Equilibrium Yield Curves and the Interest Rate Lower Bound

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December 27, 2016

[1]The views expressed herein are those of the authors and not necessarily those of the Board of Governors of the Federal Reserve System.

Question

How does the ELB constraint affect the interaction of yield dynamics and monetary policy?

Motivation

- Monetary policy constrained by the ELB is a crucial element behind recent dynamics of yields and term premiums.

- Some unconventional monetary policies are believed to stimulate the economy via the term structure of interest rates.

  e.g. "Forward guidance", purchases of longer-term government bonds.

- Policies could affect the expected path of future short rates and term premiums in importantly different ways.

  * "*When policy works by moving term premiums, as opposed to moving expectations about the path of short rates, the transmission to the real economy may be altered in subtle yet important ways that can have implications for the benefits of a policy action, its costs, and even its consequences for financial stability.*" (Stein (2012)).

- Our question is a prerequisite to answer the ultimate question: How does unconventional monetary policy affect economic activity?

What We Do

- Study the term structure of default-free interest rates in a sticky-price DSGE model with an occasionally binding ELB constraint.
- Stylized models to illustrate key forces.
- Quantitative model to examine the effects of forward guidance and interpret the recent U.S. experience.

What We Find

- The ELB constraint generates time-varying term premiums, through compressing effects and amplification effects.
  - The ELB constraint increases macroeconomic uncertainty (amplification) but decreases interest rate sensitivity (compression).
  - The compressing effect typically dominates.
  - Term premium volatility increases around the time of liftoff.
- Forward guidance reduces the absolute size of the term premiums.
  - Whether term premiums increase or decrease depends on the risk exposure of bonds to the macroeconomy.
- U.S. yield and term premium dynamics are consistent with the model.

Related Literature and Contribution

DSGE models with the ELB: Aruoba, Cuba-Borda, and Schorfheide (2016), Fernández-Villaverde et al. (2015), Gavin et al. (2015), Gust, López-Salido, and Smith (2012), Nakata (2013)

$\Longleftarrow$ Study yields and term premiums.

(Latent factor) term structure models with the ELB: Bauer and Rudebusch (2015), Christensen and Rudebusch (2013), Ichiue and Ueno (2007), Kim and Singleton (2012), Krippner (2012), Priebsch (2013), Wu and Xia (2014)

$\Longleftarrow$ Structural approach to allow for economic interpretation and policy analysis.

Equilibrium (DSGE) term structure models: Andreasen (2012a,b), Van Binsbergen et al. (2012), Campbell, Pflueger, and Viceira (2014), Dew-Becker (2014), Hsu, Li, and Palomino (2015), Kung (2015), Rudebusch and Swanson (2008, 2012), Swanson (2014), Lopez, Lopez-Salido, and Vazquez-Grande (2015)

$\Longleftarrow$ ELB constraint.

Equilibrium term structure models with the ELB: Branger et al. (2015), Sakurai (2016), Gourio and Ngo (2016)

$\Longleftarrow$ Fully structural analysis of term premium dynamics at the ELB.

Outline of the Talk

- Stylized model
  - Log utility
  - Epstein-Zin preferences (Skip)
- Quantitative model
  - Calibration and model properties
  - Effects of "forward guidance"
  - Interpretation of the recent U.S. experience

Model

Stylized Model—Representative Household

- The representative household maximizes the value function:

$$V_t = \left[U_t(C_t, N_t) + \beta_t \left\{ \mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\chi_C}{1-\gamma}} \right]^{\frac{1}{1-\chi_C}} \tag{1}$$

where:

$$U_t(C_t, N_t) = \left(C_t^{\chi_N} (1 - N_t)^{1-\chi_N} \right)^{1-\chi_C} \tag{2}$$

- subject to its budget constraint:

$$P_t C_t + \mathbb{E}_t \left[M_{t+1} \mathcal{W}_{t+1} \right] = W_t N_t + \mathcal{W}_t + \Xi_t + T_t$$

- The implied nominal pricing kernel:

$$M_{t+1} = \beta_t \left(\frac{U_{C,t+1}}{U_{C,t}} \right) \left[\frac{V_{t+1}}{\left[\mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right] \right]^{\frac{1}{1-\gamma}}} \right]^{\chi_C - \gamma} \frac{1}{\Pi_{t+1}} \tag{3}$$

- Power utility: $\gamma = \chi_C$, EZ preferences: $\gamma \neq \chi_C$. Set $\chi_C \to 1$ (log utility).

Model

Stylized Model—Intermediate Goods Producers

- Monopolistically competitive intermediate goods producers $i \in [0, 1]$ maximize profits:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} M_{t-1,t} \left[P_t(i) Y_t(i) - W_t N_t(i) - \frac{\varphi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)\bar{\Pi}} - 1 \right)^2 P_t Y_t \right]$$

with nominal rigidities based on Rotemberg adjustment costs.

- Producers are subject to the demand and production functions:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t$$

$$Y_t(i) = N_t(i)$$

Model

Stylized Model—Monetary Policy/Market Clearing/Exogenous Process

- Monetary policy is an interest rate rule with an occasionally binding ELB constraint:

$$R_t = \max\left[1, R_t^*\right]$$

where the shadow rate R_t^* follows:

$$R_t^* = \bar{R}\left[\frac{\Pi_t}{\bar{\Pi}}\right]^{\phi_\Pi}$$

- Aggregation and market clearing:

$$Y_t = N_t$$

$$Y_t = C_t + \frac{\varphi}{2}\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right)^2 Y_t$$

- Exogenous discount rate process:

$$\ln \beta_t = (1-\rho_\beta)\ln \bar{\beta} + \rho_\beta \ln \beta_{t-1} + \varepsilon_{\beta,t} \qquad \varepsilon_{\beta,t} \sim i.i.d\, \mathcal{N}(0, \bar{\sigma}_\beta^2)$$

Model

Term Structure of Interest Rates

- The *price* of a nominal default-free bond with n-period maturity is:

$$P_t^{(n)} = \mathbb{E}_t[M_{t+1} P_{t+1}^{(n-1)}] \qquad \text{with} \quad P_t^{(0)} = 1$$

- The (continuously compounded) *yield to maturity* of this bond is:

$$R_t^{(n)} = -\frac{1}{n} \ln P_t^{(n)}$$

- The *nominal term premium* for an $n-$period bond is:

$$tp_t^{(n)} \equiv R_t^{(n)} - R_t^{(n)\mathbb{Q}} = \frac{1}{n}(\ln P_t^{(n)\mathbb{Q}} - \ln P_t^{(n)})$$

Note:

$$P_t^{(n)\mathbb{Q}} = \exp(-R_t)\mathbb{E}_t[P_{t+1}^{(n-1)\mathbb{Q}}] \qquad \text{with} \quad P_t^{(0)\mathbb{Q}} = 1$$

- Real yields and term premiums are derived analogously.

Model
Solution Method

- We use a global solution method that fully accounts for the strong non-linearity of the model:
  - The occasionally binding ELB constraint
  - Time-varying term premiums
- The solution method is a time-iteration algorithm in the spirit of Coleman (1990).
  - In addition to the standard iteration on the decision rules, we also iterate on the value function due to recursive utility.

Calibration

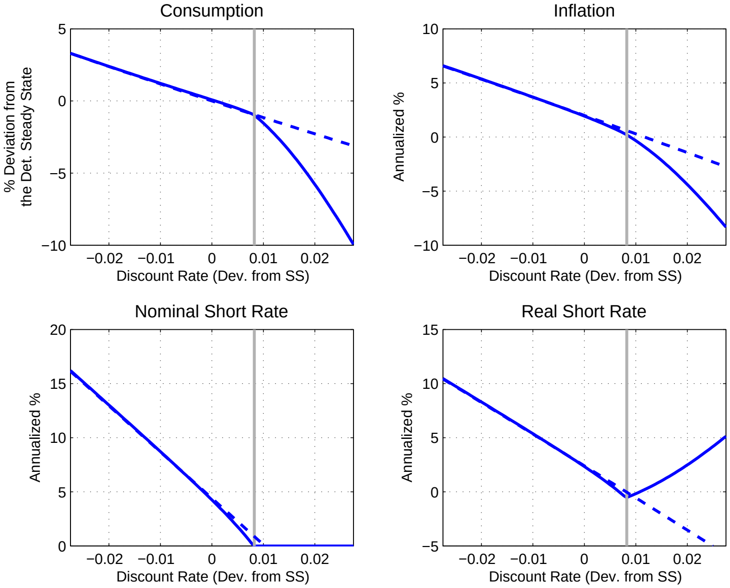
Stylized Model

- Parameter values are standard in the literature:

Parameter	Description	Parameter Value
$\bar{\beta}$	Time discount rate at steady state	$\frac{1}{1.006}$
χ_C	Inverse intertemporal elasticity of substitution	1
χ_N	Preference over consumption vs leisure	0.25
γ	Risk aversion	[1, 4]
θ	Elasticity of substitution among intermediate goods	6
φ	Price adjustment cost	75
$400(\bar{\Pi}-1)$	(Annualized) target rate of inflation	2.0
ϕ_π	Coefficient on inflation in the Taylor rule	2.5
ϕ_y	Coefficient on the output gap in the Taylor rule	0
ρ_R	Interest-rate smoothing in the Taylor rule	0
R_{ELB}	Effective interest rate lower bound	1
ρ_β	AR(1) coefficient for the discount factor shock	0.77
$\bar{\sigma}_\beta$	Standard deviation of shocks to the discount factor	$\frac{0.39}{100}$
	*Implied prob. of policy rate being at the ELB (Power U.)	9%

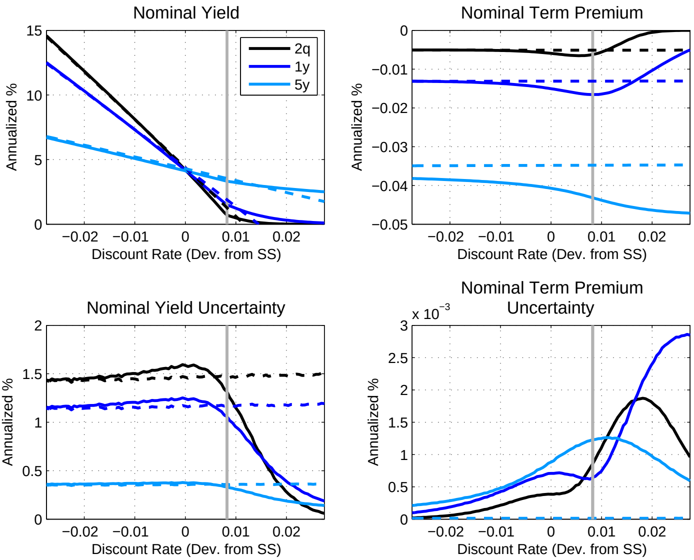
Decision Rules for Macroeconomic Variables

Log Utility



Term Structure of Interest Rates

Log Utility



Term Structure (EZ)

Mechanism of Term Premium Dynamics

Log Utility

- A decomposition of the 2-period term premium at any $\beta = \hat{\beta}$:

$$
\begin{aligned}
tp_t^{(2)}(\hat{\beta}) \equiv R_t^{(2)} - R_t^{(2)\mathbb{Q}} &\approx \frac{1}{2}\mathbb{C}ov_t(m_{t+1}, r_{t+1}) \\
&\propto -\mathbb{C}ov_t(\Delta c_{t+1}, r_{t+1}) - \mathbb{C}ov_t(\pi_{t+1}, r_{t+1}) \\
&= -\sum_{x\in\{\Delta c,\pi\}} \underbrace{\sigma_x(\hat{\beta})}_{\substack{\text{macro}\\ \text{uncertainty}}} \times \underbrace{\sigma_r(\hat{\beta}) \times \rho_{x,r}(\hat{\beta})}_{\substack{\text{interest-rate}\\ \text{sensitivity}}}
\end{aligned}
$$

- $\rho_{\Delta c,r}(\hat{\beta}) \geq 0$, $\rho_{\pi,r}(\hat{\beta}) \geq 0 \Longrightarrow tp_t^{(2)}(\hat{\beta}) \leq 0$.

- Two offsetting forces determine term premium dynamics:

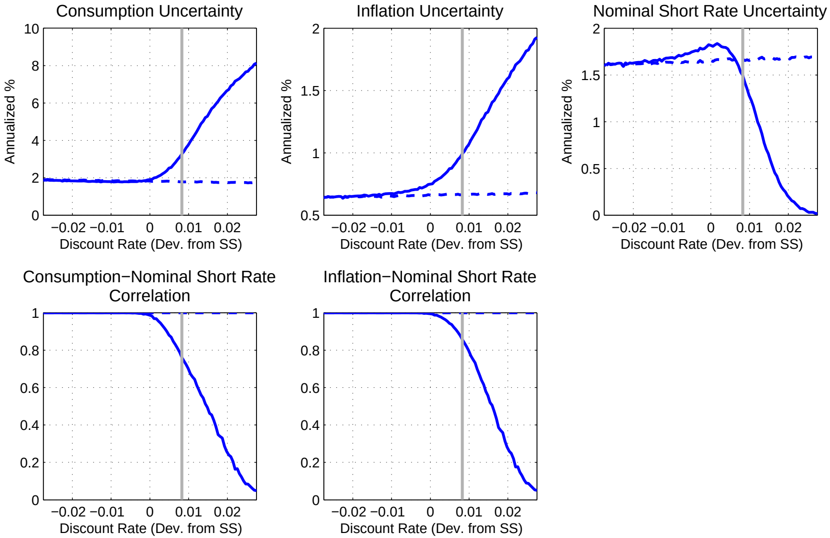
Macro uncertainty increases with β near and at the ELB
$\Longrightarrow$ **amplifies** the absolute size of term premiums

$\Updownarrow$

Interest-rate sensitivity decreases with β near and at the ELB
$\Longrightarrow$ **compresses** the absolute size of term premiums

Mechanism of Term Premium Dynamics

Log Utility

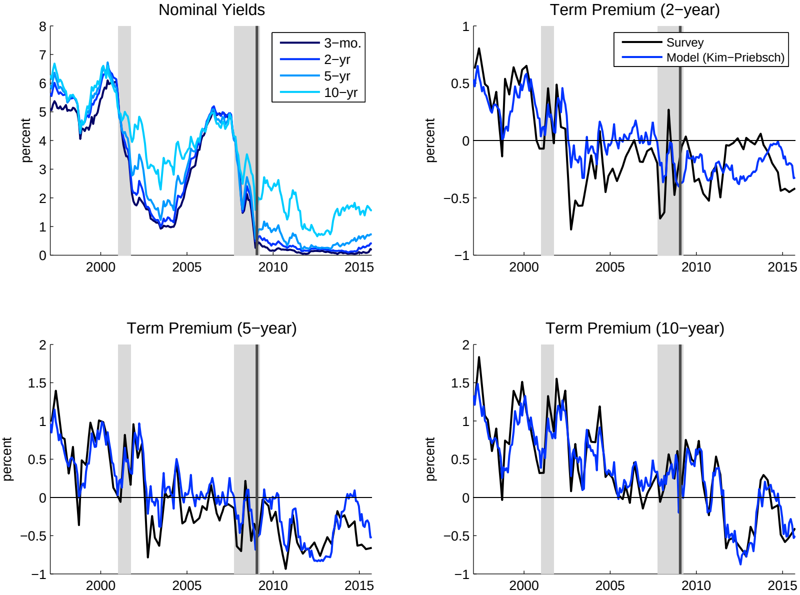


Term Structure (Pw)

Outline of the Talk

- Stylized model
  - Log utility
  - Epstein-Zin preferences (Skip)
- Quantitative model
  - Calibration and model properties
  - Effects of “forward guidance”
  - Interpretation of the recent U.S. experience

U.S. Nominal Yields and Term Premium Estimates



Additional Features in the Quantitative Model

- GHH period utility in EZ preferences
- Monetary policy rule standard in quantitative DSGE models:

$$R_t^* = \left(R_{t-1}^*\right)^{\rho_R} \left(\bar{R} \left[\frac{\Pi_t}{\bar{\Pi}} \right]^{\phi_\Pi} \left[\frac{Y_t}{\bar{Y} Z_t} \right]^{\phi_Y} \right)^{1-\rho_R}$$

- Richer exogenous processes:
  - Add stochastic TFP process A_t
  - β_t, A_t are AR(1) with **tail risk** and **time-varying volatility**:

$$\ln k_t = (1-\rho_k) \ln \bar{k} + \rho_k \ln k_{t-1} + \varepsilon_{k,t} \quad , k_t \in \{\beta_t, A_t\}$$

where

$$\tilde{\varepsilon}_{k,t} = \begin{cases} \mathcal{N}(0, \sigma_{k,t-1}^2) & \text{with prob. } 1-p_k \\ \vartheta_k & \text{with prob. } p_k \approx 0 \end{cases} \quad , \varepsilon_{k,t} \equiv \tilde{\varepsilon}_{k,t} - \mathbb{E}[\tilde{\varepsilon}_{k,t}]$$

and

$$\sigma_{k,t-1} = \frac{\theta_{ub,k}}{1 + \theta_{adj,k} \exp(\theta_{cv,k} \ln(k_{t-1}/\bar{k}))} \bar{\sigma}_k$$

Note: $\vartheta_\beta \gg \bar{\sigma}_\beta$, $\vartheta_a \ll -\bar{\sigma}_a \implies$ "Disaster" events
$\theta_{cv,\beta} < 0$, $\theta_{cv,a} > 0 \implies$ Countercyclical uncertainty

Role of Additional Features in the Quantitative Model

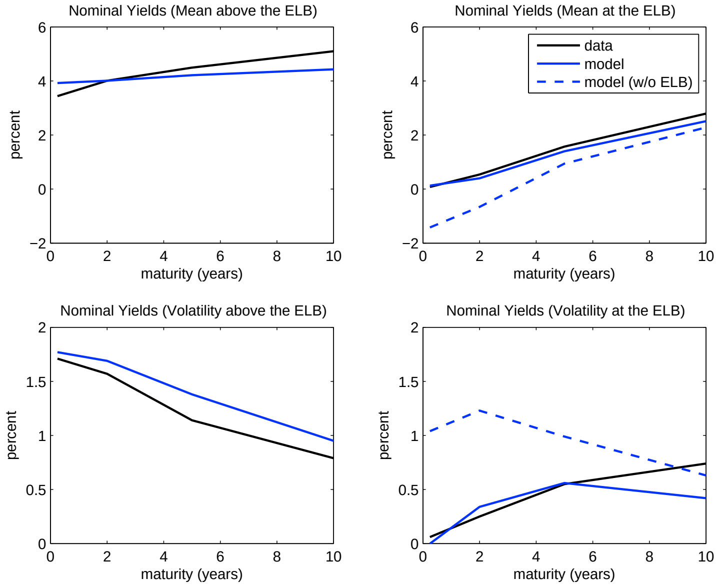
- Positive, large, volatile term premium above the ELB
  - $\Leftarrow$ GHH preferences
  - $\Leftarrow$ tail risk and time-varying volatility in TFP shocks
  - $\Leftarrow$ tail risk in demand shocks
- Negative term premium at the ELB
  - $\Leftarrow$ time-varying volatility in demand shocks

Calibration

Parameter	Description	Parameter Value
Household and Firm		
$\tilde{\beta}$	Scaled time discount rate	$\frac{1}{1.00625}$
$400(\zeta-1)$	(Annualized) deterministic trend growth in TFP	2.0
χ_c	Inverse elasticity of intertemporal substitution	9
χ_N	Inverse Frisch elasticity	$\frac{1}{3}$
α	Risk aversion ($=1-(1-\gamma)/(1-\chi_C)$)	-100
θ	Elasticity of substitution among intermediate goods	6
φ	Price adjustment cost	80
Monetary Policy		
$400(\bar{\Pi}-1)$	(Annualized) target rate of inflation	2.2
ϕ_π	Coefficient on inflation in the Taylor rule	5
ϕ_y	Coefficient on the output gap in the Taylor rule	0.5
ρ_R	Interest-rate smoothing in the Taylor rule	0.9
Discount Rate Process		
ρ_β	AR(1) coefficient for the discount rate process	0.85
$\bar{\sigma}_\beta$	The standard deviation of shocks to the discount rate process	$\frac{0.0001}{100}$
p_β	Tail risk prob. for the discount rate	10^{-30}
ϑ_β	Tail risk size for the discount rate	0.07
$\theta_{ub,\beta}$	Stochastic volatility for the discount rate (upper bound)	1050
$\theta_{cv,\beta}$	Stochastic volatility for the discount rate (curvature)	-2000
TFP Process		
ρ_a	AR(1) coefficient for the TFP process	0.93
$\bar{\sigma}_a$	The standard deviation of shocks to the TFP process	$\frac{0.1}{100}$
p_a	Tail risk prob. for TFP	$\frac{0.5}{100}$
ϑ_a	Tail risk size for TFP	-0.006
$\theta_{ub,a}$	Stochastic volatility for TFP (upper bound)	5
$\theta_{cv,a}$	Stochastic volatility for TFP (curvature)	90

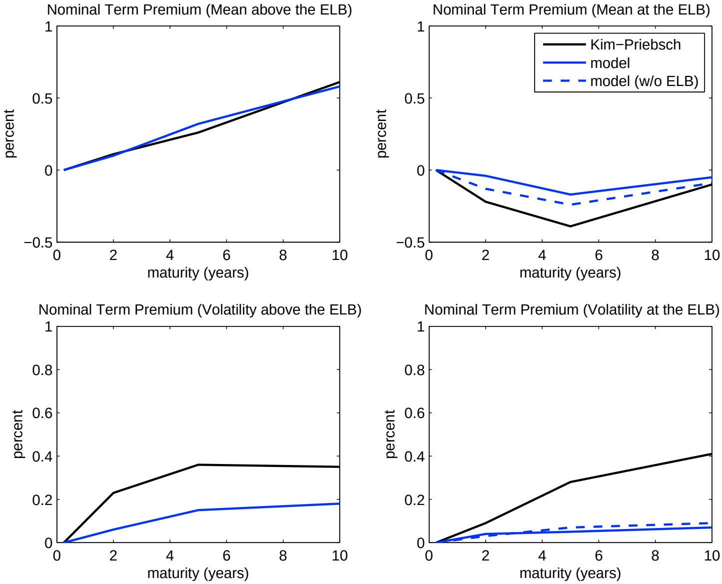
Moments above and at the ELB

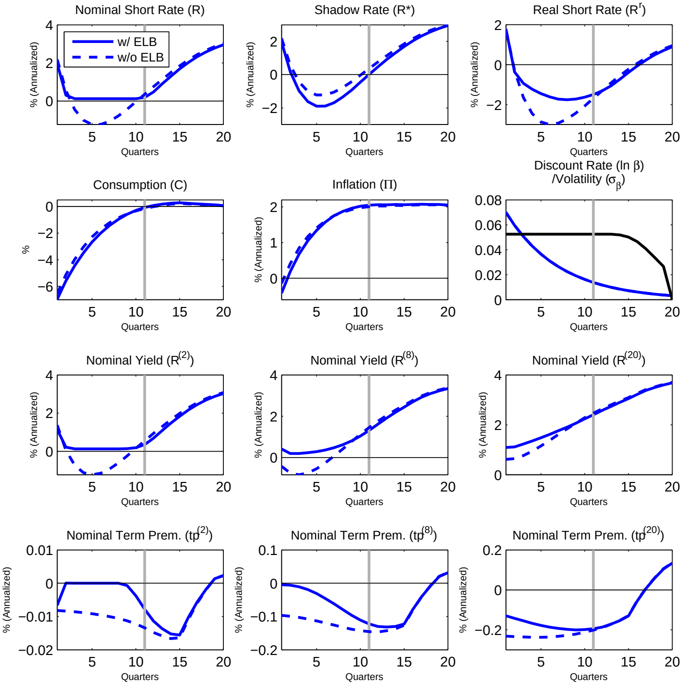
Nominal Yields



Moments above and at the ELB

Nominal Term Premiums




Nominal Short Rate (R)
w/ ELB
w/o ELB
Shadow Rate (R*)
Real Short Rate (R^r)
Consumption (C)
Inflation (Π)
Discount Rate (ln β) /Volatility (σ_β)
Nominal Yield ($R^{(2)}$)
Nominal Yield ($R^{(8)}$)
Nominal Yield ($R^{(20)}$)
Nominal Term Prem. ($tp^{(2)}$)
Nominal Term Prem. ($tp^{(8)}$)
Nominal Term Prem. ($tp^{(20)}$)
% (Annualized)
%
Quarters

Monetary Policy and Equilibrium Yield Curves at the ELB

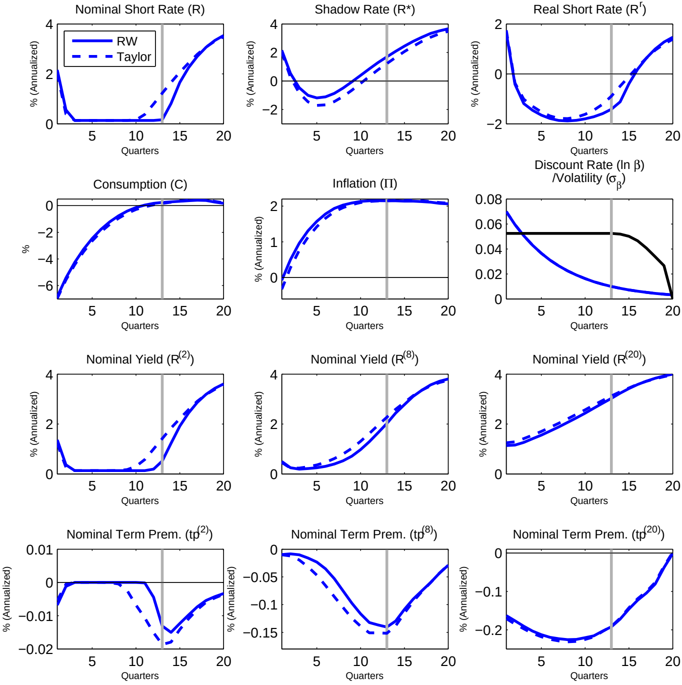
- Analyze the effects of an announcement to keep the policy rate at the ELB for longer ("forward guidance").

- Model such an announcement as an adoption of the "Reifschneider-Williams rule":

$$R_t^{(1)} = \max\left[R_{ELB}, R_t^* - \phi_{RW} J_t\right]$$
$$J_t = J_{t-1} + (R_{t-1}^{(1)} - R_{t-1}^*)$$

$\Longrightarrow$ Liftoff depends on the cumulative shortfalls in inflation and output.

- A way to capture the "signaling effect" of LSAPs.


Nominal Short Rate (R)
RW
Taylor
Shadow Rate (R*)
Real Short Rate (R^r)
Consumption (C)
Inflation (Π)
Discount Rate (ln β) /Volatility (σ_β)
Nominal Yield ($R^{(2)}$)
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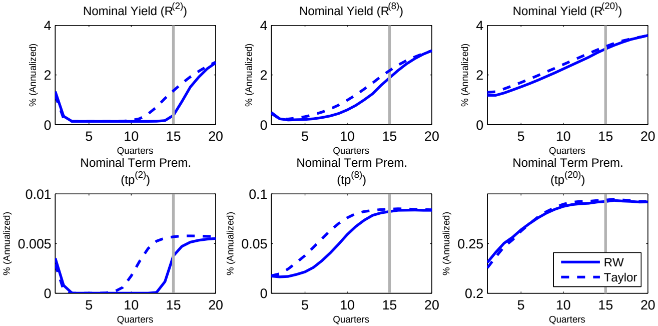
% (Annualized)
%
Quarters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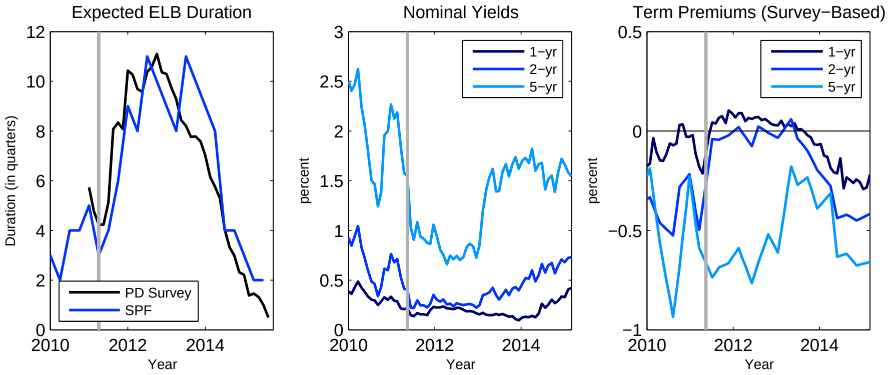
Nominal Term Prem. ($tp^{(20)}$)
% (Annualized)
Quarters
RW
Taylor

An Interpretation of the Recent U.S. Experience

- Our analysis implies a tight link between the expected time until liftoff and dynamics of shorter-maturity bonds at the ELB.
  - Expected ELB duration increases $\Longrightarrow$ yields decline and term premiums approach zero (compressing effect).
  - Liftoff approaches $\Longrightarrow$ yields increase and term premiums decline.
  - This link is weaker for longer-term bonds.

- We find some evidence supporting these implications in the recent ELB episode in the U.S.
  - Focus on the period from the introduction of "calendar-based" forward guidance in August 2011 up to liftoff in December 2015.

An Interpretation of the Recent U.S. Experience



* Expected ELB duration in the left panel is computed as the time to the first quarter when the median federal funds rate forecast exceeds 37.5 basis points. The grey vertical lines indicate the timing of the August 2011 FOMC meeting.

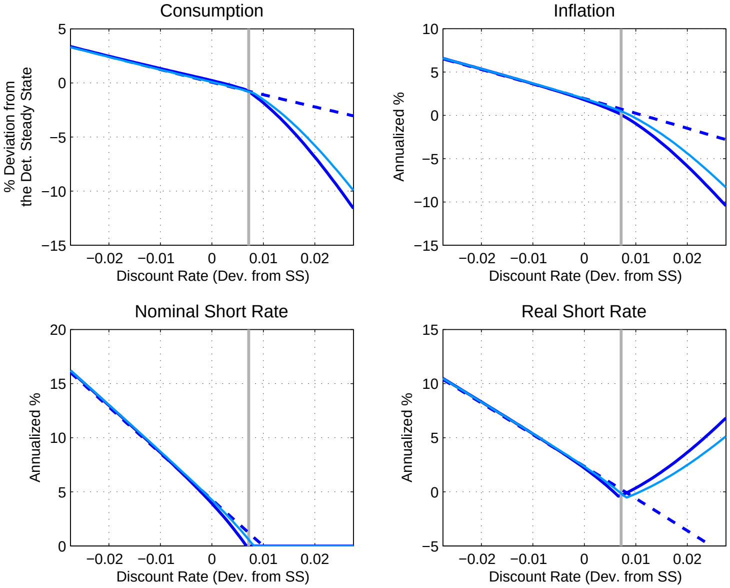
Conclusion

- Studied the term structure of interest rates in a sticky-price DSGE model with an occasionally binding ELB constraint.
- The ELB constraint generates time-varying term premiums, through compressing effects versus amplification effects.
  - The compressing effect typically dominates.
  - Term premium volatility increases around the time of liftoff.
- Forward guidance reduces the absolute size of the term premiums.
  - Whether term premiums increase or decrease depends on the risk exposure of bonds to the macroeconomy.
  - Forward guidance lowers the expected short rate path.
- U.S. yield and term premium dynamics are consistent with the model.

Additional Slides

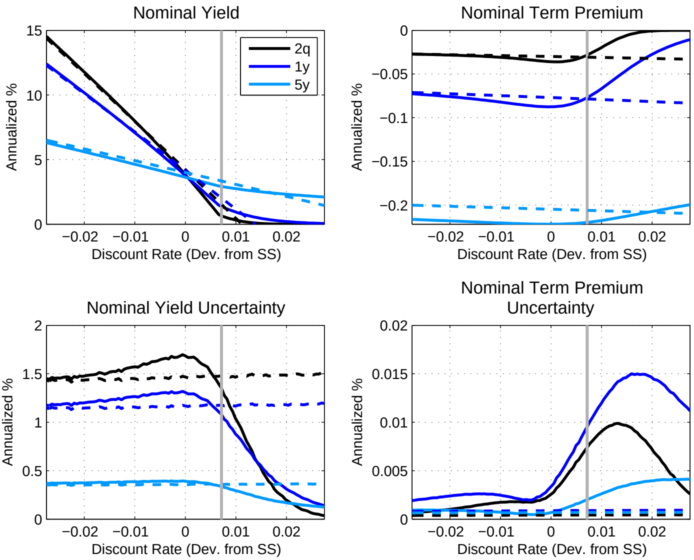
Decision Rules for Macroeconomic Variables

EZ Preferences



Term Structure of Interest Rates

EZ Preferences



▸ Outline (quantitative)

Mechanism of Term Premium Dynamics

EZ Preferences

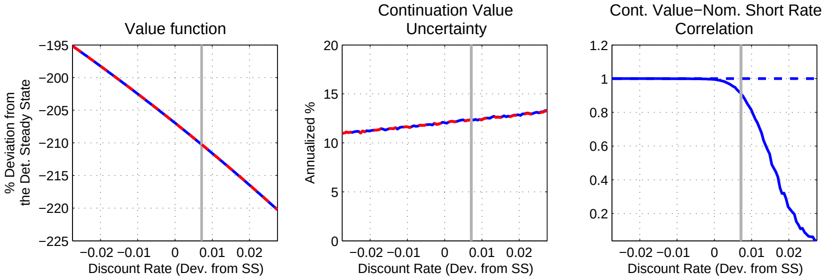
- A decomposition of the 2-period term premium at any $\beta = \hat{\beta}$:

$$
\begin{aligned}
tp_t^{(2)}(\hat{\beta}) &\approx \frac{1}{2}\mathbb{C}ov_t(m_{t+1}, r_{t+1}) \\
&\propto -\mathbb{C}ov_t(\Delta c_{t+1}, r_{t+1}) - \mathbb{C}ov_t(\pi_{t+1}, r_{t+1}) - \mathbb{C}ov_t(\tilde{v}_{t+1}, r_{t+1}) \\
&= -\sum_{x \in \{\Delta c, \pi\}} \sigma_x(\hat{\beta})\sigma_r(\hat{\beta})\rho_{x,r}(\hat{\beta}) \quad \underbrace{- \sigma_{\tilde{v}}(\hat{\beta}) \times \sigma_r(\hat{\beta}) \times \rho_{\tilde{v},r}(\hat{\beta})}_{\text{extra term with EZ preferences}}
\end{aligned}
$$

- $\rho_{\tilde{v},r} \geq 0 \implies$ extra term ≤ 0.
- $\sigma_{\tilde{v}}$ is large and roughly constant.
  - The continuation value is not much affected by the ELB, which binds only infrequently.
- Term premium varies when the ELB exists mainly due to changes in interest-rate sensitivity w.r.t. $\tilde{v}$, $\sigma_r \rho_{\tilde{v},r}$.

Mechanism of Term Premium Dynamics

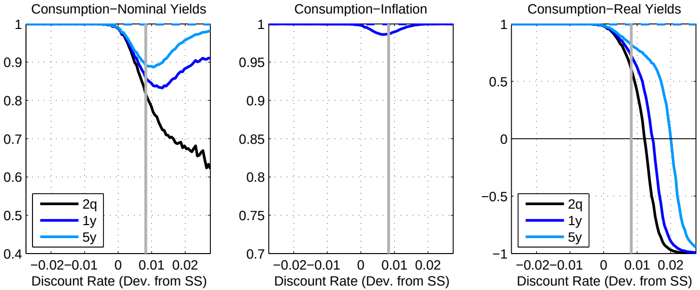
EZ Preferences



Term Structure (EZ)

Mechanism of Term Premium Dynamics

Long Term Yields—Log Utility



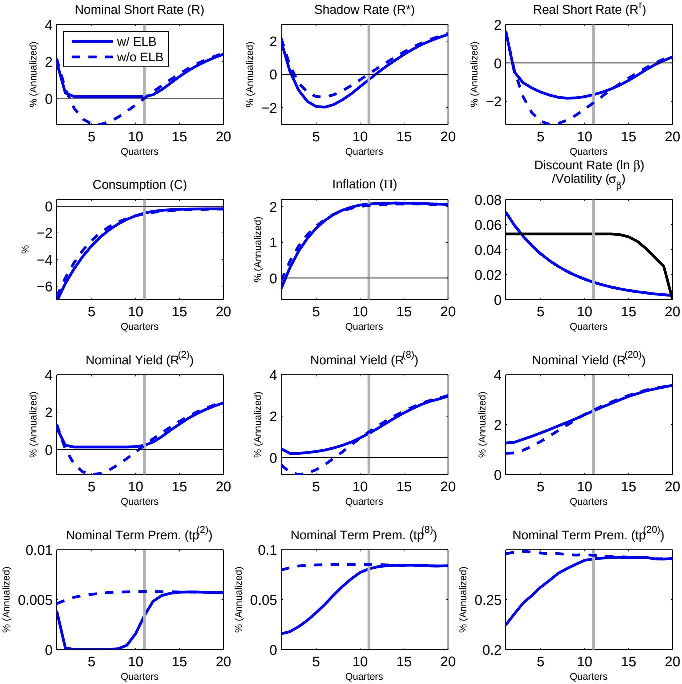
*Solid lines indicate conditional correlations (1 quarter ahead conditioned on β) of the respective variables for the model with the ELB constraint, and dashed lines indicate uncertainty for the model without the ELB constraint. The solid vertical line indicates the threshold state where the ELB binds.

◂ Term Structure Mechanism (Log)

Moments above and at the ELB

Macro Moments at and above the ELB—Model versus Data—

	Above the ELB		At the ELB		Below the ELB
	Data	Model	Data	Model	Model w/o ELB
A. Macro Variables (Mean)					
Consumption†	—	—	-2.02	-1.45	-1.06
Inflation	2.13	2.09	1.45	1.15	1.24
B. Macro Variables (Volatility)					
Consumption	2.93	2.20	4.18	8.04	6.76
Inflation	0.87	0.72	0.83	0.93	0.73



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