to be less heterogenous within a country than between countries. We control sectoral differences and time-series changes in policies, institutions and measurement errors, by using industry and year dummies. Furthermore, by using instrument variables, we aim to study if there is any causality from inflation to firm-size distributions.

Japanese Firm-level Data

As firm-level data, we use the Basic Survey of Japanese Business Structure and Activities (BSJBSA) surveyed by Ministry of Economy, Trade, and Industry in Japan. The survey covers all firms with no less than 50 employees and no less than 30 million Japanese yen in capital in Japan for 1991 and 1994-2015. We focus on manufacturing firms and calculated percentile points of nominal sales within each major division of industries[3] to see changes in firm size distribution. For inflation rates, we use industry-level input price indexes from the Producer Price Index (PPI) reported by the Bank of Japan. We consider the average input inflation during the past two years as the main explanatory variable, to be consistent with the theoretical model in the next section.

Table 1(a) shows the panel estimation results on the relationship between input inflation, $\bar{\pi}^{\text{input}}$, and sales dispersions, which are measured by the ratios of 90 percentile to median (Top/Middle) and 90 percentile to 10 percentile (Top/Bottom).

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of big firms, which may lead to the opposite result: less-developed countries have a smaller mean firm size than advanced countries. This issue is related to the so-called missing middle. Tybout (2014) calculates the deviation of employment shares from the estimated Pareto distribution by country and plant size category and find that, for India and Indonesia, the smallest and largest size categories are more populated than predicted by the Pareto distribution, whereas for the United States, the largest size category is less populated. The issue of missing middle is actually consistent with our story. In our model, a firm is an agent who makes R&D investment to invent new ideas and employs people for both production and R&D. This type of firm is not like an informal small establishment. For this reason, we exclude informal small establishments and pay our attention to the large and medium size category in Tybout (2014). This fact suggests that, in less-developed, that is, in more-inflationary countries, large firms are relatively more common than medium firms, which is consistent with our model prediction. Another important note is that cross-country differences in firm distributions are caused by many other factors as well as differences in the nominal side of the economy. Clearly, policies and institutions differ country by country.

[3]We follow industry classification used in the Producer Price Index. Because the classification has been revised a couple of times during the sample periods, we basically use the 2005 industry classification to obtain consistent price index sequences. The number of industries used in the current analysis is 14.

As seen in the basic OLS regression, columns (1) and (4), coefficients on inflation are positive and significant. Sales dispersion is increased under input inflation. For robustness, we introduce other control variables such as industry-level financing position index, D.I. from *Tankan*,[4] and industry-level real sales, industry RS, as shown in columns (2) and (5).[5] The D.I. is a proxy for the degree of financial constraint (the higher D.I., the looser constraint). We control the degree of financial constraint because it affects the size distribution if large firms have more access to finance. In the table, "D.I." is the industry-level index and "D.I. gap" is the gaps in D.I. between large- and medium-size firms (T/M) and large- and small-size firms (T/B). We include the industry-level real sales to control aggregate real factors. The table shows that coefficients on inflation are positive.

These impacts on firm size distribution from inflation may be subject to an endogeneity problem because high demand for inputs from large growing firms could drive up input prices in the industry. In columns (3) and (6), we do 2SLS with international primary commodity price as the instrument variable, which is reported by the IMF (converted to Japanese yen by the average nominal exchange rate in each year). Because the commodity price is an aggregate variable, we set the instrument as the cross terms of industry dummies and inflation in primary commodity prices during the past two years. The positive association between inflation and sales dispersion is significant again. We also see that the industry level D.I. is negatively correlated with sales dispersion, implying that less financial constraint shrinks the size gap among firms.

Table 1(b) shows an estimation result when we measure firm sizes by total employment. The result is similar to the sales dispersion. Employment dispersion is also positively associated with input inflation, and it is robust when we consider other control variables and endogeneity.

There also exists a significant relationship between firm growth and inflation. We divide firms into ten size groups according to firm size distribution in terms of sales or employment. Then, we calculate the average firm growth rates from $t-1$ to t within size groups, based on size distributions at $t-1$. The size group index is integers from

[4]The financing position index is one of the Diffusion Index, or D.I., in *Tankan*, surveyed by the Bank of Japan. It indicates the difference in the shares of firms answering (the financial position is) "easy" and "tight." Data at industry and firm-size (large, medium, and small) levels are available.

[5]Real sales are calculated by dividing nominal sales by the industry-level output price index from the PPI.

1 to 10, where groups 1 and 10 represent firms smaller than the 10 percentile point and those larger than 90 percentile point, respectively. Table 2 shows that, overall, input inflation has a negative impact on firm growth and that larger firm size leads to lower growth.[6] A more important result is seen in the cross term of inflation and size group. In all regressions, the cross term is positive and significant, implying that the negative impact of input inflation on growth is partly offset when a firm is large. This result is consistent with the increased firm size dispersion under inflation, which was observed in Table 1. Columns (3) and (6) are 2SLS with the same IV strategy as in Table 1. We confirm the robustness for the complementarity between firm size and inflation.

In summary, the results from the Japanese firm-level data suggests that there is a significant response in firm distributions to inflation. Below, we build a theoretical model that is consistent with the fact and derive growth and welfare implications of monetary policy through reallocation.

3 Model

The model is based on Lentz and Mortensen (2005) and Oikawa and Ueda (2015). A representative household consumes and supplies a fixed amount of labor. Firms, both potential entrants and incumbents, make R&D investment to develop a new product superior in quality and take over a market. Firms must pay menu costs when they revise their output price as well as when they enter the market and set an initial price. Because of the menu cost payments, some firms decide not to enter the market even if they develop a new product when their product quality is not sufficiently high. Firms are ex post heterogeneous in their product quality and, in turn, firm size.

We focus on a balanced growth path. We assume for simplicity that a central bank exogenously sets the growth rate of nominal aggregate output, E_t, at n.[7] Denoting the quality-adjusted aggregate price index and its change (inflation rate) by P_t and π, respectively, we can write the real growth rate, g, as $g = n - \pi$. For convenience, we denote the initial values in period $t = 0$ without time subscripts; for example,

[6] There is a huge literature on Gibrat's law in firm growth, stating that firm growth rate is independent of firm size. Empirically, many papers report that size has a negative impact on growth at the firm level. See Santarelli et al. (2006) for the empirical literature survey.

[7] Our results are essentially the same even if we assume that a central bank sets a certain inflation target instead of controlling n. See Oikawa and Ueda (2015).

$E_0 = E$. In the following analysis, we consider the case with $n \geq 0$ because trend nominal growth rates are usually positive. Even in the lost decades in Japan with long-term stagnation periods, the average per-capita nominal GDP growth rate is almost zero.

In the current model setting, we focus on an extensive margin to explain firm growth. By an extensive margin we mean a change in the number of product lines that firms produce. Because our model assumes the unit elasticity of substitution between different product lines and the infinite elasticity of substitution between products in the same product line, as in Aghion and Howitt (1992) and Grossman and Helpman (1991), firms' sales relative to nominal aggregate expenditure never grow as long as firms engage in production of a single product. Changes in an intensive margin, that is, an improvement of productivity or a quality update in the same product line, cannot increase firms' sales. Incumbent firms can grow their sales size only by changing an extensive margin, that is, taking over other firms' share and producing more than one product line. This is why we need multi-product firms to obtain a dispersed firm distribution.[8] Surely, we do not intend to argue that an intensive margin is unimportant. For example, the models in Luttmer (2007) and Lucas and Moll (2014) generate a firm size distribution by an intensive margin, often through heterogeneous and exogenous changes in productivity.[9]

3.1 Household

A representative household has the following preferences over all versions of $a \in \{0, 1, \cdots, A_t(j)\}$ of each product line $j \in [0, 1]$:

$$U_t = \int_t^\infty e^{-\rho(t'-t)} \log C_{t'} dt', \tag{1}$$

$$\log C_{t'} = \int_0^1 \log \left[\sum_{a=0}^{A_{t'}(j)} Q(j, a) x_{t'}(j, a) \right] dj, \tag{2}$$

[8]An extensive margin also plays an important role in reallocation in the model of Melitz (2003), where firms expand their sales via export. Using product- and firm-level data, Argente et al. (2018) show that firms grow only by adding new products.

[9]Our assumption about the elasticity of substitution is for analytical simplicity. Garcia-Macia et al. (2016) argue that one of the main sources of growth is incumbents' quality updates on their own product lines.

where ρ represents the subjective discount rate, C_t is aggregate consumption, and $x_t(j,a)$ and $Q(j,a)$ are consumption and quality of version a of product j, respectively.[10] The quality evolves as

$$Q(j,a) = \prod_{a'=0}^{a} q(j,a'), \qquad q(j,a') > 1 \quad \forall a', j,$$

where $q(j,a')$ is a quality update created by each innovation.

3.2 Firms' Decision about Price Revisions

This part is based on Oikawa and Ueda (2015). The only difference is that a firm has to pay a menu cost when it enters the market.[11] Price setting requires menu cost,

$$\kappa E_t / P_t, \tag{3}$$

at time t in the real term, which is paid by consumption goods.

Suppose that nominal prices are reset at t_i for $i = -1, 0, 1, \cdots$. If nominal price $p_{t_{-1}}$ that is posted at time t_{-1} is not revised thereafter, its real price, say ξ, decreases at the growth rate of nominal production costs (wage, W_t), n. Let $\xi_{t_{-1},\tau} \equiv p_{t_{-1}} e^{-n(t_{-1}+\tau)} \equiv \xi_{-1} e^{-n\tau}$, where τ is the elapsed time from the previous price revision and ξ_i is the revised real price at t_i (i.e., $\xi_{t_i,0} \equiv \xi_i$).[12]

Then, when the price is reset at t, the real value of each product line is expressed as

$$V_t = \max_{\{t_i, \xi_i\}_{i=1}^{\infty}} \sum_{i=0}^{\infty} \left(\int_{t_i}^{t_{i+1}} \Pi\left(\xi_{t_i, t'-t_i} e^{-n(t'-t_i)}\right) \frac{E_{t'}}{P_{t'}} e^{-(\rho+g+\delta)(t'-t)} dt' - \frac{\kappa E_{t_i}}{P_{t_i}} e^{-(\rho+g+\delta)(t_i - t)} \right), \tag{4}$$

where δ is the rate of creative destruction, $\Pi(\cdot)E_t/P_t$ represents the real period profit, and $t_0 = t$. The second term represents menu cost payments at t_i Because of the recursive structure of the price adjustment problem, this maximization problem can

[10]Extending the form of aggregate consumption to the constant elasticity of substitution (CES) form does not change our results as long as the elasticity of substitution between products in two different product lines is not too large. See Lentz and Mortensen (2008).

[11]If menu cost payment is not required at entry to each product line, firms with very low q supply goods until the first timing of price revision and exits the market without revising the initial price. Our assumption is to avoid this complication.

[12]In the current model, we do not consider indexation of final goods prices. The model works unless the nominal prices are perfectly linked to nominal growth rate, n.

be reduced to the following problem with controlling the interval of price revision, Δ, and the reset level of real price, ξ_0, where we drop the absolute time subscript from $\xi_{t_i,\tau}$. Let $\nu_\tau(q|\delta,n)$ be the normalized value of product line, P_tV_t/E_t, with elapsed time $\tau = t - t_i \in [0,\Delta]$ for some i. The problem (4) can be reduced to

$$\nu_0(q|\delta,n) = \max\left\{\tilde{\nu}_0(q|\delta,n), 0\right\}, \tag{5}$$

$$\tilde{\nu}_0(q|\delta,n) = \max_{\Delta,\xi_0}\sum_{i=0}^{\infty} e^{-(\rho+\delta)i\Delta}\left(\int_0^\Delta \Pi(\xi_0 e^{-n\tau})e^{-(\rho+\delta)\tau}d\tau - \kappa\right), \tag{6}$$

where $\tilde{\nu}_0$ is the value of a product line just before setting the price, which can be negative because of menu cost payment. If it is negative, the firm decides not to produce, or, not to enter the market. Firms choose the optimal interval of price revision Δ and real reset price ξ_0 as $\Delta(q|\delta,n)$ and $\xi_0(q|\delta,n)$, respectively. They both depend on their quality step q so that price revision behaviors are also heterogeneous among firms. We assume that each good is produced by linear technology with productivity of 1. Then, $\Pi(\xi) = (\xi - W)/\xi$.

To simplify the pricing strategy, we assume that knowledge on outdated technologies are automatically in the public domain. With this assumption, firms with secondary technology, if any, are under perfect competition and thus their prices are marginal cost, W. Hence the limit price of a leading firm is qW. From log utility, the optimal (re)set real price is

$$\xi_0(q|\delta,n) = qW. \tag{7}$$

Note that $\Delta(q|\delta,0) = \infty$ because firms do not need to reset prices when costs do not change. The optimal choice of price revision is expressed by

$$\frac{d}{d\Delta}\tilde{\nu}_0(q|\delta,n) = 0 = -(\rho+\delta)\tilde{\nu}_0 + \Pi(\xi_0 e^{-n\Delta}) + \kappa(\rho+\delta). \tag{8}$$

Substituting this into equation (6), we obtain the maximized value of a product line for type-q firm as

$$\tilde{\nu}_0(q|\delta,n) = \begin{cases} \frac{1}{\rho+\delta}\left(1 - \frac{e^{n\Delta(q|\delta,n)}}{q}\right) & \text{for } n > 0, \\ \frac{1}{\rho+\delta}\left(1 - \frac{1}{q}\right) - \kappa & \text{for } n = 0. \end{cases} \tag{9}$$

Lemma 3 in Appendix C shows that $\tilde{\nu}_0(q|\delta,n)$ is strictly increasing in q and strictly decreasing in δ and n. Moreover, the second-order cross derivative of $\tilde{\nu}_0(q|\delta,n)$ with respect to q and n is positive.

Intuitions run as follows. Rapid nominal growth (high n) implies frequent price reset (low Δ) and high menu cost payments, leading to lower value of a product line ($\tilde{\nu}_0$). The burden from a rise in menu cost payments, which is triggered by an increase in n, is relatively smaller for firms with greater q because menu costs are independent of q, while the return from the price reset is increasing in q. Higher-q firms have larger markup and choose larger time interval, Δ. This increases the sensitivity of such firms' profits to a change in n, which causes more frequent price changes when n increases (i.e., $(d^2\Delta)/(dqdn) < 0$).

As mentioned before, low-type firms cannot earn positive profits when there is positive menu costs, $\kappa > 0$. Here we denote $\underline{q}(\delta, n)$ as the threshold level of q,

$$\underline{q}(\delta, n) = \max\left\{q \,|\, \tilde{\nu}_0(q|\delta, n) \leq 0\right\}. \tag{10}$$

Proposition 1 shows that $\underline{q}(\delta, n)$ uniquely exists if κ is sufficiently small, and it is increasing in n when $n \geq 0$, implying that faster nominal growth pushes out inferior firms from the market.

Proposition 1 *If $\kappa < \frac{1}{\rho+\delta}$, then $\underline{q}(\delta, n)$ uniquely exists and $\underline{q}(\delta, n)$ is increasing in n for $n \geq 0$.*

All proofs are in Appendix C. Throughout the paper, we premise that the menu cost parameter κ is small enough for Proposition 1 to hold.

3.3 Incumbent Frims' R&D

Here we consider incumbents' R&D decision, taking the optimal pricing decision as given. This part is based on Lentz and Mortensen (2005, 2008). We assume that the size of quality improvement, q, is firm-specific and unchanged over time for each incumbent.

Suppose that an incumbent firm is type q and supplies $k \geq 1$ products. R&D cost is $kW_t c(\gamma)$, where $\gamma \geq 0$ is the R&D intensity. With probability $k\gamma$, the firm comes up with a new idea in a randomly chosen product line. The function $c(\gamma)$ is increasing and strictly convex with $c(0) = 0$. We define the real wage by $w \equiv W/E$.

The Bellman equation for the incumbent's problem is a bit complicated because profits from a product line depend on the elapsed time from the previous price revision. However, as we will see promptly, the resultant value function is similar to that in Lentz and Mortensen (2008). Let $T_k \equiv \{\tau_j\}_{j=1}^k$, the set of elapsed times from the previous price setting. The real value of the firm with q, k and T_k under the environment of (δ, w, n), say $v_k(T_k, q|\delta, w, n)$, satisfies the following Bellman equation:

$$\begin{aligned}\rho v_k(T_k, q|\delta, w, n) = \max_{\gamma} \sum_{j \notin \Omega} & \left[\Pi^0(\xi_0 e^{-n\tau_j}) + \frac{\partial v_k(T_k', q|\delta, w, n)}{\partial \tau_j}\right] \\ & + \sum_{j \in \Omega} \left[\Pi^0(\xi_0 e^{-n\tau_j}) - \kappa + \frac{\partial v_k(T_k', q|\delta, w, n)}{\partial \tau_j}\right] \\ & - kwc(\gamma) \\ & + k\gamma \left[v_{k+1}(\{T_k', 0\}, q|\delta, w, n) - v_k(T_k', q|\delta, w, n)\right] \\ & + k\delta \left[\frac{1}{k}\sum_{j=1}^{k} v_{k-1}(T'_{k-1,<j>}, q|\delta, w, n) - v_k(T_k', q|\delta, w, n)\right], \end{aligned} \tag{11}$$

where $\Omega \equiv \{j|\tau_j = \Delta(q|\delta, n)\}$ represents the set of products whose prices are to be revised, and $T_k' \equiv \{\tau_j'\}_{j=1}^k$ consists of $\tau_j' = 0$ if $j \in \Omega$ and $\tau_j' = \tau_j$ otherwise. $T_{k-1,<j>}$ is the set of elapsed time of the firm when it exits from j-th product market. In the right-hand side of equation (11), the first and second lines represent the real period profit when nominal prices are not reset and when nominal prices are reset, respectively. The real period profit decreases as τ lengthens. The third line represents R&D investment costs, which is proportional to the number of product lines k. The fourth and fifth lines represent the changes in real firm value when the firm increases and decreases its product lines by one, respectively.

Appendix A shows that the value function, v_k, is solved as

$$v_k(T_k, q|\delta, w, n) = \sum_{i=1}^{k} \nu_{\tau_i}(q|\delta, n) + k\psi(q|\delta, w, n), \tag{12}$$

where $k\psi(q|\delta, w, n)$ is the value from R&D in the future, which depends on the current number of product lines, or knowledge level, because it contributes to probability of success in R&D. The appendix also shows that incumbents make the following optimal R&D intensity choice, γ:

$$wc'(\gamma) = \max_{\gamma \in [0, \rho+\delta)} \frac{(\rho+\delta)\nu_0(q|\delta, n) - wc(\gamma)}{\rho + \delta - \gamma}. \tag{13}$$

The next proposition summarizes the properties of the optimal γ. It shows that firms with a greater q has a higher R&D intensity and there is a complementarity effect among q and n. Since a more rapid nominal growth monotonically decreases R&D intensity for any firm, the decline in R&D intensity under higher n is relatively small for firms with greater q.

Proposition 2 *Fix n and $\delta > 0$. $\gamma(q|\delta, w, n)$ uniquely exists for sufficiently large w. If such γ exists, $\gamma(q|\delta, w, n)$ is increasing in q and decreasing in n, δ, and w. Moreover, for $n \neq 0$,*

$$\frac{\partial^2 \gamma(q|\delta, w, n)}{\partial q \partial n} > 0 \quad \textit{and} \quad \frac{\partial^2 \gamma(q|\delta, w, n)}{\partial q \partial w} < 0.$$

The optimal choice $\gamma(q|\delta, w, n)$ is increasing in $\nu_0(q|\delta, n)$. Hence, a higher q implies that a higher value of a product line and, at the same time, greater R&D investment.

3.4 Firm Entry

The entry stage consists of two steps. First, a measure h of potential entrants make R&D decision before knowing its own q. The exogenous density of types of potential entrants is $\bar{\phi}(q)$ on $[1, \bar{q}]$, where $\bar{q}$ can be infinite. We assume that $\bar{\phi}(q)$ is continuous on its support. Second, some of those who observe $q < \underline{q}(\delta, n)$, defined in equation (10), call off their entry, because menu cost payment makes them worse off. We denote an ex post entry rate by η. Here, for simplicity, we do not allow for a possibility that such firms coordinate with their incumbents to enter the market. While entrants give up their entry, incumbents continue to monopolize the market. We denote the ex post distribution of q for the entrants as

$$\phi(q|\delta, n) = \begin{cases} 0 & \text{for } q < \underline{q}(\delta, n), \\ \frac{\bar{\phi}(q)}{1 - \bar{\Phi}(\underline{q}(\delta, n))} & \text{for } q \geq \underline{q}(\delta, n), \end{cases} \tag{14}$$

where $\bar{\Phi}$ is the cumulative distribution associated with $\bar{\phi}$.

We assume free entry to the market. In equilibrium, we have the following free entry (FE) condition,

$$\int_1^\infty \bar{\phi}(q)v_1(\{0\}, q|\delta, w, n)dq = wc'\left(\gamma_\eta(\delta, w, n)\right), \tag{15}$$

$$\gamma_\eta(\delta, w, n) \equiv \frac{\eta(\delta, w, n)}{1 - \bar{\Phi}(\underline{q}(\delta, n))}\frac{1}{h}, \tag{16}$$

where $v_1(\{0\}, q|\delta, w, n)$ is the firm value of an entrant and $\gamma_\eta(\delta, w, n)$ is entrants' R&D intensity to have the ex post entry rate of η. The R&D cost function c for potential entrants is the same as that for incumbents.

3.5 Firm Distribution

3.5.1 Price Distribution

Because of the Ss pricing rule due to menu costs, firms are heterogeneous in prices even within firms with the same q. From Oikawa and Ueda (2015), the stationary distribution of real prices among type-q firms is

$$f(\xi(\tau)|q, n) = \frac{\delta e^{-\delta\tau}}{1 - e^{-\delta\Delta(q|\delta,n)}} \qquad \text{for } \tau \in [0, \Delta(q|\delta, n)]. \tag{17}$$

3.5.2 Firm Size and Quality

Let $M_k(q|\delta, w, n)$ be the measure of type-q firms that produce k products under the environment (δ, w, n). Following Lentz and Mortensen (2005), a stationary distribution satisfies

$$\begin{aligned}\gamma(q|\delta, w, n)(k-1)M_{k-1}(q|\delta, w, n) + \delta(k+1)M_{k+1}(q|\delta, w, n)\\ = (\gamma(q|\delta, w, n) + \delta)kM_k(q|\delta, w, n) \qquad \text{for } k \geq 2,\end{aligned} \tag{18}$$

with

$$\phi(q|\delta, n)\eta = \delta M_1(q|\delta, w, n), \tag{19}$$

$$\phi(q|\delta, n)\eta + 2\delta M_2(q|\delta, w, n) = (\gamma(q|\delta, w, n) + \delta)M_1(q|\delta, w, n). \tag{20}$$

Appendix B shows that we have the following equation:

$$1 = \eta \int_{\underline{q}(\delta,n)}^{\infty} \frac{\phi(q|\delta,n)}{\delta - \gamma(q|\delta,w,n)} dq. \tag{21}$$

Equation (21) pins down the entry rate in equilibrium, $\eta(\delta,w,n)$. Because the measure of products supplied by type-q firms is represented by

$$K(q|\delta,w,n) \equiv \frac{\eta(\delta,w,n)\phi(q|\delta,n)}{\delta - \gamma(q|\delta,w,n)},$$

condition (21) is equivalent to the condition that the total measure of product lines must be one.

The equilibrium entry rate, $\eta(\delta,w,n)$, has the following properties. First, η is increasing in w because $\partial\gamma/\partial w < 0$ for any $q > \underline{q}(\delta,n)$ so that the integral in equation (21) decreases with w. Second, the relationship between δ and η is ambiguous. In response to an increase in δ, $\delta - \gamma(q|\delta,w,n)$ increases for any $q > \underline{q}(\delta,n)$ because $\partial\gamma/\partial\delta < 0$, but at the same time, $\phi(q|\delta_1,n)$ stochastically dominates $\phi(q|\delta_2,n)$ if $\delta_1 > \delta_2$. The former effect reduces the integral in equation (21) but the latter distribution effect enlarges it. Although it is possible that the distribution effect dominates and η is decreasing in δ if we consider ad-hoc distribution for $\bar{\phi}$,[13] we premise that the distribution effect does not dominate in the following analyses, meaning, roughly speaking, that we narrow down the class of $\bar{\phi}$ to sufficiently smooth density functions.

Substituting $\eta(\delta,w,n)$ into the free entry condition, equation (15), we have Proposition 3 that shows that the free entry condition is depicted as a downward-sloping curve on the δ-w space unless the distribution effect is too strong.

Proposition 3 *For given δ, the wage w that satisfies the free entry condition uniquely exists. Moreover, as long as $\eta(\delta,w,n)$ is increasing in δ, the free entry condition implies a negative association between δ and w.*

[13] It can occur when the increase in $\underline{q}(\delta,n)$ in response to an increase in δ eliminates large mass of firms, as indicated by the following example. Suppose that $\bar{\phi}$ is defined on a discrete space and has two mass points with equal probabilities at $\{q_1,q_2\}$ and there are negligible probabilities on other q. Suppose $\underline{q}$ is very close to q_1 so that $\gamma(q_1)$ is almost zero. And suppose $\gamma(q_2) = \frac{\delta}{2}$. Now consider the impact of an increase in δ to δ', where the increase is tiny but enough to eliminate q_1-type firms from the market. Before the change, the integral in equation (21), the expected value of $(\delta - \gamma(q))^{-1}$, is approximately $\frac{1.5}{\delta}$. After the change, it becomes approximately $\frac{2}{\delta'}$. If δ'/δ is sufficiently small, then η is decreasing in δ around the current δ.

Intuitively, a greater creative destruction rate should be at least partly driven by a higher entry rate although it implies a lower expected firm value after entry, which dampens innovation incentives. A wage reduction is required to fill in the gap because it increases post-entry firm value and, at the same time, it also leads to greater incumbents' R&D which implies a lower share of entry to creative destruction rate.

3.6 Labor Market

Labor consists of three groups of workers: production workers L_X, R&D workers hired by incumbents $L_{R,inc}$, and R&D workers hired by entrants $L_{R,ent}$. There is no skill difference across sectors.

Production workers are represented by

$$L_X(\delta, w, n) = \int_{\underline{q}(\delta,n)}^{\infty} K(q|\delta, w, n) L_{X,q}(q|\delta, w, n) dq, \tag{22}$$

where $L_{X,q}(q|\delta, w, n)$ is production labor demanded by type-q firms, which is derived from the price density function (17) such as

$$L_{X,q}(q|\delta, w, n) = \int_0^{\Delta(q|\delta,n)} f(\xi(\tau)) \frac{1}{\xi(\tau)} d\tau = \frac{\delta}{\delta - n} \frac{1}{Ewq} \frac{1 - e^{-(\delta-n)\Delta(q|\delta,n)}}{1 - e^{-\delta\Delta(q|\delta,n)}}. \tag{23}$$

The two types of R&D labor demands are written as

$$L_{R,inc}(\delta, w, n) = \int_{\underline{q}(\delta,n)}^{\infty} K(q|\delta, w, n) c(\gamma(q|\delta, w, n)) dq. \tag{24}$$

$$L_{R,ent}(\delta, w, n) = hc\left(\gamma_\eta(\delta, w, n)\right), \tag{25}$$

The labor market clearing (LMC) condition is defined as

$$L = L_X(\delta, w, n) + L_{R,inc}(\delta, w, n) + L_{R,ent}(\delta, w, n) \equiv L_D(\delta, w, n). \tag{26}$$

As discussed in Appendix D, we consider it is natural to have

$$\frac{\partial L_D}{\partial \delta} > 0 \quad \text{and} \quad \frac{\partial L_D}{\partial w} < 0.$$

This yields a positive association between δ and w for the LMC condition as depicted in Figure 1.

4 Stationary Equilibrium

4.1 Equilibrium

Given exogenous $\{n, \bar{\phi}(q)\}$, a stationary equilibrium consists of constant endogenous variables $\{\nu_0(q|\delta, n), \Delta(q|\delta, n), \xi_0(q, \delta, w, n), \gamma(q|\delta, w, n), w, \delta, \eta\}$ and they satisfy producers' optimal Ss-pricing (7) and (8), the firm value (9), incumbents' optimal R&D intensity choice (13), the free-entry condition (15), the entry/exit rate equation (21), and the labor-market clearing condition (26) as well as stationary distributions $\{M_k(q|\delta, w, n), \phi(q|\delta, n)\}$ that satisfy equations (14) and (18)-(20).

Figure 1 illustrates a typical loci of FE and LMC curves with the curve of $\underline{w}(\delta, n)$, defined in Appendix B, above which a nondegenerated quality distribution exists in the stationary state. The left edge of the LMC curve should coincide with a point on the curve of $\underline{w}(\delta, n)$. This is because $L_D(\delta, \underline{w}(\delta, n), n)$ is continuous in δ and

$$\lim_{\delta \to 0} L_D(\delta, \underline{w}(\delta, n), n) = 0 < L < \lim_{\delta \to \frac{1}{\kappa} + \rho} L_D(\delta, \underline{w}(\delta, n), n),$$

where the upper bound of δ is defined to have the finite $\underline{q}$.[14]

Since the locus of FE is always above $\underline{w}(\delta, n)$, we can show the existence of a stationary equilibrium as in the next proposition.

Proposition 4 *For given $n \geq 0$, there exists a stationary equilibrium with a positive entry rate.*

Because uniqueness of the equilibrium depends on monotonicity of labor demand function, multiple equilibria may occur under some kind of potential distribution, $\bar{\phi}$. However, we do not argue on this possibility anymore because, according to our simulations with smooth potential distributions, this nonmonotonicity is not relevant.

4.2 Welfare

Using the above endogenous variables, we can determine other endogenous variables as well: $\{C, P, \pi, g, U\}$.

[14]Note that $\lim_{\delta \to 0} \underline{w}(\delta) = \infty$ and $\lim_{\delta \to \frac{1}{\kappa} + \rho} L_{R,ent} = \infty$ because $\underline{q}(\frac{1}{\kappa} + \rho, n) = \infty$.

The goods market is cleared as

$$
\begin{aligned}
E = &PC + \kappa E \int dq \sum_{k=1}^{\infty} M_k(q|\delta, w, n) k f(\xi_0(q, n)) \\
\Rightarrow \quad & C = \frac{E}{P} \left(1 - \kappa\delta \int \frac{K(q|\delta, w, n)}{1 - e^{-\delta\Delta(q|\delta, n)}} dq \right),
\end{aligned} \tag{27}
$$

where the second term in the parenthesis is the total menu cost payments.

Appealing to Oikawa and Ueda (2015), the aggregate price level and the inflation rate are given by

$$
\log P = \int K(q|\delta, w, n) \left\{ \log \xi_0(q, n) + n\Delta \frac{e^{-\delta\Delta(q|\delta, n)}}{1 - e^{-\delta\Delta(q|\delta, n)}} - \frac{n}{\delta} \right\} dq, \tag{28}
$$

$$
\pi = n - \delta \int K(q|\delta, w, n) \log q \, dq. \tag{29}
$$

Then, the real growth rate g as

$$
g = n - \pi = \delta \int_{\underline{q}(\delta, n)}^{\infty} K(q|\delta, w, n) \log q \, dq. \tag{30}
$$

Welfare at $t = 0$ is given by

$$
U = \int_0^{\infty} e^{-\rho t} \log C_t dt = \frac{g}{\rho^2} + \frac{\log C}{\rho}. \tag{31}
$$

5 The Impacts of Nominal Growth

In the current model, a monetary policy is described as a change in the nominal growth rate, n. We simply assume that the central bank chooses n and compare the stationary states across different n.[15]

5.1 Reallocation Effects on the Real Growth Rate

A nonzero nominal growth has two kinds of reallocation effects. First, it pushes low-type firms out of the market because of greater burden of menu cost payments

[15] The current model conforms to Lentz and Mortensen (2008) (although distributions are continuous here) if there is no menu cost, or $\kappa = 0$. When $n = 0$, the current model becomes Lentz and Mortensen (2008) with fixed cost of entry to product line.

($\partial \underline{q}/\partial n > 0$). Second, it causes higher type firms to hold more product lines because $\partial^2 \gamma/\partial n \partial q > 0$: the decline in R&D intensity caused by an upward shift of n is relatively small for firms with greater q. Moreover, the increase in the product line share owned by high-q firms benefits efficiency in R&D investment because the probability of success is $k\gamma$. These reallocation effects are interpreted as the cleansing effect named by Caballero and Hammour (1994). The reallocation effects are positive for the real growth rate, g.

At the same time, there is a negative impact on g from faster nominal growth, which is examined in the Oikawa and Ueda (2015) model that abstracts ex ante firm heterogeneity. When the economy grows faster in the nominal term, firms revise their prices more frequently and pay more menu costs, leading to lower firm values and lower innovation incentives.

The overall impact of faster nominal growth on real growth is determined by the balance between these effects. As shown in equation (30), the real growth rate is determined by the creative destruction rate, δ, multiplied by the weighted average of quality gaps, $\log q$. Even though the creative destruction rate decreases with n, when the reallocation effect is sufficiently large, the increase in the weighted average of quality gap, $\int K(q|\delta, w, n) \log q \, dq$, dominates the negative effect on δ, and then the real growth rate, g, becomes increasing in the nominal growth rate, n.

5.2 Reallocation Effects on Welfare

The impacts on welfare consist of several factors. As seen in equations (27) and (31), the impacts should be divided into the real growth channel, the menu cost channel, and the initial price channel. The real growth channel consists of two elements. The one is simply from the above mentioned real growth effect. Since the market equilibrium is suboptimal, more rapid growth improves welfare if positive externality of R&D is sufficiently strong. The other element stems from reduction in negative externality in R&D, or the business stealing effect (Aghion and Howitt (1992)). It comes from the real product line value "stolen" from the former leading firm. Thus, innovations by high-q firms tend to be accompanied with low business stealing effects relative to the social benefits of their innovations. In other words, menu costs can improve welfare by automatically hindering entry of low-quality firms. Thus, nominal rigidity improves welfare through reallocation.

The menu cost channel basically has an ambiguous effect on welfare. An increase in n imposes more frequent menu cost payments for all firms. But at the same time, an increase in n tends to bring less creative destruction (and less entry) that reduces the total menu cost. Moreover, a greater n makes $K(q|\delta, w, n)$ be more dense on higher q. Because the increase in frequency of price revisions caused by an increase in n is relatively small for high-q firms, the total menu cost payments may be decreased under a rapid nominal growth.

The price channel is two-fold and both have ambiguous effects. The first one comes from a change in markups. The markup rate at price revision is high among the products supplied by high-q firms. Because distribution $K(q|\delta, w, n)$ becomes more dense on higher q when we have a higher n, the real prices tend to be high. However, this impact is partly or fully canceled by a lower rate of entry and a rapid decline in real prices, caused by an increase in n. The second is from a change in wage, w. The price level P is monotonically increasing in w but the equilibrium value of w can be decreasing and increasing in n, according to the labor market conditions. If a smaller w realizes by an increase in n, it positively affects welfare.

6 Simulations

In this section, we present the results of numerical simulations.

6.1 Parameter Setting

We set the total number of workers, L, at one. As for the parameter of the menu cost, we use $\kappa = 0.022$ from the work of Midrigan (2011). As for the other parameters, we follow the work of Lentz and Mortensen (2008), whose model is equivalent to our model with $n = 0$ and $\kappa = 0$. We calibrate the model to the Denmark economy. The discount rate ρ is set at 0.0361, which equals the interest rate 0.05 minus the real growth rate $g = 0.0139$ in Lentz and Mortensen (2008). The R&D cost function is given by $c(\gamma) = c_0 \times \gamma^{c_1}$, where $c_1 = 3.728$. As for c_0 and a measure of potential entrants, h, we calibrate the values so that the rate of creative destruction, δ, equals 0.071 and the rate of entry, η, equals 0.045 in the case of $n = 0$. We then obtain $c_0 = 1.020 \cdot 10^5$ and $h = 1.667$.

Following the previous studies,[16] we assume that the distribution of innovation ability, $\bar{\phi}(q)$, obeys the Pareto distribution, where the Pareto coefficient is calibrated so as to be consistent with Lentz and Mortensen (2008). They assume a simple discrete distribution for q, which takes only three values, that is, $q = 1$ (no quality growth), $q_L > 1$, and $q_H > 1$ and then estimate the probability for each q. Here, it is important to note that firms with $q = 1$ never enter into the market because of menu cost payments in our model, whereas 85% of firms have the quality of $q = 1$ in the estimated model of Lentz and Mortensen (2008). Thus, we calibrate the Pareto coefficient by targeting the variance of q in Lentz and Mortensen (2008), which leads to 17.50. In what follows, we discuss the robustness of our results to changes in those parameter values as well as the shape of a quality distribution, $\bar{\phi}(q)$.

6.2 Simulation Results

Sales and Employment Dispersion Figure 2 shows the simulation result that is analogous to our empirical findings reported in Section 2. In the left (right) panel, we calculate the ratio of sales (employment) per firm for top 0.1%, 1%, or 10% firm to those for a median firm for different n.[17] The lines have positive slopes with respect to n, suggesting that larger firms tend to grow more rapidly than smaller firms, when the nominal growth rate, n, increases.

This simulation result confirms Proposition 2. In the proposition, we showed that the ratio of R&D investment by high-q firms to that by low-q firms widens as n increases. This results in the widening of the ratio of sales for large firms to those for small firms.

Furthermore, this simulation result is consistent with what we found from the Japanese firm-level data in Section 2: the growth difference between large and small firms becomes larger as the rate of changes in firms' input price increases. Note that the rate of changes in the input price is analogous to the rate of changes in firms' production costs n, or more precisely, $\pi = n - g$.

[16] Lucas (1978) uses Pareto distribution for managerial talent. Melitz (2003) uses Pareto distribution for potential productivity of firms. Luttmer (2007) considers that entrants imitate randomly matched incumbent's productivity, whose distribution is also Pareto.

[17] In our calibrated model, bottom half firms produce only one product ($k = 1$). Thus, for example, the ratio of sales for top 1% firm to bottom 1% firm is almost equal to the ratio of sales for top 1% firm to a median firm.

Aggregate Variables To understand mechanism how changes in the nominal growth rate n influence the economy, we draw Figure 3. It shows that the real growth rate, g, is increasing with n. Behind this, there are two opposing effects, as we discussed in Section 5.

A negative effect of an increase in n is seen as a decrease in the creative destruction rate, δ. Because a higher n implies greater burden of menu cost payments and, thus, aggregate menu cost payments increase, incentives to innovate decrease (innovation incentive effect). As a result, the entry rate, η, as well as the R&D investment, $\gamma(q)$, with low q firms decreases with n.

However, there is a positive reallocation effect. The average $\log q$ increases, because, under a greater n, $K(q|\delta, w, n)$ has more weights for higher q, while firms with relatively small q cannot survive. Indeed, the lower threshold of q for the firm entry, $\underline{q}$, increases with n. Thus, the average firm size, k, increases with n, while the gap of $k(q)$ widens for firms with different q's. Despite greater menu cost payments, the R&D investment, $\gamma(q)$, with high q firms does not decrease with n. As a result, the average $\log q$ increases, while the average markup also increases. This markup increase decreases demand for production workers. Combined with a decrease in demand for R&D workers for possible entrants, $L_r(ent)$ in the figure, this result in an increase in demand for R&D workers for incumbents, $L_r(inc)$ in the figure, because of the labor market clearing condition. In this simulation, we find that the latter reallocation effect dominates the former innovation incentive effect. Thus, the real growth rate increases with the nominal growth rate.

Furthermore, Figure 3 shows that the welfare, U, increases with n. This result makes a sharp contrast to the model without firm reallocation. In our previous study (Oikawa and Ueda (2015)), we showed that real growth is maximized at $n = 0$ and, thus, the growth-maximizing inflation rate is negative. By contrast, in the model with firm reallocation, the welfare, U, increases with n, mainly because the real growth rate, g, increases with n.

Firm Distributions In this subsection, we discuss the reallocation effect by showing how distributions change depending on the value of the nominal growth rate, n. Figure 4 shows changes in the distribution of realized q, $\phi(q)$ (note it is not the exogenous $\bar{\phi}(q)$), changes. As n increases, $\underline{q}$ increases, which causes firms with low q to exit from the market. Thus, the average quality improves with n.

Next, Figure 5 shows the cumulative distribution for sales and employment per firm. The sales distribution in the model coincides with the distribution of the number of products, k, a firm produces. Employment includes both that for production and for R&D investment. Following the convention of empirical studies, we depict the distribution on the log-log scale and calculate the tail distribution (1 minus cumulative distribution) of sales or employment. We find from the figure that both sales and employment per firm have heavier tails under faster nominal growth, and they increase on average as nominal growth rate increases. In other words, firms tend to become bigger.

6.3 Menu Cost May Improve Welfare

> It is sometimes said that it would be illogical for labour to resist a reduction of money-wages but not to resist a reduction of real wages. For reasons given below, this might not be so illogical as it appears at first; and, as we shall see later, **fortunately so** [emphasis added].
>
> — Keynes (1936)

Because of the reallocation effect, price stickiness may improve welfare. To see this, in Figure 6, we calculate the effects of nominal growth on welfare by varying menu cost parameter values, κ, from 0.022 to 0, 0.002, or 0.012.[18] The figure shows that price stickiness indeed improves welfare. Even with a slight menu cost value ($\kappa > 0$), welfare improves compared with a case with no menu cost ($\kappa = 0$) for the case of $n > 0$. As Keynes says, it is fortunate to have nominal rigidity.[19]

6.4 Growth Decomposition

Here we decompose the impact of nominal growth on the real growth rate, i.e. $g(n) - g(0)$ into four components. First, following Lentz and Mortensen (2008), we examine the composition of the real growth rate at $n = 0$, as the reference point, into the

[18] The welfare levels for each κ are slightly different at $n = 0$ because we assume that firms pay menu cost at the first price setting.

[19] We thank Prof. Katsuhito Iwai for pointing out the last two words in the quote.

entry/exit effect, the selection effect, and the within (residual) effect:

$$\begin{aligned} g(0) = &\int_{\underline{q}(0)}^{\infty} \eta(0)\phi(q|0)\log q\,dq \\ &+ \int_{\underline{q}(0)}^{\infty} [K(q|0) - \phi(q|0)]\,\gamma(q|0)\log q\,dq \\ &+ \int_{\underline{q}(0)}^{\infty} \phi(q|0)\gamma(q|0)\log q\,dq. \end{aligned} \tag{32}$$

The first term represents the net entry effect by entrants. The second term represents the selection effect, which indicates the contribution to real growth through heterogenous firm growth which depends on quality. The last term represents the within effect, which Lentz and Mortensen (2008) call the residual effect: the contribution to real growth under the assumption that the share of products by firms is the same as that at entry.

In our calibrated model, the entry/exit effect, the selection effect, and the within effect account for 55.1%, 7.4%, and 37.5%, of the real growth at $n = 0$, respectively. The contribution of the selection effect is the smallest, whereas that of the entry/exit effect is the largest. This result is in sharp contrast with that in Lentz and Mortensen (2008), where the selection effect accounts for the largest share (52.8%), followed by the within effect (26.8%) and the entry/exit effect (21.1%). This difference arises from the fact that our model incorporates menu costs. Even under no nominal growth, our model has $\underline{q}(0) > 1$, that is, entrants need to pay menu costs, which serves as an entry barrier. This generates another selection effect at the time of entry: only sufficiently good firms decide to enter the market, which instead weakens the selection effect that works through second term in equation (32).

Having positive nominal growth, $n > 0$, we can write down the change in the real growth as follows:

$$\begin{aligned} g(n) - g(0) = &-\delta(0)\int_{\underline{q}(0)}^{\underline{q}(n)} K(q|0)\log q\,dq \\ &+ \int_{\underline{q}(n)}^{\infty} \{\eta(n)\phi(q|n) - \eta(0)\phi(q|0)\}\log q\,dq \\ &+ \int_{\underline{q}(n)}^{\infty} \{[K(q|n) - \phi(q|n)]\,\gamma(q|n) - [K(q|0) - \phi(q|0)]\,\gamma(q|0)\}\log q\,dq \\ &+ \int_{\underline{q}(n)}^{\infty} \{\phi(q|n)\gamma(q|n) - \phi(q|0)\gamma(q|0)\}\log q\,dq. \end{aligned} \tag{33}$$

Equation (33) shows the growth decomposition of the real growth rate at n minus that at $n = 0$, that is, $g(n) - g(0)$. A new component appears in the first term on the right-hand side of the equation, which we call the entry barrier effect. This effect is one of the two reallocation effects in our model. An increase in n causes low-type firms out of the market because of greater burden of menu cost payments ($\partial \underline{q}/\partial n > 0$). Because these firms would contribute to the economic growth if they were in the market, the sign of the entry barrier effect becomes negative. The second to fourth effects are labelled in the same way as before: the entry/exit effect, the selection effect, and the within effect. However, it differs in that the effects are measured relative to those under no nominal growth, $n = 0$.

Consider how these effects change when the nominal growth rate, n, increases. The selection effect increases real growth because the gap between the ex post distribution has more density for greater q than in the ex ante distribution. The within effect on real growth is positive because the average q as well as the gap of R&D intensity between high- and low-q firms increases with n. The entry/exit effect on real growth is ambiguous because the entry rate, η decreases with n, while the average q rises.

Figure 7 shows the growth decomposition of $g(n) - g(0)$. The contribution of the selection effect to the change in real growth is positive and the largest, followed by the within effect. Both the entry/exit effect and the entry barrier effect influence the real growth rate negatively. This figure suggests the importance of the selection effect when considering the effect of monetary policy on real growth, although the other effects such as the within and the entry/exit effects are more important when nominal rigidity is absent at $n = 0$.

6.5 Further Analyses

Sensitivity to the Shape of Distributions In the baseline simulation, we assume a Pareto distribution for the shape of the distribution of innovation ability, $\bar{\phi}(q)$. To see the robustness of the result, in Figure 8, we show the effects of nominal growth n on real growth g and welfare U, by assuming a uniform or log-normal distribution. In the case of a uniform distribution, the band for q is from 1 to $\bar{q}$, where $\bar{q}$ is selected so that the variance of q coincides with that of q in Lentz and Mortensen (2008), as we calculated a Pareto coefficient in the bencmark simulation. In the case of a log-normal distribution, we set the mean and variance of q so that both the mean and the variance of q coincide with those of q in Lentz and Mortensen (2008).

Figure 8 shows that both the real growth rate and welfare are increasing with n in the case of a log-normal distribution, whereas they are decreasing with n in the case of a uniform distribution. In the latter case, the positive reallocation effects of nominal growth is considered to be weak, because q does not have a large mass of low-q firms, which drop out from the market with high n, and there is an upper bound of q.

Comparison with Denmark and Japan Our baseline simulation is based on parameters calibrated to the Dannish economy by Lentz and Mortensen (2008). For comparison, we conduct another simulation using parameters calibrated to the Japanese economy using the work by Murao and Nirei (2011) who apply a model based on Lentz and Mortensen (2008) to Japan. Our calibration approach is the same. The discount rate ρ is set at 0.0385, which equals the interest rate 0.05 minus the real growth rate $g = 0.0115$ in Lentz and Mortensen (2008). The R&D cost function is given by $c(\gamma) = c_0 \times \gamma^{c_1}$, where $c_1 = 1.923$. As for c_0 and a measure of potential entrants, h, we calibrate the values so that the rate of creative destruction, δ, equals 0.085 and the rate of entry, η, equals 0.079 in the case of $n = 0$. We then obtain $c_0 = 6.028 \cdot 10^3$ and $h = 11.682$. We assume the distribution of innovation ability, $\bar{\phi}(q)$, obeys the Pareto distribution, where the Pareto coefficient is calibrated as 4.821.

Figure 9 shows simulation results for Japan. Now the effects of nominal growth on real growth and welfare are negative. The real growth rate and welfare are the highest when $n = 0$. While there are many differences in parameter values, the most important difference seems to be a measure of potential entrants, h: 11.682 for Japan and $h = 1.667$ for Denmark. This stems from a difference in the entry rate, η: 0.079 for Japan and 0.045 for Denmark, whereas the rate of creative destruction, δ, is not much different. In other words, entrants are relatively more important in Japan than in Denmark, while incumbents are relatively more important in Denmark. This weakens the reallocation effects for Japan, which should generate positive impacts of nominal growth on real growth and welfare if they were strong.

This exercise does not necessarily mean that inflation is good for Denmark but bad for Japan. It is important to emphasize that the measurement of entry and exit rates are subject to errors and, thus, the parameters we used are indecisive. An important implication is that the effects of nominal growth depend on circumstances. Particularly, in the economy where there are a great mass of potential entrants, low inflation is desirable to ensure frequent turnovers and minimize menu cost payments. By contrast, in the economy where incumbents play a big role in innovations, high inflation is desirable to maximize reallocation effects.

7 Concluding Remarks

In this study, we built an endogenous growth model with firm heterogeneity and nominal rigidity to analyze the impacts of monetary policy on long-run economic growth and welfare through reallocation. Our result showed that nominal growth can enhance growth and welfare if the reallocation effect is sufficiently strong. Beyond theoretical possibility, we found that this is indeed the case in the model calibrated to Denmark. According to the model, the optimal nominal growth rate is strictly positive, whereas it is zero in standard New Keynesian models. Menu cost burdens suppress entry of least innovative firms and, moreover, reallocate R&D resources from low-quality to high-quality firms among survivors.

It is worth noting that this reallocation effect works selectively, while nominal growth or inflation through monetary policy is purely aggregate phenomena. Selection occurs because menu costs at each price revision is independent of firm quality, whereas the return from the price reset is increasing in firm quality.

Our model postulates non-directed R&D and thus incumbents do not invest in quality improvement of the products they currently produce. As Garcia-Macia et al. (2016) point out, a significant share of innovations is attributed to incumbents' quality updates of their own products. We consider that the reallocation effect in the current model holds at least partly even after introducing incumbents' quality updates in their own product lines because such extension does not directly change the impact of nominal growth on product or firm values. In the context of data fitting, another deficiency of our model is the unit elasticity of substitution among products. This assumption significantly simplifies the price-setting rule and its influence to the firm value for the model to be tractable. Rigorous quantitative analyses with those extensions are our future research topics.

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A Derivation of Equation (13)

Now suppose that the firm value is profit flows from each product plus the return from R&D that depends on k. Let the present-discount value of the sum of profit flows for a product with τ_i be $\nu_{\tau_i}(q|\delta, n)$. So our guess for the value function is

$$v_k(T_k, q|\delta, w, n) = \sum_{i=1}^{k} \nu_{\tau_i}(q|\delta, n) + k\psi(q|\delta, w, n). \tag{34}$$

Based on the guess of equation (34), we have

$$\frac{1}{k}\sum_{i=1}^{k} v_{k-1}(T'_{k-1,<i>}, q|\delta, w, n) = \left(1 - \frac{1}{k}\right)\sum_{i=1}^{k} \nu_{\tau'_i}(q|\delta, n) + (k-1)\psi(q|\delta, w, n) \tag{35}$$

and, thus,

$$\frac{1}{k}\sum_{i=1}^{k} v_{k-1}(T'_{k-1,<i>}, q|\delta, w, n) - v_k(T'_k, q|\delta, w, n) = -\frac{1}{k}\sum_{i=1}^{k} \nu_{\tau'_i}(q|\delta, n) - \psi(q|\delta, w, n). \tag{36}$$

Equation (11) is then written as

$$\begin{aligned}
&\rho \sum_{i=1}^{k} \nu_{\tau_i}(q|\delta, n) + \rho k \psi(q|\delta, w, n) \\
&\quad = \sum_{i=1}^{k} \left[\Pi^0(\xi_0 e^{-n\tau_i}) - I\{\tau_i = \Delta(q|\delta, n)\}\kappa + \frac{\partial \nu_{\tau_i}(q|\delta, n)}{\partial \tau} \right] \\
&\qquad - \delta \sum_{i=1}^{k} \nu_{\tau'_i}(q|\delta, n) - k\delta\psi(q|\delta, w, n) \\
&\qquad + k \max_{\gamma} \left\{ \gamma \left[v_{k+1}(\{T'_k, 0\}, q|\delta, w, n) - v_k(T'_k, q|\delta, w, n) \right] - wc(\gamma) \right\}.
\end{aligned} \tag{37}$$

Noticing that

$$\rho \nu_{\tau_i}(q|\delta, n) = \begin{cases} \Pi^0(\xi_0 e^{-n\tau_i}) + \frac{\partial \nu_{\tau_i}(q|\delta, n)}{\partial \tau} - \delta \nu_{\tau_i}(q|\delta, n) & \text{for } \tau_i \in [0, \Delta(q|\delta, n)), \\ \Pi^0(\xi_0 e^{-n\tau_i}) - \kappa + \frac{\partial \nu_0(q|\delta, n)}{\partial \tau} - \delta \nu_0(q|\delta, n) & \text{for } \tau_i = \Delta(q|\delta, n), \end{cases} \tag{38}$$

we have

$$(\rho + \delta)\psi(q|\delta, w, n) = \max_{\gamma} \left\{ \gamma[\nu_0(q|\delta, n) + \psi(q|\delta, w, n)] - wc(\gamma) \right\}. \tag{39}$$

The R&D intensity is determined at

$$\nu_0(q|\delta, n) + \psi(q|\delta, w, n) = wc'(\gamma). \tag{40}$$

Using equation (39) to eliminate ψ, this first-order condition can be rewritten as

$$wc'(\gamma) = \max_{\gamma \in [0, \rho+\delta)} \frac{(\rho + \delta)\nu_0(q|\delta, n) - wc(\gamma)}{\rho + \delta - \gamma}, \tag{41}$$

where the constraint $\gamma < \rho + \delta$ should hold because ψ is not well-defined otherwise.

B Firm Size and Quality

Equation (18) leads to

$$M_k(q|\delta, w, n) = \frac{\phi(q|\delta, n)\eta}{\delta k}\left(\frac{\gamma(q|\delta, w, n)}{\delta}\right)^{k-1}. \tag{42}$$

The mass of type-q firms in the stationary state, $M(q|\delta, w, n)$, is $\sum_{k=1}^{\infty} M_k(q|\delta, w, n)$. If $\gamma(q|\delta, w, n) < \delta$ for almost all q, then $M(q|\delta, w, n)$ is well-defined such as

$$M(q|\delta, w, n) = \frac{\eta}{\delta}\left[\log\left(\frac{\delta}{\delta - \gamma(q|\delta, w, n)}\right)\right]\frac{\delta\phi(q|\delta, n)}{\gamma(q|\delta, w, n)}. \tag{43}$$

The condition $\sup_q \gamma(q|\delta, w, n) < \delta$ is supported when w is sufficiently large. The threshold level of w is determined by the first-order condition of the maximization in equation (13). Let $\underline{w}(q|\delta, n)$ be the individual threshold wage level such that $\gamma(q|\delta, w, n) < \delta$ for $w > \underline{w}(q|\delta, n)$. The threshold wage in the whole economy is $\underline{w}(\delta, n) \equiv \sup_q \underline{w}(q|\delta, n)$. Since $\nu(q|\delta, n)$ is monotonically increasing in q and $\lim_{q\to\infty} \nu(q|\delta, n) = \frac{1}{\rho+\delta} - \kappa$, we have

$$\underline{w}(\delta, n) = \begin{cases} \frac{1-(\rho+\delta)\kappa}{c(\delta)+\rho c'(\delta)} & \text{if } \bar{q} \to \infty, \\ \frac{(\rho+\delta)\nu(\bar{q}|\delta,n)}{c(\delta)+\rho c'(\delta)} & \text{if } \bar{q} \text{ is finite.} \end{cases} \tag{44}$$

Let us make two remarks. First, one can easily show that $\underline{w}$ is decreasing in the creative destruction rate, δ, in both cases. This is because declines in the creative destruction rate and wage both stimulate incumbents' R&D. Second, $\underline{w}$ is independent of n if the support of $\bar{\phi}$ is not bounded, whereas greater nominal growth enlarges the admissible set otherwise.

Any equilibrium has $w > \underline{w}(\delta, n)$. Because the total mass of products is one, the creative destruction rate must be equal to the sum of the entry rate and the creation rates of all the incumbents. As long as $w > \underline{w}(\delta, n)$, we have

$$\begin{aligned} \delta &= \eta + \int_{\underline{q}(\delta,n)}^{\infty} dq \sum_{k=1}^{\infty} kM_k(q|\delta, w, n)\gamma(q|\delta, w, n) \\ &= \eta \int_{\underline{q}(\delta,n)}^{\infty} \frac{\delta\phi(q|\delta, n)}{\delta - \gamma(q|\delta, w, n)} dq, \end{aligned}$$

which leads to the following equation:

$$1 = \eta \int_{\underline{q}(\delta,n)}^{\infty} \frac{\phi(q|\delta,n)}{\delta - \gamma(q|\delta,w,n)} dq. \tag{45}$$

C Proofs

Lemma 1 *For $n \neq 0$, $\Delta(q|\delta,n)$ is increasing in q and decreasing in $|n|$. $|n|\Delta$ is increasing in $|n|$. Moreover, $\Delta(q|\delta,n)$ is increasing (decreasing) in δ for $n > 0$ ($n < 0$).*

Proof of Lemma 1 In this paper, we sketch the proof in the case of $n > 0$ only. See Oikawa and Ueda (2015) for the case of $n < 0$. From equations (8) and (9), we obtain

$$q\kappa = \frac{e^{n\Delta} - e^{-(\rho+\delta-n)\Delta}}{\rho+\delta} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{\rho+\delta-n}. \tag{46}$$

The total differentiation of equation (46) is

$$\begin{aligned}
\kappa dq - &\frac{ne^{n\Delta}(1 - e^{-(\rho+\delta)\Delta})}{\rho+\delta} d\Delta \\
&= \left[e^{n\Delta} \left(\frac{\Delta e^{-(\rho+\delta)\Delta}}{\rho+\delta} - \frac{1 - e^{-(\rho+\delta)\Delta}}{(\rho+\delta)^2} \right) - \left(\frac{\Delta e^{-(\rho+\delta-n)\Delta}}{\rho+\delta-n} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{(\rho+\delta-n)^2} \right) \right] d\delta \\
&\quad + \left[\frac{\Delta e^{n\Delta}(1 - e^{-(\rho+\delta)\Delta})}{\rho+\delta} + \frac{\Delta e^{-(\rho+\delta-n)\Delta}}{\rho+\delta-n} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{(\rho+\delta-n)^2} \right] dn \\
&= e^{-(\rho+\delta-n)\Delta} \left[\left(\frac{\Delta}{\rho+\delta} - \frac{e^{(\rho+\delta)\Delta} - 1}{(\rho+\delta)^2} \right) - \left(\frac{\Delta}{\rho+\delta-n} - \frac{e^{(\rho+\delta-n)\Delta} - 1}{(\rho+\delta-n)^2} \right) \right] d\delta \\
&\quad + \left[q\kappa\Delta + \left(\frac{\Delta}{\rho+\delta-n} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{(\rho+\delta-n)^2} \right) \right] dn \qquad (47) \\
&= e^{-(\rho+\delta-n)\Delta} \left[h_2(\rho+\delta,\Delta) - h_2(\rho+\delta-n,\Delta) \right] d\delta + \left[q\kappa\Delta + h_1(\rho+\delta-n,\Delta) \right] dn,
\end{aligned}$$

where we substituted equation (46) in the second equality. Functions h_1 and h_2 are defined in Lemma 2 below. Owing to Lemma 2, the signs of the coefficients of $d\delta$ and dn are determined uniquely and we have $d\Delta/dq > 0$, $d\Delta/d\delta > 0$, and $d\Delta/dn < 0$. ■

Lemma 2 *Define the following functions over $x \neq 0$ with $y \geq 0$ by*

$$h_1(x,y) = \frac{y}{x} - \frac{1-e^{-xy}}{x^2},$$
$$h_2(x,y) = \frac{y}{x} - \frac{e^{xy}-1}{x^2}.$$

Then, for any $x \neq 0$ and $y \geq 0$, the following relationships hold:

$$h_1(x,y) \geq 0, \quad h_2(x,y) \leq 0,$$

$$\frac{\partial h_1(x,y)}{\partial x} \leq 0, \quad \frac{\partial h_2(x,y)}{\partial x} \leq 0,$$

with equalities only when $y = 0$.

Proving the signs of the functions is straightforward:

$$h_1(x,y) = \frac{xy - (1-e^{-xy})}{x^2} \geq 0,$$
$$h_2(x,y) = \frac{xy - (e^{xy}-1)}{x^2} \leq 0.$$

About the derivative of h_1, we have

$$\frac{\partial h_1(x,y)}{\partial x} = -\frac{2(1+e^{-xy})}{x^3}\left[\frac{xy}{2} - \frac{1-e^{-xy}}{1+e^{-xy}}\right].$$

Since $y \geq 0$ and

$$\frac{xy}{2} \geq \frac{1-e^{-xy}}{1+e^{-xy}} \quad \text{if and only if } xy \geq 0,$$

we have $\frac{\partial h_1(x,y)}{\partial x} \leq 0$.

Last, the partial derivative of h_2 is expressed as

$$\frac{\partial h_2(x,y)}{\partial x} = -\frac{2(1+e^{xy})}{x^3}\left[\frac{xy}{2} - \frac{1-e^{-xy}}{1+e^{-xy}}\right].$$

Thus, the sign of $\partial h_2(x,y)/\partial x$ is equivalent to that of $\partial h_1(x,y)/\partial x$. ∎

Lemma 3 *$\tilde{\nu}_0(q|\delta,n)$ is strictly increasing in q and strictly decreasing in δ. For $n \neq 0$,*

$$\frac{\partial \tilde{\nu}_0(q|\delta,n)}{\partial n} < 0, \qquad \frac{\partial^2 \tilde{\nu}_0(q|\delta,n)}{\partial q \partial n} > 0.$$

Proof of Lemma 3 It is obvious to see the former of the proposition: $\partial\tilde{\nu}_0(q|\delta,n)/\partial q > 0$ and $\partial\tilde{\nu}_0(q|\delta,n)/\partial\delta < 0$. As for the latter of the proposition, first suppose $n > 0$.

$$\begin{aligned}
\frac{\partial^2\tilde{\nu}_0(q|\delta,n)}{\partial q\partial n} &= \frac{\partial(n\Delta)}{\partial(n)}\frac{\partial^2\tilde{\nu}_0(q|\delta,n)}{\partial q\partial(n\Delta)} = -\frac{1}{\rho+\delta}\frac{\partial(n\Delta)}{\partial(n)}\frac{e^{n\Delta}}{q}\left(n\frac{\partial\Delta}{\partial q}-\frac{1}{q}\right) \\
&= -\frac{1}{\rho+\delta}\frac{\partial(n\Delta)}{\partial(n)}\frac{e^{n\Delta}}{q}\frac{(\rho+\delta)q\kappa-\left(e^{n\Delta}-e^{-(\rho+\delta-n)\Delta}\right)}{q\left(e^{n\Delta}-e^{-(\rho+\delta-n)\Delta}\right)} \\
&= \frac{1}{\rho+\delta}\frac{\partial(n\Delta)}{\partial(n)}\frac{e^{n\Delta}}{q}\frac{1}{q\left(e^{n\Delta}-e^{-(\rho+\delta-n)\Delta}\right)}\frac{\rho+\delta}{\rho+\delta-n}\left(1-e^{-(\rho+\delta-n)\Delta}\right) \\
&> 0.
\end{aligned}$$

For the derivation of the last line, we use Lemma 1. Similarly, if $n < 0$, we have

$$\begin{aligned}
\frac{\partial^2\tilde{\nu}_0(q|\delta,n)}{\partial q\partial n} &= \frac{\partial(n\Delta)}{\partial(n)}\frac{\partial^2\tilde{\nu}_0(q|\delta,n)}{\partial q\partial(n\Delta)} = -\frac{1}{\rho+\delta}\frac{\partial(n\Delta)}{\partial(n)}\frac{e^{-n\Delta}}{q}\left(n\frac{\partial\Delta}{\partial q}+\frac{1}{q}\right) \\
&= -\frac{1}{\rho+\delta}\frac{\partial(n\Delta)}{\partial(n)}\frac{e^{-n\Delta}}{q}\frac{(\rho+\delta)q\kappa+e^{-n\Delta}-e^{(\rho+\delta-n)\Delta}}{q\left(e^{-n\Delta}-e^{(\rho+\delta-n)\Delta}\right)} \\
&= -\frac{1}{\rho+\delta}\frac{\partial(n\Delta)}{\partial(n)}\frac{e^{-n\Delta}}{q}\frac{1}{q\left(e^{(\rho+\delta-n)\Delta}-e^{-n\Delta}\right)}\frac{\rho+\delta}{\rho+\delta-n}\left(e^{(\rho+\delta-n)\Delta}-1\right) \\
&< 0.
\end{aligned}$$

■

Proof of Proposition 1 From equation (9), $\kappa \geq \frac{1}{\rho+\delta}$ implies that $\underline{q}(\delta,n) \to \infty$ and no firm can yield profits. Thus, we assume $\kappa < \frac{1}{\rho+\delta}$ below.

When $n = 0$, then $\underline{q}(\delta,0) = \frac{1}{1-(\rho+\delta)\kappa}$.

When $n > 0$, $\tilde{\nu}_0(q|\delta,n) < 0$ implies that $\Delta(q|\delta,n) > \frac{\log q}{n} \equiv \bar{\Delta}$. From the first-order condition when choosing Δ:

$$q\kappa = \frac{e^{n\Delta}-e^{-(\rho+\delta-n)\Delta}}{\rho+\delta} - \frac{1-e^{-(\rho+\delta-n)\Delta}}{\rho+\delta-n},$$

$\underline{q}(\delta,n)$ is q satisfying

$$\kappa = \frac{1-e^{-(\rho+\delta)\bar{\Delta}}}{\rho+\delta} - \frac{1/q-e^{-(\rho+\delta)\bar{\Delta}}}{\rho+\delta-n} = \frac{1}{\rho+\delta} - \frac{q^{-1}-\frac{n}{\rho+\delta}q^{-\frac{\rho+\delta}{n}}}{\rho+\delta-n}. \tag{48}$$

Let $A_1(q,n)$ be the right-hand side of equation (48). Note that $A_1(1,n) = 0$ and $\lim_{q\to\infty} A_1(q,n) = \frac{1}{\rho+\delta}$.

Suppose $n \neq \rho+\delta$. Since

$$\frac{\partial A_1(q,n)}{\partial q} = \frac{\rho+\delta}{q^2}\frac{1-q^{1-\frac{\rho+\delta}{n}}}{\rho+\delta-n} > 0,$$

$\underline{q}(\delta,n)$ uniquely exists. Next, $\partial\underline{q}/\partial n < 0$ comes from

$$\frac{\partial A_1(q,n)}{\partial n} = \frac{q^{-\frac{\rho+\delta}{n}}}{(\rho+\delta-n)^2}\left[1+\log q^{\frac{\rho+\delta}{n}-1} - q^{\frac{\rho+\delta}{n}-1}\right] < 0 \quad \text{for } q > 1.$$

Moreover, since $A_1(q,n)$ is continuous as $n \downarrow 0$, $\underline{q}(\delta,n) > 1/[1-(\rho+\delta)\kappa]$ for $n > 0$. Thus, $\partial\underline{q}(\delta,n)/\partial n > 0$.

For the case of $n = \rho+\delta$, $A_1(q,n) = \frac{1-1/q}{\rho+\delta}$, which implies $A_1(q,n)$ is strictly increasing in q. Further, we have

$$\lim_{n\to\rho+\delta}\frac{\partial A_1(q,n)}{\partial n} = \lim_{n\to\rho+\delta} -\frac{1}{2}\frac{\rho+\delta}{n^2}q^{-\frac{\rho+\delta}{n}}(\log q)^2 = -\frac{(\log q)^2}{2q(\rho+\delta)} < 0.$$

Thus, $\underline{q}(\delta,n)$ uniquely exists and $\partial\underline{q}(\delta,n)/\partial n > 0$.

When $n < 0$, similarly, $\underline{q}(\delta,n)$ is obtained from

$$\kappa = -\frac{\epsilon}{\rho+\delta} + \frac{q^{-1} - \frac{\epsilon n}{\rho+\delta}(q\epsilon)^{-\frac{\rho+\delta}{n}}}{\rho+\delta-n}, \qquad \epsilon \equiv 1-(\rho+\delta)\kappa > 0. \tag{49}$$

Let $A_2(q,n)$ be the right-hand side of equation (49). Note that $A_2(1/\epsilon,n) = 0$ and $\lim_{q\to\infty} A_2(q,n) = \infty$. And,

$$\frac{\partial A_2(q,n)}{\partial q} = \frac{q^{-2}}{\rho+\delta-n}\left[(q\epsilon)^{-\frac{\rho+\delta}{n}+1} - 1\right] > 0 \quad \text{for } q > \frac{1}{\epsilon},$$

$$\frac{\partial A_2(q,n)}{\partial n} = -\frac{\epsilon(q\epsilon)^{-\frac{\rho+\delta}{n}}}{(\rho+\delta-n)^2}\left[1+\log(q\epsilon)^{\frac{\rho+\delta}{n}-1} - (q\epsilon)^{\frac{\rho+\delta}{n}-1}\right] > 0 \quad \text{for } q > \frac{1}{\epsilon}.$$

Therefore, $\underline{q}(\delta,n)$ uniquely exists in $(1/\epsilon,\infty)$ and $\partial\underline{q}(\delta,n)/\partial n < 0$ for $n < 0$. ■

Proof of Proposition 2 From equation (13), the optimal γ satisfies

$$\frac{(\rho+\delta)\nu_0(q|\delta,n)}{w} = c(\gamma) + (\rho+\delta-\gamma)c'(\gamma), \qquad \gamma \in [0, \rho+\delta).$$

From the strict convexity of $c(\gamma)$ and the assumption of $c(0) = 0$, γ has a unique interior solution if w is sufficiently large. $\partial\gamma/\partial|n| < 0$ and $\partial\gamma/\partial w < 0$ are obvious from the first-order condition. About the impact of δ, $(\rho+\delta)\nu_0(q|\delta,n)$ is strictly decreasing in δ for $n \geq 0$ from Lemma 1. Thus, $\partial\gamma/\partial\delta < 0$. And

$$\frac{\partial\gamma}{\partial q} = \frac{1}{wc''(\gamma)}\frac{\rho+\delta}{\rho+\delta-\gamma}\frac{\partial\nu_0(q|\delta,n)}{\partial q} > 0, \tag{50}$$

$$\frac{\partial^2\gamma}{\partial q\partial n} = \frac{1}{wc''(\gamma)}\frac{\rho+\delta}{\rho+\delta-\gamma}\frac{\partial^2\nu_0(q|\delta,n)}{\partial q\partial n}\begin{cases} > 0 & \text{for } n > 0 \\ < 0 & \text{for } n < 0, \end{cases} \tag{51}$$

$$\frac{\partial^2\gamma}{\partial q\partial w} = -\frac{1}{w^2c''(\gamma)}\frac{\rho+\delta}{\rho+\delta-\gamma}\frac{\partial\nu_0(q|\delta,n)}{\partial q} < 0. \tag{52}$$

■

Proof of Proposition 3 The free entry condition, equation (15), can be rewritten as

$$\int_{\underline{q}(\delta,n)}^{\infty} \bar{\phi}(q)c'(\gamma(q|\delta,w,n))dq = c'(\gamma_\eta(\delta,w,n)), \tag{53}$$

by substituting ψ from equation (13).

Fix $\delta > 0$ arbitrarily. The right-hand side of equation (53) is increasing in w because $\eta(\delta,w,n)$ is increasing in w and $c'(\gamma)$ is positive. On the other hand, its left-hand side is decreasing in w because $\gamma(q|\delta,w,n)$ is decreasing in w for any $q > \underline{q}(\delta,n)$ in the admissible set. Hence, if w satisfying the FE condition exists for a given δ, it is unique. Its existence is guaranteed if we can prove that the right-hand side is zero at $\underline{w}(\delta,n)$, because the left-hand side is strictly positive at $\underline{w}(\delta,n)$ and both the left- and right-hand sides are continuous for $w \geq \underline{w}(\delta,n)$. The right-hand side is shown to be zero at $\underline{w}(\delta,n)$ because equation (42) suggests that, in the limit of $\delta = \gamma$, η has to converge to zero to make the total measure of firms finite.

As long as η is positively correlated with δ in condition (21), an increase in δ drives up $\gamma_\eta(\delta,w,n)$ for given w and n while it reduces the left-hand side of equation

(53) by an increase in $\underline{q}(\delta, n)$ and declines in $\gamma(q|\delta, w, n)$ for any $q > \underline{q}(\delta, n)$. Thus, when δ increases, it is required to have a smaller w to satisfy the FE because γ is decreasing in w and γ_η is increasing in w. ■

Proof of Proposition 4 Define $\delta_{FE}(w, n)$ as δ that satisfies equation (21) and the FE condition, (53). Consider $\eta(\delta_{FE}(w, n), w, n)$ by substituting $\delta_{FE}(w, n)$ into equation (21). As $w \to \infty$, $\underline{w}^{-1}(w, n)$ and the left-hand side of equation (53) go to 0. To make the right-hand side of equation (53), η (and δ) must be zero. Then, $\lim_{w\to\infty} \eta(\delta_{FE}(w, n), w, n) = 0$. On the other hand, when $w \to 0$, $\delta_{FE}(w, n) \to \infty$ because $\underline{w}^{-1}(w, n) \to \infty$. Thus, $\lim_{w\to 0} \eta(\delta_{FE}(w, n), w, n) = \infty$. In terms of employment, when $w \to \infty$, we can see

$$\lim_{w\to\infty} L_{R,ent}(\delta_{FE}(w, n), w, n) = 0, \qquad \lim_{w\to\infty} L_{R,inc} = \lim_{w\to\infty} L_X = 0,$$

which leads to $\lim_{w\to\infty} L_D(\delta_{FE}(w, n), w, n) = 0$ for any $n \geq 0$. When $w \to 0$, $\lim_{w\to 0} L_{R,ent}(\delta_{FE}(w, n), w, n) = \infty$ for any $n \geq 0$, which implies $\lim_{w\to 0} L_D(\delta_{FE}(w, n), w, n) = \infty$.

Because L_D is continuous in w, there exists at least one w that satisfies $L_D(\delta_{FE}(w, n), w, n) = L$ for any $n \geq 0$. ■

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Lemma 4 *Fix $\delta > 0$ and $n \geq 0$. The cumulative distribution of $K(q|\delta, w_1, n)$ stochastically dominates that of $K(q|\delta, w_2, n)$ if $w_1 < w_2$.*

Proof of Lemma 4 From equation (21) and the definition of K, $K(q|\delta, w, n)$ is increasing in w if and only if

$$\int_{\underline{q}(\delta,n)}^{\infty} -\frac{\partial\gamma(q'|\delta, w, n)}{\partial w} \frac{K(q'|\delta, w, n)}{\delta - \gamma(q'|\delta, w, n)} dq' > -\frac{\partial\gamma(q|\delta, w, n)}{\partial w} \frac{1}{\delta - \gamma(q|\delta, w, n)}.$$

The left-hand side is a positive constant for any q, while the right-hand side is zero at $q = \underline{q}(\delta, n)$ and monotonically increasing in q from Proposition 2. To keep $\int_{\underline{q}}^{\infty} K(q|\delta, w, n)dq = 1$, there exists $\hat{q} > \underline{q}$ such that $\partial K/\partial w > 0$ if and only if $q < \hat{q}$.

Since $\frac{\partial}{\partial w}\int_{\underline{q}}^{\infty} K(q'|\delta, w, n)dq'$ must be zero by definition,

$$0 = \frac{\partial}{\partial w}\int_{\underline{q}}^{\hat{q}} K(q'|\delta, w, n)dq' + \frac{\partial}{\partial w}\int_{\hat{q}}^{\infty} K(q'|\delta, w, n)dq'.$$

Hence, for any $q \in (\hat{q}, \infty)$,

$$\frac{\partial}{\partial w}\int_{\underline{q}}^{\hat{q}} K(q'|\delta, w, n)dq' > -\frac{\partial}{\partial w}\int_{\hat{q}}^{q} K(q'|\delta, w, n)dq'.$$

Therefore, $\frac{\partial}{\partial w}\int_{\underline{q}}^{q} K(q'|\delta, w, n)dq' > 0$ for any finite q, which implies the stated stochastic dominance. ■

We assume that this effect does not dominate.

Proposition 5 *$L_{R,ent}$ is increasing in w. $L_{R,inc}$ is decreasing in w. L_X is decreasing in w if the distribution effect is sufficiently weak.*

Proof of Proposition 5 Fix $\delta > 0$ and n arbitrarily. $L_{R,ent}$ is increasing in w because

$$\frac{\partial \gamma_\eta(\delta, w, n)}{\partial w} \propto \frac{\partial \eta(\delta, w, n)}{\partial w} > 0.$$

Next, the response of $L_{R,inc}$ to an increase in w is

$$\begin{aligned}\frac{\partial L_{R,inc}}{\partial w} &= \int_{\underline{q}(\delta,n)}^{\infty} \frac{\partial K(q|\delta, w, n)}{\partial w} c(\gamma(q|\delta, w, n))dq \\ &\quad + \int_{\underline{q}(\delta,n)}^{\infty} K(q|\delta, w, n)c'(\gamma(q|\delta, w, n))\frac{\partial \gamma(q|\delta, w, n)}{\partial w}dq \qquad (54)\end{aligned}$$

The last term is negative from Proposition 2. The first term in the right hand side depends on $\partial K/\partial w$. Because $c(\gamma(q))$ is an increasing function of q, Lemma 4 implies that the first term in the right-hand side of equation (54) is negative. Hence, R&D labor demand from incumbents is decreasing in w.

The impact of an increase in w on L_X is

$$\frac{\partial L_X}{\partial w} = -\frac{L_X}{w} + \int_{\underline{q}(\delta,n)}^{\infty} \frac{\partial K(q|\delta, w, n)}{\partial w} L_{X,q}(q|\delta, w, n)dq \qquad (55)$$

The last term is positive because $L_{X,q}(q|\delta, w, n)$ is a decreasing function of q and K

is stochastically dominated when w increases as show in Lemma 4.[20] Thus, to have L_X decreasing in w, the distribution effect is sufficiently small. ■

The LMC condition does not necessarily imply a monotonic relationship between δ and w. Suppose that $w > \underline{w}(\delta, n)$ and $\eta(\delta, w, n)$ is increasing in δ in condition (21). Then we have

$$\frac{\partial L_D}{\partial \delta} = \underbrace{\frac{\partial L_X}{\partial \delta}}_{(+)} + \underbrace{\frac{\partial L_{R,inc}}{\partial \delta}}_{-} + \underbrace{\frac{\partial L_{R,ent}}{\partial \delta}}_{+} \tag{56a}$$

$$\frac{\partial L_D}{\partial w} = \underbrace{\frac{\partial L_X}{\partial w}}_{(-)} + \underbrace{\frac{\partial L_{R,inc}}{\partial w}}_{-} + \underbrace{\frac{\partial L_{R,ent}}{\partial w}}_{+} \tag{56b}$$

The signs of responses of L_X in equations (56) are in parentheses because it does not hold in general. In equation (56a), a rise in creative destruction rate, firms extend the interval between price resets, leading to lower markups under positive nominal growth. This is the primary factor to increase production employment. However, a rise in δ also brings more takeover of product lines with the highest markup at takeover. Further, there is a reallocation effect: $K(q|\delta, w, n)$ becomes less concentrated in higher q because firm growth tends to be low under high frequency of creative destruction[21] but we have higher $\underline{q}(\delta, n)$.

In equation (56b), $\partial L_X/\partial w$ could be positive when the reallocation effect is too strong. We can show that $K(q|\delta, w, n)$ stochastically dominates that with higher w from the following Lemma. This Lemma implies that higher w leads to a reduction in average markup and thus an increase in production.

[20] $L_{X,q}(q|\delta, w, n)$ is decreasing in q because of the following. The value of product line obtained from price revision increases with q, and thus a firm with greater q does not wait decline in real price lower than the optimal lower bound for firms with smaller q. Hence, the lower bound of real price, $e^{n\Delta(q|\delta,n)}$, is increasing in q. The average real price is definitely higher for high type firms.

[21] If δ is extremely high, $K(q|\delta, w, n)$ is close to $\phi(q|\delta, n)$.

E Other Properties

E.1 Firm Size and Quality

Relation to Market Concentration Firm size and firm quality are positively correlated. The expected number of product lines supplied by a type-q firm is

$$E\left[k|q\right] = \frac{K(q,\delta,w,n)}{M(q,\delta,w,n)} = \frac{\gamma(q|\delta,w,n)}{\delta-\gamma(q|\delta,w,n)} \frac{1}{\log\left(\frac{\delta}{\delta-\gamma(q|\delta,w,n)}\right)}, \tag{57}$$

which is strictly increasing in q from Proposition 2. This relationship also implies that the firm size and the average markup are also positively correlated because q determines the maximum markup rate.

Another important implication from equation (57) is about concentration in the market. If there are more high type firms, more product lines are occupied by those firms and the degree of concentration becomes higher.

Nominal Sales Distribution Because of log utility, nominal sales in each product line equals to nominal income, E_t, independent of prices. Hence, the sales distribution across firms is the same as the distribution of the number of products, k. This is the reason why we examined the relationship between sales distribution and inflation in Section 2.

Let $R(k|\delta,w,n)$ be the density around the sales of kE_t,

$$\begin{aligned} R(k|\delta,w,n) &\equiv \int_{\underline{q}(\delta,n)}^{\infty} \phi(q|\delta,n) \frac{M_k(q|\delta,w,n)}{M(q|\delta,w,n)} dq \\ &= \frac{1}{k} \int_{\underline{q}(\delta,n)}^{\infty} \phi(q|\delta,n) \left(\frac{\gamma(q|\delta,w,n)}{\delta}\right)^k \left[\log\frac{\delta}{\delta-\gamma(q|\delta,w,n)}\right]^{-1} dq. \end{aligned} \tag{58}$$

Table 1: Inflation and Reallocation

(a) Sales distribution:

	Top/Middle ratio			Top/Bottom ratio		
	(1)	(2)	(3)	(4)	(5)	(6)
	OLS	OLS	2SLS	OLS	OLS	2SLS
$\bar{\pi}^{\text{input}}$	100.6***	102.7***	194.6***	282.7***	289.0***	620.5***
	(20.16)	(20.58)	(27.73)	(63.48)	(64.53)	(87.52)
D.I. gap (T/M or T/B)		-0.0454	0.135		-0.134	0.365
		(0.18)	(0.18)		(0.49)	(0.53)
D.I.		-0.358	-0.547**		-1.220*	-1.876**
		(0.22)	(0.23)		(0.70)	(0.73)
Industry RS		2.168	3.437		-6.377	-5.679
		(5.69)	(6.08)		(17.85)	(19.26)
Constant	4.591	-20.99	-33.84	17.9800	104.8	106.2000
	(5.40)	(72.16)	(77.19)	(17.00)	(226.40)	(244.40)
Year/Industry FE	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	322	316	302	322	316	302
$\bar{R}^2$	0.509	0.507	0.483	0.679	0.679	0.653
Underidentification			164.1			165.4
Weak identification			22.97			23.37

(b) Employment distribution:

	Top/Middle ratio			Top/Bottom ratio		
	(1)	(2)	(3)	(4)	(5)	(6)
	OLS	OLS	2SLS	OLS	OLS	2SLS
$\bar{\pi}^{\text{input}}$	3.484***	3.343***	5.903***	11.24***	11.80***	16.73***
	(1.027)	(0.992)	(1.309)	(2.734)	(2.674)	(3.497)
D.I. gap (T/M or T/B)		-0.0215***	-0.0116		0.0242	0.0436**
		(0.008)	(0.009)		(0.020)	(0.021)
D.I.		-0.0245**	-0.0279***		-0.0529*	-0.0588**
		(0.011)	(0.011)		(0.029)	(0.029)
Industry RS		1.097***	1.290***		2.735***	3.172***
		(0.273)	(0.284)		(0.737)	(0.764)
Constant	4.510***	-9.090***	-10.95***	11.21***	-23.19**	-28.16***
	(0.275)	(3.455)	(3.608)	(0.732)	(9.337)	(9.661)
Year/Industry FE	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	322	316	302	322	316	302
$\bar{R}^2$	0.7	0.729	0.723	0.776	0.791	0.786
Underidentification			163.7			165.1
Weak identification			22.84			23.27

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Inflation and firm growth

	Sales growth			Employment growth		
	(1)	(2)	(3)	(4)	(5)	(6)
	OLS	OLS	2SLS	OLS	OLS	2SLS
$\bar{\pi}^{\text{input}}$	-0.592***	-0.628***	-0.623***	-0.0464**	-0.0611***	-0.0408
	(0.0792)	(0.0797)	(0.1500)	(0.0191)	(0.0187)	(0.0354)
Size group	-0.00687***	-0.00838***	-0.00837***	-0.00245***	-0.00301***	-0.00296***
	(0.0007)	(0.0008)	(0.0009)	(0.0002)	(0.0002)	(0.0002)
$\bar{\pi}^{\text{input}} \times$ Size group	0.0524***	0.0570***	0.0564***	0.0148***	0.0170***	0.0145***
	(0.0118)	(0.0118)	(0.0195)	(0.0028)	(0.0028)	(0.0046)
Average D.I.		0.00156***	0.00155***		0.000528***	0.000516***
		(0.0004)	(0.0004)		(0.0001)	(0.0001)
Industry RS		(0.0024)	(0.0024)		-0.00766**	-0.00768**
		(0.0144)	(0.0143)		(0.0034)	(0.0034)
Constant	0.0881***	0.133	0.133	0.0211***	0.123***	0.123***
	(0.0125)	(0.1830)	(0.1810)	(0.0030)	(0.0429)	(0.0426)
Year/Industry FE	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	2940	2880	2880	2940	2880	2880
$\bar{R}^2$	0.179	0.181	0.181	0.27	0.289	0.289
Underidentification			798.2			796.9
Weak identification			83.43			83.25

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

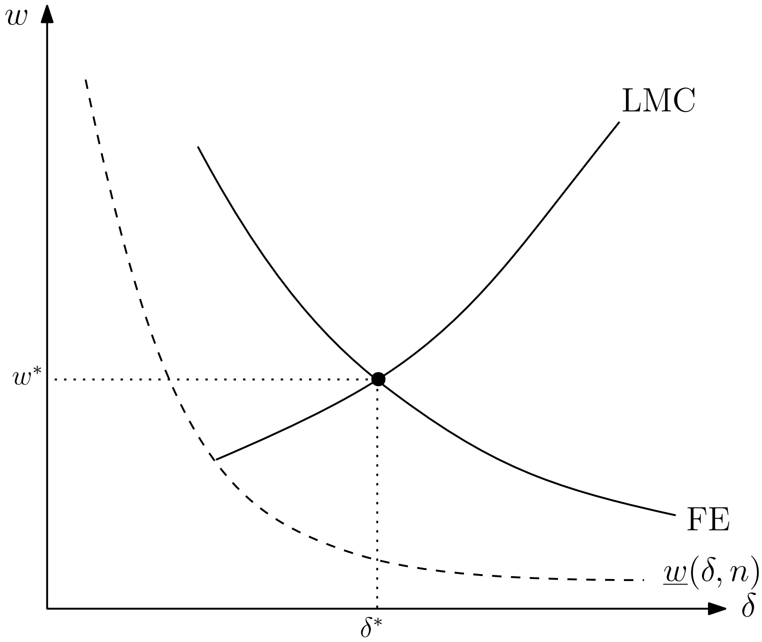

Figure 1: Stationary equilibrium. FE and LMC stand for the free entry condition and the labor market clearing condition, respectively.

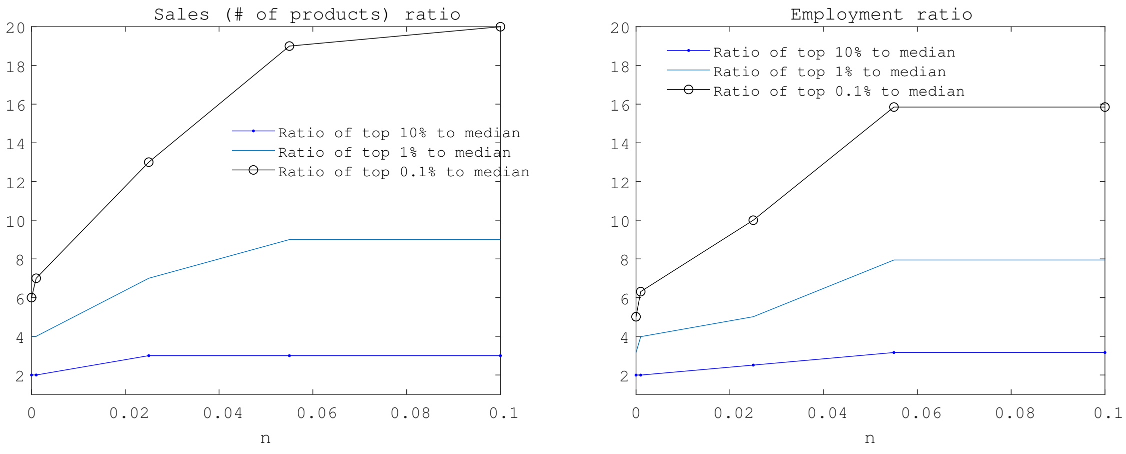

Figure 2: Sales and Employment Dispersion for Changes in the Nominal Growth Rate

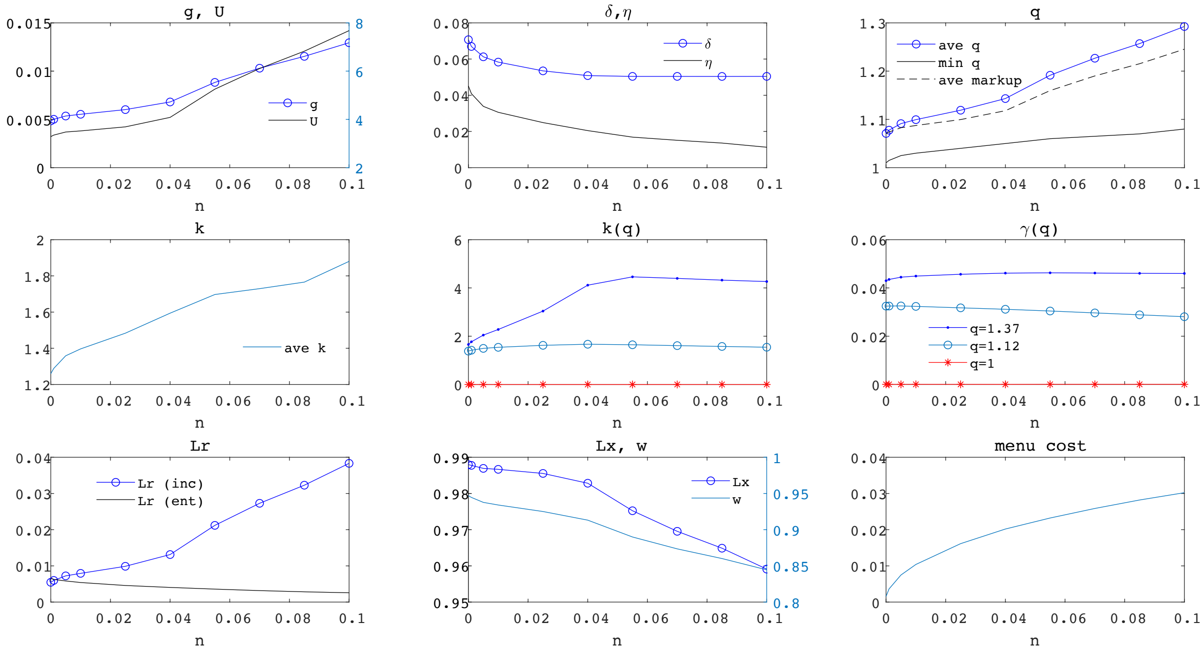

Figure 3: Effects of Nominal Growth
Note: The figures show changes in economic variables when the nominal growth rate, n, changes.

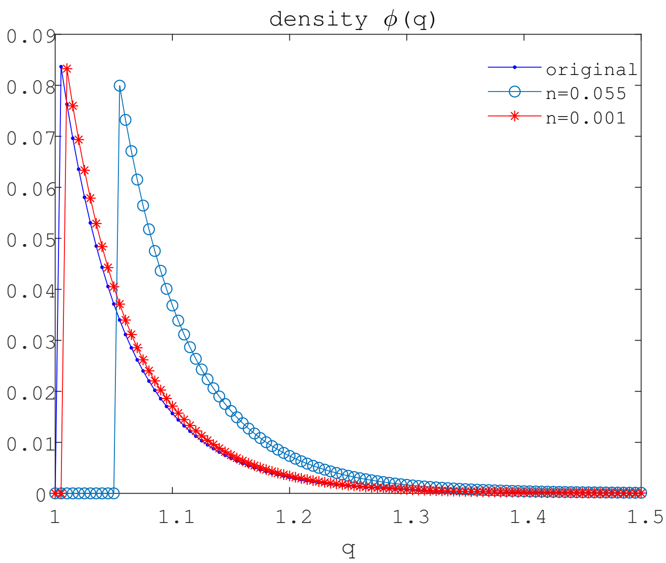

Figure 4: Changes in the Quality Distribution for Changes in the Nominal Growth Rate

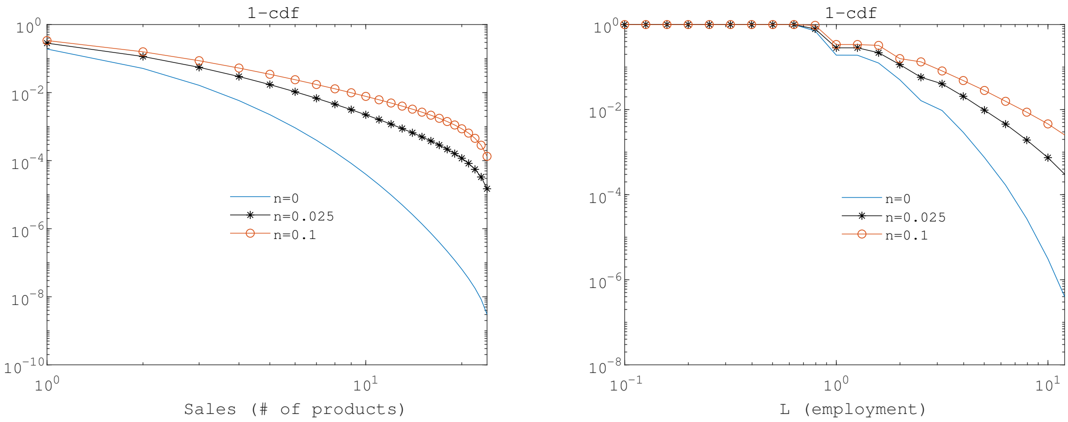

Figure 5: Changes in the Tail Distributions of Sales and Employment for Changes in the Nominal Growth Rate

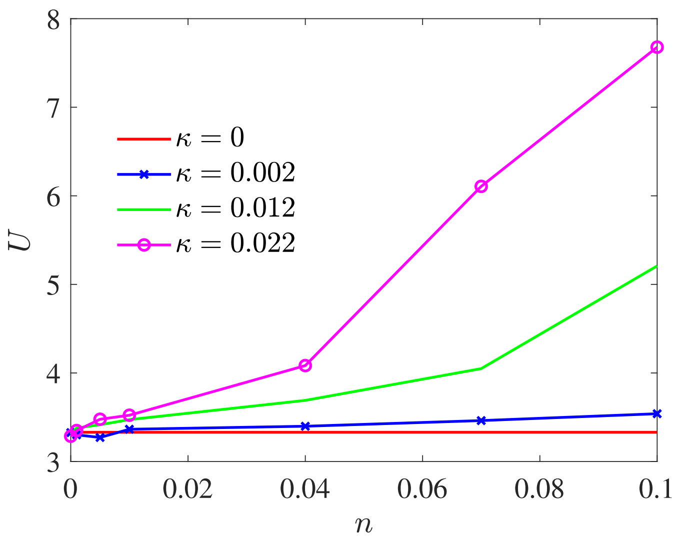

Figure 6: Effects of Nominal Growth on Welfare for Different Menu Cost Parameter Values

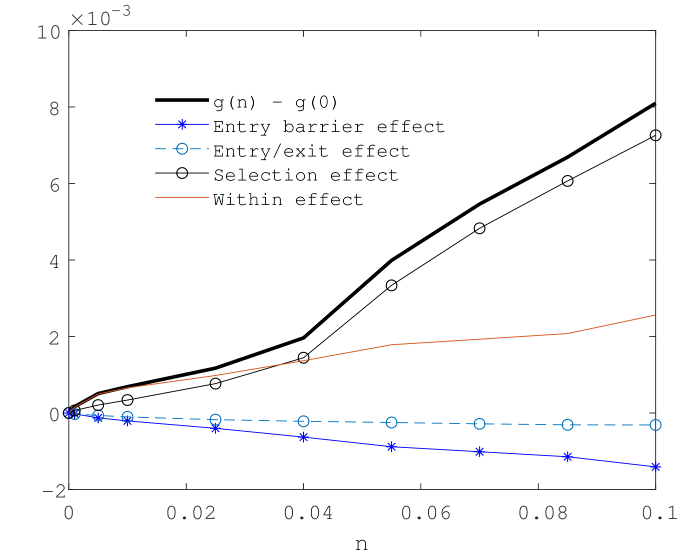

Figure 7: Growth Decomposition of $g(n) - g(0)$

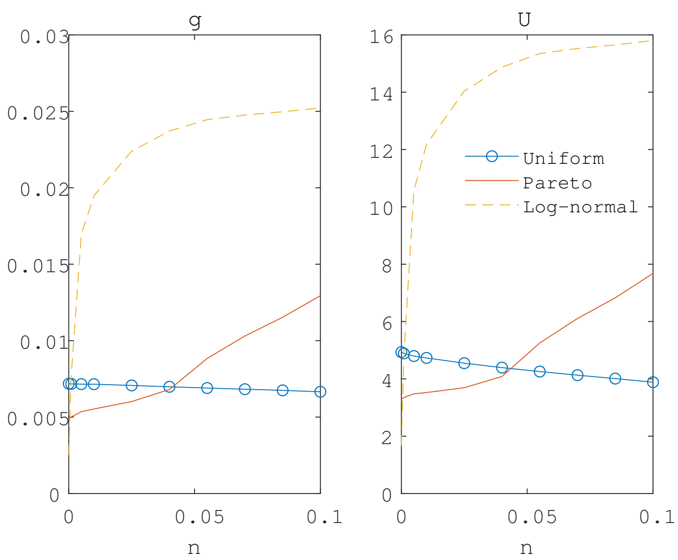

Figure 8: Effects of Nominal Growth under Different Distributions $\bar{\phi}(q)$

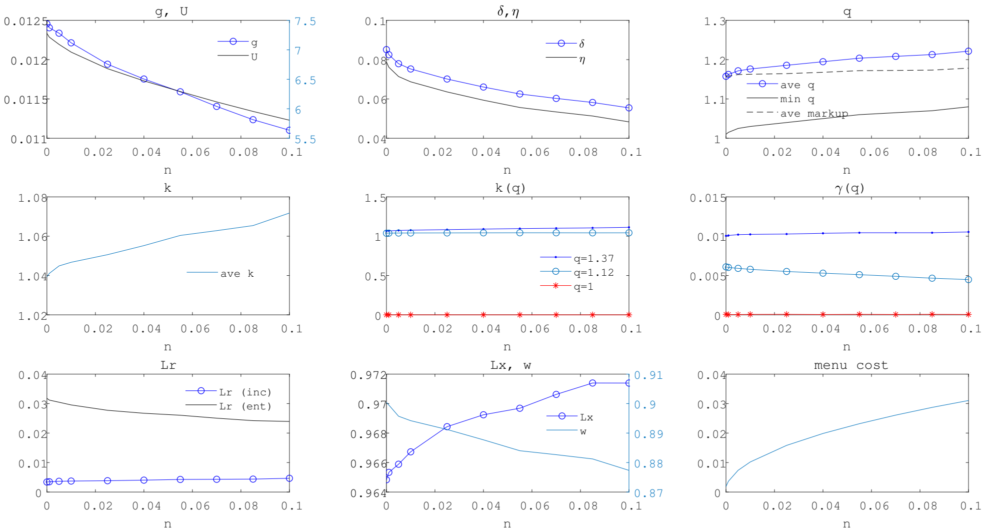

Figure 9: Effects of Nominal Growth for Japan. Note: The figures show changes in economic variables when the nominal growth rate, n, changes.