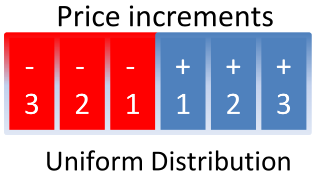
経済・社会への分野横断的研究会
主催：キャノングローバル戦略研究所

共催：文部科学省　ポスト「京」萌芽的課題
「多層マルチ時空間スケール社会・経済シミュレーション技術の研究・開発」
サブ課題「マクロ経済シミュレーション」
2017年9月25・26日

Quantized volatility model for transaction data

by

Hiroyuki Moriya
Quasars22,Singapore



Price Movements

Random walk hypothesis

$$P_t = P_{t-1} + w_t = P_0 + \sum_{j=1}^{t} w_j ,$$

where w_j is a ramdon variable,

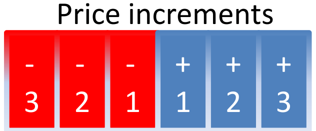
$E(w_j) = 0,$

$E(w_j w_k) = 0 \; (j \neq k)$, and

$E(w_j w_j) = \text{constant}$

Fama,E(1965)"Random walks in stock market prices"
Samuelson,P(1965)"Proof that properly anticipated prices fluctuate randomly"
Black,F.,Scholes,M.(1973)"The pricing of options and corporate Liabilities"
Cox,J.Leland,H."(2000)On dynamic investment strategies"

Price increments



Uniform Distribution

Price Movements

A random walk hypothesis

$$\log(P_t) = \log(P_0) + \sigma B_t + \left(\mu + \frac{\sigma^2}{2} \right) t$$

where B_t is a wiener process, μ_t is a drift rate, and σ_t is a volatility.

Black,F.,Scholes,M.(1973)"The pricing of options and corporate Liabilities"

A non-random walk hypothesis

$$P_t = \mu_t \mathrm{d}t + P_{t-1} + \sigma_t \mathrm{d}B_t$$

Lo,A.(2005)"The adoptive markets hypothesis"

Price increments

-3 -2 -1 +1 +2 +3

Uniform Distribution

Price Movements

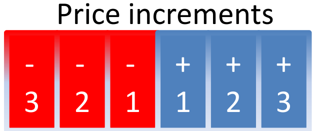
Short-term seasonality vs long-term stable volatility

Random walk hypothesis

Vs

Non-random walk hypothesis

Price increments



Uniform Distribution

Price Movements

Price Movements with ticks

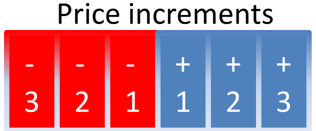
$$P_t = P_{t-1} + \varepsilon_t$$

where

$$\varepsilon_i = \pm \varepsilon_0 \times i,$$

ε_0 is the minimum size of price increment specified by the stock exchange, *i* is an integer.

Price increments



Uniform Distribution

Price Movements

Price Movements with ticks

$$P_t = P_{t-1} + \varepsilon_t$$

	売気配		買気配	
		成行		
最良気配外指値注文	641	17140		
最良気配指値注文	331	17135		即時約定指値注文
即時約定指値注文		17130	138	最良気配指値注文
		17125	499	最良気配外指値注文

Price increments

-3	-2	-1	+1	+2	+3

Uniform Distribution

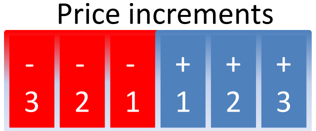
Price Movements

Price Movements with ticks

$$P_t = P_{t-1} + \varepsilon_t$$

	売気配	17:55 成行	買気配
	361	16555	
	208	16550	
		16545	296
		16540	369
時刻	現在値	前回比	出来数量
17:55	16550	0	4
17:55	16550	0	8
17:54	16550	0	1
17:54	16550	0	5
17:54	16550	0	6
17:54	16550	0	2
17:54	16550	0	80

Price increments

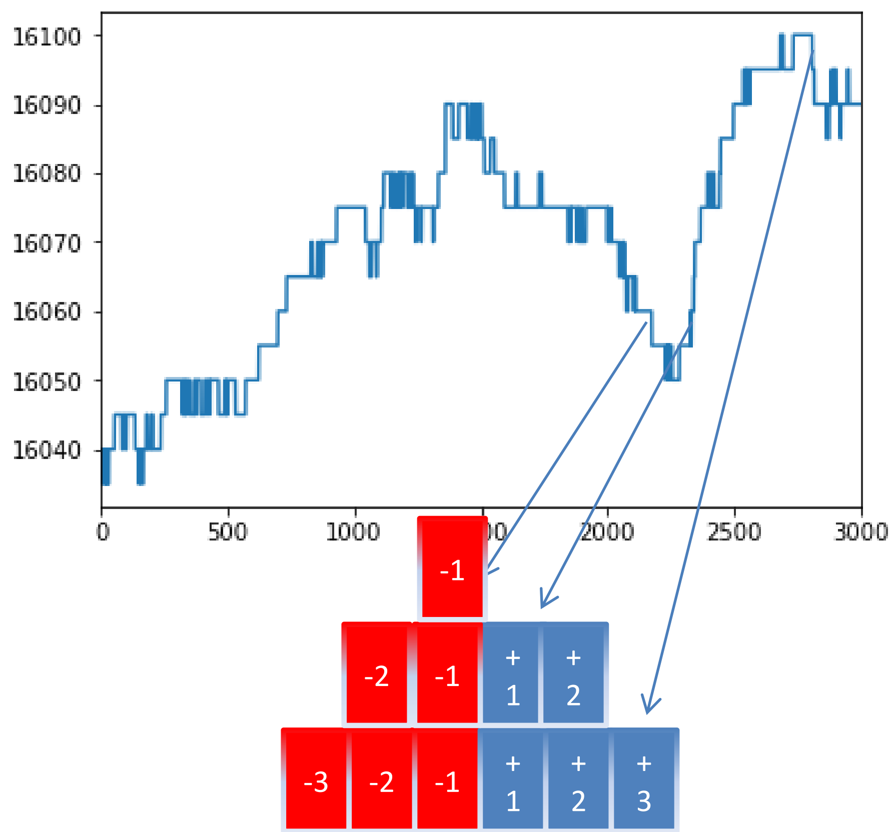


Uniform Distribution

Price Movements

Price Movements with ticks

$$P_t = P_{t-1} + \varepsilon_t$$



The sum of squared price increments

Squared price increments

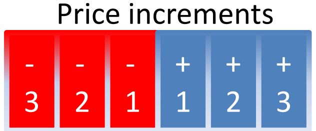
$$e_i = (\pm \varepsilon_i)^2$$

where $\varepsilon_i = \pm \varepsilon_0 \mathrm{x}\ i$

Sum of squared price increments

$$E = \sum_{i=1}^{I} N_i e_i$$

Price increments



Uniform Distribution

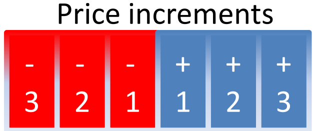
The sum of squared price increments

Sum of squared price increments

$$E = \sum_{i=1}^{I} N_i e_i$$

must be stable for a long-term.

Price increments



Uniform Distribution

The sum of squared price increments

Sum of squared price increments

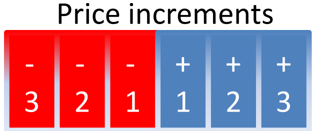
must be stable for a long-term.

But why?

Markets balance the interests between

Investors and market makers.

Price increments



Uniform Distribution

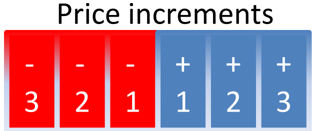
The sum of squared price increments

Sum of squared price increments
must be stable for a long-term, but why?

Investors want to
minimize the bid-ask spread and
have homogeneous transaction price.

Market makers cover their losses from
adverse selections.

Price increments

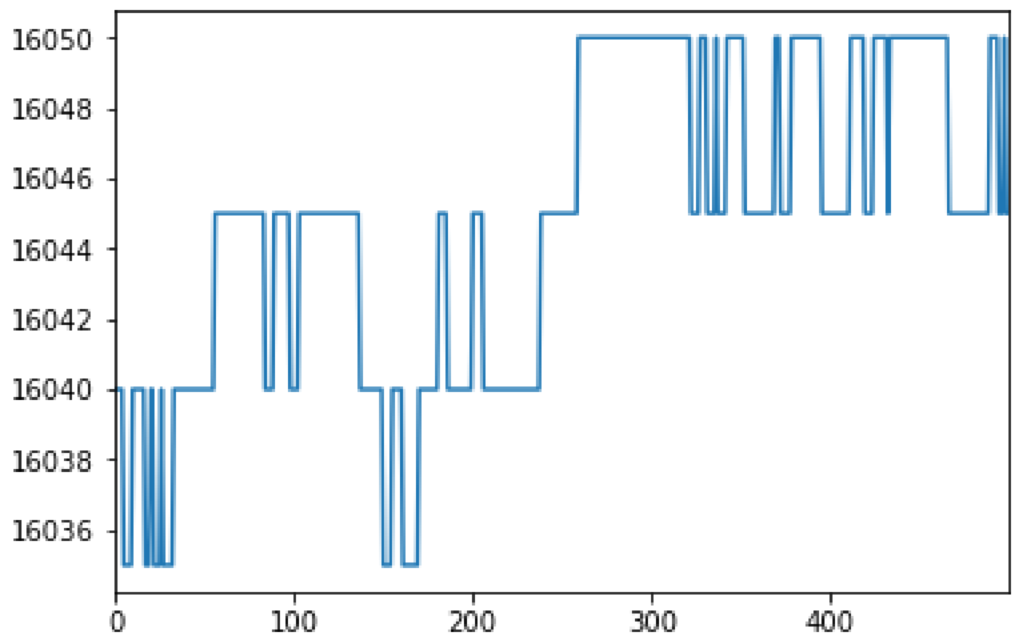


Uniform Distribution

The sum of squared price increments

Darkice, Iceberg algorithms and **stealth trading** strategies are implemented to reduce market impacts.

$$E = \sum_{i=1}^{I} N_i e_i$$



The sum of squared price increments

How to obtain the sum of squared price increments?

Remove all transactions without price movements and bid-ask bounce effects.

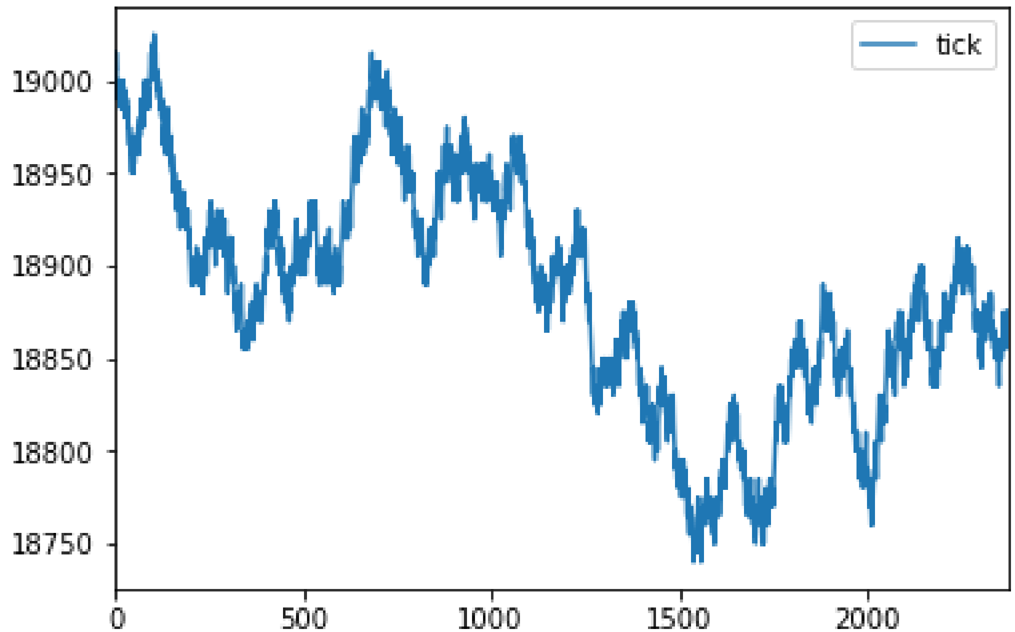
It is called an **immediacy trades**.

In Nikkei 225 mini, 98% of transactions are not immediacy trades.

$$E = \sum_{i=1}^{I} N_i e_i$$

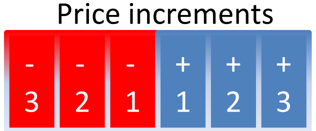
The sum of squared price increments

How does a price of immediacy trade move?



$$E = \sum_{i=1}^{I} N_i e_i$$

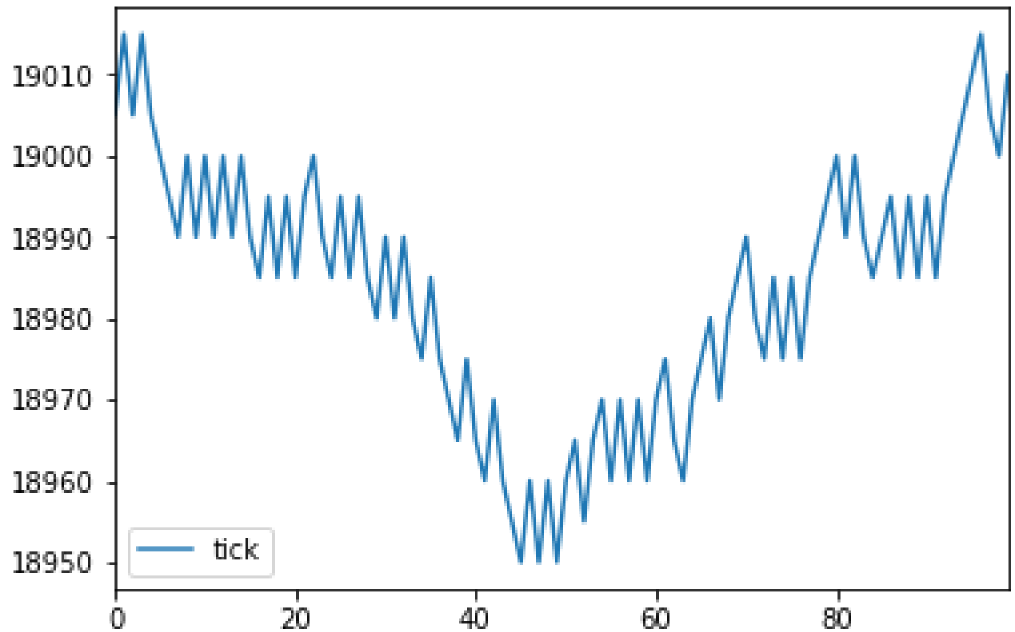
Price increments



Uniform Distribution

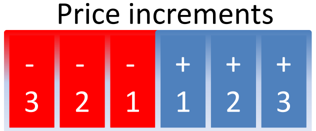
The sum of squared price increments

How does a price of immediacy trade move?



$$E = \sum_{i=1}^{I} N_i e_i$$

Price increments



Uniform Distribution

The sum of squared price increments

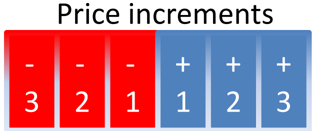
If prices of immediacy trade follow a random walk,

MMs and investors prefer stable markets.

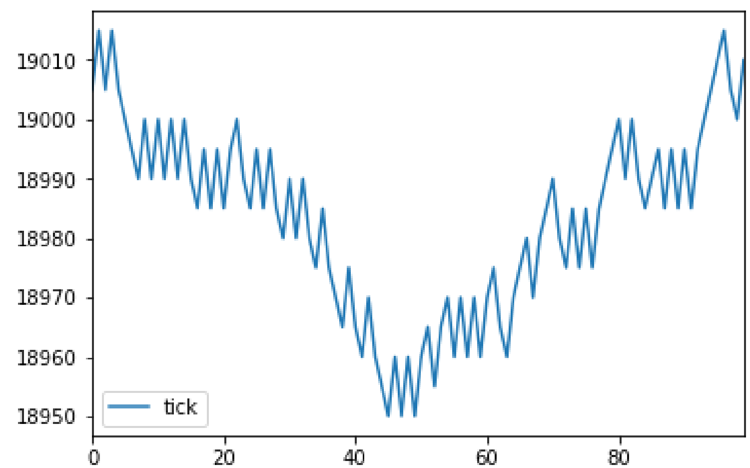
Use the runs test and the Durbin-Watson test.

$$E = \sum_{i=1}^{I} N_i e_i$$

Price increments



Uniform Distribution

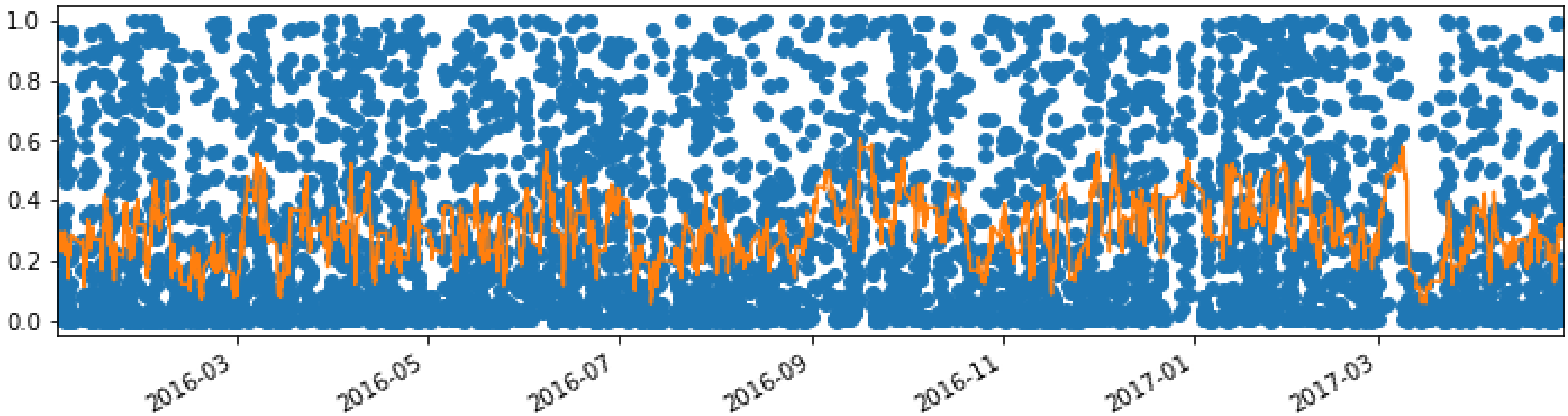


The sum of squared price increments

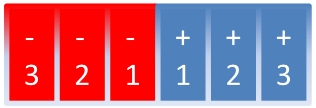
The runs test p-value

The probability of p-value>0.1 is 0.59.

From 2016.01 to 2017.04 (hourly analysis)



Price increments



Uniform Distribution

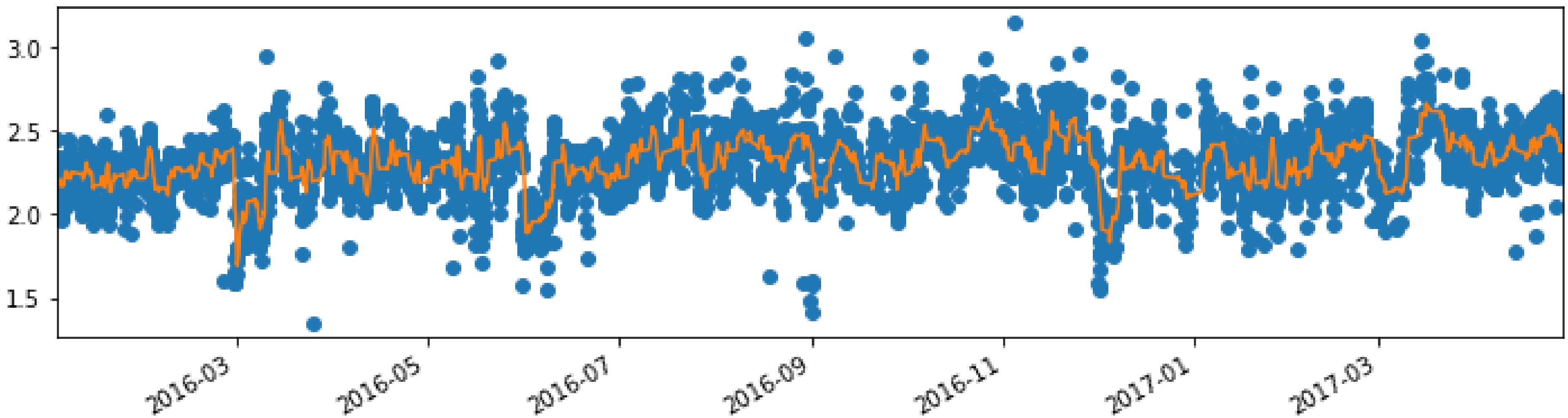
$$E = \sum_{i=1}^{I} N_i e_i$$

The sum of squared price increments

The Durbin-Watson test

Average dw=2.3

From 2016.01 to 2017.04 (hourly analysis)



$$E = \sum_{i=1}^{I} N_i e_i$$

The sum of squared price increments

Immediacy trades may follow a random walk process.

Price increments

-3	-2	-1	+1	+2	+3

Uniform Distribution

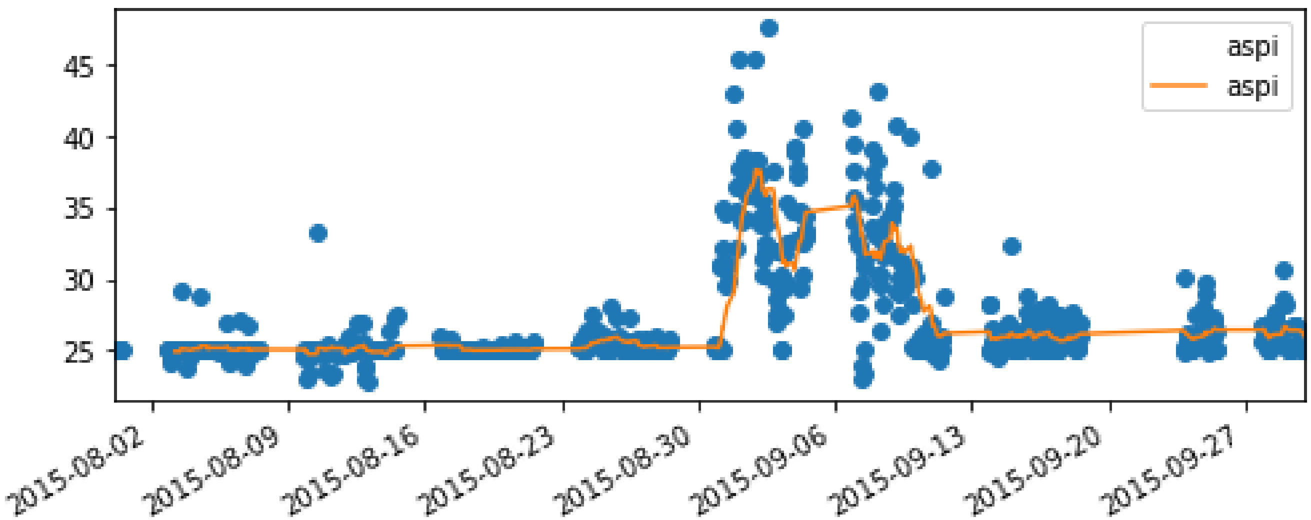
$$E = \sum_{i=1}^{I} N_i e_i$$

The sum of squared price increments

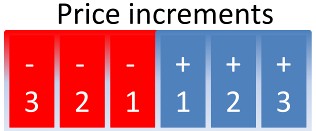
How the sum of squared price increments moves?

The average sspi=27.01

from 2015.08 to 2015.09



Price increments



Uniform Distribution

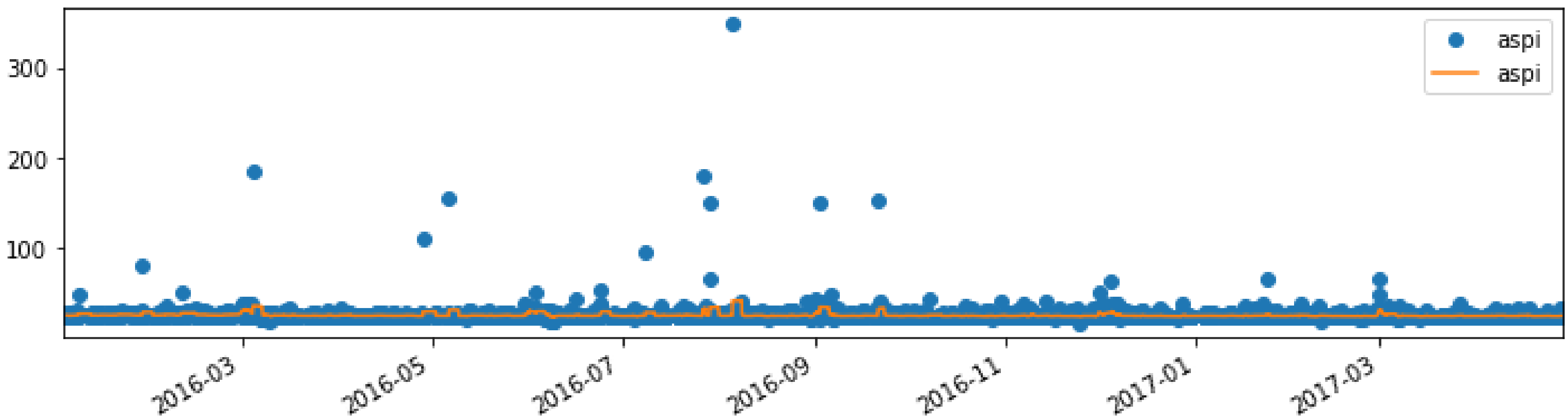
$$E = \sum_{i=1}^{I} N_i e_i$$

The sum of squared price increments

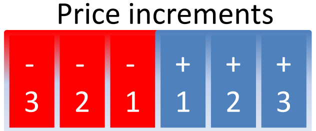
How the sum of squared price increments moves?

The average sspi=26.1 f

rom 2016.01 to 2017.04



Price increments



Uniform Distribution

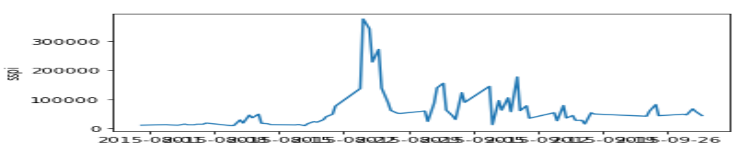
$$E = \sum_{i=1}^{I} N_i e_i$$

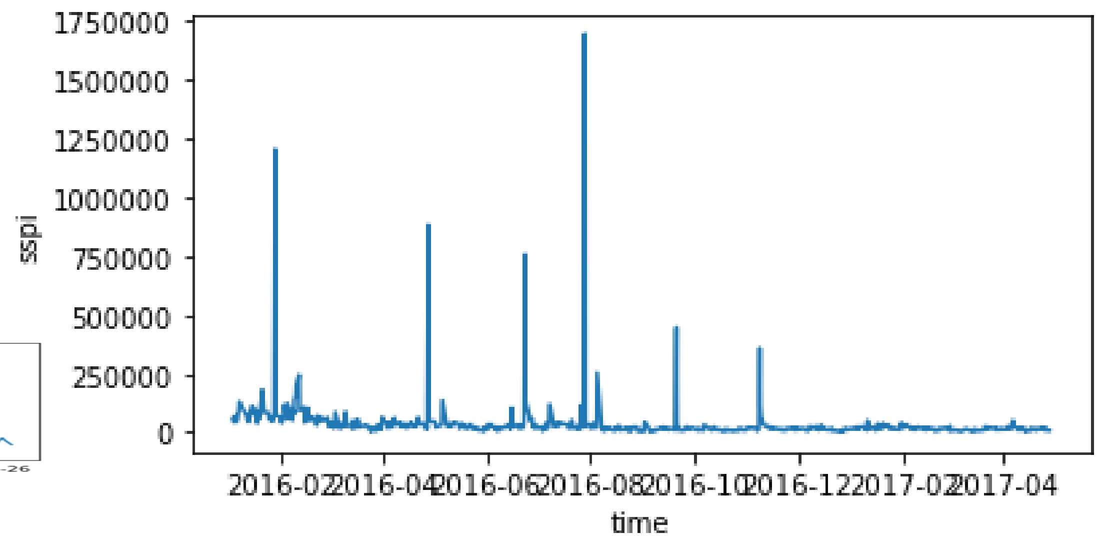
The sum of squared price increments

How the sum of squared price increments moves?

From 2015.08 to 2015.09

from 2016.01 to 2017.04





Price increments

-3	-2	-1	+1	+2	+3

Uniform Distribution

$$E = \sum_{i=1}^{I} N_i e_i$$

The sum of squared price increments

The sum of squared price increments

Might be stable for a long-term,

but

have a seasonality in a short-term.

Price increments

-3	-2	-1	+1	+2	+3

Uniform Distribution

$$E = \sum_{i=1}^{I} N_i e_i$$

The sum of squared price increments

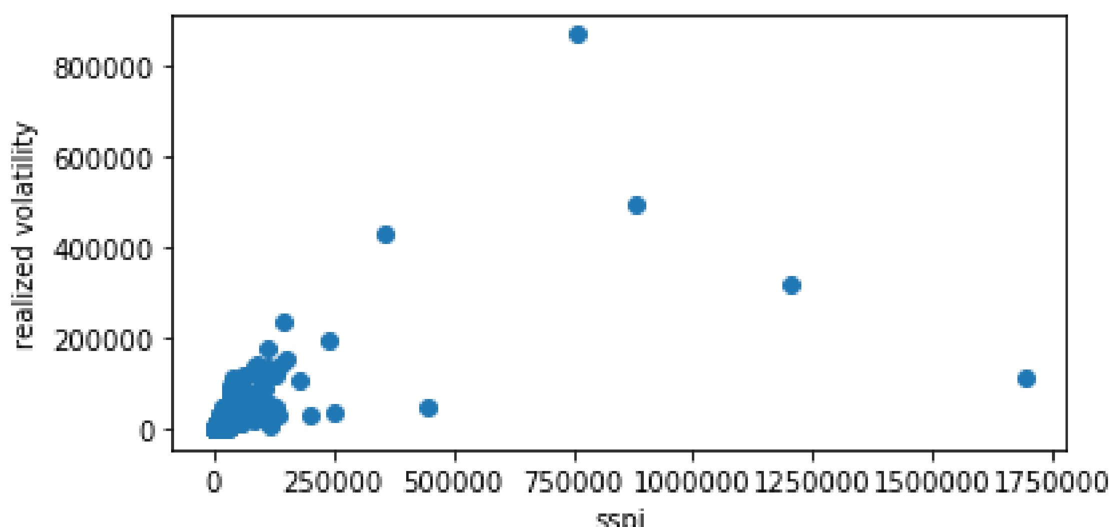
The sum of squared price increments

Vs

The realized volatility (1hour interval)

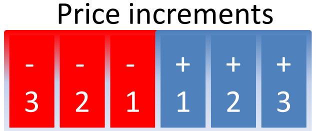
rom 2016.01 to 2017.04

Corr=0.62



$$E = \sum_{i=1}^{I} N_i e_i$$

Price increments



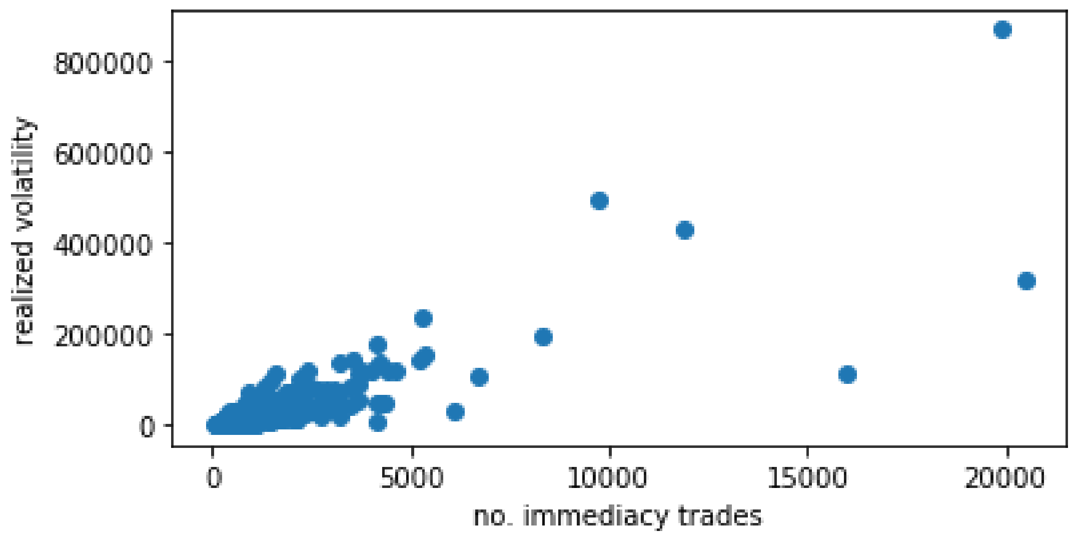
Uniform Distribution

The sum of squared price increments

The number of immediacy trades
Vs
The realized volatility (1hour interval)

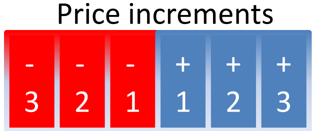
rom 2016.01 to 2017.04

Corr=0.84



$$E = \sum_{i=1}^{I} N_i e_i$$

Price increments



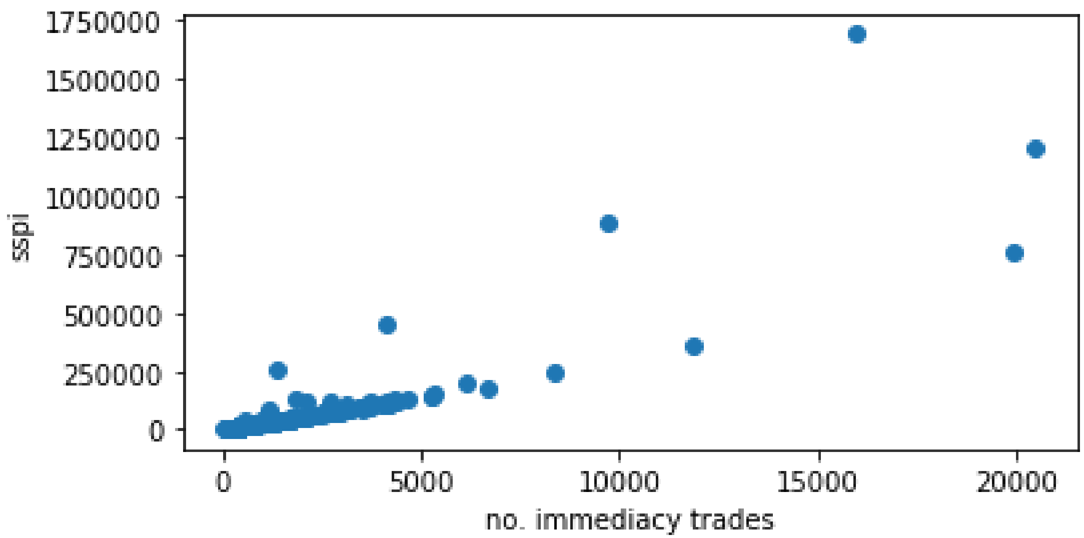
Uniform Distribution

The sum of squared price increments

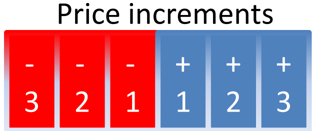
The number of immediacy trades
Vs
The sspi (1hour interval)

from 2016.01 to 2017.04

Corr=0.87



Price increments



Uniform Distribution

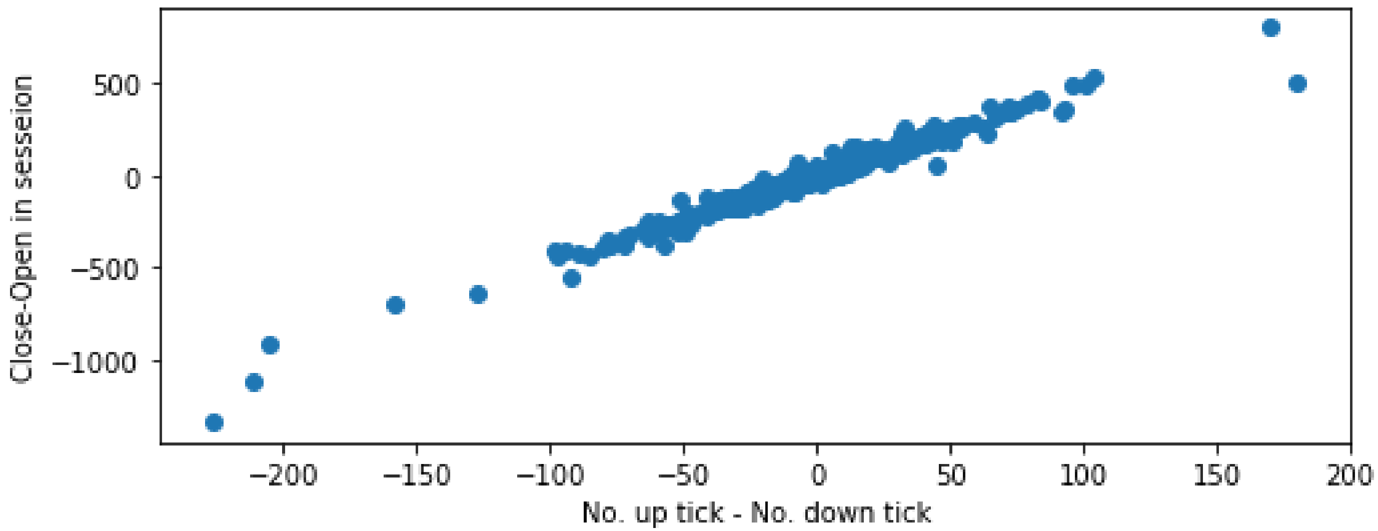
$$E = \sum_{i=1}^{I} N_i e_i$$

The sum of squared price increments

Close price – Open price in session
Vs
The difference between no. up ticks and no. down ticks

rom 2016.01 to 2017.04

Corr=0.98



$$E = \sum_{i=1}^{I} N_i e_i$$

The sum of squared price increments

Conclusions

1. Transaction prices may follow a random walk process.
   1. Market makers prefer the markets that the price movements are stable over time. Thus it is easy for them to cover the losses from adverse selections.
   2. Investors prefer the trades that minimize the market impacts.
2. The sum of squared price increments is fixed where the market makers and investors interest could be balanced.
3. Is it reasonable to analyze risky asset markets based on a financial return?

$$E = \sum_{i=1}^{I} N_i e_i$$