The Natural Rate of Interest in a Nonlinear DSGE Model

Yasuo Hirose¹  Takeki Sunakawa²

¹Keio University
²Kobe University

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Background

- Since Wicksell (1898), the natural rate of interest has been one of the key concepts of monetary policy analyses.

- Modern New Keynesian (NK) framework makes the concept of the natural rate relevant for welfare (Woodford, 2003; Galí, 2008).
  - Provides a useful benchmark for policymakers to measure the stance of monetary policy.
The natural rate is unobservable and must be estimated.

**Short-run estimates based on DSGE models:**


- Employ linearized DSGE models that abstract from the ZLB

**Long-run estimates based on semi-structural/reduced-form models:**

What we do

- Estimate the natural rate of interest in the U.S. using a nonlinear New Keynesian DSGE model with the ZLB.

- Examine how and to what extent nonlinearities affect the estimates of the natural rate.

  - The recent experience of the low interest rates has led researchers to conduct an empirical analysis that takes account of the ZLB:

What we do: Estimation strategy

- Estimating the nonlinear model is very costly.

- The two-step approach as in Aruoba, Cuba-Borda, and Schorfheide (2017):
  1. Estimate a linearized model using data prior to the date when the nominal interest rate is bounded at zero.
  2. Given the estimated parameters, we solve a fully nonlinear model with the ZLB and apply a nonlinear filter to extract the sequence of the natural rate for a full sample.

- For comparison, the natural rate based on its linear counterpart is also computed.
What we found

- The estimated natural rate based on the nonlinear model is higher than that based on the linearized model in the periods when the nominal interest rate is close to or bounded at zero.

- This difference is mainly ascribed to a contractionary effect of the ZLB which is considered only in the nonlinear model.

- Other nonlinearities, such as price and wage dispersion, have a limited effect on the estimate of the natural rate.
Outline

1. Introduction
2. The Model
3. Estimation Procedure
4. Results
5. Conclusion
Overview

- A standard empirical NK model with sticky wages and no capital
  - Households (labor unions, labor packers)
  - Intermediate-good firms
  - Final-good firm
  - Central bank

  - Calvo (1983)-type stickiness in prices and nominal wages.

  - Stochastic trend in TFP.

  - Exogenous shocks to: Productivity, Discount factor, Price markup, and Monetary Policy.

- The natural rate of interest is defined as the real interest rate that would prevail if prices and wages were fully flexible with no price markup shocks.
Overview, cont’d

Households: make consumption & saving decisions
Central bank: set the nominal interest rate
Source of nonlinearity: Price and wage dispersions

- The final-good market clearing condition is
  \[ Y_t = C_t. \]

- The labor market clearing condition is
  \[ l_t = \frac{\Delta_{w,t} \Delta_{p,t} Y_t}{A_t}, \]
  where \( l_t = \int_0^1 \int_0^1 l_{f,h,t} df dh \) is the aggregate labor input.

  - \( \Delta_{w,t} = \int_0^1 (W_{h,t} / W_t)^{-\theta_w} dh > 1: \) wage dispersion
  - \( \Delta_{p,t} = \int_0^1 (P_{f,t} / P_t)^{-\theta_p} df > 1: \) price dispersion
Source of nonlinearity: ZLB

- Monetary policy rule is given by:

\[ R_t^n = \max \left[ \hat{R}_t^n, 1 \right], \]

\[ \hat{R}_t^n = \left( \hat{R}_{t-1}^n \right)^{\phi_r} \left[ \hat{R} \bar{\Pi} \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} \left( \frac{Y_t}{Y_t^*} \right)^{\phi_y} \right]^{1-\phi_r} \exp(\varepsilon_{r,t}). \]

- The output gap is defined as the ratio of the actual output to the natural output.

- The ZLB has a contractionary effect, not only when it is binding but also when there is uncertainty about whether it will bind or not in the future.
The natural rate of interest

- The natural output and the natural rate are defined as the ones that would prevail if both prices and wages are perfectly flexible with no price markup shocks:

\[
(\gamma Y_t^* - \gamma Y_{t-1}^*) \left( \frac{Y_t^*}{A_t} \right)^\eta = \mu A_t,
\]

\[
R_t^* = \beta^{-1} d_t \left( \mathbb{E}_t \left\{ \frac{Y_t^* - \gamma Y_{t-1}^*}{Y_{t+1}^* - \gamma Y_t^*} \right\} \right)^{-1}.
\]

- \( Y_t^* \) is determined by the sequence of \( A_t \).
- \( R_t^* \) is determined by the sequences of \( Y_t^* \) and \( d_t \).
- The sequences of \((A_t, d_t)\) is indentified by the whole model including other equilibrium conditions.
Outline

1. **INTRODUCTION**
2. **THE MODEL**
3. **ESTIMATION PROCEDURE**
4. **RESULTS**
5. **CONCLUSION**
Strategy for estimation

- The two-step approach as in Aruoba, Cuba-Borda, and Schorfheide (2017)
  - Estimate a linearized model using data prior to the date when the nominal interest rate is bounded at zero.
  - Given the estimated parameters, we solve a fully nonlinear model with the ZLB and apply a nonlinear filter to extract the sequence of the natural rate for a full sample.
- For comparison, the natural rate based on the linearized model is also computed.
Estimation of parameters

- Data: the growth rate of per capita real GDP ($\Delta \log GDP_t$); the inflation rate of GDP deflator ($\Delta \log PGDP_t$); the federal funds rate ($FF_t$); the log of hours worked ($\log H_t$).

- Sample: 1983:I–2007:IV, up to the date when the nominal interest rate is bounded at zero.

- Observation equations ($y_t = Y_t / A_t$ is the detrended output):

$$
\begin{bmatrix}
\Delta \log GDP_t \\
\Delta \log PGDP_t \\
FF_t \\
\log H_t
\end{bmatrix}
= 100
\begin{bmatrix}
\bar{a} \\
\bar{\pi} \\
\bar{\pi} + \bar{r} \\
\bar{l}
\end{bmatrix}
+ 100
\begin{bmatrix}
\log(y_t / y_{t-1}) + a_t \\
\log(\Pi_t / \Pi) \\
\log(R^n_t / R^n) \\
\log(l_t / l)
\end{bmatrix}.
$$

- Estimated by Bayesian methods.

- In the subsequent analysis, parameters are fixed at the posterior mode.
Nonlinear solution

- The policy functions:
  \[ S = h(S_{-1}, \tau), \]
  where \( S_{-1} = (y_{-1}, \Pi_{-1}, w_{-1}, \hat{R}_{n-1}, \Delta p_{-1}, \Delta w_{-1}, y^*_1); \tau = (d, a, z, \varepsilon_r). \]

- Computed using the time-iteration method (c.f., Christiano and Fisher 2000; Gust et al. 2017) with a Smolyak approximation of sparse grids.
  - Apply more efficient algorithm developed by Judd, Maliar, Maliar, and Valero (2014).
  - Very accurate, albeit with the reduced number of grid points.
Nonlinear solution

According to samples simulated from the nonlinear solution, the economy is at the ZLB for 11.8% of quarters, and the average duration of the ZLB spells is 4.3 quarters.

- Fernández-Villaverde et al. (2015): ZLB prob. = 5.5%; ZLB duration = 2.1 qtrs.
- Gust et al. (2017): ZLB prob. = 4%; ZLB duration = 3.5 qtrs.
Filtering


- The data is the same as those used for the parameter estimation but the sample period is extended to 2016:III.

- The observation equations are the same but with measurement errors. Measurement errors of output growth, inflation, the nominal interest rate, and hours worked are respectively set to 20%, 10%, 5% and 5% of their standard deviations in the data.

- The number of particles is set $N = 20000$. 
Data vs. Smoothed estimates

Output growth

Inflation

Hours

Nominal interest rate
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Natural rate of interest

Graph showing the natural rate of interest over time from 1985 to 2015. The graph compares a nonlinear model (blue line) and a linear model (green line). The x-axis represents years from 1985 to 2015, and the y-axis represents the natural rate of interest ranging from -25 to 20. The graph indicates fluctuations in the natural rate of interest with the nonlinear model generally showing more volatile changes compared to the linear model.
Some notes on the estimates

- Our estimate of the natural rate ranges from -12% to 15% in the nonlinear model (from -20% to 15% in the linear model).

- Even though the timing of peaks and troughs is similar, the magnitude is a bit larger than the ones in previous studies.
  - Barsky, Justiniano and Melosi (2014): -6% to 12%
  - Del Negro, Giannone, Ginannoni and Tambalotti (2017): -6% to 12%
Identification of the natural rate of interest

- The natural output and the natural rate are determined by:

\[
(Y_t^* - \gamma Y_{t-1}^*) \left( \frac{Y_t^*}{A_t} \right)^{\eta} = \mu A_t, \\
R_t^* = \beta^{-1} d_t \left( \mathbb{E}_t \left\{ \frac{Y_t^* - \gamma Y_{t-1}^*}{Y_{t+1}^* - \gamma Y_t^*} \right\} \right)^{-1}.
\]

- \(Y_t^*\) is determined by the sequence of \(A_t\).
- \(R_t^*\) is determined by the sequences of \(Y_t^*\) and \(d_t\).
- Therefore, the natural rate is pinned down by identifying \(A_t\) and \(d_t\).
Identification of productivity shock $a_t$

- In the linearized model, $a_t = \log(A_t/A)$ is identified by the data of output growth and hours worked.
  - Labor market clearing condition: $\tilde{l}_t = \tilde{y}_t$.
  - Observation equations: $\frac{\Delta \log GDP_t}{100} = \bar{a} + \tilde{y}_t - \tilde{y}_{t-1} + a_t; \frac{\log H_t}{100} = \bar{l} + \tilde{l}_t$.

- In the nonlinear model, the labor market clearing condition is
  
  $$l_t = \Delta p, t \Delta w, t y_t,$$

  where $\Delta p, t \geq 1$ and $\Delta w, t \geq 1$.

  - $y_t$ becomes smaller and $a_t$ is identified to be larger, which results in the higher natural rate.
Identification of discount factor shock $d_t$

- In the nonlinear model, the ZLB has a contractionary effect:
  - not only when the ZLB is binding,
  - but also when there is uncertainty about whether the ZLB will bind or not in the future (c.f., Hills, Nakata and Schmidt, 2016).

- While such an effect lowers expected output and inflation, actual output and inflation are pegged to the data in the filtering process.
  - $d_t$ must increase, or expectation terms must decrease, to satisfy the consumption Euler equation (as well as the other optimality conditions):

\[
\hat{d}_t \approx \hat{R}_t^n - \mathbb{E}_t \hat{\Pi}_{t+1} + \hat{y}_t - \mathbb{E}_t \hat{y}_{t+1} \\
+ \text{(other terms pegged to the data & at higher order).}
\]
Estimated Shocks

Discount factor shock

Productivity shock

Nonlinear model
Linear model
Expected inflation and output

**Expected inflation**

- Nonlinear model
- Linear model

**Expected output**
Price and wage dispersions

**Price Dispersion**

- 1985: 0.03
- 1990: 0.02
- 1995: 0.01
- 2000: 0.00
- 2005: 0.02
- 2010: 0.03
- 2015: 0.04

**Wage Dispersion**

- 1985: 0.1
- 1990: 0.08
- 1995: 0.06
- 2000: 0.04
- 2005: 0.02
- 2010: 0.1
- 2015: 0.08
Consider a quasi-linear model, in which the ZLB constraint is imposed but all the equilibrium conditions are linearized.

Monetary policy rule is replaced with (\(\tilde{\cdot}\) stands for log-deviation from the deterministic steady state)

\[
\begin{align*}
\tilde{R}_t^n &= \max \left[ \tilde{R}_t^n, -\log R^n \right], \\
\tilde{\tilde{R}}_t^n &= \phi_r \tilde{R}_{t-1} + (1 - \phi_r) \left( \phi_\pi \tilde{\Pi}_t + \phi_y (\tilde{y}_t - \tilde{y}_t^*) \right) + \epsilon_{r,t}.
\end{align*}
\]
The natural rate of interest: Nonlinear vs. Quasi-linear
Estimated Shocks: Nonlinear vs. Quasi-linear

Discount factor shock

-5 0 5

Nonlinear model
Quasi-linear model

Productivity shock

-1 -0.5 0 0.5 1

Nonlinear model
Quasi-linear model
Outline

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Concluding Remarks

- The estimated natural rate is very different from the one estimated with its linear counterpart.

- This difference is mainly explained by a contractionary effect of the ZLB.

- Other nonlinearities than the ZLB, such as price and wage dispersions, have a limited effect.

- Our analysis could be extended by exploiting a medium-scale model and/or estimating the original nonlinear model.
Household

Each household \( h \in [0, 1] \) maximizes the utility function

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} D_{0,t} \left\{ \log (C_{h,t} - \gamma C_{t-1}) - \frac{l_{h,t}^{1+\eta}}{1+\eta} \right\},
\]

subject to the budget constraint

\[
P_tC_{h,t} + B_{h,t} \leq W^n_{h,t} l_{h,t} + R^n_{t-1} B_{h,t-1} + T_{h,t},
\]

where \( D_{t,t+j} = \prod_{k=t+1}^{t+j} \beta / d_k \) is the cumulative discount factor.

Discount factor shock:

\[
\log d_t = \rho_d \log d_{t-1} + \epsilon_{d,t},
\]

where \( \epsilon_{d,t} \sim N(0, \sigma^2_d) \).
Labor packer

A labor packer resells the labor package \( l_{f,t} \) to intermediate-good firm \( f \in [0,1] \) by collecting each differentiated labor \( \{l_{f,h,t}\} \) from households so as to maximize profit

\[
W^n_t l_{f,t} - \int_0^1 W^n_{h,t} l_{f,h,t} dh,
\]

subject to a CES aggregator

\[
l_{f,t} \equiv \left( \int_0^1 l_{f,h,t}^{\theta_w-1} dh \right)^{\frac{\theta_w}{\theta_w-1}}.
\]
Labor union

  - In each period, a fraction \(1 - \xi_w \in (0, 1)\) labor unions reoptimize nominal wages, while the remaining fraction \(\xi_w\) index nominal wages (adjusted by the economy's deterministic trend \(\gamma_a \equiv e^{\bar{a}}\)) to a weighted average of \(\Pi_{t-1}\) and \(\bar{\Pi}\).

- The unions that reoptimize their nominal wages then maximize expected utility

\[
E_t \sum_{j=0}^{\infty} \xi_w^j D_{t,t+j} \left( \frac{\gamma_a W_{h,t}^n}{P_{t+j}} \prod_{k=1}^{j} \left( \Pi_{t+k-1}^{\Pi_{1-\xi_w}} \right) \Lambda_{h,t+j} l_{h,t+j} - \frac{l_{h,t+j}^{1+\eta}}{1 + \eta} \right),
\]

where \(l_{h,t} \equiv \int_0^1 l_{f,h,t} df\), subject to the labor demand

\[
l_{f,h,t+j} = \left[ \frac{\gamma_a W_{h,t}^n}{W_{t+j}} \prod_{k=1}^{j} \left( \Pi_{t+k-1}^{\Pi_{1-\xi_w}} \right) \right]^{-\theta_w} l_{f,t+j}.
\]
Final-Good Firm

The representative final-good firm produces output $Y_t$ by choosing intermediate inputs $\{Y_{f,t}\}$ so as to maximize profit

$$P_t Y_t - \int_0^1 P_{f,t} Y_{f,t} df,$$

subject to a CES production technology

$$Y_t = \left( \int_0^1 Y_{f,t}^{\frac{\theta_p - 1}{\theta_p}} df \right)^{\frac{\theta_p}{\theta_p - 1}}.$$
Intermediate-Good Firm

- Each intermediate-good firm $f$ produces differentiated good $Y_{f,t}$ by cost-minimizing labor input $l_{f,t}$ (purchased from labor packer $f$) subject to the production function

$$Y_{f,t} = A_t l_{f,t}.$$  

- $A_t$ is represents total factor productivity which follows

$$\log A_t = \log \gamma_a + \log A_{t-1} + a_t,$$

where $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$ and $\varepsilon_{a,t} \sim N(0, \sigma_a^2)$. 

Intermediate-Good Firm, cont’d


- The firms that reoptimize their prices then maximize

\[
\mathbb{E}_t \sum_{j=0}^{\infty} \xi_j D_{t,t+j} \frac{\Lambda_{t+j}}{\Lambda_t} \left[ \frac{P_{f,t}}{P_{t+j}} \prod_{k=1}^{j} \left( \Pi_{t+k-1}^{l_p} \Pi_{t+k-1}^{1-l_p} \right) - \frac{W_{t+j}}{A_{t+j}} z_{t+j} \right] Y_{f,t+j},
\]

subject to the final-good firm’s demand

\[
Y_{f,t+j} = \left[ \frac{P_{f,t}}{P_{t+j}} \prod_{k=1}^{j} \left( \Pi_{t+k-1}^{l_p} \Pi_{t+k-1}^{1-l_p} \right) \right]^{-\theta_p} Y_{t+j}.
\]

- Cost-push shock:

\[
\log z_t = \rho_z \log z_{t-1} + \varepsilon_{z,t},
\]

where \( \varepsilon_{z,t} \sim N(0, \sigma_z^2) \).
Market clearing conditions

- The final-good market clearing condition is
  \[ Y_t = C_t. \]

- The labor market clearing condition is
  \[ l_t = \frac{\Delta_p, t \Delta_w, t Y_t}{A_t}, \]
  where \( l_t = \int_0^1 \int_0^1 l_{f, h, t} df dh \) is aggregate labor input,
  - \( \Delta_p, t = \int_0^1 \left( \frac{P_{f, t}}{P_t} \right)^{-\theta_p} df > 1 \) represents price dispersion, and
  - \( \Delta_w, t = \int_0^1 \left( \frac{W_{h, t}}{W_t} \right)^{-\theta_w} dh > 1 \) represents wage dispersion.
Closing the model

- Monetary policy rule is given by:

\[ R^n_t = \max \left( \hat{R}^n_t, 1 \right), \]

\[ \hat{R}^n_t = \left( \hat{R}^n_{t-1} \right)^{\phi_r} \left( \frac{\bar{\Pi}}{\bar{\Pi}} \right)^{\phi_{\pi}} \left( \frac{Y_t}{Y^*_t} \right)^{\phi_y} \exp(\varepsilon_{r,t}), \]

where monetary policy shock is \( \varepsilon_{r,t} \sim N(0, \sigma_r^2) \).

- Since \( A \) is nonstationary, some variables are detrended such as \( y = Y / A \), \( w = W / A \) and \( y^* = Y^* / A \).
Linearized model

\[ \tilde{y}_t = \frac{\gamma a}{\gamma a + \gamma} (\mathbb{E}_t \tilde{y}_{t+1} + a_{t+1}) + \frac{\gamma}{\gamma a + \gamma} (\tilde{y}_{t-1} - a_t) \]

\[ - \frac{\gamma a - \gamma}{\gamma a + \gamma} (\tilde{R}_t - \mathbb{E}_t \tilde{\Pi}_{t+1} - \tilde{a}_t), \]

\[ \tilde{\Pi}_t = \frac{\beta}{1 + \beta \ell_p} \mathbb{E}_t \tilde{\Pi}_{t+1} + \frac{\ell_p}{1 + \beta \ell_p} \tilde{\Pi}_{t-1} + \kappa_p (\tilde{w}_t + \tilde{z}_t), \]

\[ \tilde{\Pi}_{w,t} = \beta (E_t \tilde{\Pi}_{w,t+1} + a_{t+1} - \ell_w \tilde{\Pi}_t) + \ell_w \tilde{\Pi}_{t-1} - a_t \]

\[ + \kappa_w \left[ \left( \eta + \frac{\gamma a}{\gamma a - \gamma} \right) \tilde{y}_t - \frac{\gamma}{\gamma a - \gamma} \tilde{y}_{t-1} + \frac{\gamma}{\gamma a - \gamma} a_t - \tilde{\omega}_t \right], \]

\[ \tilde{\omega}_t = \tilde{\Pi}_{w,t} - \tilde{\Pi}_t + \tilde{w}_{t-1}, \]

\[ \tilde{R}_t^n = \phi_r \tilde{R}_{t-1}^n + (1 - \phi_r) \left( \phi_{\pi} \tilde{\Pi}_t + \phi_{\gamma} (\tilde{y}_t - \tilde{y}_t^*) \right) + \varepsilon_{r,t}, \]

where \( \kappa_p \equiv \frac{(1 - \xi_p)(1 - \xi_p \beta)}{\xi_p (1 + \beta \ell_p)} \) and \( \kappa_w \equiv \frac{(1 - \xi_w)(1 - \beta d^{-1} \xi_w)}{\xi_w (1 + \eta \theta_w)}. \)
## Priors and posteriors

<table>
<thead>
<tr>
<th>Param.</th>
<th>Dist.</th>
<th>Mean</th>
<th>S.D.</th>
<th>Mode</th>
<th>Mean</th>
<th>90%</th>
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<td>0.515</td>
<td>[0.429, 0.596]</td>
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## Priors and posteriors, cont’d

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<th>Prior Distribution</th>
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<th>Priors</th>
<th>Mode</th>
<th>Mean</th>
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