Equilibrium Yield Curves and the Interest Rate Lower Bound*

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Abstract

We study the term structure of default-free interest rates in a sticky-price model with an occasionally binding effective lower bound (ELB) constraint on interest rates and recursive preferences. The ELB constraint induces state-dependency in the dynamics of term premiums by affecting macroeconomic uncertainty and interest-rate sensitivity to economic activities. In a model calibrated to match key features of the aggregate economy and term structure dynamics in the U.S. above and at the ELB, we find that the ELB constraint typically lowers the absolute size of term premiums at the ELB and increases their volatility around the time of liftoff. The central bank's announcement to keep the policy rate at the ELB for longer than previously expected lowers the expected short rate path, but its effect on term premiums depends on the risk exposure of bonds to the macroeconomy; while the announcement increases term premiums if bonds are a hedge against economic downturns, it decreases them otherwise.

JEL: E12, E32, E43, E44, E52, G12

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1 Introduction

Since the onset of the recent global recession, many central banks have implemented various unconventional policies to stimulate economic activities, as short-term nominal interest rates—the conventional tool of monetary policy—hit the effective lower bound (ELB) constraint. Some of these policies, such as policies aimed at providing more transparency in the likely path of the policy rate (so-called "forward guidance" policies) and purchases of long-term government bonds, are widely believed to stimulate the economy through their effects on the term structure of interest rates. Accordingly, it is important to have a coherent understanding of the interaction between the macroeconomy, the term structure of interest rates, and monetary policy actions if we are to understand the recent dynamics of yield curves and evaluate the efficacy of these unconventional policies.[1] The ELB can affect this interaction in important ways.

In this paper, we study how the ELB constraint jointly affects the macroeconomy, the term structure of interest rates, and monetary policy in a structural general equilibrium (DSGE) model. Our model is a variant of a standard New Keynesian model featuring monopolistic competition in the product market, sticky-prices, and an interest-rate feedback rule. Explicit micro-foundations set our model apart from the existing term structure models with the ELB—which are mostly statistical models with limited economic structure, as reviewed shortly. Our analysis proceeds in two steps. In the first part of the paper, we analyze how the ELB constraint affects the dynamics of term premiums using stylized versions of our model. In the second part of the paper, we augment the stylized model with additional features to match key features of macroeconomic data and the term structure of interest rates in the U.S. and analyze how an alternative monetary policy strategy affects the dynamics of the macroeconomy and term premiums at and away from the ELB.

Our main finding from the stylized models is that the ELB constraint generates state-dependency in term premiums through two key factors affecting term premiums—macroeconomic uncertainty and the sensitivity of interest rates to macroeconomic fluctuations. On the one hand, macroeconomic uncertainty is higher when the policy rate is constrained by the ELB than when it is not, as the ELB constraint prevents the central bank from counteracting the effects of exogenous shocks to demand on consumption and inflation. This increased macroeconomic uncertainty at and near the ELB constraint is a force that pushes up the absolute size of term premiums. On the other hand, the sensitivity of interest rates to macroeconomic fluctuations is smaller when the policy rate is at or near the ELB than when it is not, as the central bank faces restrictions in adjusting its policy rate in the near term under such a circumstance. This reduced

[1] Policymakers have also reiterated the importance of understanding term premiums in the current environment. For example, former Fed Governor Jeremy Stein stated in a speech, "*When policy works by moving term premiums, as opposed to moving expectations about the path of short rates, the transmission to the real economy may be altered in subtle yet important ways that can have implications for the benefits of a policy action, its costs, and even its consequences for financial stability.*" (Stein (2012)). Also, former Fed Chairman Ben Bernanke points to the decline in term premiums as an important factor behind the decline in longer-term U.S. interest rates (Bernanke (2015)).

sensitivity of interest rates to macroeconomic fluctuations at and near the ELB constraint is a force that compresses the absolute size of term premiums.

Which of these two effects dominates depends on the state of the economy and yield maturity. When the economy is in a deep recession and the policy rate is expected to be at the ELB for a long period of time, the second compression effect typically dominates the first amplification effect, and the size of term premium is lower than when the policy rate is comfortably above the ELB. When the economy is in a mild recession and the policy rate is expected to be at the ELB for a short period of time, the first effect often dominates the second and the size of term premiums is higher than when the policy rate is comfortably above the ELB. The compressing effects of the ELB on term premiums induced by the reduced sensitivity of interest rates is stronger for shorter-maturity yields that are more strongly affected by the presence of the ELB than longer-maturity yields. Finally, the compressing effects of the ELB are also stronger in the model with Epstein-Zin preferences as term premiums in that model depend less on macroeconomic uncertainty and more on the sensitivity of interest rates to fluctuations in the continuation value. Overall, the absolute size of term premiums is on average lower at the ELB than away from the ELB for most variants of the stylized model.

This time-variation in term premiums implies that term premium uncertainty—conditional volatility of term premiums—is particularly high when the policy rate is currently at the ELB but is expected to be positive in the near future, or when the policy rate is currently positive but is near the ELB. That is, term premium uncertainty is particularly high around the time of liftoff. In contrast, term premium uncertainty is low when the policy rate is currently at the ELB and is expected to stay at the ELB for a long period of time, as the interest rate is not likely to move at all under such a circumstance. This result suggests that the dynamics of term premiums may warrant more attention in a severe recession involving the ELB than under normal circumstances.

After analyzing yield curve dynamics in the stylized model, we extend the model in several directions to improve its ability to quantitatively match key features of consumption, inflation, and the term structure of interest rates in the U.S. We introduce TFP shocks with tail risk and stochastic volatility, which makes the conditional moments of our model while the policy rate is away from the ELB consistent with those in the data. In particular, yield curves are on average upward sloping and term premiums are positive, large, and volatile while the policy rate is away from the ELB in our model, as in recent DSGE term structure models. Our yield curve is steeper when the policy rate is constrained at the ELB than otherwise, as in the data. By introducing stochastic volatility in the demand shock process, we also generate term premiums that are on average small and negative at the ELB; Small and negative term premiums are consistent with survey-based estimates of term premiums—as well as term premium estimates based on some available term structure models—from the period in which the federal funds rate was at the ELB.

A key benefit of working with fully structural models is that they allow us to conduct counterfactual policy experiments that are robust to the Lucas critique. In the final exercise of the paper, we use our quantitative model to study the effects on the macroeconomy and the term structure of interest rates of the central bank's announcement to keep the policy rate at the ELB for longer than previously expected. We model the effects of this announcement ("forward guidance") by comparing the economy under the baseline monetary policy rule with the economy under an alternative monetary policy rule, called "the Reifschneider-Williams rule," in which the timing of liftoff depends on the cumulative shortfall in inflation and output in the past and the policy rate is expected to stay at the ELB for longer. We find that this forward guidance not only reduces the expected short-rate path, but also the absolute size of term premiums. In our benchmark quantitative model in which term premiums are on average negative at the ELB, this finding means that the forward guidance increases term premiums, partially offsetting the declines in the expected short rate path. In an alternative quantitative model in which term premiums are on average positive even at the ELB, this finding means that the forward guidance reduces term premiums, amplifying the decline in the expected short-rate path.

One caveat in our analysis is that our model does not have an explicit role for the central bank balance sheet, an ingredient that is likely to be important in understanding the effects of unconventional policy measures such as large-scale asset purchases (LSAPs). Introducing such an ingredient into our model is an important next step in our research agenda. We believe that, since unconventional policy measures were typically taken while the policy rate was constrained at the ELB, understanding how the ELB constraint affects the dynamics of the term structure of interest rates is an essential first step towards understanding how unconventional policies affect term structure of interest rates and the economy. Moreover, if balance sheet policies work mainly by signaling to the private sector about the central bank's commitment for accommodative policy stance and thus affecting the expected path of short-term nominal interest rate, as suggested by some recent studies, the analysis of forward guidance provided in our paper can be seen as capturing some portions of the effects of balance sheet policies.

Literature Review

Our contribution is to present the first analysis of the yield curve and term premiums using a DSGE model with an occasionally binding ELB constraint that is calibrated to match key features of U.S. data including the recent ELB episode. Naturally, our work is related to several strands of the literature.

First, our work builds on the rapidly expanding literature analyzing the implications of the ELB constraint on the macroeconomy. In particular, our paper is closely related to a set of papers that analyze the implication of the ELB constraint in fully nonlinear New Keynensian models with an occasionally binding ELB constraint. Examples are Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramirez (2015), Gavin, Keen, Richter, and Throckmorton (2015), Gust, Herbst, Lopez-Salido, and Smith (2016) and Nakata (2013), among many others. The key difference between these papers and our paper is that they analyze the implications of the ELB constraint on macroeconomic variables, while we focus on the term structure of interest rates. Reflecting this difference and our desire to build a quantitative model, our model includes some features absent in these papers, such as recursive preferences, tail risks and stochastic volatilities in exogenous shocks.

Second, our work is related to the relatively recent literature on term structure models with the ELB. Examples are Bauer and Rudebusch (2015), Christensen and Rudebusch (2013), Ichiue and Ueno (2007), Kim and Singleton (2012), Krippner (2012), Kim and Priebsch (2013) and Wu and Xia (2014). As we noted above, the existing models are reduced-form in the sense that that they impose very limited economic structure, such as lack of arbitrage opportunities.[2] Although the flexibility afforded by the sparse economic structure allows these models to fit yield data quite well, the driver of asset prices in these models consists of latent factors that are not economically interpretable. In contrast, our general-equilibrium term-structure model features explicit microfoundations and includes a description of how the central bank conducts monetary policy, allowing us to give economic interpretations to the term structure of interest rates.

Third, our paper extends the literature on equilibrium term structure models to incorporate the ELB constraint. In particular, our paper is closely related to recent papers analyzing the term structure of interest rate in a production economy. Examples are Andreasen (2012a,b), Van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012), Campbell, Pflueger, and Viceira (2014), Dew-Becker (2014), Hsu, Li, and Palomino (2015), Kung (2015), Lopez, Lopez-Salido, and Vazquez-Grande (2015), Rudebusch and Swanson (2008, 2012) and Swanson (2014). These papers in turn build on earlier work by Piazzesi and Schneider (2007) and many others that analyze equilibrium yield curves in endowment economies. Our main departure from these models is to incorporate an occasionally binding ELB constraint on interest rates. We note that, even though these papers have made substantial progress in fitting macro and yield data jointly, they usually find it difficult to generate endogenous volatility in term premiums. In our model, endogenous volatility in term premiums arises naturally from the presence of the ELB constraint.

Before closing, we mention some recent developments of equilibrium term structure models with the ELB constraint. Branger, Schlag, Shaliastovich, and Song (2015) introduce the ELB constraint into an endowment-economy model and estimate it using U.S. data. In their model, consumption and inflation dynamics are exogenously specified; we study a production economy in which consumption and inflation are endogenous variables and there is a description for monetary policy. Sakurai (2016) develops a term structure model with the ELB constraint that combines a linearized New Keynesian economy and a reduced-form pricing kernel; the pricing kernel in our model is internally consistent with the macroeconomic structure. Finally, most closely related to our paper is a contemporaneous study by Gourio and Ngo (2016) that examines asset-pricing

[2] These papers, in turn, build on the vast literature on term structure models without the ELB constraint, which we do not review here. Readers may refer to literature reviews such as Gürkaynak and Wright (2012) and references therein for more information.

implications of a DSGE model with the ELB constraint. One key distinction between our model and theirs is that the differences in the dynamics of term premiums at and away from the ELB are substantially more pronounced in our model than in their model.[3]

The structure of the paper is as follows. Section 2 presents the model in general form. We start our discussion of equilibrium yield dynamics above and at the ELB in Section 3 using stylized versions of the model. We then proceed to explore the quantitative potential of our model to match certain features of U.S. data using a more careful calibration in Section 4. In Section 5, we conduct monetary policy experiments at the ELB based on this model. Section 6 collects some further discussion of our results. Section 7 concludes.

2 The Model

In this section, we lay out our general equilibrium model of default-free interest rates.

There will be broadly two versions of the model—one stylized and one quantitative. The stylized model is a variation of a plain-vanilla New Keynesian model with a single discount rate shock and the ELB constraint; we will study a specification with power utility as well as one with Epstein-Zin preferences, both of which will be used to describe key features of equilibrium yield curves in a transparent way. The quantitative model introduces additional features—period utility à la Greenwood, Hercowitz, and Huffman (1988) that features non-separability between consumption and leisure ("GHH utility"), a more realistic monetary policy rule, and richer shock structures—to the stylized model. This model will be calibrated to match key features of the term structure of interest rates in the data, and will be used to conduct couterfactual monetary policy experiments. Many elements of the model are the same across the two versions. The differences are summarized in Table 1. We will discuss details of each feature in the following sections.

Table 1: **Summary of Model Features**

	Stylized Model	Quantitative Model
Utility	Separable labor + Pw or EZ	GHH+EZ
Policy rule	Inflation response	Inflation/output response+policy inertia†
Shocks	Discount rate	Discount rate+TFP
Shock features	Normal+homoskedastic	Tail risk+heteroskedastic

† We further augment this rule following Reifschneider and Williams (2000) to conduct monetary policy experiments.

2.1 Households

The representative household's value function V_t takes the following recursive form originally proposed by Epstein and Zin (1989) ("Epstein-Zin (EZ) preferences"):

[3] While we focus on the term structure of interest rates, they focus on the correlation between equity prices and inflation.

$$V_t = \left[U_t(C_t, N_t) + \beta_t \left\{ \mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\chi_C}{1-\gamma}} \right]^{\frac{1}{1-\chi_C}} \tag{1}$$

$U_t(C_t, N_t)$ is the period utility function that satisfies $U_C \geq 0$, $U_N \leq 0$, twice differentiability and strict concavity. In the stylized models considered in Section 3, the period utility function is given by the standard log-separable form:

$$U_t(C_t, N_t) = \left(C_t^{\chi_N} (1 - N_t)^{1-\chi_N} \right)^{1-\chi_C} \tag{2}$$

where $\chi_C > 0$ is the inverse elasticity of intertemporal substitution and $\chi_N > 0$. In the quantitative model used in Sections 4 and 5, the period utility function is given by the following specification originally proposed by Greenwood, Hercowitz, and Huffman (1988) ("GHH utility"):

$$U_t(C_t, N_t) = \frac{1}{1-\chi_C} \left(C_t - Z_t \frac{N_t^{1+\chi_N}}{1+\chi_N} \right)^{1-\chi_C} \tag{3}$$

where $\chi_C > 0$ captures the attitude towards intertemporal substitution of the consumption-labor composite and $\chi_N \in (0, 1)$ is now the inverse Frisch elasticity. Z_t is a deterministic trend in total factor productivity. The scaling of labor disutility by Z_t ensures the existence of a balanced growth path in equilibrium.

C_t is the household's aggregate consumption of final goods based on a CES aggregator of intermediate goods $C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$ where $\theta > 1$ is the elasticity of demand across the intermediate goods. $N_t = \int_0^1 N_t(i) di$ denotes the household's total supply of labor, which is the integral of labor $N_t(i)$ supplied to each intermediate good producer i in a perfectly competitive labor market given nominal wage W_t. β_t is the stochastic time discount rate. We will specify the dynamics of β_t in detail below (Section 2.5). $\gamma > 0$ parameterizes the household's risk aversion. Note that $\gamma = \chi_C$ corresponds to the important special case of power utility.

The household maximizes (1) by choosing state contingent paths for C_t, N_t and asset holdings subject to its initial wealth and the following sequence of flow budget constraints:

$$P_t C_t + \mathbb{E}_t \left[M_{t+1} \mathcal{W}_{t+1} \right] \leq W_t N_t + \mathcal{W}_t + \Xi_t + T_t$$

where the aggregate price level of the consumption basket $P_t \equiv \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ is implied by the household's cost minimization problem (or equivalently, the optimization of a perfectly competitive representative final good producer combining intermediate goods). Assuming complete financial markets, $\mathcal{W}_{t+1}$ is the household's wealth portfolio of state contingent claims chosen by the end of period t. These claims are priced by the unique nominal pricing kernel M_{t+1} implied by the household's problem:

$$M_{t+1} = \beta_t \left(\frac{U_{C,t+1}}{U_{C,t}}\right) \left[\frac{V_{t+1}}{\left[\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]\right]^{\frac{1}{1-\gamma}}}\right]^{\chi_C-\gamma} \frac{1}{\Pi_{t+1}} \tag{4}$$

where $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is the gross (aggregate) inflation rate. The term with squared brackets is the additional term that appears by assuming EZ preferences instead of power utility, implying that the household is sensitive to the distribution of future consumption (and labor supply) on top of current consumption growth. Ξ_t is firms' profit rebated back to the household. T_t denotes lump-sum government taxes and/or transfers.

2.2 Intermediate Goods Producers

There are a continuum of monopolistically competitive intermediate goods producers indexed by $i \in [0,1]$. Each producer faces costly price adjustments, which is introduced via an adjustment cost function proposed by Rotemberg (1982). The expected discounted value of each producer i's profit stream is:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} M_{t-1,t} \left[P_t(i)Y_t(i) - W_t N_t(i) - \frac{\varphi}{2}\left(\frac{P_t(i)}{P_{t-1}(i)\bar{\Pi}} - 1\right)^2 P_t Y_t\right] \tag{5}$$

where $\varphi > 0$ measures the degree of costly price adjustment. We assume the producers adopt a simple price indexation scheme where they index on steady state inflation $\bar{\Pi}$ (or the central bank's inflation target) when setting prices. Each producer maximizes (5) by choosing a state contingent path of $\{P_t(i), Y_t(i), N_t(i)\}$ subject to the demand and production functions:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$$

$$Y_t(i) = A_t Z_t N_t(i)$$

where A_t and Z_t are two components of total factor productivity (TFP) that are both treated as exogenous. We assume A_t is stationary and Z_t is a deterministic trend which grows at a rate of ζ (i.e., $\zeta = \frac{Z_t}{Z_{t-1}}$). This assumption implies that TFP in our model, $A_t Z_t$, features trend stationarity. Trend-stationarity of TFP is critical in obtaining an upward sloping nominal and real term structure in our simple model. This modeling choice is also adopted by Rudebusch and Swanson (2008, 2012).[4] We will describe the exact specification of the process in Section

[4] We could alteratively assume that TFP growth is difference stationary, but our modeling choice helps to fit the upward sloping term structure we observe in U.S. data on average. Since we focus on a relatively short sample of the past two decades, it does not appear unreasonable to abstract from a stochastic trend compared to when we analyze a longer sample period. The financial crisis may have shifted the perception towards long-run growth, but if the shift is more in terms of the deterministic trend, our main results will be largely unaffected.

2.5.

2.3 Monetary Policy

The central bank sets the nominal one-period interest rate, $R_t^{(1)}$, following a Taylor rule with an occasionally binding ELB constraint:

$$R_t^{(1)} = \max\left[R_{ELB}, R_t^*\right] \tag{6}$$

where R_{ELB} is the ELB on the nominal short rate and the nominal *shadow rate* R_t^* follows:

$$R_t^* = \left(R_{t-1}^*\right)^{\rho_R} \left(\bar{R}\left[\frac{\Pi_t}{\bar{\Pi}}\right]^{\phi_\Pi} \left[\frac{Y_t}{\bar{Y} Z_t}\right]^{\phi_Y}\right)^{1-\rho_R} \tag{7}$$

where $\bar{R}$ and $\bar{Y}$ denote the steady state of $R_t^{(1)}$ and normalized output $\hat{Y}_t \equiv \frac{Y_t}{Z_t}$, respectively.[5] In the two stylized models considered in Section 3, $\phi_R = \phi_Y = 0$ so that there are neither interest-rate smoothing nor response to the output gap. In the quantitative model, $\phi_R, \phi_Y > 0$.

For the monetary policy experiment in Section 5, we further augment this rule based on the work of Reifschneider and Williams (2000)—the "RW rule"—as follows:

$$\begin{aligned} R_t^{(1)} &= \max\left[R_{ELB}, R_t^* - \phi_{RW} J_t\right] \\ J_t &= J_{t-1} + (R_{t-1}^{(1)} - R_{t-1}^*) \end{aligned} \tag{8}$$

where the shadow rate R_t^* follows the same feedback rule as in (7), and $\phi_{RW} \geq 0$ controls the degree of extra accommodation at the ELB. Note when $\phi_{RW} = 0$, the RW rule collapses to the standard Taylor rule with the ELB constraint (6). J_t is the cumulative past deviation of the policy rate from the shadow rate. Further details are deferred to Section 5.

2.4 Market Clearing

In equilibrium, the goods market, labor market, and asset market must clear at all dates and states. The clearing condition for final goods is:

$$Y_t = C_t + \frac{\varphi}{2}\left[\int_0^1 \left(\frac{P_t(i)}{P_{t-1}(i)\bar{\Pi}} - 1\right)^2 di\right] Y_t$$

[5] The inertial rule is specified such that the shadow policy rate today depends on the lagged shadow policy rate, as opposed to the lagged actual policy rate. The degree of history dependence in the policy rate is much more pronounced under the lagged-shadow-rate specification than under the lagged-actual-rate specification, allowing us to increase the expected ELB duration. For more detailed discussions on the distinction between the lagged-shadow-rate and the lagged-actual-rate specifications, see Hills and Nakata (2014) and Gust, Herbst, Lopez-Salido, and Smith (2016).

Note the equilibrium is necessarily symmetric. We can aggregate the supply of intermediate goods by integrating each producer's supply to obtain:

$$Y_t = A_t Z_t N_t$$

For the asset market, we make a standard assumption that state contingent claims are in zero net supply.

2.5 Exogenous Stochastic Processes

In the stylized model, A_t is constant and the discount rate β_t is the only exogenous process. The discount rate process is given by:

$$\ln \beta_t = (1 - \rho_\beta) \ln \bar{\beta} + \rho_\beta \ln \beta_{t-1} + \varepsilon_{\beta,t} \tag{9}$$

where $\varepsilon_{\beta,t}$ is i.i.d. normal with standard deviation of $\bar{\sigma}_\beta$.

In the quantitative model, we assume both A_t and β_t follow AR(1) processes with time-varying volatility and tail risk. For each exogenous process $k \in \{\beta, A\}$, the process is given by

$$\ln k_t = (1 - \rho_k) \ln \bar{k} + \rho_k \ln k_{t-1} + \varepsilon_{k,t} \tag{10}$$

We assume that $\varepsilon_{k,t}$ is drawn from an i.i.d. normal distribution with time-varying standard deviation $\sigma_{k,t-1}$ with probability $1 - p_k$ (the "normal" state) and takes an extreme value ϑ_k with a small probability p_k (the "crisis" state). That is, for each k we define,

$$\tilde{\varepsilon}_{k,t} = \begin{cases} \mathcal{N}(0, \sigma^2_{k,t-1}) & \text{with prob. } 1 - p_k \\ \vartheta_k & \text{with prob. } p_k \end{cases}$$

and construct $\varepsilon_{k,t}$ as $\varepsilon_{k,t} = \tilde{\varepsilon}_{k,t} - \mathbb{E}[\tilde{\varepsilon}_{k,t}]$ such that the innovation has mean zero.[6] We further specify time-varying volatility $\sigma_{k,t-1}$ as a logistic function of k as follows:

$$\sigma_{k,t-1} = \frac{\theta_{ub,k}}{1 + \theta_{adj,k} \exp(\theta_{cv,k} \ln(k_{t-1}/\bar{k}))} \bar{\sigma}_k \tag{11}$$

where we define $\bar{\sigma}_k$ as the standard deviation of $\tilde{\varepsilon}_{k,t}$ conditional on the realization of the normal state and when k_{t-1} is at its average level $\bar{k}$. The term that multiplies $\bar{\sigma}_k$ controls how much it is scaled depending on the deviation of k_{t-1} from its average. $\theta_{ub,k}$ is the upper bound of the scaling and $\theta_{cv,k}$ controls the curvature of the function. Once $\theta_{ub,k}$ and $\theta_{cv,k}$ are chosen, $\theta_{adj,k}$ is set to guarantee that the scaling term equals 1, and hence $\sigma_{k,t-1} = \bar{\sigma}_k$ when $\ln k = \ln \bar{k}$.[7] Our

[6] The general formulation of tail risk is similar to Andreasen (2012b), but our model focuses on discount factor shocks as well as TFP shocks, and simultaneously accounts for time-varying volatility.

[7] Note that $\theta_{ub,k}$ and $\bar{\sigma}_k$ are separately identified as we set $\theta_{adj,k}$ as a function of $\theta_{ub,k}$ and not $\bar{\sigma}_k$. However, this formulation is simply for the sake of exposition. In essence two free parameters are added to the model when

structure is flexible and useful in jointly fitting macroeconomic data and features of the term structure in the U.S. including the recent ELB period. We provide further motivation of this specification and the exact parameterization in Sections 4.2 and 4.3.

2.6 Term Structure of Default-free Interest Rates

Given the equilibrium under complete markets, we can price the term structure of default-free interest rates under our model economy using the pricing kernel. The equilibrium price of a $n-$period zero-coupon nominal bond that pays one dollar at maturity $P_t^{(n)}$ can be derived recursively using the nominal stochastic discount factor from the DSGE model:

$$P_t^{(n)} = \mathbb{E}_t[M_{t+1} P_{t+1}^{(n-1)}].$$

where $P_t^{(0)} = 1$ for $\forall t$. The continuously compounded yield to maturity of this bond follows directly from its price:

$$R_t^{(n)} = -\frac{1}{n} \ln P_t^{(n)}$$

Following the large existing literature on the term structure of interest rates, we further define the term premium of this bond as the difference between the yield and its “risk-neutral” counterpart $R_t^{(n)\mathbb{Q}}$:

$$tp_t^{(n)} \equiv R_t^{(n)} - R_t^{(n)\mathbb{Q}} = \frac{1}{n}(\ln P_t^{(n)\mathbb{Q}} - \ln P_t^{(n)})$$

where the risk-neutral price of a $n-$period zero-coupon nominal bond $P_t^{(n)\mathbb{Q}}$ can be derived similarly as:

$$P_t^{(n)\mathbb{Q}} = \exp(-R_t^{(1)})\mathbb{E}_t[P_{t+1}^{(n-1)\mathbb{Q}}].$$

where again, $P_t^{(0)\mathbb{Q}} = 1 \; \forall t$. Note that the yield to maturity and the term premium of a $n-$period zero-coupon real bond can be derived analogously, by simply replacing the nominal stochastic discount factor and the nominal one-period interest rate used for discounting the risk-neutral prices with their real counterparts.

2.7 Equilibrium Characterization

Given the initial condition $\{R_{-1}^*, J_{-1}\}$ and the exogenous processes $\{\beta_t, A_t, Z_t\}_{t\geq 0}$, a monopolistically competitive equilibrium is defined in a standard way as a set of stochastic processes $\{C_t(i), N_t(i), Y_t(i), C_t, N_t, Y_t, W_t, P_t, P_t(i), R_t^{(n)}, R_t^*, J_t\}_{t\geq 0}$ such that (1) households maximize utility, (2) firms maximize profits, (3) monetary policy follows the interest rate rule, (4) fiscal policies satisfy the budget constraint and (5) goods, labor and asset markets clear.

we introduce heteroskedacticity, which can be defined in multiple ways.

To obtain a stationary equilibrium we follow the standard procedure of normalizing all relevant variables by the (deterministic) trend growth Z_t. Defining the normalized variables using hats and letting $\tilde{\beta}_t \equiv \beta_t \zeta^{-\chi_C}$, the normalized equilibrium conditions, excluding the equations for the term structure, are (for $U_t \geq 0$):

$$\hat{V}_t = \left[\hat{U}_t(\hat{C}_t, N_t) + \zeta \tilde{\beta}_t \left\{\mathbb{E}_t\left[\hat{V}_{t+1}^{1-\gamma}\right]\right\}^{\frac{1-\chi_C}{1-\gamma}}\right]^{\frac{1}{1-\chi_C}} \tag{12}$$

$$M_{t+1} = \tilde{\beta}_t \left(\frac{\hat{U}_{C,t+1}}{\hat{U}_{C,t}}\right)\left[\frac{\hat{V}_{t+1}}{\left[\mathbb{E}_t\left[(\hat{V}_{t+1})^{1-\gamma}\right]\right]^{\frac{1}{1-\gamma}}}\right]^{\chi_C-\gamma} \frac{1}{\Pi_{t+1}} \tag{13}$$

$$\mathbb{E}_t\left[M_{t+1}R_t^{(1)}\right] = 1$$

$$\hat{w}_t = \frac{-\hat{U}_{N,t}}{\hat{U}_{C,t}} \tag{14}$$

$$\hat{Y}_t\left[\varphi\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right)\frac{\Pi_t}{\bar{\Pi}} - (1-\theta) - \theta\frac{\hat{w}_t}{A_t}\right] = \mathbb{E}_t\left[\zeta M_{t+1}\Pi_{t+1}\hat{Y}_{t+1}\varphi\left(\frac{\Pi_{t+1}}{\bar{\Pi}} - 1\right)\frac{\Pi_{t+1}}{\bar{\Pi}}\right]$$

$$R_t^{(1)} = \max\left[R_{ELB}, R_t^* - \phi_{RW} J_t\right]$$

$$R_t^* = \left(R_{t-1}^*\right)^{\rho_R}\left(\bar{R}\left[\frac{\Pi_t}{\bar{\Pi}}\right]^{\phi_\Pi}\left[\frac{\hat{Y}_t}{\bar{Y}}\right]^{\phi_Y}\right)^{1-\rho_R}$$

$$J_t = J_{t-1} + (R_{t-1}^{(1)} - R_{t-1}^*)$$

$$\hat{Y}_t = A_t N_t$$

$$\hat{Y}_t = \hat{C}_t + \frac{\varphi}{2}\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right)^2 \hat{Y}_t$$

Equations (12) to (14) rely on the functional form of utility, and we spell them out in Appendix A.

2.8 Solution Method

We solve the model globally using a time-iteration method in the spirit of Coleman (1991). It is especially important to use a global solution method for our analysis, as opposed to a local approximation method such as perturbation, since the model exhibits a strong non-linearity around the ELB constraint, which, in turn, generates endogenous time-varying volatility as well as time-varying term premiums. Similar methods are used in recent studies of the occasionally binding ELB constraint, but in addition to the standard iteration on decision rules, we also iterate on the value function due to recursive utility. The details of the solution method are described in Appendix B.

3 Equilibrium Yield Curves in the Stylized Models

In this section, we analyze the dynamics of equilibrium yield curves in two stylized models. The first stylized model features power utility (i.e. $\gamma = \chi_C$ in the value function (1)) with period utility (2). We further set $\chi_C = 1$, which results in a standard preference with log separability between consumption and labor. The setup is a plain-vanilla New Keynesian model with the discount rate being the only exogenous process. Its macroeconomic dynamics at the ELB has been studied in detail by Gavin, Keen, Richter, and Throckmorton (2015) and Nakata (2013), among many others in a similar setting. The second stylized model features Epstein-Zin preferences (i.e. $\gamma \neq \chi_C$), but is otherwise identical to the first model.[8] The goal of this section is to describe how the ELB constraint affects equilibrium yield curves in a transparent way. For the sake of brevity, we focus our discussion on the nominal term structure and relegate the discussion on the real term structure as well as inflation compensation/risk premiums to Appendices F and G. Parameter values for the stylized models are shown in Table 2. All values are within the range found in the literature.

Table 2: **Parameter Values for the Stylized Model**

Parameter	Description	Parameter Value
β	Time discount rate at steady state	$\frac{1}{1.006}$
χ_C	Inverse intertemporal elasticity of substitution	1
χ_N	Preference over consumption vs leisure	0.25
γ	Risk aversion	[1, 4]
θ	Elasticity of substitution among intermediate goods	6
φ	Price adjustment cost	75
$400(\bar{\Pi} - 1)$	(Annualized) target rate of inflation	2.0
ϕ_π	Coefficient on inflation in the Taylor rule	2.5
ϕ_y	Coefficient on the output gap in the Taylor rule	0
ρ_R	Interest-rate smoothing in the Taylor rule	0
R_{ELB}	Effective interest rate lower bound	1
ρ_β	AR(1) coefficient for the discount factor shock	0.77
$\bar{\sigma}_\beta$	Standard deviation of shocks to the discount factor	$\frac{0.39}{100}$
	*Implied prob. of policy rate being at the ELB (Power U.)	9%

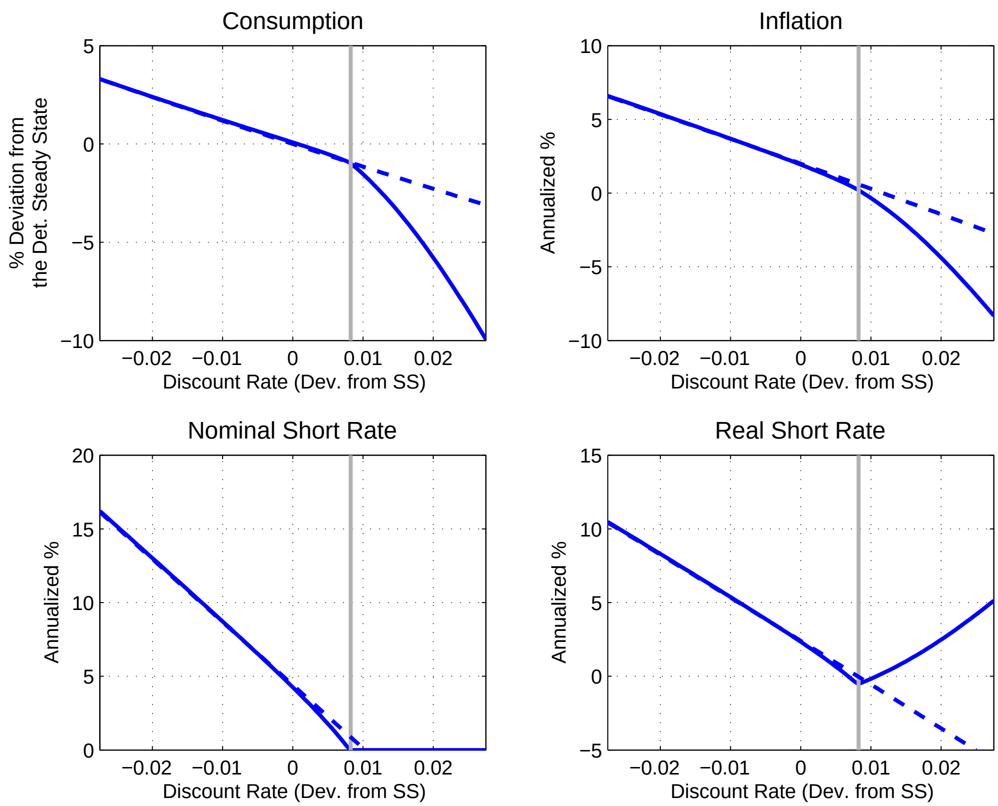
3.1 Stylized Model with Power Utility

Figure 1 shows the equilibrium decision rules for consumption, inflation, the nominal short rate, and the real rate as functions of the time discount rate β, which is the single state variable in the model. In all panels, solid and dashed lines are for the models with and without the ELB constraint. As shown in the top-left, top-right, and bottom-left panels, consumption, inflation, and the policy rate decline as β increases. An increase in the discount rate means that households want to save more for tomorrow and spend less today. Lower demand for the final good by the households leads to lower marginal costs and inflation. According to our interest

[8] This requires taking the limit $\chi_C \to 1$ of equation (1) and boils down to the recursive specification used in Tallarini (2000). See Appendix A for details.

rate feedback rule, the policy rate declines in response to lower inflation (we set the coefficient on output to zero), partially offsetting the contractionary effects of the discount rate increase. For a sufficiently large realization of β, the policy rate hits the ELB. When the policy rate is constrained at the ELB, the contractionary effects of an increase in β cannot be offset by a decline in the policy rate. As a result, an additional increase in the discount rate leads to larger declines in consumption and inflation when the policy rate is at the ELB than when it is not. Note that the real short rate increases with β at the ELB.

Figure 1: **Decision Rules for Macroeconomic Variables—Power Utility—**



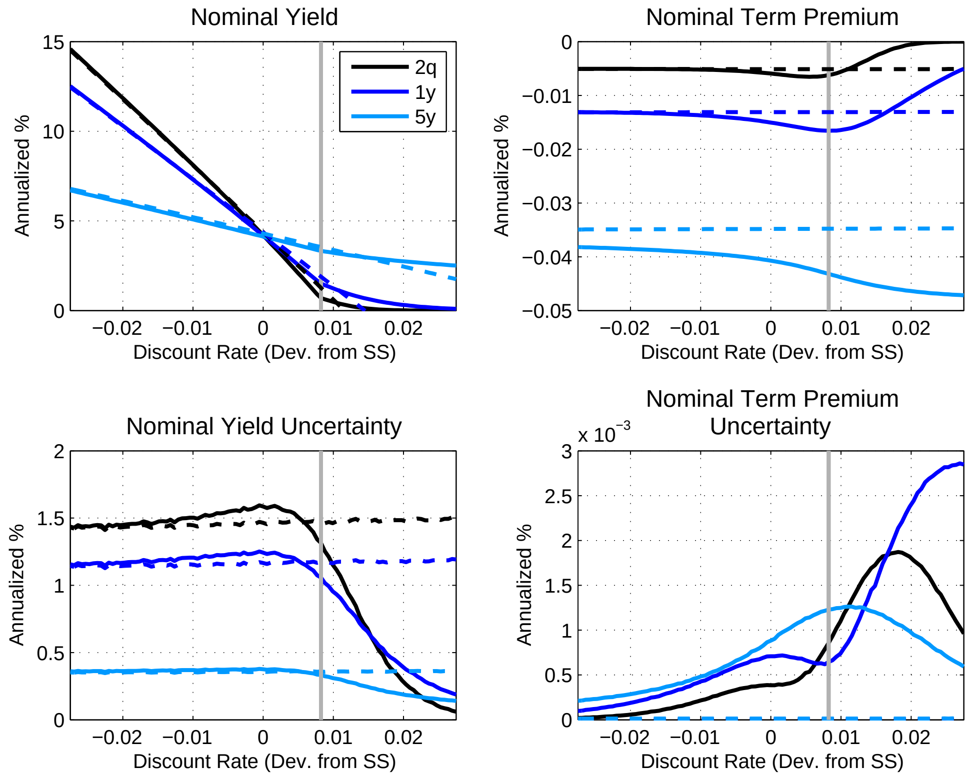
*Solid lines indicate decision rules of the model with the ELB constraint, and dashed lines indicate decision rules of the model without the ELB constraint. The solid vertical line indicates the threshold state where the ELB binds.

Figure 2 shows the decision rules for nominal yields, nominal term premiums, and their conditional volatilities—or, "uncertainties."[9] As in the previous figure, solid and dashed lines are for the models with and without the ELB constraint.

For the equilibrium nominal yields shown in the top-left panel of Figure 2, three features are worth highlighting. The first feature is that for all maturities, nominal yields decrease with the discount rate β. The second feature is that the longer the maturity is, the less sensitive nominal

[9]We use this terminology for conciseness. To avoid any confusion, we note that, within our rational expectations framework, this usage coincides with, for example, Jurado, Ludvigson, and Ng (2015) who define "uncertainty" as the conditional volatility of the purely unforecastable component of the future value of a time series.

Figure 2: **Equilibrium Term Structure—Power Utility—**



*Solid lines indicate results from the model with the ELB constraint, and dashed lines indicate results from the model without the ELB constraint. The solid vertical line indicates the threshold state where the ELB binds.

yields are to changes in β, reflecting the natural consequence of stationarity that is built into the model; In other words, the term structure of yield volatility is downward sloping. These two features imply a third feature that the slope of the nominal yield curve is countercyclical: in a boom when the discount rate is below its steady state, the nominal yield curve is downward sloping with respect to *maturity* (as opposed to β).[10] In a recession when the discount rate is above its steady state, it is upward sloping. All of these features are well-documented features of U.S. Treasury yield data.[11]

The ELB constraint affects yield dynamics for all maturities. Not surprisingly, shorter-maturity yields are more affected by the presence of the ELB than longer-maturity yields. Notice that, even when a longer-maturity yield is not constrained at the ELB, it can be influenced by the ELB constraint if the shorter-maturity yield is constrained or is expected to be constrained at the ELB. For example, the 5-year yield is above the ELB at the highest discount rate shown in the figure. However, at high realizations of β the 5-year yield is somewhat higher in the model with the ELB constraint than in the model without the ELB, reflecting the fact that the

[10]In other words, the level of yields are decreasing (increasing) with maturity for any $\ln(\beta/\bar{\beta}) \geq 0 (< 0)$.

[11]To be precise, although the slope of the yield curve in the data is countercyclical, the slope is not as downward sloping in the data as in the model in booms.

short-rate in the model with the ELB does not fall as much as it would in the model without the ELB.

The top-right panel of Figure 2 plots nominal term premiums. Here, we highlight three features. First, nominal term premiums are negative regardless of the level of the discount rate for both models with and without the ELB and for all maturities. Second, for any given discount rate, the absolute size of term premiums increases with maturity. Third, while nominal term premiums are virtually constant in the model without the ELB constraint, they are state-dependent (or time-varying) in the model with the ELB constraint.

The nature of state-dependency in term premiums depends on maturity. For shorter maturities, nominal term premiums exhibit an interesting non-monotonicity. As the discount rate increases, term premiums decline away from the ELB and become most negative around the state where the economy enters into or exits from the ELB state. At the ELB, term premiums increase in response to a further increase in the discount rate and eventually reach zero. Note that the ELB constraint affects term premiums even when the policy rate is away from the ELB because it binds stochastically in our model. For the region shown in the figure, term premiums for longer maturities decline monotonically as the discount rate increase, but would exhibit a similar pattern if we were to include a state space larger than the one shown in the figure. Interestingly, even when the discount rate is low and the policy rate is far above the ELB constraint, long-maturity term premiums in the model with the ELB are different from those in the model without the ELB.

The nonlinearities in yields and term premiums observed in the model with the ELB constraint imply state-dependence in their uncertainties of yields and term premiums. Naturally, yield uncertainty is on average lower while the policy rate is at the ELB than when it is not, as shown in the bottom-left panel of Figure 2. Meanwhile, the state-dependence of term premium uncertainty is quite different. For shorter maturities, term premium uncertainty increases as the discount rate increases from the steady state, but starts declining when the discount rate is sufficiently high and the policy rate is expected to be at the ELB for a long period of time. That is, term premium uncertainty peaks when the short rate is currently at the ELB but is expected to be positive in the near future. For longer maturities, uncertainty peaks around the discount rate where the ELB starts binding. In contrast, yield uncertainty is essentially constant and term premium uncertainty is virtually zero in the model without the ELB, reflecting the near linearity of decision rules for yields and term premiums, as shown by the dashed lines.

3.1.1 Mechanism

To understand the force behind the dynamics of equilibrium term premiums just described, it is useful to decompose term premiums into their macroeconomic determinants. By taking the first-order Taylor expansion of decision rules around an arbitrary $\beta = \hat{\beta}$, the 2-period nominal term premium can be written as:

$$
\begin{aligned}
tp_t^{(2)}(\hat{\beta}) \equiv R_t^{(2)} - R_t^{(2)\mathbb{Q}} &\approx \frac{1}{2}\mathbb{C}ov_t(m_{t+1}, r_{t+1}) \\
&\propto -\mathbb{C}ov_t(\Delta c_{t+1}, r_{t+1}) - \mathbb{C}ov_t(\pi_{t+1}, r_{t+1}) \\
&= -\sum_{x\in\{\Delta c,\pi\}} \underbrace{\sigma_x(\hat{\beta})}_{\substack{\text{macro}\\ \text{uncertainty}}} \times \underbrace{\sigma_r(\hat{\beta}) \times \rho_{x,r}(\hat{\beta})}_{\substack{\text{interest-rate}\\ \text{sensitivity}}}
\end{aligned} \tag{15}
$$

where lower case variables (m, Δc, r, π) correspond to the natural logarithms of the upper case counterparts, σ_x and $\rho_{x,y}$ denote the 1-period ahead conditional standard deviation of variable x and conditional correlation of variable x and y, respectively.[12]

This expression decomposes term premiums into two components—macroeconomic uncertainty and the sensitivity of interest rates to macroeconomic fluctuations.[13] The sensitivity of interest rates to macroeconomic fluctuations can further be decomposed into interest-rate uncertainty and the correlation between interest rates and a macro variable (consumption growth or inflation).

According to this decomposition, the sign of the term premium is determined by the correlation between the policy rate and inflation/consumption growth. As shown in the bottom panels of Figure 3, both correlations are positive for any discount rate in both models with and without the ELB constraint.[14] As a result, the term premium is negative for any discount rate in both models with and without the ELB constraint.

The negative term premium can also be understood intuitively through the risk exposure of bonds. Suppose we expect a positive shock to β next period. A positive β shock implies a decrease in consumption growth and inflation next period. According to our monetary policy rule, the expected decline in consumption and inflation means that the short rate is expected to fall, or equivalently, the two-period bond price next period is expected to rise. Thus, the two-period bond price next period is expected to rise when consumption is expected to fall, implying that the two-period bond works as a hedge and the term premium must be negative.

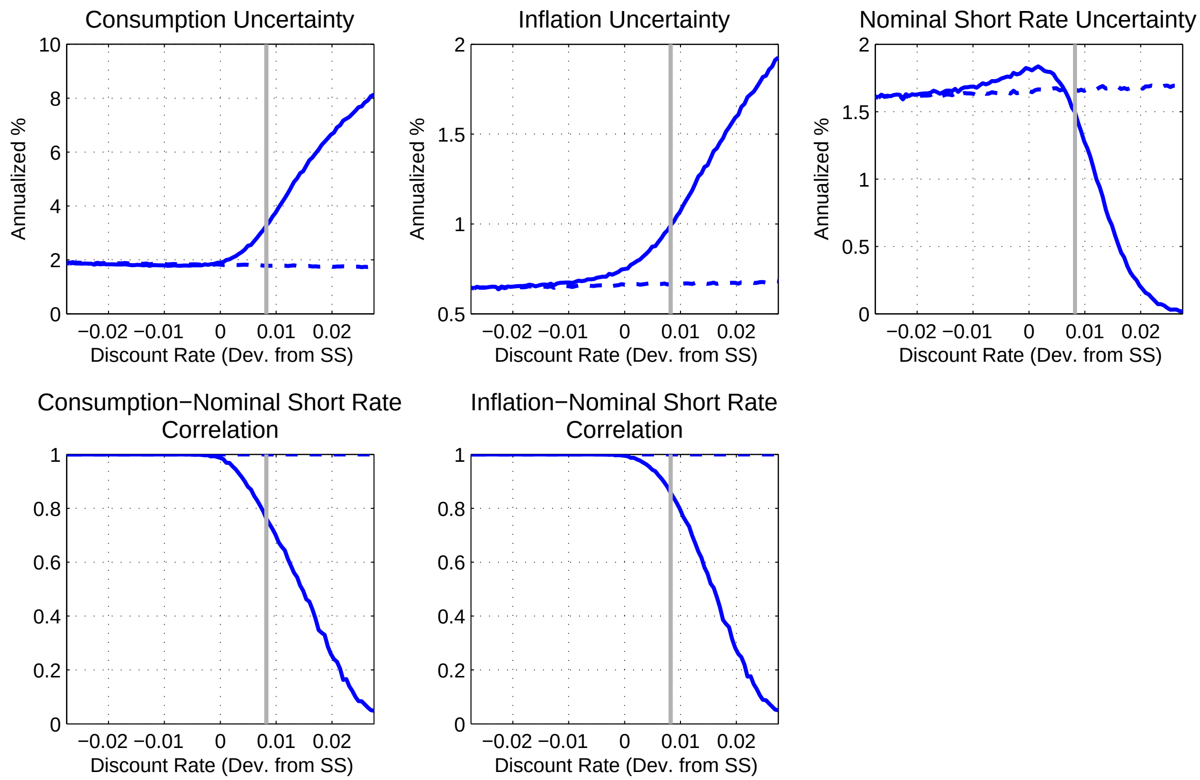
We can also understand the state-dependency of nominal term premiums through this decomposition by analyzing how macroeconomic uncertainty and the sensitivity of interest rates to macroeconomic fluctuations vary with the underlying shock β. As shown in the top left

[12]The first relation follows from the normality of the logarithm of the decision rules, implied by the log-linear approximation. Despite the ELB constraint that produces a kink in the policy rate decision rule, we could think of approximating it with a polynomial of arbitrary order (the Weierstrass Theorem). In fact, popular numerical solution techniques such as projection methods solve for the equilibrium in this way.

[13]Although we believe this decomposition is intuitive, alternatives are certainly conceivable. For instance, as common in the asset pricing literature, we could decompose the term premium into "risk price" and "risk exposure of bonds". One case in which such a decomposition may be suitable is where one assumes an important role of time-varying preference parameters. However, since our stylized model features log preferences, the price of risk, as defined by the volatility of the real stochastic discount factor, would simply coincide with consumption growth uncertainty.

[14]Note that the conditional second moments of consumption and consumption growth one-period ahead coincide.

Figure 3: **Conditional Volatilities and Correlations of Macroeconomic Variables —Power Utility—**



*Solid lines indicate either 1-quarter ahead standard deviation ('uncertainty') or correlation conditional on β of the respective variables for the model with the ELB constraint, and dashed lines indicate those for the model without the ELB constraint. The solid vertical line indicates the threshold state where the ELB binds.

and middle panels of Figure 3, two ingredients of macroeconomic uncertainty—consumption (growth) and inflation uncertainties—do not depend on the level of β as along as the policy rate is sufficiently above zero. However, when the policy rate is near or at the ELB, consumption (growth) and inflation uncertainties increase with β, as the policy rate cannot be adjusted to mitigate the effects of shocks. Thus, macro-uncertainty components of nominal term premiums increase near and at the ELB.[15]

On the other hand, the sensitivity of interest rates to macro fluctuations decreases near and at the ELB, as both interest-rate uncertainty and the correlation between the interest rate and macro variables decrease—as shown in the top-right and bottom panels of Figure 3. As a result, the reduction in the sensitivity of interest rates to macro fluctuations near and at the ELB acts as a force that reduces the absolute size of nominal term premiums.

Whether nominal term premiums increase or decrease at and near the ELB thus depends on which of the two conflicting forces—an increase in macro uncertainty versus a decrease in

[15]Plante, Richter, and Throckmorton (2014) point out the increased macro uncertainty at and near the ELB and use this feature of the model to explain the negative correlation between macroeconomic uncertainty and real GDP growth in the data. However, they do not analyze asset-pricing implications of time-varying volatilities.

interest-rate sensitivity—dominates. In the stylized model with power utility, the amplifying effect of higher macro uncertainty dominates the compressing effect of lower interest-rate sensitivity near the ELB, and the size of the nominal term premiums increases for a certain range of β. However, for a sufficiently large β, the compressing effect dominates the amplifying effect, and the size of the nominal term premiums decreases and approaches zero.[16]

The forces that shape term premium dynamics of longer maturities are more complicated than those of 2-period maturities. Yet, the basic tension is similar. As discussed in detail in Appendix C, the nominal term premium for longer maturities is given approximately by the product of macro uncertainty as well as the sensitivity of longer-term yields to macro fluctuations. Since the ELB is typically not a binding constraint for longer-term yields, the ELB constraint reduces the sensitivity of longer-term yields by less than the sensitivity of short rates. For the 5-year term premium shown in the bottom-left panel of Figure 2, the compressing effect of a reduction in the yield sensitivity is dominated by the amplifying effect of an increase in macro uncertainty, and the size of the term premium is higher at the ELB than away from the ELB. Only when the shock size is sufficiently large does the former effect dominate the latter and the size of the term premium declines.

3.2 Stylized Model with Epstein-Zin Preferences

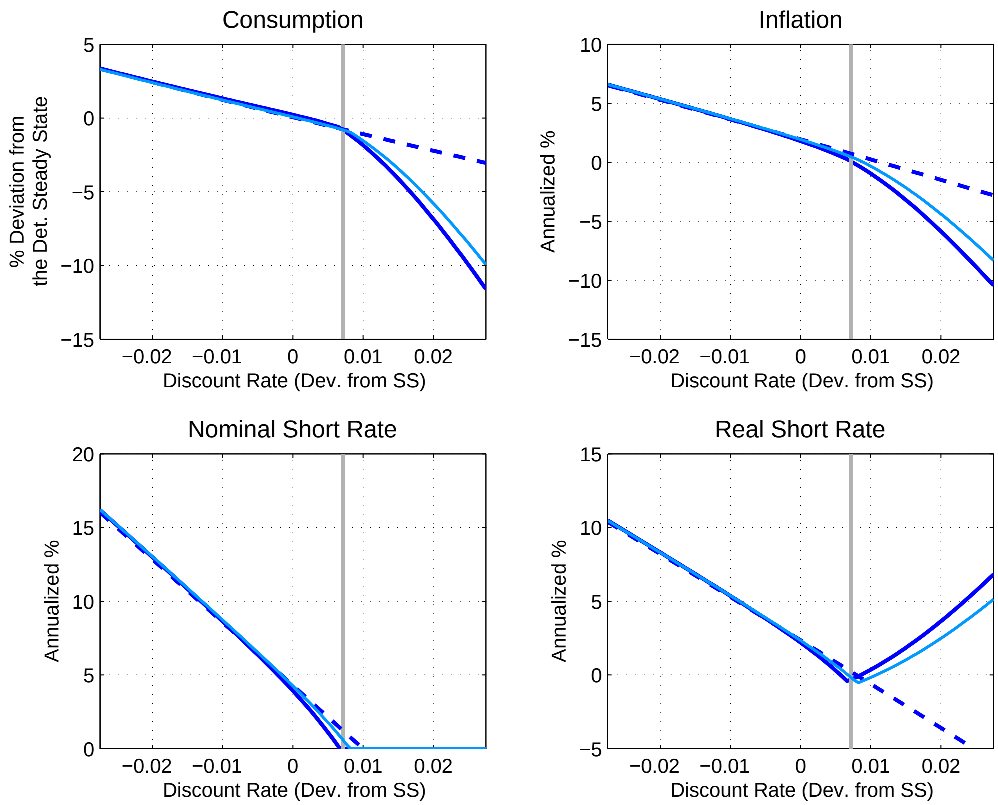
One feature of the term structure model with power utility is that it generates very small (in absolute terms) term premiums. The literature on equilibrium term structure models has demonstrated that the introduction of EZ preferences can magnify the size of term premiums.[17] Accordingly, we now analyze the implications of EZ preferences on macro variables, yields and term premiums in the model with the ELB constraint.

Figure 4 presents the decision rules for consumption, inflation, and the policy rate, and the real rate from the stylized model with EZ preferences with $\gamma = 4$. Solid and dashed blue lines are for models with and without the ELB constraint. The thin light blue lines are the decision rules from the model with power utility shown in Figure 1. According to the figure, the decision rules in the model with EZ preferences are similar to those in the model with power utility when the policy rate is above the ELB constraint. However, the difference between these two economies can be noticeable when the policy rate is constrained. In particular, consumption and inflation

[16] Note this clear tension between macro uncertainty and interest-rate sensitivity depends somewhat on our assumption that the stylized model contains only a "demand" shock. Although the literature largely agrees such a shock is key in accounting for the recent ELB episode in the U.S., consumption and inflation near and at the ELB will respond differently to a "supply" shock. For instance, consider an increase in TFP which boosts consumption but also sends the economy towards the ELB as monetary policy accommodates the ensuing deflationary pressure. As the economy approaches the ELB, the increase in consumption is curbed as limited monetary policy accommodation leads to a rise in real rates. This can end up reducing consumption uncertainty. However, it is unclear if such a mechanism, which implies higher productivity becoming contractionary at the ELB, is empirically relevant. We discuss the implication of the model with both discount rate and TFP shocks in Section 4.

[17] Piazzesi and Schneider (2007) highlighted this point in an early contribution. See also references in the literature review.

Figure 4: **Decision Rules for Macroeconomic Variables —Epstein-Zin Preferences—**



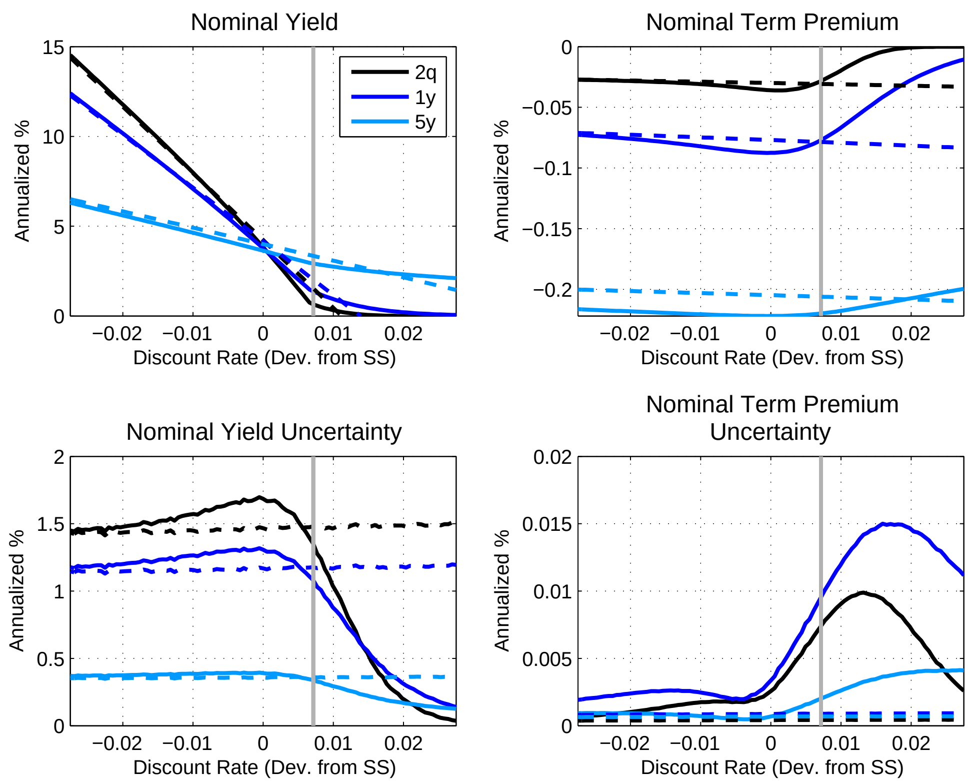
*Solid blue lines indicate decision rules of the model with the ELB constraint, and dashed blue lines indicate decision rules of the model without the ELB constraint. For reference, the light blue lines indicate decision rules of the model with the ELB constraint under power utility. The solid vertical line indicates the threshold state where the ELB binds.

are lower with EZ preferences than with power utility, and their differences increase with the discount rate β.

The responses of nominal yields and their uncertainty in the model with EZ preferences—shown in top-left and bottom-left panels in Figure 5—are qualitatively similar to those in the model with power utility—shown in top-left and bottom-left panels in Figure 2. As long as the ELB constraint is not binding, nominal yields decrease with β for all maturities and longer-maturity yields are less sensitive to β than shorter-maturity yields. For all maturities, nominal yield uncertainty is not state-dependent in the model without the ELB, while it is state-dependent in the model with the ELB. Nominal yields under EZ preferences are qualitatively similar to those under power utility. However, there are non-trivial differences due to the differences in term premiums between the two preference specifications, a topic we shall turn to now.

The top-right panel in Figure 5 shows the term premium dynamics in the model with EZ preferences. One important difference between EZ preferences and power utility is that the absolute size of term premiums is substantially larger under EZ preferences than under power

Figure 5: **Equilibrium Term Structure—Epstein-Zin Preferences—**



*Solid lines indicate results of the model with the ELB constraint, and dashed lines indicate results of the model without the ELB constraint. The solid vertical line indicates the threshold state where the ELB binds.

utility. Focusing our attention to the 5-year maturity in the model with the ELB, the term premium is -22 basis points at the steady state under EZ preferences, versus -4 basis points under power utility, as shown in the top-right panel of Figure 5. At the highest discount rate shown in the figure where the policy rate is constrained at the ELB, the term premium is -20 basis points at the steady state under EZ preferences, versus -4.7 basis points under power utility. As discussed in Section 4, the average 5-year term premium estimates at the ELB are about 40 basis points. So, the amplification in the size of term premiums due to EZ preferences is non-trivial.

Qualitatively, the behavior of term premiums under EZ preferences share many properties with those under power utility. With or without the ELB constraint, nominal term premiums are negative across states and maturities and the degree of negativity increases with maturity at any given state.[18] While term premiums are essentially constant without the ELB, they

[18] We recognize that the negativity result depends on our choice of calibration, in particular the EIS ($1/\chi_C$). In our exercise, term premiums become positive if the EIS is sufficiently greater than 1, consistent with the claim by Albuquerque, Eichenbaum, Luo, and Rebelo (2015) that discount rate shocks combined with EZ preferences can generate positive term premiums if the EIS is larger than one. However, even in a calibration with positive term premiums, the key result of our paper—the ELB constraint creates state-dependency in the dynamics of term premiums—survives. We also note that our choice of EIS=0.5 is closer to the substantial empirical evidence documenting a very small EIS. The mechanism that generates positive term premiums with EIS$>$ 1 is quite

are state-dependent due to the ELB constraint. For shorter maturities, term premiums in the model with EZ preferences are non-monotonic as in the model with power utility, albeit less pronounced. One key difference shows up in the 5-year maturity; while the term premiums are more negative at the ELB than above the ELB under power utility, they are less negative under EZ preferences.

According to the bottom-right panel, the conditional volatilities of term premiums for all maturities peak when the policy rate is currently at the ELB, but is expected to be above the ELB in the near future. For shorter maturities, this feature of the peak is consistent with that under power utility. The peak for longer-maturity bonds occur at a larger discount rate—where the expected duration until liftoff is longer— than that for shorter maturities. As in the model with power utility, these dynamics of term premium uncertainty are distinct from those of yield uncertainty, particularly near the entry to and exit from the ELB.

We can understand the effects of EZ preferences on term premiums by decomposing them into their macroeconomic determinants, similar to what we did for the case of power utility. The 2-quarter term premium under EZ preferences can be written as:

$$\begin{aligned} tp_t^{(2)}(\hat{\beta}) &\approx \frac{1}{2}\mathbb{C}ov_t(m_{t+1}, r_{t+1}) \\ &\propto -\mathbb{C}ov_t(\Delta c_{t+1}, r_{t+1}) - \mathbb{C}ov_t(\pi_{t+1}, r_{t+1}) - \mathbb{C}ov_t(\tilde{v}_{t+1}, r_{t+1}) \\ &= -\sum_{x\in\{\Delta c, \pi\}} \sigma_x(\hat{\beta})\sigma_r(\hat{\beta})\rho_{x,r}(\hat{\beta}) \quad \underbrace{-\,\sigma_{\tilde{v}}(\hat{\beta}) \times \sigma_r(\hat{\beta}) \times \rho_{\tilde{v},r}(\hat{\beta})}_{\substack{\text{extra term} \\ \text{with EZ preferences}}} \end{aligned} \tag{16}$$

where $\tilde{v}_t$ denotes the continuation value relative to its risk-adjustment.[19]

Comparing (16) with (15) we observe that EZ preferences affects the 2-quarter term premium through the additional term $\sigma_{\tilde{v}}\sigma_r\rho_{\tilde{v},r}$, where following previous notation, $\sigma_{\tilde{v}}$ is the uncertainty of $\tilde{v}_t$, and $\rho_{\tilde{v},r}$ is the conditional correlation between $\tilde{v}_t$ and the nominal short-rate.

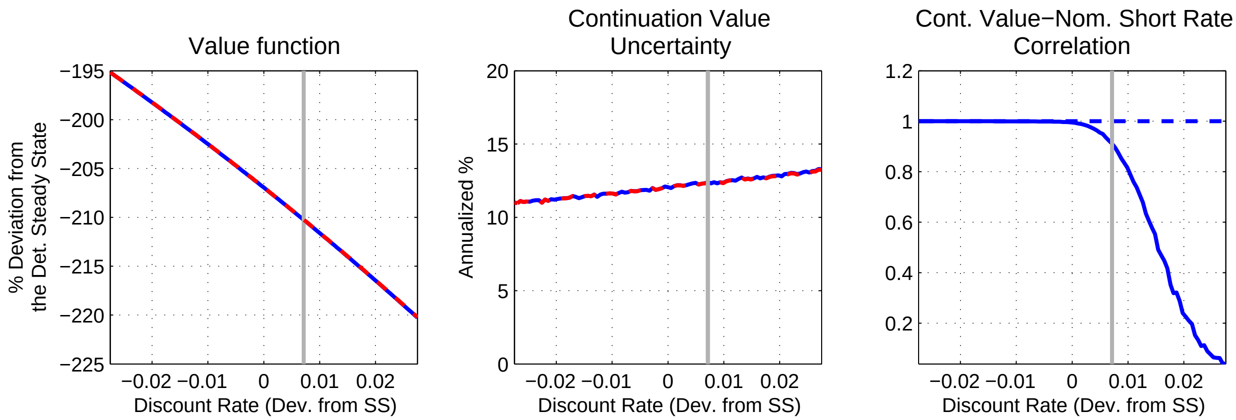
To understand the behavior of this additional term, it is useful to investigate the value function of the model. The value function—shown by the solid blue line in the left panel of Figure 6—decreases with the discount rate. The negative relationship between the discount rate and the value implies a positive correlation between the policy rate and the continuation value ($\rho_{\tilde{v},r} \geq 0$). Perhaps somewhat surprisingly, the value function is relatively linear, regardless of whether the economy is above or at the ELB. This near linearity is in stark contrast to the nonlinearity seen in the decision rules of consumption and inflation. The near linearity of the value function implies that the conditional volatility of $\tilde{v}_t$ is relatively constant, as shown in the middle panel of Figure 6.

The value function is relatively linear compared to other macro variables because it is not only a function of the current state, but also a function of the states of the economy into the

different from ours. It is interesting and worth further investigation.

[19]The actual expression is $\tilde{v}_t \equiv \ln\left[\exp(\xi V_{t+1})/\mathbb{E}_t\left[\exp(\xi V_{t+1})\right]\right]$ after $\ln V_{t+1}$ is substituted by its linear approximation. We will occasionally refer to this term simply as "continuation value" for brevity.

Figure 6: **Value Function—Epstein-Zin Preferences—**



*Left panel: Value function. Middle and right panels: 1-quarter ahead standard deviation ('uncertainty') and 1-quarter ahead correlation of the continuation value and the nominal short rate conditional on β. Note for all panels the solid lines are results for the model with the ELB constraint, and the dashed lines are for the model without the ELB constraint, respectively. The solid vertical line indicates the threshold state where the ELB binds.

infinite future. Given the low frequency at which the ELB binds and the short duration of the ELB states in our stylized model, the effects of the ELB on the value function are small even when the policy rate is currently constrained, making the value function relatively linear. The absence of noticeable impacts of the ELB on the value function at the ELB just described is consistent with the fact that the value function of the model with the ELB is virtually equivalent to that of the model without the ELB, shown by the dashed red line lying on top of the blue line in the left panel.

The positive correlation between the policy rate and the continuation value ($\rho_{\tilde{v},r} > 0$) means that the additional term in equation (16) is negative and that term premiums under EZ preferences is more negative than under power utility. That is, nominal bonds provide an additional hedge against changes in the continuation value for EZ households. Quantitatively, the size of this additional term is much larger than the size of the first term. As a result, the time-variation in term premiums in the model with EZ preferences is mainly determined by the time-variation in the additional term. The quantitative importance of this term would increase further with the degree of risk aversion.

Since the volatility of the continuation value $\sigma_{\tilde{v}}$ is nearly constant, the decomposition in (16) tells us that, under EZ preferences, the reduction in the sensitivity of interest rates to macroeconomic fluctuations is the key driver of term premium dynamics near and at the ELB. Indeed, as shown in the right panel of Figure 6, the correlation of the continuation value and the policy rate declines towards zero as the discount rate increases, and policy rate uncertainty declines similar to the case of power utility near and at the ELB (not shown). As a result, the non-monotonicity of the decision rule for shorter maturities under power utility is less pronounced under EZ preferences. For the 5-year maturity, the term premium increases as the discount rate increases under EZ preferences.

The implications of EZ preferences on the dynamics of n-period term premiums can be understood by a similar decomposition: they are discussed in the Appendix.

4 Equilibrium Yield Curves in the Quantitative Model

We now turn to the analysis of a quantitative model that incorporates additional features into the stylized models so as to better capture key features of U.S. data. Our analysis proceeds in two steps. First, we calibrate the quantitative model to match some key features of consumption, inflation, yield data and term premium *estimates*[20] at and away from the ELB (this section). The exercise offers a coherent interpretation of macro and term structure dynamics of the past two decades including the recent U.S. ELB episode through the lens of our model. Second, we examine how alternative monetary policy strategies affect the dynamics of yield curves and their decomposition into the expected short-rate path and term premiums at the ELB (Section 5).

We start this section by discussing key features of nominal yields and term premium estimates which we are interested in fitting our model to.[21] We then explain how the additional features in the quantitative model help match key features of macro variables and yield curves, and present the parameter values chosen. Finally, the various results will be reported.

4.1 Nominal Yields and the Estimates of Term Premiums

The top-left panel of Figure 7 shows 3-month, 2-year, 5-year, and 10-year Treasury yields over the last two decades, from Jaunary 1997 until September 2015.[22] Nominal yields are procyclical. They are higher during the economic booms preceding the 2001 recession and the 2007-2009 recession than during those recessions and the subsequent periods of sluggish recovery from them. The slope of the yield curve is countercyclical. During recessions and their aftermath, the difference between the short-rate and the 10-year rate is large, while it is small, or even negative, during economic booms. Finally, the term structure of yield volatility is downward sloping. The volatility of shorter-maturity yields is generally higher than that of longer-maturity yields.

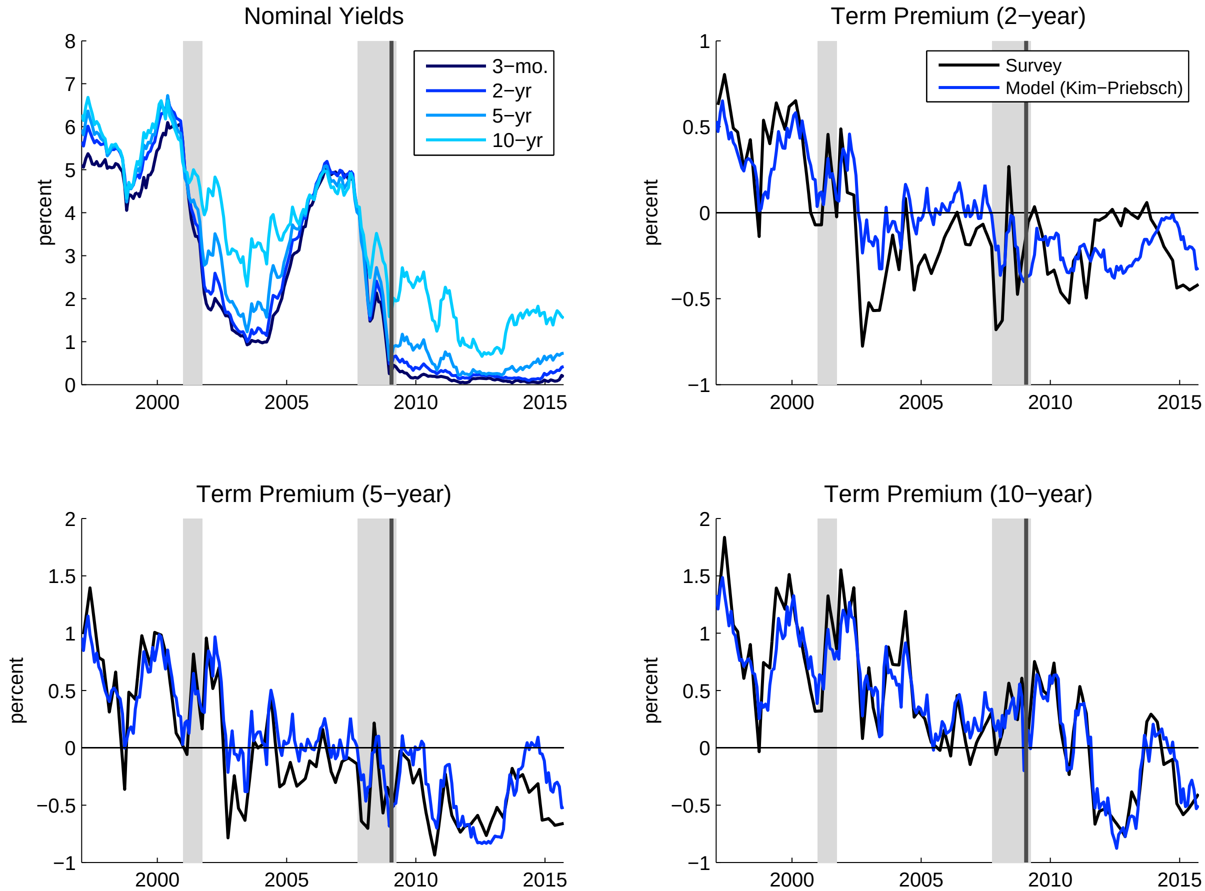
The top-right, bottom-left, and bottom-right panels of Figure 7 show estimates of term premiums for 2-year, 5-year, and 10-year bonds, respectively. Since term premiums are unobserved, there are various estimates for them. We show two estimates in this figure. One is calculated using 3-month T-bill forecasts from Blue chip surveys as a proxy for future short-rate expectations and is shown by black lines. The other is based on a term structure model developed by Priebsch (2013) and Kim and Priebsch (2013), which modifies the widely-used no-arbitrage

[20] Term premiums are unobserved, and must be estimated using either model- or survey-based expectations.

[21] We defer details of other U.S. data used for our calibration, including consumption and inflation data, to the Appendix.

[22] See the Appendix for details of the data. The somewhat recent starting date is motivated by (i) empirical evidence suggesting distinct changes in U.S. Treasury bond risk with respect to the aggregate stock market since the late 1990s (Campbell, Sunderam, and Viceira (2013)) and (ii) the relative stabilization of trend inflation around a similar timing (for example, Mertens (2015)). Restricting our analysis to this shorter period makes it less susceptible to misspecification from structural breaks in the economy.

Figure 7: **Nominal Yields and Term Premium Estimates**



* For term premium estimates, survey-based estimates (black lines) are computed using 3-month T-bill forecasts from Blue Chip surveys and the model-based estimates (blue lines) are from the 3-factor no-arbitrage term structure model with the ELB of Kim and Priebsch (2013). For both yields and term premium panels, the solid vertical line indicates January 2009 (roughly the start of the ELB period), and the shaded grey areas correspond to NBER recession periods. Data are at a monthly frequency for yields and model-based term premium estimates, and roughly quarterly for the survey-based estimates.

affine term structure model of Kim and Wright (2005) to incorporate the ELB constraint, and is shown by blue lines.[23] The model- and survey-based estimates broadly comove, though they diverge at times.

Term premium estimates were positive, large and were generally increasing in maturity in the early part of the sample. Throughout the sample, term premiums appear to be following a secular downward trend amid significant volatility. In the most recent years—when the ELB was a binding constraint—the 2-year and 5-year term premiums became increasingly negative. The 10-year term premium, which remained positive throughout most of the sample period, has become smaller, and have frequently been negative in the last several years. All told, both

[23]These models contain latent factors as drivers of term structure dynamics, which make them very flexible and offer excellent fit to yield data. The estimates from Kim and Priebsch (2013) above the ELB are similar to the estimates from the well-known Kim and Wright (2005) model.

survey-based and model-based estimates were on average positive across maturities during the pre-ELB era, while negative—particularly up to intermediate maturities—during the ELB era. One criterion of success for previous equilibrium term structure models has been to generate realistically large, and volatile term premiums for longer-maturity bonds observed during the pre-ELB era.

4.2 Model Features Elaborated

As discussed in Section 2 and summarized in Table 1, the additional features of the quantitative model, relative to the stylized models, are GHH utility, interest-rate smoothing in the policy rate, and richer shock processes—the introduction of an AR(1) TFP process, tail risks and stochastic volatility in both TFP and discount rate shocks.

We use GHH utility to help the model generate sufficiently large and volatile nominal term premiums. As discussed in Guvenen (2009), the non-separability of consumption and labor and the absence of wealth effects on labor supply in the GHH specification mitigate the dampening effect of endogenous labor supply on risk prices by preventing labor to serve as an effective hedge against consumption fluctuations.[24]

An interest-rate-smoothing term is introduced to the policy rule to make it empirically plausible. Most estimates of the interest-rate feedback rule find large weight on the lagged interest rate.[25] The introduction of policy inertia is also important in generating ELB episodes that are ceteris paribus longer, and thus more realistic.

We introduce an AR(1) TFP process and tails risk in TFP to make the nominal term premium sufficiently positive on average while the ELB is not binding. A TFP shock generates negative comovement in consumption growth and inflation. As we have seen earlier, discount rate shocks, or other "demand" shocks, do not have this feature, and hence the term premiums are negative in the model with demand shock only. The use of TFP shocks to generate positive term premiums is common in existing studies of term structures of interest rates based on DSGE models.[26]

We include tail risk in TFP because the presence of the ELB constraint puts an important restriction on how much we can rely on normally distributed shocks alone to generate positive term premiums away from the ELB. If we were to simply increase the standard deviation of a (normal) TFP shock to increase the size of term premiums, then at some point positive realizations of TFP will become likely to push the policy rate to the ELB by lowering inflation.

[24]The combination of GHH period utility with EZ preferences has been also used by a few papers such as Guvenen (2009) and Blanco (2016). However, to the best of our knowledge this combination has not been used for DSGE term structure models. It is also worth mentioning that this specification does not add any endogenous state variables, which reduces computational burden; a particularly convenient feature for our application.

[25]See for example, Smets and Wouters (2007) and Gust, Herbst, Lopez-Salido, and Smith (2016) (especially the 2012 version that uses a model closer to ours).

[26]See, for example, Andreasen (2012a,b), Van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012), Dew-Becker (2014), Hsu, Li, Palomino (2015), and Rudebusch and Swanson (2008, 2012). Note that some recent papers such as Albuquerque, Eichenbaum, Luo, and Rebelo (2015) and Creal and Wu (2015), emphasize the role of discount rate shocks in explaining asset pricing facts. See also our discussion in Section 3.2.

In ELB episodes induced by TFP shocks, inflation is lower but output is higher at the ELB than above the ELB, contrary to the fact that both inflation and output were low during the most recent ELB episode in the U.S. [27] Meanwhile, a negative tail risk in TFP that would lead to higher inflation and lower output can increase the negative correlation among consumption growth and inflation and push up the term premium without making the ELB episodes counterfactual. We believe that the assumption of negative tail risk in TFP is sensible, as we can tie it to the Great Inflation episode of the 1970s, for instance.[28]

We further introduce stochastic volatility in the TFP process to generate plausible volatility in term premiums while the policy rate is away from the ELB. The use of stochastic volatility to amplify volatilities in term premiums is also common in the literature.[29]

Now we turn to the specification of the discount rate process. We introduce tail risk in the discount rate shock as a device to push the policy rate to the ELB without making average term premiums negative while the policy rate is away from the ELB. As described in the previous section and as we will elaborate more in Section 6.1, the presence of demand shocks pushes down term premiums substantially when the household has EZ preferences and is highly risk averse. If the main driver to push the policy rate to the ELB is a normally distributed demand shock with a high standard deviation, it is difficult to keep the term premium positive when the policy rate is away from the ELB. With the use of tail risk in the discount rate shock, we can maintain positive term premiums away from the ELB, while making both average inflation and consumption lower at the ELB than away from the ELB.

Finally, stochastic volatility in the discount rate process is introduced to account for the difference between the average levels of empirical term premium estimates away and at the ELB. As discussed earlier, the term premium estimates are on average negative at the ELB. By allowing the variance of the discount rate shock to increase in the face of the crisis shock that pushes the policy rate to the ELB, we can temporarily weaken the negative correlation of consumption and inflation, and thus make the term premiums negative, while at the ELB. Note that our modelling choice in this dimension is consistent with recent empirical evidence showing that economic uncertainty increases in recessions.

4.3 Parameter Values

Parameter values for the quantitative model are summarized in Table 3.

The scaled time discount rate ($\tilde{\beta} \equiv \bar{\beta}\zeta^{-\chi_C}$) is set to 1/1.00625, implying a steady state real short rate of 2.5% (annualized).[30] The deterministic trend growth rate of TFP, ζ, is 2 percent

[27] Another restriction is that the volatilities of other equilibrium objects in our model—such as consumption and inflation—need to be consistent with those in the data.

[28] This argument is somewhat heuristic, as we focus on a sample period that does not include the Great Inflation episode. However, it is unclear that strictly relying on observed disaster events during the short sample period we consider is a more sensible approach to calibrate the model.

[29] See, for example, Andreasen (2012b), Kung (2015) and Hsu, Li, and Palomino (2015).

[30] To calibrate this parameter, we first choose ζ and χ_C (as below), and set $\bar{\beta}$ to be consistent with our target $\tilde{\beta}$.

Table 3: **Parameter Values for the Quantitative Model**

Parameter	Description	Parameter Value
Household and Firm		
$\tilde{\beta}$	Scaled time discount rate	$\frac{1}{1.00625}$
$400(\zeta - 1)$	(Annualized) deterministic trend growth in TFP	2.0
χ_C	Inverse elasticity of intertemporal substitution	9
χ_N	Inverse Frisch elasticity	$\frac{1}{3}$
α	Risk aversion ($= 1 - (1-\gamma)/(1-\chi_C)$)	-100
θ	Elasticity of substitution among intermediate goods	6
φ	Price adjustment cost	80
Monetary Policy		
$400(\bar{\Pi} - 1)$	(Annualized) inflation target parameter	2.2
ϕ_π	Coefficient on inflation in the Taylor rule	5
ϕ_y	Coefficient on the output gap in the Taylor rule	0.5
ρ_R	Interest-rate smoothing in the Taylor rule	0.9
Discount Rate Process		
ρ_β	AR(1) coefficient for the discount rate process	0.85
$\bar{\sigma}_\beta$	The standard deviation of shocks to the discount rate process	$\frac{0.0001}{100}$
p_β	Tail risk prob. for the discount rate	10^{-30}
ϑ_β	Tail risk size for the discount rate	0.07
$\theta_{ub,\beta}$	Stochastic volatility for the discount rate (upper bound)	1050
$\theta_{cv,\beta}$	Stochastic volatility for the discount rate (curvature)	-2000
TFP Process		
ρ_a	AR(1) coefficient for the TFP process	0.93
$\bar{\sigma}_a$	The standard deviation of shocks to the TFP process	$\frac{0.1}{100}$
p_a	Tail risk prob. for TFP	$\frac{0.5}{100}$
ϑ_a	Tail risk size for TFP	-0.006
$\theta_{ub,a}$	Stochastic volatility for TFP (upper bound)	5
$\theta_{cv,a}$	Stochastic volatility for TFP (curvature)	90

(annualized), which is consistent with the average per capita consumption growth over the last two decades.

The inverse Frisch elasticity χ_N, the inverse of the elasticity of intertemporal substitution χ_C, and the coefficient of relative risk aversion α are set to 1/3, 9, and -100, respectively.[31] They are chosen so that macro and term structure moments away from the ELB are broadly in line with those in the data. The value for χ_N is well in line with many macroeconomic studies.[32] Our values for χ_C and α are similar to those used in other studies of the term structure using DSGE models.[33] The parameters governing the production side of the economy ($\theta = 6$ and $\varphi = 80$) are also standard.

Moving on to the parameters governing the monetary policy rule, the inflation-target parameter is set to 2.2 percent, so that the model generates the average inflation rate prior to the

[31] Here we use $\alpha \equiv 1 - (1-\gamma)/(1-\chi_C)$ as the measure of risk aversion for ease of comparison with previous papers in the literature such as Rudebusch and Swanson (2012).

[32] Most studies report numbers around 0.25 to 0.5. For example, King and Rebelo (1999) uses 0.25.

[33] For studies that use a high χ_C (low IES), see Hall (1988), Campbell (2003) and Beeler and Campbell (2012). Our choice for the value of risk aversion α is very similar to Rudebusch and Swanson (2012). The equilibrium term structure literature typically uses a risk aversion parameter that is substantially larger than those used when the DSGE model is used only to examine macroeconomic variables.

beginning of the ELB policy in our sample. The coefficient on inflation is set to 5. This high value allows the model to fit the relatively stable inflation since mid-1990s.[34] The coefficient on output and the interest-rate smoothing parameters are 1 and 0.9, which are in line with the values used in the literature.

For parameters governing the TFP process, we choose the persistence parameter ρ_a to be a high value of 0.93, in order to generate volatile longer-tem yields and large and volatile term premiums away from the ELB. $\bar{\sigma}_a$, the conditional volatility at the steady state, is chosen to match the volatility of consumption and inflation away from the ELB. We choose the size and probability of the tail events (p_a and ϑ_a) to be 0.5% and -0.006 such that it is a 6 standard deviation negative shock occurring every 50 years on average. Stochastic volatility parameters $(\theta_{ub,a}, \theta_{cv,a})$ are chosen to generate nominal term premium volatility of a plausible magnitude without severely affecting other moments away from the ELB. Note that $\theta_{cv,a} > 0$ implies countercyclical uncertainty in TFP, which in turns implies countercyclical term premiums, features that are widely accepted in the literature.[35]

Moving on to the parameters for the discount rate process, the persistence parameter ρ_β is set to 0.85, which implies an expected ELB duration of about 10 quarters. The size of the crisis (ϑ_β) is chosen to broadly capture the magnitude of average consumption and inflation as well as their volatilities at the ELB. The probability of a crisis (p_β) is set to 10^{-30}, an extremely small value. We made this parameter choice because we find that even a tiny possibility of a large increase in the discount rate reduces the average term premiums substantially below zero while the policy rate is above the ELB (see Section 6.1 for more details). The extremely low probability of a large discount rate shock makes our model closer to the perfect-foresight model often used in the ELB literature in which the possibility of hitting the ELB is zero and the liquidity trap episodes are completely unanticipated (Eggertsson and Woodford (2003) and Christiano, Eichenbaum, and Rebelo (2011)).

We set the conditional volatility of β at the steady state, $\bar{\sigma}_\beta$, to be a very small number such that the main drivers of economic dynamics away from the ELB are TFP shocks. This emphasis on TFP shocks is consistent with other DSGE term structure studies in the literature. We set the stochastic volatility parameters ($\theta_{cv,\beta}$ and $\theta_{ub,\beta}$) such that the discount rate volatility evolves as if it follows a two-state Markov process, taking a negligible value in the "normal" state when β is near its steady state value (the policy rate will be sufficiently away from the ELB in this case) and a positive value in the "crisis" state when β is large (the policy rate will be at or near the ELB in this case). The combination of $\bar{\sigma}_\beta$ and $\theta_{ub,\beta}$ determines the average volatility

[34] The estimate of this parameter in the 2012 version of Gust, Herbst, Lopez-Salido, and Smith (2016), whose model is closer to ours, is also about 5.

[35] A convenient feature of our logistical specification of volatility (eqn. (11)) is that the conditional volatility of TFP becomes exponentially small as TFP increases under this specification, making it unlikely that a large *increase* in TFP pushes the policy rate to the ELB. As discussed earlier, a TFP-driven ELB episode in which consumption is higher and inflation is lower than at the steady state is empirically uninteresting. Our specification of the volatility is similar to an exponential function used in Hsu, Li, and Palomino (2015). Ours makes conditional volatility bounded at the tails and is therefore better behaved numerically.

of the discount rate at the ELB, which is set such that the 10-year nominal term premium is on average slightly negative at the ELB, similar to the estimate of Kim and Priebsch (2013).[36] To clarify the role of stochastic volatility in the discount rate, we also consider an alternative calibration without stochastic volatility in Appendix E.

4.4 Results

4.4.1 Moments above and at the ELB

We now describe the ability of our model to fit the data by looking at selected statistical moments of (detrended) consumption, inflation and the term structure of interest rates both above and at the ELB. The moments of interest are averages and standard deviations for "above the ELB" and "at the ELB" samples.[37] To compute moments for the data counterparts, we define the ELB period as starting from 2009:Q1, as the effective federal funds rate hit the ELB in the middle of December 2008.

Table 4 presents the moments of consumption, inflation, and the term structure of interest rates from the model and the data, while Table 5 presents the moments of the term premium estimates based on Kim and Priebsch (2013) and the term premiums from our model. In each table, the first two columns are for the moments when the policy rate is above the ELB and the next two columns are for the moments when the policy rate is at the ELB. The final column shows the moments from a version of our quantitative model without the ELB constraint when the policy rate is below the ELB.

As discussed earlier, our ELB episodes are driven by a large increase in the discount rate shock. This shock is also associated with an increase in the volatility of the discount rate. As a result, the difference between the moments above the ELB and those at the ELB are not only driven by whether the ELB constraint is binding or not, but also by the fact that the moments at the ELB are computed conditional on the economy being hit by a crisis shock, as well as by the difference in the degree of uncertainty regarding the future path of the discount rate. The comparison of this column with the fourth column helps us isolate the effects of the ELB constraint on the model's moments from the effects associated with the large crisis shock in the

[36] Since our model does not include capital, a shock to the discount rate may, under some additional assumptions, also be interpreted as a shock to demand for particularly safe and liquid assets such as short-term Treasuries, as suggested by Fisher (2015). This safety demand, in turn, could be influenced by some of the balance sheet policies implemented during the crisis period, and may be an important driver of term premiums. Our calibration strategy attempts to capture such dynamics as well, albeit in a reduced form manner.

[37] The model implied moments at the ELB is computed by averaging over many simulated ELB episodes. In particular, the model is simulated 500 times with the tail shock in the discount rate, and averaged over all episodes where the shock actually took the economy to the ELB. The initial values of the state variables at the time of the shock are drawn from their stationary distributions. A more natural way to compute moments at the ELB is to simply simulate the model and compute moments of interest conditional on the economy being at the ELB, i.e. $\mathbb{E}[X_t \mid R_t = 1]$, where X_t is the equilibrium variable of interest. We do not take this approach for the following two reasons. First, since the ELB binds only with an extremely small probability in our model, it is computationally infeasible to implement this approach. Second, with some probability, a large positive TFP shock(s) can also take the economy to the ELB. As discussed earlier, a TFP-driven liquidity trap is an empirically uninteresting situation and our approach focuses on the average dynamics of demand-driven ELB episodes.

Table 4: **Macro and Yield Curve Moments at and above the ELB —Model versus Data—**

	Above the ELB		At the ELB		Below the ELB
	Data	Model	Data	Model	Model w/o ELB
A. Macro Variables (Mean)					
Consumption†	—	—	-2.02	-1.45	-1.06
Inflation	2.13	2.09	1.45	1.15	1.24
B. Macro Variables (Volatility)					
Consumption	2.93	2.20	4.18	8.04	6.76
Inflation	0.87	0.72	0.83	0.93	0.73
C. Nominal Yields (Mean)					
3-month	3.44	3.92	0.08	0.13	-1.42
2-year	4.01	4.01	0.54	0.40	-0.66
5-year	4.49	4.21	1.57	1.40	0.94
10-year	5.10	4.43	2.79	2.51	2.28
D. Nominal Yields (Volatility)					
3-month	1.71	1.77	0.06	0.00	1.04
2-year	1.57	1.69	0.25	0.34	1.23
5-year	1.14	1.38	0.55	0.56	0.99
10-year	0.79	0.95	0.74	0.42	0.63

*This table contains summary statistics for selected macroeconomic and term structure variables comparing data versus model counterparts. The sample period is 1997:Q1-2008:Q4 for data above the ELB, and 2009:Q1-2015:Q3 for data at the ELB.

†The "mean" reported for consumption at/below the ELB is the average deviation from either the trend (for data) or the model implied consumption away from the ELB (for the model) in percentage points. We do not report statistics away from the ELB as the model implied average deviation is zero by construction.

discount rate per se.

Moments above the ELB: As shown in the first two columns of the top two panels of Table 4, the average inflation as well as volatilities of consumption and inflation from our model are in line with those in the data while the policy rate is above the ELB. According to the first column of Panels C and D, yields are upward sloping and the term structure of yield volatility is downward sloping on average when the policy rate is above the ELB. Our model successfully replicates these patterns, as shown in the second column of panels C and D. The moments of the yield curve from our model are not only qualitatively consistent with, but also quantitatively close to, those from the data.

According to the first column of the top panel of Table 5, the empirical estimates of term premiums are positive for all maturities and increases with maturity. According to the first column of panel B, the volatility of the empirical estimates of term premiums increase with

Table 5: **Term Premium Moments at and above the ELB —Model versus Estimates—**

	Above the ELB		At the ELB		Below the ELB
	Estimates	Model	Estimates	Model	Model w/o ELB
	A. Nominal Term Premium(Mean)				
2-year	0.11	0.10	-0.22	-0.04	-0.13
5-year	0.26	0.32	-0.39	-0.17	-0.24
10-year	0.61	0.58	-0.10	-0.05	-0.09
	B. Nominal Term Premium (Volatility)				
2-year	0.23	0.06	0.09	0.04	0.03
5-year	0.36	0.15	0.28	0.05	0.07
10-year	0.35	0.18	0.41	0.07	0.09

*This table contains summary statistics for selected term premiums comparing estimates versus model counterparts. The sample period is 1997:Q1-2008:Q4 for estimates above the ELB, and 2009:Q1-2015:Q3 for estimates at the ELB. All variables are in annualized percentage terms.

maturity. Term premiums from our model are consistent with these patterns. Quantitatively, the average level of term premiums from our model is close to the empirical counterparts, while the volatility of term premiums from our model is somewhat lower than the empirical estimates. However, our model's term premiums are quantitatively reasonable given that any estimates of term premiums are surrounded by a nontrivial amount of uncertainty.

Moments at the ELB: We turn to the comparison of our model with the data at the ELB. According to the first and third columns of the top panel of Table 4, consumption is 2 percent lower and inflation is about 70 basis points lower on average at the ELB than above the ELB in the data. These declines in consumption and inflation observed at the ELB are broadly in line with those in our model. According to the first and third columns of panel B, the volatility of consumption is higher at the ELB than away from the ELB, while the volatility of inflation at the ELB is similar to that above the ELB. In our model, consumption volatility is substantially higher at the ELB than above the ELB and inflation volatility is moderately higher at the ELB than above the ELB. Consumption volatility from our model is about twice as large as that from the data and inflation volatility from our model is only 10 basis point above that of the data. The comparison of the fourth column with the fifth (last) column where we show the moments from the model without the ELB shows that the existence of the ELB makes the recession caused by the discount rate shock more severe.

According to the first and third column of panel C of Table 4, nominal yields are lower across maturities and the yield curve is steeper on average at the ELB than above the ELB in the data. According to the first and third column of panel D, the volatilities of nominal yields are lower at the ELB than above the ELB across maturities in the data. Also, while the nominal yield volatility declines with maturity when the policy rate is above the ELB, the nominal yield volatility increases with maturity when the policy rate is at the ELB. Our model captures all of these features reasonably well.

Comparison of the moments of nominal yields from our model with those from our model without the ELB constraint—shown in the last column—shows that nominal yields are on average higher in the presence of the ELB constraint than in the absence of it across all maturities. Also, this comparison shows that the upward slope of the volatility term structure is a consequence of the ELB. In the absence of the ELB constraint, the volatility decreases with maturity during a severe recession in which the policy rate is negative.

As discussed earlier and as shown in the third column of panel A of Table 5, existing estimates of nominal term premiums are negative on average while the policy rate is constrained at the ELB. Our model similarly generates term premiums that are negative at the ELB (seen in the fourth column of panel A).

Term premiums are largely a product of two forces in our quantitative model. The first force is our assumption that the discount rate volatility increases with the size of the demand shock as discussed in Section 4.2. With elevated demand uncertainty, correlation between consumption growth and inflation becomes less negative and, as a result, term premiums are reduced. In addition, bonds become a better hedge against future real uncertainty (Appendix F). This force is present regardless of whether there is an ELB constraint in the model or not. Indeed, the last column of panel A shows that term premiums become negative in the face of the crisis shock even in the model without the ELB constraint.

The second force is the ELB constraint. As discussed in detail in Section 3, the ELB constraint can either increase or decrease the size of term premiums depending on how much the ELB constraint decreases the sensitivity of interest rates to macroeconomic fluctuations and how much it increases macroeconomic uncertainty. According to the fourth and last columns of panel A, the compressing effect of the ELB associated with reduced sensitivity of interest rates dominates the amplifying effect associated with increased macroeconomic uncertainty and the ELB constraint reduces the absolute size of term premiums. In our model, in which term premiums are negative at the ELB, the reduction in the absolute size of term premiums means an increase in term premiums.

In an alternative calibration discussed in Appendix E, we assume the conditional volatility of the discount rate shock is homoskedastic and fixed at its level away from the ELB. Since the volatility of the shock is assumed to be negligible away from the ELB, this specification amounts to one in which TFP is essentially the only priced risk. This allows us to study an economy in which term premiums are positive at the ELB, and also isolate the effect of the ELB on the transmission of TFP shocks. In this economy, the reduction in the absolute size of term premiums due to the compression effect of the ELB means a *decrease* in term premiums. However, quantitatively, this effect appears to be masked by the effect of the ELB on term premiums through the discount rate shock.

We close our discussion of moments by briefly discussing the volatility of term premium dynamics at the ELB. As shown in the first and third column of panel B of Table 5, the volatilities of the empirical estimates of term premiums up to 5-year maturities are lower at the ELB than above the ELB, albeit by a small amount for longer maturities. Our model is consistent with this feature that the ELB constraint reduces the volatilities of term premiums, though the size of the term premium volatility at the ELB is somewhat lower in our model than the empirical estimates. The last column of this panel shows that the volatilities of term premiums are small in a severe recession even in the absence of the ELB constraint.

4.4.2 Dynamics

We now describe the dynamics of our model in response to a crisis shock that sends the policy rate to the ELB to gain further insights on how the ELB constraint affects the term structure of interest rates. We are particularly interested in obtaining insights on how the term structure behaves when the economy is near the entry to or exit from the ELB state, insights that cannot be gleaned from comparing moments above and at the ELB.

Figure 8 plots the median responses of macroeconomic and nominal term structure variables to a positive discount rate tail shock. For each variable, solid and dashed blue lines are for the models with and without the ELB constraint.

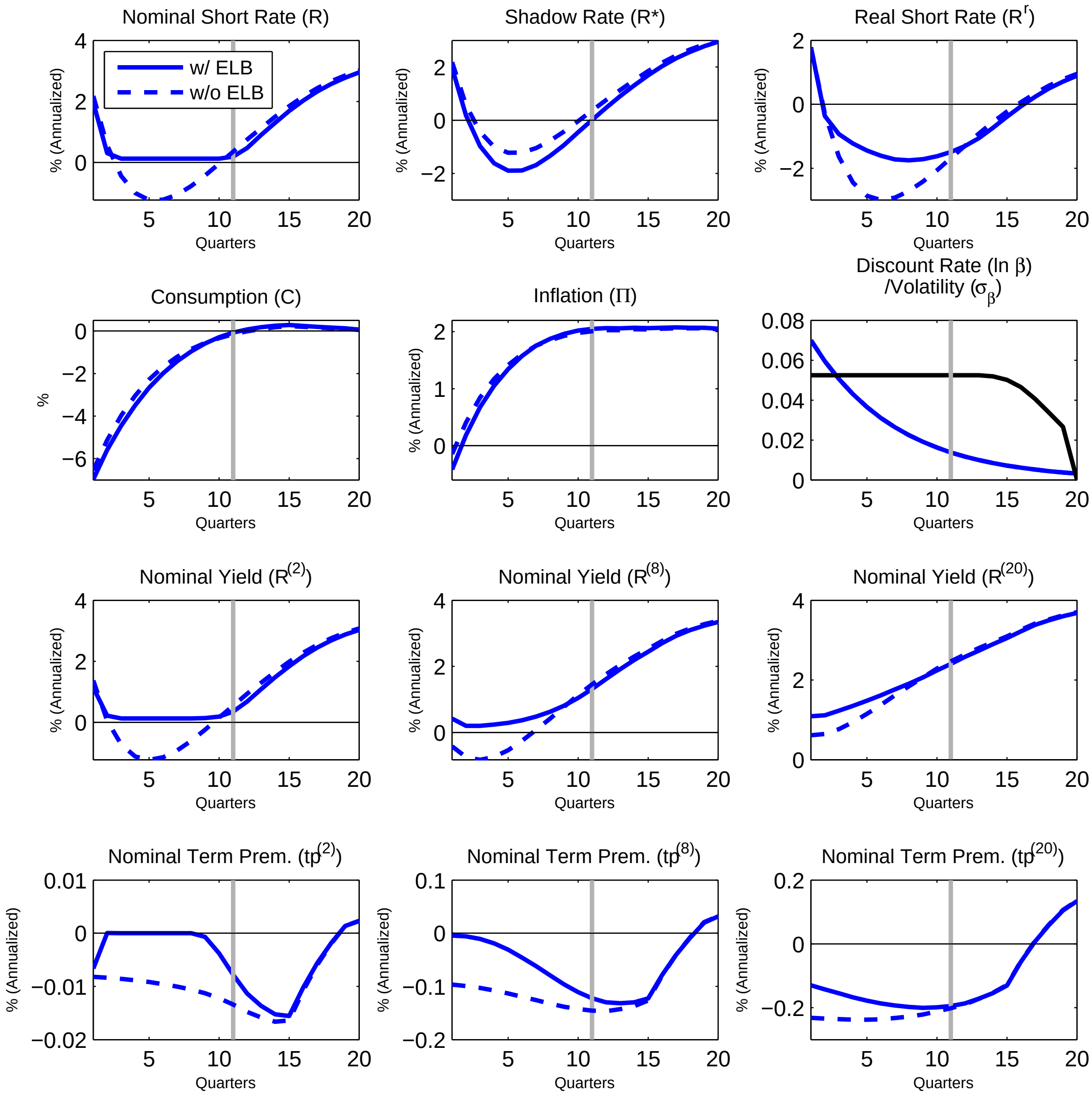
As shown in the top left panel of Figure 8, in response to a large increase in the discount rate (second row, right, blue line), the nominal short rate falls from the steady state level. Due to the interest-rate smoothing in the policy rule, it takes some time—two quarters—for the short rate to reach the ELB constraint. Consumption (second row, left) and inflation (second row, center) initially drop by 7 percent and 3 percentage points, respectively. As the crisis shock fades, consumption and inflation gradually return to their steady state. The recovery is supported by an accommodative monetary policy wherein the central bank keeps the policy rate lower than its steady state level even after consumption and inflation reach their steady-state values. Indeed, the real short rate (top right) drops to negative territory and stays there for a prolonged period even after policy rate liftoff. The liftoff occurs at quarter 12. As explained in Section 3, the declines in consumption and inflation are larger in the presence of the ELB constraint than in the absence of it.[38]

The responses of select nominal yields are shown in the third row.[39] In the wake of the

[38]Compared to the model without the ELB, annualized inflation as well as the drop in consumption in the model with the ELB are about 0.2% and 0.5% lower on average, over the first four quarters after the crisis shock hits. Although there are very few studies that rigorously quantify the effect of the ELB on macroeconomic variables, Gust, Herbst, Lopez-Salido, and Smith (2016) offers one such analysis. According to the mean estimates of GHLSS, inflation and consumption were 0.3% and 1% lower on average, respectively, over the 2008-2012 period due to the ELB. Our estimates of the effect of the ELB appear lower. However, note the estimates from our impulse response exercise cannot be directly compared with those from GHLSS, which takes into account the subsequent shocks that are more consistent with the data. Also, GHLSS reports fairly wide confidence intervals around the effects, which encompasses a possibility of almost no effect of the ELB on consumption.

[39]We present results up to 5 years, which are most interesting in our model. Given the expected duration of

Figure 8: **Impulse Responses (to β shock)**



*We plot $median[X_{t+h} \mid \varepsilon_t^d = +\vartheta_\beta, \bar{X}]$ for a given variable X_t. The solid blue lines are the responses from the model with the ELB constraint, and the dashed blue lines are the responses from the model without the ELB constraint. The vertical grey bars indicate the timing of liftoff from the ELB. The black line in the second row right panel indicates the path of volatility $\sigma_{\beta,t}$.

crisis, nominal yields decline for all maturities. The 2-quarter yield is at the ELB until quarter 10, the 2-year, or 8-quarter yield stays below 25 basis point until quarter 4, and the 5-year, or 20-quarter yield gradually increases from about 1 percent at quarter 1 throughout the horizon. Note that, while the 5-year yield is comfortably above the ELB throughout the horizon, it is still affected by the presence of the ELB constraint; were it not for the ELB constraint, the 5-year yield would be lower than otherwise, as shown in the dashed line (third row, right).

Turning to nominal term premium dynamics, we first note that due to our specification of time-varying volatility in the discount rate, the tail shock increases its own conditional volatility in the initial period, which stays constant at a high level throughout and even a bit after the period when the economy is at the ELB (second row, right, black line). The hightened volatility of the discount rate induces negative term premium for an extended period of time after the tail shock hits, as nominal bonds become more attractive as hedging instruments when uncertainty in "demand" increases. This can be seen from the responses of term premiums in the economy without the ELB (bottom row, dashed blue lines). However, according to the bottom-left panel, the 2-quarter term premium rises to zero at period 2 and stays there during most of the time when the policy rate is constrained at the ELB. The near-zero term premium arises because the sensitivity of the policy rate is minimal when the rate is at the ELB and is expected to remain at the ELB for a while, as seen in the stylized model. When liftoff approaches, the term premium starts falling back to normal and the decline continues until the policy rate is sufficiently above the ELB. During this period, the policy rate can react to macroeconomic risk and hence bonds become a better hedging instrument. Up to this point, the dynamics are consistent with that for the stylized models in Section 3. Later on, when the discount rate is sufficiently close to its steady state, the discount rate volatility declines and the effect of TFP shocks starts to dominate. As a result, later in the recovery, nominal bonds become riskier and the term premium increases.[40]

The 2-year term premium, shown in the middle panel, shows a qualitatively similar response, but due to its longer maturity, the term premium starts to decrease at an even earlier stage of the ELB period. The 5-year term premium (right panel) remains relatively stable at a negative value after the initial shock as longer-term forward rates are less affected by the ELB given its expected duration. We revisit these results in the context of the recent U.S. experience in Section 6.2.

During the periods in which the policy rate is at the ELB, term premiums in the model with the ELB constraint are more variable than they would be without the ELB constraint. In the model without the ELB, there are no compressing effects of the ELB constraint on term premiums during the periods of crisis. As a result, term premiums are negative and do not move much in the crisis period. In the model with the ELB constraint, the compressing effects are

the ELB in our model, it will not have much bite at the 10-year horizon.

[40] We could reparameterize the discount rate volatility function to make the decline in discount rate volatility and the increase in term premiums later in the recovery smoother (see Section 4.3). Our calibration, however, provides a cleaner analysis of endogenous uncertainty due to the ELB.

strong initially when the liftoff is not imminent. As time rolls on and the liftoff approaches, the compressing effects of the ELB constraint slowly dissipate and term premiums decline, making term premiums more volatile than they would otherwise be in the absence of the ELB constraint.

5 Monetary Policy and Equilibrium Yield Curves at the ELB

Our general equilibrium term structure model is a natural framework to study how monetary policy affects the dynamics of yields and term premiums in a coherent manner. In this section, we focus on how the central bank's announcement to keep the policy rate at the ELB for longer than previously expected (or, accommodative "forward guidance") affects the dynamics of the term structure of interest rates in our quantitative model.

We analyze the effects of the announcement to keep the policy rate at the ELB for longer by examining the effects of adopting a policy rule in which the timing of liftoff depends on the cumulative shortfalls in inflation and output more strongly than our baseline rule. The alternative policy rule is given by equation (8) and is called the "Reifschneider-Williams rule" (RW rule) in the macro literature due to Reifschneider and Williams (2000).[41] In the model without the ELB constraint, this rule is identical to our baseline policy rule. The parameter ϕ_{RW} measures the additional degree of policy accommodation. We set $\phi_{RW} = 0.5$ in our experiment, a value taken from Reifschneider and Roberts (2006).

Figure 9 shows the dynamics of our model under the two rules. The responses of the model's key variables under the baseline rule and the RW rule are shown in dashed and solid blue lines, respectively.

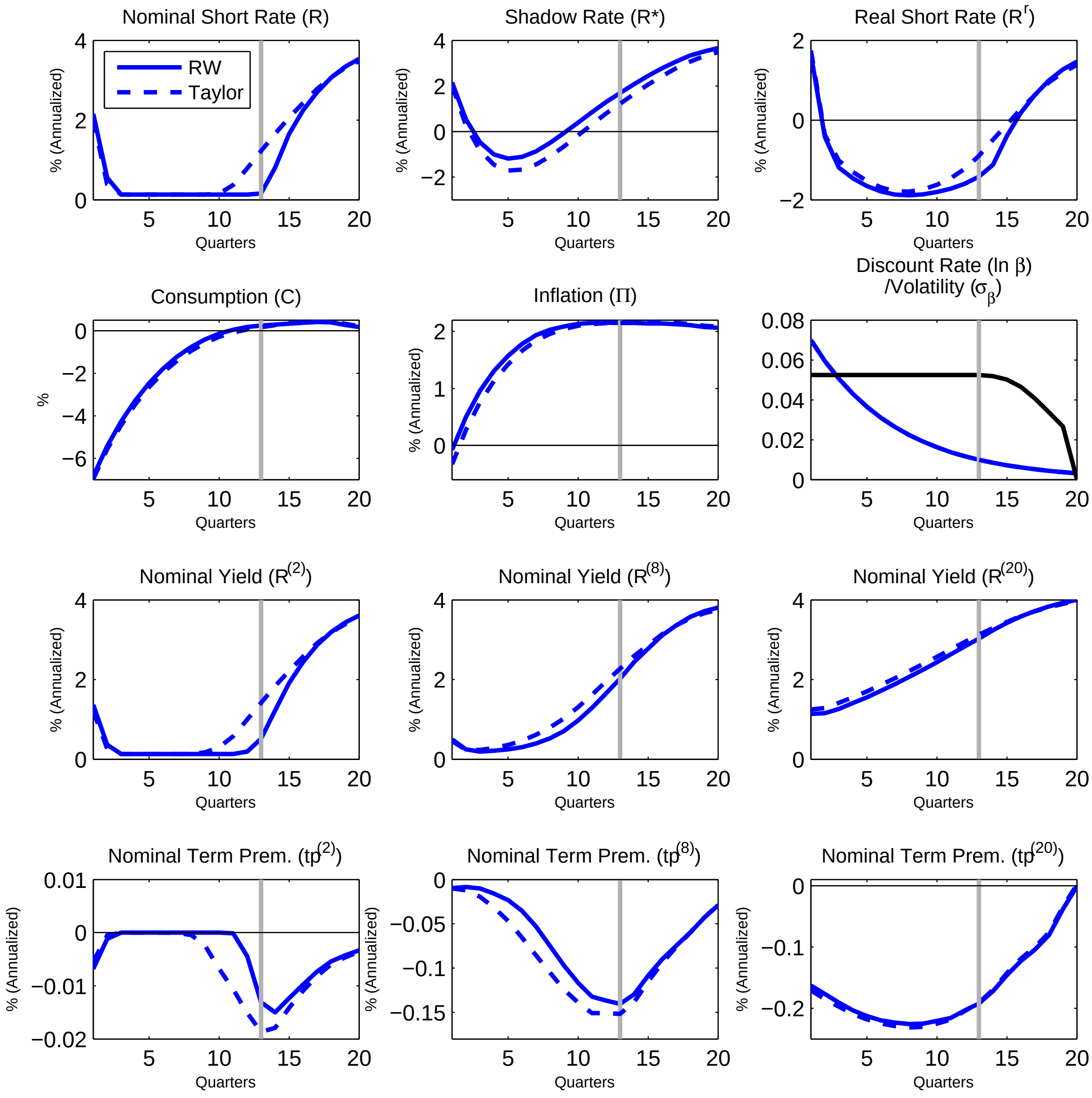
As shown in the top-left panel of Figure 9, the policy rate is kept at the ELB for longer—by about 3 quarters—under the RW rule than under the baseline rule. With the nominal rate staying at the ELB for longer, the path of real rates is lower, as shown in the top-right panel. Since households are forward-looking, the lower path of future real rates stimulates consumption (second-row left). An increase in consumption is associated with an increase in output, which raises marginal costs of production and thus inflation (second row center). The mechanism through which the RW rule stimulates economic activities at the ELB is the same as the mechanism though which the central bank stimulates economic activities at the ELB under optimal commitment policy, and is well known in the macro literature.[42]

Consistent with the lower path of short-term nominal interest rates, the paths of other

[41] A number of recent papers on the effects of forward guidance (e.g. Campbell, Evans, Fisher, and Justiniano (2012), Del Negro, Giannoni, and Patterson (2015)) model forward guidance as exogenous "news" shocks to the policy rule. We prefer our formulation for two reasons. One is that the timing of liftoff and the evolution of the policy rate thereafter are state-contingent under the RW rule, consistent with the data-dependent nature of the policy decisions repeatedly emphasized by FOMC participants. The other is a computational advantage of our formulation; modeling forward guidance about the policy rate path by news shocks adds many state variables to an already computationally burdensome model, whereas the RW rule adds only one state variable (J_{t-1}).

[42] Classic references discussing optimal commitment policy at the ELB include Eggertsson and Woodford (2003) and Adam and Billi (2006). Jung, Teranishi, and Watanabe (2005) and Nakov (2008) compare the performance of the RW rule and optimal commitment policy.

Figure 9: **Effect of Accommodative "Forward Guidance" at the ELB**

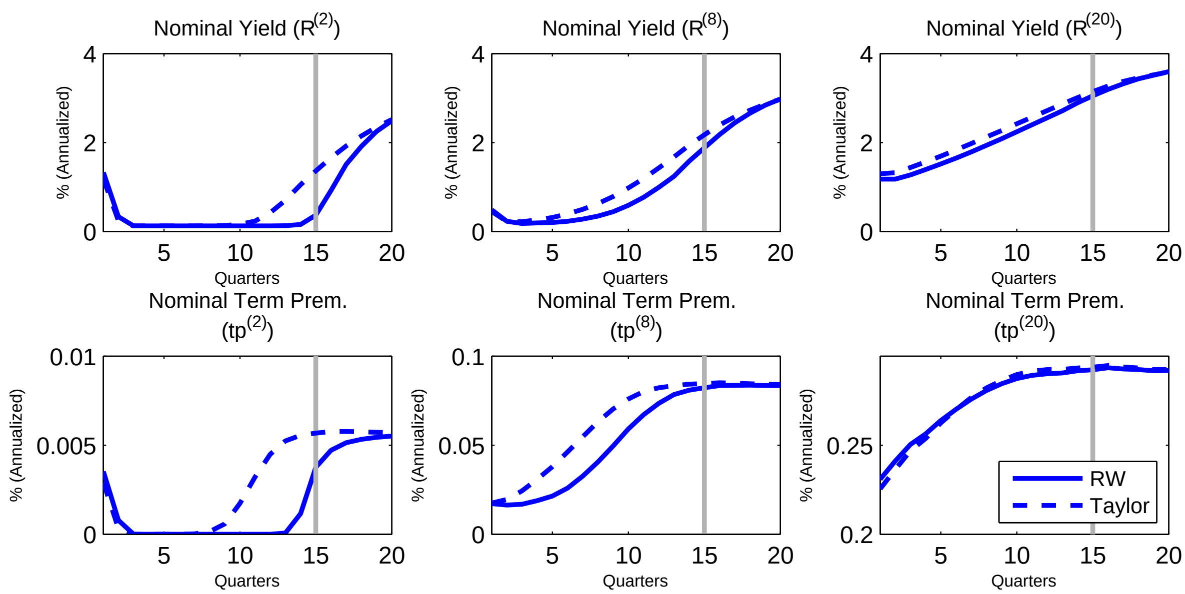


*We plot $median[X_{t+h} \mid \varepsilon_t^d = +\vartheta_\beta, \bar{X}]$ for a given variable X_t. The solid blue lines are the responses from the model under the RW rule, and the dashed blue lines are the responses from the model under the Taylor rule with the ELB constraint. The vertical grey bars indicate the timing of liftoff from the ELB. The black line in the second row right panel indicates the path of volatility $\sigma_{\beta,t}$.

nominal yields are lower under the RW rule than under the baseline rule for all maturities. Notice that, while the difference in the short-term nominal rate between these two policy rules emerges well after the initial shock, the difference in longer-maturity yields shows up at the onset of the crisis, reflecting the forward-looking nature of longer-maturity yields.

The panels in the last row of Figure 9 plot the effects of adopting the RW rule on nominal term premiums. In contrast to the effects on yields and the expected path of future short rates, nominal term premiums at the ELB are higher under the RW rule than under the baseline rule. For shorter maturities, the difference is most pronounced around the time of liftoff. As discussed in Section 3, the ELB constraint reduces the absolute size of term premiums at the time of the crisis by reducing the sensitivity of interest rates to macroeconomic fluctuations. The RW rule extends the period at which the policy rate is at the ELB and thus the interest-rate sensitivity is low. Accordingly, the absolute size of term premiums is lower, or in this case, term premiums are less negative, under the RW rule than under the baseline rule.

Figure 10: **Effect of Accommodative "Forward Guidance" at the ELB —Case of Positive Term Premiums at the ELB—**



*We plot $median[X_{t+h} \mid \varepsilon_t^d = +\vartheta_\beta, \bar{X}]$ for a given variable X_t. The solid blue lines are the responses from the model under the RW rule, and the dashed blue lines are the responses from the model under the Taylor rule with the ELB constraint. The vertical grey bars indicate the timing of liftoff from the ELB.

For all maturities, term premiums do not decline under the RW rule as much as under the baseline rule as the liftoff approaches. As a result, term premiums are less volatile under the RW rule than under the standard rule. For shorter maturities, the compressing effects of the ELB are notably stronger under the RW rule than under the baseline rule around the time of liftoff. Thus, these term premiums are particularly less volatile under the RW rule around the time of liftoff.

In our baseline model in which term premiums at the ELB are negative to begin with, lower absolute sizes mean higher term premiums. In an alternative calibration in which term premiums are positive at the ELB, the RW rule decreases term premiums. This case is shown in Figure 10. Under this specification, term premiums will be near zero for the shortest maturities and generally positive for the longer ones. Thus, the flattening effects of "forward guidance" on the expectations of short-rate path can be either mitigated or amplified by changes in term premiums, depending crucially on the risk exposure of bonds.

We used the forward guidance policy to motivate our analysis of the effects of adopting the RW rule. However, our analysis may be useful in understanding the effects of the central bank's other unconventional policies, such as large-scale asset purchases (LSAPs). Some have argued that LSAPs stimulate economic activities partly by signaling the central bank's commitment to an extended period of more accommodative policies in the future (Bhattarai, Eggertsson, and Gafarov (2015); Bauer and Rudebusch (2014); Woodford (2012)). If that is the case, then the RW rule can be seen as capturing this signaling effect of LSAPs.

6 Discussion

In this section, we present further results. We first discuss the quantitative importance of a key parameter of our model—tail risk probability for the discount rate. Second, we argue that the dynamics of our model are consistent with some aspects of the evolution of the yield curve and term premiums during the recent ELB episode in the U.S.

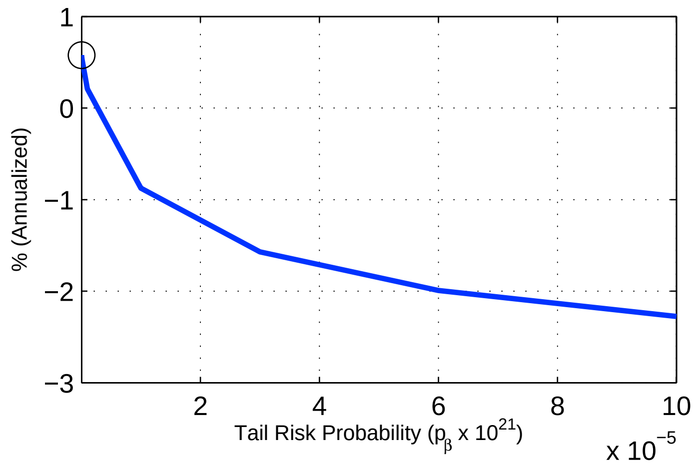
6.1 Tail Risk and Term Premiums

Figure 11 shows how the tail risk probability of the discount rate affects the average nominal term premium of the 10-year bond while the policy rate is above the ELB. According to the figure, term premiums away from the ELB are quite sensitive to the probability of discount rate tail risk. As the probability increases from the baseline value of 10^{-30} to 10^{-25}—both extremely small numbers—the 10-year nominal term premium drops from 60 basis points to less than -200 basis points on average. Term premiums away from the ELB are very responsive to the tail risk probability because investors in our economy are very risk averse ($\alpha = -100$). We need this high degree of risk aversion to match the large positive term premium observed in the pre-ELB sample.[43] Note that, given the extremely small possibility of the tail event, the dynamics of consumption and inflation at and away from the ELB are virtually unchanged in response to the same parameter changes.

This result—that even a small increase in the crisis probability can lower the nominal term premiums substantially and make them negative while the policy rate is away from the ELB constraint—is interesting in light of the current uncertainty regarding long-run interest rates and thus the frequency of hitting the ELB constraint in the future. It is possible that, while market

[43] Since average term premiums away from the ELB are positive in the data, this consideration forces us to assign an extremely small value to the probability of tail risk.

Figure 11: **Term Premium Sensitivity with respect to Tail Risk Probability for β**



*The black circle indicates the value using the parameters of our quantitative model.

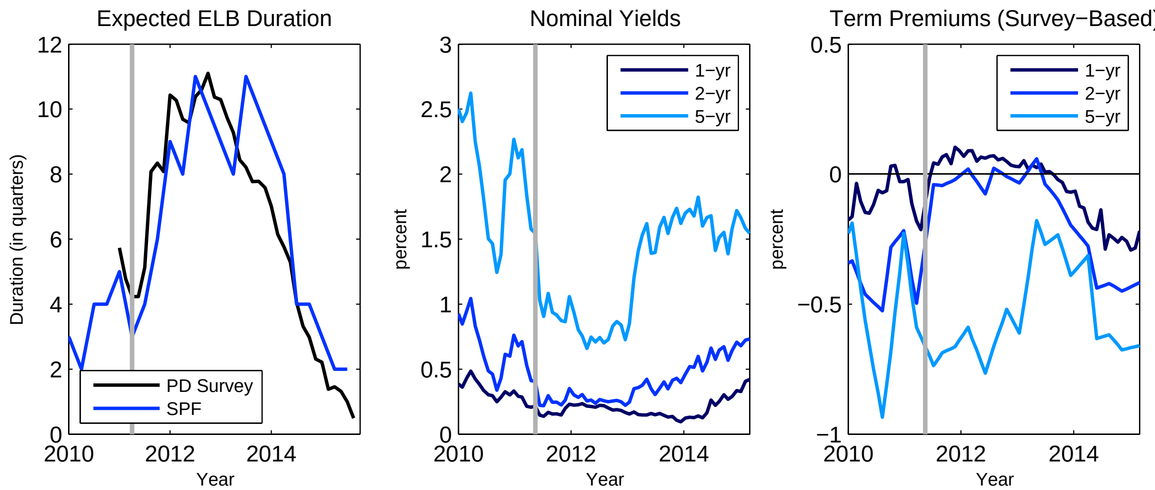
participants saw the probability of the ELB as being negligible prior to the Great Recession, the recent ELB episode has led them to revise up their estimates of the ELB frequency. If that is the case, this analysis suggests that term premiums may stay at current low levels going forward and that long-term interest rates may converge to a level lower than the average level prior to the Great Recession.

6.2 An Interpretation of the Recent U.S. Experience

The analyses in Section 4 and 5 point to a tight link between the expected time until liftoff and term premiums for short-maturity bonds at the ELB. When the policy rate is expected to remain at the ELB for a long time, term premiums are close to zero due to the compressing effect from reduced interest-rate sensitivity. As liftoff approaches, the compressing effect fades and term premiums decline. The link between the expected time until liftoff and term premiums for longer-term bonds is weaker, as the sensitivity of longer-maturity yields is less affected by the ELB.

We find some evidence supporting this link in the recent ELB episode in the U.S. Figure 12 shows the expected time until liftoff, nominal yields, and term premium estimates of 1-year, 2-year, and 5-year maturities since 2010, a bit more than one year before the introduction of "calendar-based" forward guidance in August 2011. As shown in the left panel, from the beginning of 2010 to July 2011, market participants expected that the liftoff would occur within one year or so. After the FOMC meeting in August 2011, indicated by the grey vertical line, the expectations changed dramatically as the FOMC explicitly stated that it was likely to keep the policy rate at the ELB at least through mid-2013. From that point on to mid-2013, the market expected that the policy rate would be at the ELB for at least two additional years. Since then, the expected duration has gradually come down. The Federal Reserve eventually raised the policy rate off the ELB in December 2015. Consistent with this evolution of the expected ELB duration, the 1-year, 2-year and 5-year nominal yields declined in August 2011 and remained low for a period of time as the expected duration was elevated around its peak. The 1-year yield started to increase around mid-2014, while the 2-year yield was gradually rising since as early as mid- 2013. Meanwhile, the 5-year yield increased sharply circa mid-2013 (middle panel).

Figure 12: **Nominal Yields and Term Premium Estimates at the ELB**



* Expected ELB duration in the left panel is computed as the time to the first quarter when the median federal funds rate forecast exceeds 37.5 basis points. Data are at a monthly frequency for yields and 1-year survey-based term premium estimates, quarterly for expected ELB duration, and roughly quarterly for survey-based term premium estimates of longer maturities. The grey vertical lines indicate the timing of the August 2011 FOMC meeting.

According to the right panel of the figure, 1-year term premium estimates were on average slightly negative until August 2011, while 2-year term premium estimates were comfortably below zero until August 2011. After the introduction of the calendar-based forward guidance, both 1-year and 2-year term premium estimates increased to slightly above zero and around zero, respectively, as the expected duration of the ELB jumped. Both term premium estimates hovered around zero until mid-2013, after which they gradually declined into negative territory as the expected duration of the ELB decreased. The joint dynamics of the expected duration of the ELB and term premiums of short-term bonds observed in the data is broadly consistent with the predictions of our model. For the 5-year term premium estimate, the link was absent: regardless of the expected time until liftoff, the 5-year term premium estimate remained comfortably negative since 2010, despite a few episodes of high volatility.[44] This lack of a link between the expected ELB duration and term premiums of longer maturity is also consistent with the prediction of our model.[45]

[44]For instance, the sharp rise in the 5-year term premium observed after mid-2013 corresponds to the "taper tantrum" episode where term premiums of longer-term bonds reportedly became volatile as a result of uncertainty about the tapering of LSAPs amplified by reduced market liquidity.

[45]We acknowledge such an interpretation of events comes with caveats. The quantitative variation in term premiums from our model is small compared to that from the survey estimates. The estimates themselves are subject to uncertainty and indeed the pattern is not as clear from the model-based estimates. The relationship

7 Conclusion

We have studied the term structure of default-free interest rates in a New Keynesian model with an occasionally binding ELB constraint. Using stylized models, we show that the ELB constraint induces state-dependency in the dynamics of term premiums by affecting macroeconomic uncertainty and the sensitivity of interest rates to macroeconomic fluctuations. In particular, the absolute level of term premiums are on average lower at the ELB than away from the ELB and the volatility of term premiums increases during the transition period when the policy rate declines to the ELB or when the policy rate lifts off from the ELB. We find that these implications of the ELB survive in a quantitative version of our model calibrated to match key features of consumption, inflation, yield curves, and term premium estimates in the U.S.

We have also investigated how the central bank's announcement to keep the policy rate at the ELB longer than previously expected affects the term structure of interest rates. We demonstrate that such an announcement reduces the expected short-rate path and the absolute size of term premiums. That is, if bonds are a hedge against economic downturns and term premiums are negative at the time of announcement, then the announcement increases term premiums. Otherwise, it decreases term premiums.

As discussed in the introduction, central banks in the advanced economies have been providing various unconventional policies at a time when the short-term nominal interests are at or near the ELB. Some of these policies, especially involving the central bank's balance sheet adjustments, are considered to stimulate economic activities by reducing term premiums on longer-term bonds. Thus, we believe that understanding how the ELB constraint affects the dynamics of the term structure of interest rates is an essential step towards understanding how these unconventional policies affect the expected path of interest rates, term premiums and the economy. Extending our model to explicitly include the central banks balance sheet is an interesting avenue for future research.

References

Adam, K., and R. M. Billi (2006): "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking*, 38(7), 1877–1905.

Albuquerque, R., M. S. Eichenbaum, V. Luo, and S. Rebelo (2015): "Valuation Risk and Asset Pricing," NBER Working Papers 18617, National Bureau of Economic Research.

Andreasen, M. M. (2012a): "An Estimated DSGE Model: Explaining Variation in Nominal Term Premia, Real Term Premia, and Inflation Risk Premia," *European Economic Review*, 56(8), 1656–1674.

between the estimates of term premiums and the expected ELB duration in 2009 does not conform to the prediction of our model.

——— (2012b): "On the Effects of Rare Disasters and Uncertainty Shocks for Risk Premia in Non-linear DSGE Models," *Review of Economic Dynamics*, 15(3), 295–316.

Bauer, M. D., and G. D. Rudebusch (2014): "The Signaling Channel for Federal Reserve Bond Purchases," *International Journal of Central Banking*, 10(3), 233–289.

——— (2015): "Monetary Policy Expectations at the Zero Lower Bound," *Available at SSRN 2334540*.

Beeler, J., and J. Y. Campbell (2012): "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment," *Critical Finance Review*, 1(1), 141–182.

Bernanke, B. S. (2015): "Why are Interest Rates so Low, Part 4: Term Premiums," *Ben Bernanke's Blog, April 13*.

Bhattarai, S., G. B. Eggertsson, and B. Gafarov (2015): "Time Consistency and the Duration of Government Debt: A Signalling Theory of Quantitative Easing," NBER Working Papers 21336, National Bureau of Economic Research.

Blanco, J. A. (2016): "Optimal Inflation Target in an Economy with Menu Costs and an Occasionally Binding Zero Lower Bound," *Unpublished Manuscript, University of Michigan.*

Branger, N., C. Schlag, I. Shaliastovich, and D. Song (2015): "Macroeconomic Bond Risks at the Zero Lower Bound," *Working Paper*.

Campbell, J. R., C. L. Evans, J. D. Fisher, and A. Justiniano (2012): "Macroeconomic Effects of Federal Reserve Forward Guidance," *Brookings Papers on Economic Activity*, pp. 1–80.

Campbell, J. Y. (2003): "Consumption-Based Asset Pricing," *Handbook of the Economics of Finance*, 1, 803–887.

Campbell, J. Y., C. Pflueger, and L. M. Viceira (2014): "Monetary Policy Drivers of Bond and Equity Risks," NBER Working Papers 20070, National Bureau of Economic Research.

Campbell, J. Y., A. Sunderam, and L. M. Viceira (2013): "Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds," NBER Working Papers 14701, National Bureau of Economic Research.

Christensen, J. H., and G. D. Rudebusch (2013): "Modeling Yields at the Zero Lower Bound: Are Shadow Rates the Solution?," *Mimeo*.

Christiano, L., M. Eichenbaum, and S. Rebelo (2011): "When Is the Government Spending Multiplier Large?," *Journal of Political Economy*, 119(1), 78–121.

Coleman, W. J. (1991): "Equilibrium in a Production Economy with an Income Tax," *Econometrica*, 59(4), 1091–1104.

Creal, D. D., and J. C. Wu (2015): "Bond Risk Premia in Consumption-based Models," *Working Paper*.

D'Amico, S., D. H. Kim, and M. Wei (2014): "Tips from TIPS: the Informational Content of Treasury Inflation-Protected Security Prices," Finance and Economics Discussion Series 2014-24, Board of Governors of the Federal Reserve System (U.S.).

Del Negro, M., M. P. Giannoni, and C. Patterson (2015): "The Forward Guidance Puzzle," *FRB of New York Staff Report*, (574).

Dew-Becker, I. (2014): "Bond Pricing with a Time-Varying Price of Risk in an Estimated Medium-Scale Bayesian DSGE Model," *Journal of Money, Credit and Banking*, 46(5), 837–888.

Eggertsson, G. B., and M. Woodford (2003): "Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003(1), 139–233.

Epstein, L. G., and S. E. Zin (1989): "Substitution, Risk aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, pp. 937–969.

Fernández-Villaverde, J., G. Gordon, P. Guerrón-Quintana, and J. F. Rubio-Ramirez (2015): "Nonlinear Adventures at the Zero Lower Bound," *Journal of Economic Dynamics and Control*, 57, 182–204.

Fisher, J. D. (2015): "On the Structural Interpretation of the Smets–Wouters Risk Premium Shock," *Journal of Money, Credit and Banking*, 47(2-3), 511–516.

Gavin, W. T., B. D. Keen, A. W. Richter, and N. A. Throckmorton (2015): "The Zero Lower Bound, the Dual Mandate, and Unconventional Dynamics," *Journal of Economic Dynamics and Control*, 55, 14–38.

Gourio, F., and P. Ngo (2016): "Risk Premia at the ZLB: a macroeconomic interpretation," *Unpublished Manuscript*.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988): "Investment, Capacity Utilization, and the Real Business Cycle," *The American Economic Review*, pp. 402–417.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2007): "The US Treasury Yield Curve: 1961 to the Present," *Journal of Monetary Economics*, 54(8), 2291–2304.

——— (2010): "The TIPS Yield Curve and Inflation Compensation," *American Economic Journal: Macroeconomics*, pp. 70–92.

Gürkaynak, R. S., and J. H. Wright (2012): "Macroeconomics and the Term Structure," *Journal of Economic Literature*, 50(2), 331–367.

Gust, C., E. Herbst, D. Lopez-Salido, and M. Smith (2016): "The Empirical Implications of the Interest-Rate Lower Bound," Finance and Economics Discussion Series 2012-83, Board of Governors of the Federal Reserve System (U.S.).

Guvenen, F. (2009): "A Parsimonious Macroeconomic Model for Asset Pricing," *Econometrica*, 77(6), 1711–1750.

Hall, R. E. (1988): "Intertemporal Substitution in Consumption," *The Journal of Political Economy*, pp. 339–357.

Hansen, L. P., and T. J. Sargent (1995): "Discounted Linear Exponential Quadratic Gaussian Control," *IEEE Transactions on Automatic control*, 40(5), 968–971.

Hills, T., and T. Nakata (2014): "Fiscal Multipliers at the Zero Lower Bound: The Role of Policy Inertia," Finance and Economics Discussion Series 2014-107, Board of Governors of the Federal Reserve System (U.S.).

Hsu, A. C., E. X. Li, and F. Palomino (2015): "Real and Nominal Equilibrium Yield Curves with Endogenous Inflation: A Quantitative Assessment," *Available at SSRN 2517572*.

Ichiue, H., and Y. Ueno (2007): "Equilibrium Interest Rate and the Yield Curve in a Low Interest Rate Environment," *Mimeo*.

Jung, T., Y. Teranishi, and T. Watanabe (2005): "Optimal Monetary Policy at the Zero-Interest-Rate Bound," *Journal of Money, Credit, and Banking*, 37(5), 813–835.

Jurado, K., S. C. Ludvigson, and S. Ng (2015): "Measuring Uncertainty," *American Economic Review*, 105(3), 1177–1216.

Kim, D., and M. Priebsch (2013): "Estimation of Multi-Factor Shadow-Rate Term Structure Models," *Unpublished Manuscript*.

Kim, D. H., and K. J. Singleton (2012): "Term Structure Models and the Zero Bound: an Empirical Investigation of Japanese Yields," *Journal of Econometrics*, 170(1), 32–49.

Kim, D. H., and J. H. Wright (2005): "An Arbitrage-Free Three-Factor Term Structure Model and the Recent Behavior of Long-Term Yields and Distant-Horizon Forward Rates," *FEDS Working Paper*.

King, R. G., and S. T. Rebelo (1999): "Resuscitating Real Business Cycles," *Handbook of Macroeconomics*, 1, 927–1007.

Krippner, L. (2012): "Modifying Gaussian Term Structure Models when Interest Rates are near the Zero Lower Bound," *Reserve Bank of New Zealand Discussion Paper*, (2012/02).

Kung, H. (2015): "Macroeconomic Linkages between Monetary Policy and the Term Structure of Interest Rates," *Journal of Financial Economics*, 115(1), 42–57.

Lopez, P., D. Lopez-Salido, and F. Vazquez-Grande (2015): "Nominal Rigidities and the Term Structures of Equity and Bond Returns," Finance and Economics Discussion Series 2015-64, Board of Governors of the Federal Reserve System (U.S.).

Mertens, E. (2015): "Measuring the Level and Uncertainty of Trend Inflation," *Review of Economics and Statistics (forthcoming).*

Nakata, T. (2013): "Uncertainty at the Zero Lower Bound," Finance and Economics Discussion Series 2013-09, Board of Governors of the Federal Reserve System (U.S.).

Nakov, A. (2008): "Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate," *International Journal of Central Banking*, 4(2), 73–127.

Piazzesi, M., and M. Schneider (2007): "Equilibrium Yield Curves," in *NBER Macroeconomics Annual 2006, Volume 21*, pp. 389–472. MIT Press.

Plante, M., A. W. Richter, and N. A. Throckmorton (2014): "The Zero Lower Bound and Endogenous Uncertainty," *Available at SSRN 2456425.*

Priebsch, M. (2013): "Computing Arbitrage-Free Yields in Multi-Factor Gaussian Shadow-Rate Term Structure Models," Finance and Economics Discussion Series 2013-63, Board of Governors of the Federal Reserve System (U.S.).

Reifschneider, D., and J. C. Williams (2000): "Three Lessons for Monetary Policy in a Low-Inflation Era," *Journal of Money, Credit and Banking*, pp. 936–966.

Reifschneider, D. L., and J. M. Roberts (2006): "Expectations Formation and the Effectiveness of Strategies for Limiting the Consequences of the zero Bound," *Journal of the Japanese and International Economies*, 20(3), 314–337.

Rudebusch, G. D., and E. T. Swanson (2008): "Examining the Bond Premium Puzzle with a DSGE Model," *Journal of Monetary Economics*, 55, S111–S126.

——— (2012): "The Bond Premium in a DSGE Model with Long-Run Real and Nominal," *American Economic Journal: Macroeconomics*, 4(1), 105–143.

Sakurai, Y. (2016): "How does the Bond Market Perceive Macroeconomic Risks under Zero Lower Bound?," *Working Paper.*

Smets, F., and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

Stein, J. C. (2012): "Evaluating Large-Scale Asset Purchases," *Remarks at the Brookings Institution, Washington D.C.*

Swanson, E. (2014): "A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt," *Unpublished Manuscript, University of California, Irvine.*

Tallarini, T. D. (2000): "Risk-sensitive Real Business Cycles," *Journal of Monetary Economics*, 45(3), 507–532.

Van Binsbergen, J. H., J. Fernández-Villaverde, R. S. Koijen, and J. Rubio-Ramírez (2012): "The Term Structure of Interest Rates in a DSGE model with Recursive Preferences," *Journal of Monetary Economics*, 59(7), 634–648.

Woodford, M. (2012): "Methods of Policy Accommodation at the Interest-Rate Lower Bound," in *The Changing Policy Landscape: 2012 Jackson Hole Symposium*. Federal Reserve Bank of Kansas City.

Wu, J. C., and F. D. Xia (2014): "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," NBER Working Papers 20117, National Bureau of Economic Research.

For Online Publication: Appendix

A Model Specific Equilibrium Conditions

Here we state the equilibrium conditions that rely on functional forms specific to the version of the model.

Stylized Models: For the stylized models, the value function takes the form:

$$V_t = \chi_N \ln(C_t) + (1-\chi_N)\ln(1-N_t) + \beta_t \xi^{-1} \ln \mathbb{E}_t\left[\exp(\xi V_{t+1})\right]$$

where $\xi \equiv (1-\gamma)(1-\bar{\beta})$. This "risk-sensitive recursion" follows Hansen and Sargent (1995) and Tallarini (2000) while we also allow for a time-varying discount rate process. Note this value function can be derived by taking the limit $\chi_C \to 1$ of a slightly modified version of the value function (1):

$$\tilde{V}_t = \left[(1-\bar{\beta})\left(C_t^{\chi_N}(1-N_t)^{1-\chi_N}\right)^{1-\chi_C} + \bar{\beta}\left\{\mathbb{E}_t\left[\tilde{V}_{t+1}^{1-\gamma}\right]\right\}^{\frac{1-\chi_C}{1-\gamma}}\right]^{\frac{1}{1-\chi_C}}$$

and augmenting the continuation value with a discount rate shock. The nominal pricing kernels and optimal labor supplies are characterized as:

$$M_{t+1} = \beta_t \left(\frac{C_{t+1}}{C_t}\right)^{-1}\left[\frac{\exp(\xi V_{t+1})}{\mathbb{E}_t\left[\exp(\xi V_{t+1})\right]}\right]\frac{1}{\Pi_{t+1}}$$

$$w_t = \frac{1-\chi_N}{\chi_N}\frac{C_t}{1-N_t}$$

The model with power utility sets $\gamma = \chi_C = 1$ and the model with Epstein-Zin preferences sets $\gamma \neq \chi_C = 1$.

Quantitative Model: For the quantitative model, the value function, the nominal pricing kernel and optimal labor supply are characterized as:

$$\hat{V}_t = \frac{\hat{X}_t^{1-\chi_C}}{1-\chi_C} - \tilde{\beta}_t \zeta \left\{\mathbb{E}_t\left[(-\hat{V}_{t+1})^{1-\alpha}\right]\right\}^{\frac{1}{1-\alpha}}$$

$$M_{t+1} = \tilde{\beta}_t \left(\frac{\hat{X}_{t+1}}{\hat{X}_t}\right)^{-\chi_C}\left[\frac{-\hat{V}_{t+1}}{\left[\mathbb{E}_t\left[(-\hat{V}_{t+1})^{1-\alpha}\right]\right]^{\frac{1}{1-\alpha}}}\right]^{-\alpha}\frac{1}{\Pi_{t+1}}$$

$$\hat{w}_t = N_t^{\chi_{N1}}$$

where $\hat{X}_t \equiv \frac{C_t}{Z_t} - \chi_{N_0}\frac{N_t^{1+\chi_{N_1}}}{1+\chi_{N_1}}$, $\hat{V}_t \equiv \left(\frac{V_t}{Z_t}\right)^{1-\chi_C}$, $\tilde{\beta}_t \equiv \beta_t \zeta^{-\chi_C}$, and $\alpha \equiv 1-(1-\gamma)/(1-\chi_C)$.

B Details of the Solution Method

We solve the model globally using a time-iteration method in the spirit of Coleman (1991). Similar methods are used in recent studies of the occasionally binding ELB constraint, such as Gavin, Keen, Richter, and Throckmorton (2015) and Nakata (2013). In addition to the iteration on decision rules conducted by these papers, we also iterate on the value function due to recursive utility.

The equilibrium conditions of our model laid out in Section 2.7 in the main text can be cast in the general form: $\mathbb{E}_t[f(\mathbf{Y}_{t+1}, \mathbf{Y}_t, \mathbf{X}_{t+1}, \mathbf{X}_t)] = 0$, where $\mathbf{Y}_t$ is a vector of non-predetermined variables and $\mathbf{X}_t$ is a vector of predetermined variables. Note $\mathbf{X}_t$ can be decomposed into an endogenous state vector $\mathbf{X}_{t-1}^{END}$ and an exogenous state vector $\mathbf{X}_t^{EXO}$ such that $\mathbf{X}_t \equiv \{\mathbf{X}_{t-1}^{END}, \mathbf{X}_t^{EXO}\}$. In our quantitative model, $\mathbf{X}_{t-1}^{END} \equiv \{R_{t-1}^*(\mathbf{X}_{t-1}), J_{t-1}(\mathbf{X}_{t-1})\}$ and $\mathbf{X}_t^{EXO} \equiv \{\beta_t, A_t, Z_t\}$. To solve our model with recursive utility, it is convenient to expand the set of non-predetermined states $\mathbf{Y}_t$ to include the value function V_t. Thus, $\mathbf{Y}_t \equiv \{C_t(\mathbf{X}_t), Y_t(\mathbf{X}_t), N_t(\mathbf{X}_t), \Pi_t(\mathbf{X}_t), W_t(\mathbf{X}_t), R_t(\mathbf{X}_t), V_t(\mathbf{X}_t)\}$.[46]

The time iteration starts by discretizing the state space and providing an initial guess of the decision rules and the value function for each node. The normally distributed shocks are discretized using the Gaussian quadrature rule. We use the deterministic steady state as an initial guess when we solve for the model with power utility, and the decision rules under power utility for the model with EZ preferences. Via piecewise linear interpolation across the nodes and linear extrapolation beyond, we obtain a guess for the continuous decision rule vector $\mathbf{Y}_t^{(1)}$. Given $\mathbf{Y}_t^{(1)}$, $\mathbf{X}_t^{EXO}$ and $\mathbf{X}_{t-1}^{(1)}$ we can solve for $\mathbf{Y}_{t-1}^{(1)}$ and $\mathbf{X}_{t-1}^{END,(1)}$ through $\mathbb{E}_{t-1}[f(\mathbf{Y}_t^{(1)}, \mathbf{Y}_{t-1}^{(1)}, \mathbf{X}_t^{(1)}, \mathbf{X}_{t-1}^{(1)})] = 0$ for each node in the discretized state space. This functional equation is solved by combining numerical integration with a non-linear equation solver. We iterate this procedure of using a new guess $\mathbf{Y}_t^{(\tau+1)} = \mathbf{Y}_{t-1}^{(\tau)}$ to obtain $\{\mathbf{Y}_{t-1}^{(\tau+1)}, \mathbf{X}_{t-1}^{END,(\tau+1)}\}$ for $\tau = 1, 2, 3, ...$ until convergence.

Once the solution to the macroeconomic model is obtained, we can solve for the decision rules of the stochastic discount factor. Under the assumption of complete markets, the solution for the stochastic discount factor allows us to solve for equilibrium yield curves and term premiums.

We assess the numerical accuracy of our solution through the standard practice of examining Euler equation errors (for the consumption Euler equation). The Euler equation error is defined in the common form, which is normalized with respect to consumption: $EE_t \equiv \log_{10}|1 - \tilde{C}_t/\hat{C}_t|$ where $\tilde{C}_t$ denotes consumption (potentially normalized by trend) *implied* by the Euler equation and the solutions to the decision rules other than $\hat{C}_t$. Table B.1 summarizes the mean and 99.9 percent quantile of the error distribution generated from simulating the two stylized models and the quantitative model respectively. Since the probability of the ELB binding due to a large discount rate shock in the quantitative model is extremely low, the final row reports statistics from an alternative simulation of the quantitative model where disaster shocks to the discount rate (i.e. an episode where the economy hits the ELB) are forced to occur relatively frequently within the sample.[47] We find that the errors are sufficiently small for the stylized models. In addition, although the errors from the quantitative model—which features high risk aversion, tail risk and stochastic volatility along with the ELB—are somewhat larger, the accuracy of the solution is still comparable to what is reported in the literature.[48]

[46] We exclude such variables as the stochastic discount factor and the term structure here since they are unnecessary to obtain other equilibrium objects.

[47] In this simulation, the ELB binds about 10 percent of the time.

[48] For example, Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramirez (2015) reports errors with a mean of -3.3 and 99.9 percent quantile of -2.2 for their New-Keynesian model with an occasionally binding

Table B.1: **Euler Equation Errors**

	Mean	99.9 Percentile
Stylized (Log)	-6.5	-4.6
Stylized (EZ)	-6.2	-4.5
Quantitative	-5.6	-4.5
Quantitative (ELB)	-5.4	-3.7

*Errors are for the consumption Euler equation and expressed in base 10 logarithms. "Quantitative (ELB)" reports errors from a simulation of the quantitative model where disaster shocks to the discount rate are forced to occur within the sample.

C $n-$period Term Premiums

In this section, we illustrate the mechanism of how the $n-$period term premiums are determined following a similar logic we used to understand the 2-period case. We focus here on nominal term premiums under power utility since real term premiums and the case under EZ preferences could be understood analogously. For this, we log-linearize the equilibrium n-period yields $R_t^{(n)}(\beta_t)$ at $\bar{\beta}$ and consider an approximation of the nominal term premium similar to (15). The generalization of (15) to the n-period term premium is:

$$\begin{aligned} tp_t^{(n)}(\bar{\beta}) \equiv R_t^{(n)} - R_t^{(n)\mathbb{Q}} &= \mathbb{E}_t\left[\sum_{i=0}^{n-2}\frac{n-i-1}{n}\mathbb{C}ov_{t+i}(m_{t+i+1}, r_{t+i+1}^{(n-i-1)})\right] + u_t^{(n)} \\ &= \frac{n-1}{2}\sum_{i=0}^{n-2}\omega_{n-1-i}(-\rho_{\Delta c, r^{(n-1-i)}}\sigma_{r^{(n-1-i)}}\sigma_{\Delta c} - \rho_{\pi, r^{(n-1-i)}}\sigma_{r^{(n-1-i)}}\sigma_{\pi}) + u_t^{(n)} \\ &\equiv -\varrho_{\Delta c, r^{(1\to n-1)}}\sigma_{\Delta c} - \varrho_{\pi, r^{(1\to n-1)}}\sigma_{\pi} + u_t^{(n)} \end{aligned} \tag{C.1}$$

where $\sigma_{r^{(n-i-1)}}$ is the conditional volatility of the $(n-i-1)$-period yield and $\rho_{x,r^{(n-1-i)}}$ is the conditional correlation of the $(n-i-1)$-period yield and x. The second equality slightly rewrites this by using ω_{n-1-i}, which are weights assigned to each $\rho_{\Delta c, r^{(n-1-i)}}\sigma_{r^{(n-i-1)}}$ and $\rho_{\pi, r^{(n-1-i)}}\sigma_{r^{(n-1-i)}}$ that sum up to 1 and increasing in maturity. The last line simply defines $\varrho_{x,r^{(1\to n-1)}} \equiv \frac{n-1}{2}\sum_{i=0}^{n-2}\omega_{n-1-i}\rho_{x,r^{(n-1-i)}}\sigma_{r^{(n-1-i)}}$. Finally, $u_t^{(n)}$ collects all terms arising from a Jensen's inequality effect, which we ignore here.[49] According to equation (C.1), we can obtain the (approximate) n-period nominal term premium by replacing the product of the nominal short rate volatility and the correlation of macro-variables and the nominal short rate ($\rho_{x,r}\sigma_r$) in (15) with a weighted average of analogous products for yields across the maturity spectrum, scaled by a constant term that linearly increases with maturity ($\varrho_{x,r^{(1\to n-1)}}$). Given $\rho_{\Delta c, r^{(1\to n-1)}}$, $\rho_{\pi, r^{(1\to n-1)}} \geq 0$ as implied by the left and right panels of Figure C.1,[50] the negative term premiums of n-period maturities follow naturally.

In terms of the dynamics of n-period term premiums, the bottom left panel of Figure 2 points to a difference in the shape of longer-maturity term premiums and shorter-maturity term premiums. For instance, the fall in the 5-year nominal term premium is larger and more

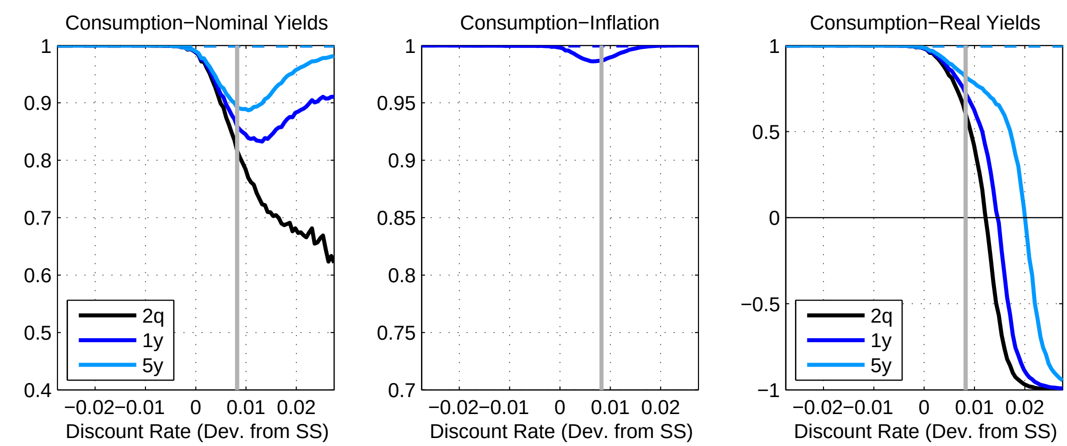
ELB constraint.

[49] $u_t^{(2)} = 0$, hence this term did not appear in (15).

[50] Since 1-step ahead consumption growth is almost perfectly correlated with inflation (right panel of Figure C.1), it is clear that interest-rate sensitivity to consumption growth and inflation are very similar.

consistent as the discount factor increases compared to its 2-quarter counterpart. Recall the non-monotonicity observed for term premiums of shorter maturities is due to the offsetting effect of decreasing policy rate uncertainty and sensitivity as the economy moves further into the ELB state. Instead, (C.1) implies that what matter for longer-maturity term premiums are the uncertainty and sensitivity of longer-term yields.[51] Compared to shorter-term yields, conditional moments of longer-term yields depend less on the current state since they largely reflect the economy's eventual convergence to its ergodic distribution. This can be seen in the bottom-left panel of Figure 2 and the left panel of Figure C.1 where the 5-year yield uncertainty and correlation with consumption are relatively unchanged at the ELB. This, in turn, implies that the change in longer-maturity term premiums near and at the ELB will be characterized mostly by the increase in macroeconomic uncertainty until the economy hits an extremely severe state where even longer-term yields cannot escape the influence of the ELB.

Figure C.1: **Conditional Correlations—Power Utility—**



*Solid lines indicate conditional correlations (1 quarter ahead conditioned on β) of the respective variables for the model with the ELB constraint, and dashed lines indicate uncertainty for the model without the ELB constraint. The solid vertical line indicates the threshold state where the ELB binds.

D Data

For the short-term nominal interest rate, we use the 3-month T-bill rates from the Federal Reserve Board's H.15 statistical release. For nominal yields of 2-, 5- and 10-year maturities, we use the zero-coupon yields from Gürkaynak, Sack, and Wright (2007). In Figure 7, we show monthly averages of daily data. In Table 4 we use quarterly averages from 1997:Q1 to 2015:Q3 to calibrate our model. For 5- and 10-year real yields, we use zero-coupon yields interpolated from TIPS by Gürkaynak, Sack, and Wright (2010). These yield data are widely used in the literature. When we compare our model implied real term estimates away from the ELB with those from empirical estimates, we use those derived from the 4-factor affine term structure model of D'Amico, Kim, and Wei (2014).

For consumption data, we compute per capita consumption from Personal Consumption Expenditures (nondurables+services, seasonally adjusted) and detrend it with an HP-filter for the period away from the ELB. For the ELB period, we detrend it using a linear trend that

[51] To be more precise, the weighted average of changes in uncertainty of longer-term yields matter. However, the coefficients ωs overweight longer-term maturities.

implies the gap between consumption and its trend has about closed in 2015:Q3.[52] We use the quarterly change in the GDP deflator for our measure of inflation. Our choice of consumption and inflation data are standard.

E Alternative Calibration with Positive Term Premiums at the ELB

E.1 Moments at and above the ELB

In this section, we discuss an alternative calibration of our quantitative model where we assume the conditional volatility of the discount rate shock is homoskedastic and fixed at its level away from the ELB, i.e. $\sigma_{\beta,t} = \bar{\sigma}_\beta$ in equation (11). Since the volatility of the shock is assumed to be negligible away from the ELB, this specification amounts to one in which TFP is essentially the only priced risk. This allows us to study an economy in which term premiums are positive at the ELB, and also isolate the effect of the ELB on the transmission of TFP shocks. Table E.1 shows averages and standard deviation of consumption, inflation, and nominal yields, while Table E.2 shows those of term premiums.

Comparison of the second column in Table E.1 with that in Table 4 in the main text shows that, regardless of whether or not we have stochastic volatility in the discount rate process, quantitative implications of our model for macro variables and nominal yields are similar when the policy rate is above the ELB constraint. This result is intuitive since the tail risk probability of the discount rate shock, which increases the shock volatility under our specification of stochastic volatility, is extremely small. Similarly, comparison of the second column in Table E.2 with that in Table 5 in the main text shows that the absence of stochastic volatility does not alter the behavior of term premiums materially when the policy rate is above the ELB constraint.

Comparison of the fourth column in Table E.1 with that in Table 4 in the main text shows that, even when the policy rate is constrained at the ELB, the stochastic volatility in the discount rate does not alter the quantitative performance of the model substantially for macro variables and nominal yields. A key difference between the models with and without stochastic volatility is in term premiums. While term premiums are on average negative at the ELB in the model with stochastic volatility (the fourth column of Table 5), they are on average positive at the ELB in the model without stochastic volatility, which is essentially a model with only TFP as a source of risk (the fourth column of Table E.2). Note that in the model without stochastic volatility, term premiums are lower in the model with the ELB than they would otherwise be in the absence of the ELB, implying that the compression effect of the ELB that decreases the absolute size of term premiums also *decreases* the level of term premiums that arise from TFP risk. However, in our benchmark model where variation in the time discount rate coexists with that in TFP, it appears that the ELB constraint ends up increasing the level of term premiums at the ELB, on average.

E.2 Dynamics

Figure E.1 shows impulse responses of the endogenous variables from the model without stochastic volatility. Impulse responses of macro variables and nominal yields are not substantially affected by the absence of stochastic volatility. However, unlike in the model with stochastic

[52] We assume the HP filtered consumption trend shifts to this linear trend continuously at 2009:Q1. Our trend at the ELB is admittedly adhoc, but is simple and appears sensible.

Table E.1: **Macro and Yield Curve Moments at and above the ELB –Case of Positive Term Premiums at the ELB, Model versus Data–**

	Above the ELB		At the ELB		Below the ELB
	Data	Model	Data	Model	Model w/o ELB
A. Macro Variables (Mean)					
Consumption†	—	—	-2.02	-1.64	-1.24
Inflation	2.13	2.09	1.45	1.25	1.33
B. Macro Variables (Volatility)					
Consumption	2.93	2.21	4.18	7.66	6.60
Inflation	0.87	0.72	0.83	0.88	0.72
C. Nominal Yields (Mean)					
3-month	3.44	3.90	0.08	0.13	-1.45
2-year	4.01	4.00	0.54	0.42	-0.58
5-year	4.49	4.20	1.57	1.62	1.21
10-year	5.10	4.42	2.79	2.91	2.71
D. Nominal Yields (Volatility)					
3-month	1.71	1.77	0.06	0.00	1.06
2-year	1.57	1.69	0.25	0.34	1.23
5-year	1.14	1.38	0.55	0.59	1.00
10-year	0.79	0.95	0.74	0.44	0.63

*This table contains summary statistics for selected macroeconomic and term structure variables comparing data versus model counterparts. The sample period is 1997:Q1-2008:Q4 for data above the ELB, and 2009:Q1-2015:Q3 for data at the ELB.

†The "mean" reported for consumption at/below the ELB is the average deviation from either the trend (for data) or the model implied consumption away from the ELB (for the model) in percentage points. We do not report statistics away from the ELB as the model implied average deviation is zero by construction.

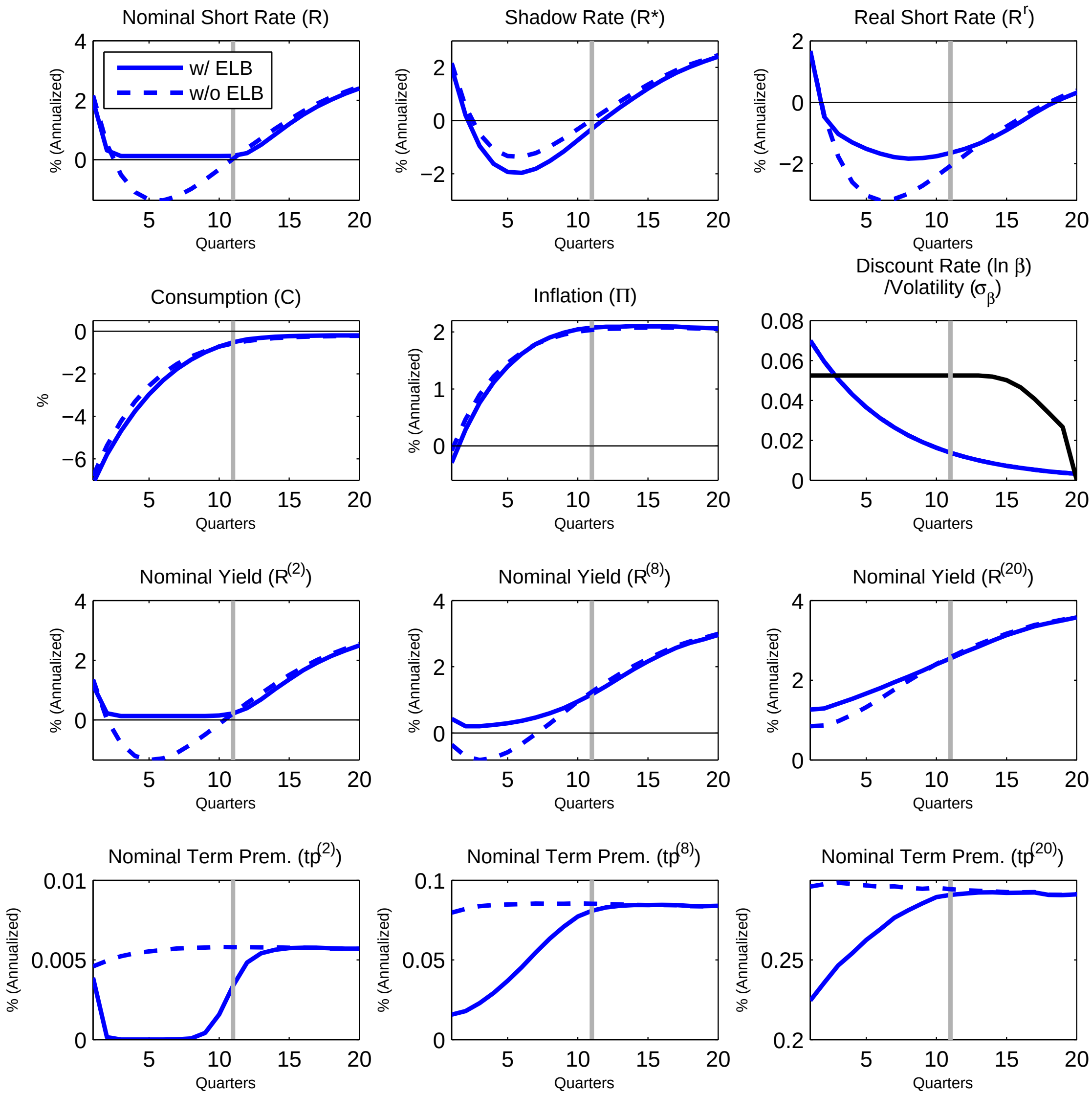
Table E.2: **Term Premium Moments at and above the ELB –Case of Positive Term Premiums at the ELB, Model versus Estimates–**

	Above the ELB		At the ELB		Below the ELB
	Estimates	Model	Estimates	Model	Model w/o ELB
A. Nominal Term Premium(Mean)					
2-year	0.11	0.10	-0.22	0.04	0.07
5-year	0.26	0.33	-0.39	0.23	0.26
10-year	0.61	0.59	-0.10	0.49	0.51
B. Nominal Term Premium (Volatility)					
2-year	0.23	0.06	0.09	0.03	0.02
5-year	0.36	0.15	0.28	0.07	0.06
10-year	0.35	0.18	0.41	0.08	0.08

*This table contains summary statistics for selected term premiums comparing estimates versus model counterparts. The sample period is 1997:Q1-2008:Q4 for estimates above the ELB, and 2009:Q1-2015:Q3 for estimates at the ELB. All variables are in annualized percentage terms.

volatility, nominal term premiums are positive throughout the simulation in the model without stochastic volatility. For all maturities, the path of term premiums are lower in the model with the ELB constraint than in the model without the ELB constraint, as seen in the final row of the figure.

Figure E.1: **Impulse Responses (to β shock)**
—Case of Positive Term Premiums at the ELB—



*We plot $median[X_{t+h} \mid \varepsilon_t^d = +\vartheta_\beta, \bar{X}]$ for a given variable X_t. The solid blue lines are the responses from the model with the ELB constraint, and the dashed blue lines are the responses from the model without the ELB constraint. The vertical grey bars indicate the timing of liftoff from the ELB.

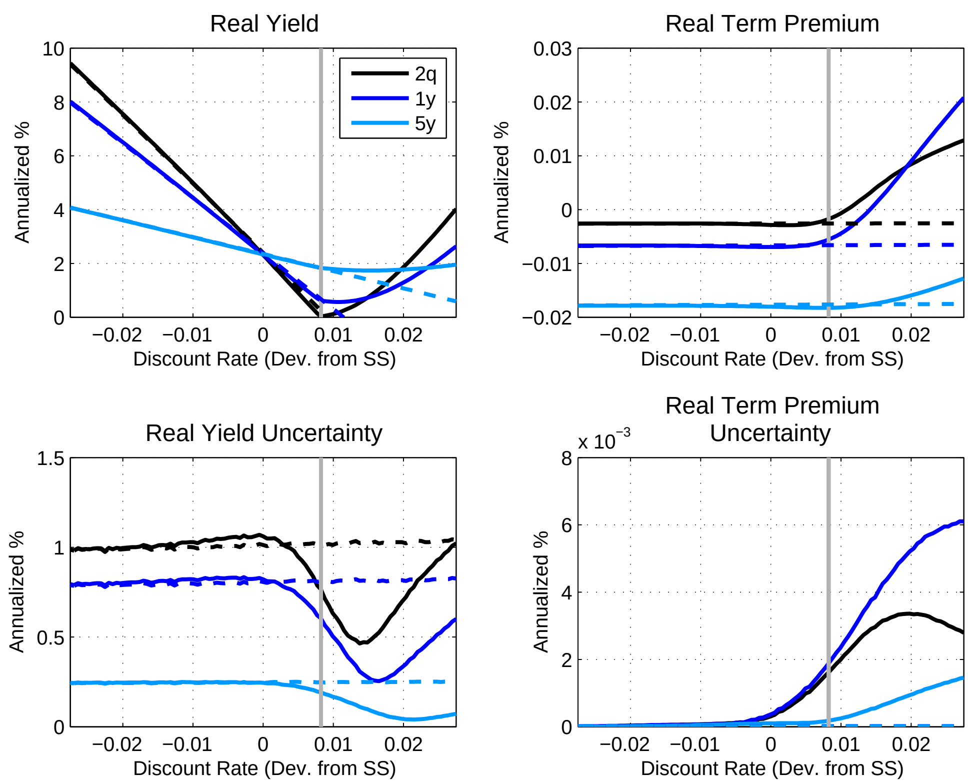
F The Real Term Structure

In this section, we analyze the term structure of real interest rates both in the stylized and quantitative models.

F.1 The Real Term Structure in the Stylized Model

F.1.1 Power Utility

Figure F.1: **Equilibrium Real Term Structure—Power Utility—**

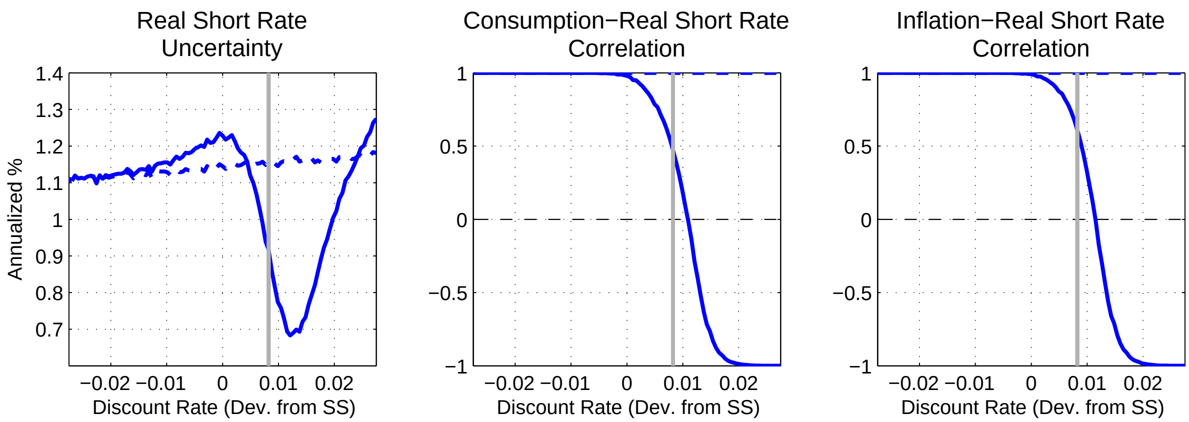


*Solid lines indicate results from the model with the ELB constraint, and dashed lines indicate results from the model without the ELB constraint. The solid vertical line indicates the threshold state where the ELB binds.

When the policy rate is above the ELB constraint, or in the model without the ELB constraint, the dynamics of real yields—shown in the top-left panel of Figure F.1—are qualitatively similar to those of nominal yields. Real yields decline with β. The longer the maturity is, the less sensitive the yield is to fluctuations in β. The term structure of yield volatility is downward sloping and the real yield curve is countercyclical. Key differences with nominal yields show up when the nominal short rate is at the ELB. When the nominal short rate is at the ELB, an increase in β leads to a decline in inflation, pushing up real yields of short maturity bonds. Real yields of longer maturities decline, but do not decline as much as they would in the model without the ELB in response to an increase in β.

As shown in the top-right panel of Figure F.1, similarly to nominal term premiums, real term premiums are state-dependent in the model with the ELB, but not in the model without the ELB. When the policy rate is above the ELB, real term premiums are roughly constant at negative values. When the policy rate is at the ELB, an increase in β pushes up real term premiums. For shorter maturities, the correlation between real yields and consumption growth is negative while at the ELB, as shown in the right panel of Figure C.1. As a result, for a sufficiently large β, real term premiums are positive. For longer maturities, the correlation between real yields tends to remain positive at the ELB (right panel of Figure C.1) and thus real term premiums remain negative at the ELB.

Figure F.2: **Conditional Volatilities and Correlations of Macroeconomic Variables —Power Utility—**



*Solid lines indicate either 1-quarter ahead standard deviation ('uncertainty') or correlation conditional on β of the respective variables for the model with the ELB constraint, and dashed lines indicate those for the model without the ELB constraint. The solid vertical line indicates the threshold state where the ELB binds.

According to the bottom-left panel of Figure F.1, the behavior of real yield uncertainty is similar to that of nominal yield uncertainty when the policy rate is above the ELB or is expected to be above the ELB in the near future. However, when the policy rate is at the ELB and is expected to remain at the ELB for a while, real yield uncertainty increases with β. Finally, real term premium uncertainty is higher when the ELB is binding than when it is not, similarly to nominal term premium uncertainty, as shown in the bottom-right panel.

The dynamics of real term premiums can be intuitively understood by decomposing real term premiums into macro uncertainty and real interest-rate sensitivity. We can decompose the 2-period real term premium as follows:

$$
\begin{aligned}
tp_t^{r(2)}(\bar{\beta}) \equiv R_t^{r(2)} - R_t^{r(2)\mathbb{Q}} &\approx \frac{1}{2}\mathbb{C}ov_t(m_{t+1}^r, r_{t+1}^{r(1)}) \\
&\propto -\mathbb{C}ov_t(\Delta c_{t+1}, r_{t+1}^{r(1)}) \\
&= - \underbrace{\sigma_{\Delta c}}_{\substack{\text{macro} \\ \text{uncertainty}}} \times \underbrace{\rho_{\Delta c, r^r} \times \sigma_{r^r}}_{\substack{\text{real-rate} \\ \text{sensitivity}}}
\end{aligned}
\tag{F.1}
$$

where σ_{r^r} and $\rho_{\Delta c, r^r}$ are the conditional volatility of the real short rate and the conditional correlation of consumption growth and the real short rate, respectively.

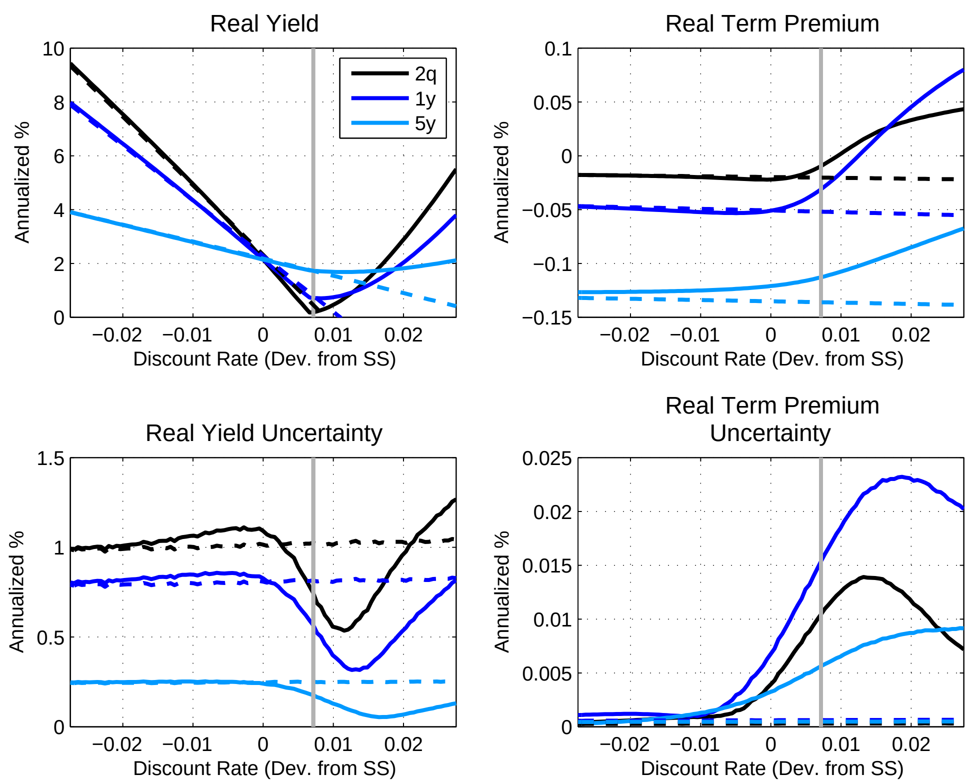
This decomposition is analogous to the one derived for nominal term premiums in (15). As described above and shown in the top-right panel of Figure F.1, a key feature of real term premiums is that they could turn positive from negative at the ELB, especially for shorter maturities. Through the lens of equation (F.1), this feature must come from the change in $\rho_{\Delta c, r^r}$. Indeed, the conditional correlation reported in the middle panel of Figure F.2 confirms this. Above the ELB, the reason why term premiums are negative is similar to that for nominal term premiums, as accommodative monetary policy lowers the real rate as well as the nominal rate. At the ELB, the rise in the real rates (or lower bond prices) described above implies that holding a real bond will only increase the volatility of the household's wealth. Thus, real bonds are risky, implying a positive term premium.

The non-monotonic pattern of shorter-maturity nominal term premiums is less pronounced for real term premiums. Following an argument similar to the one we used to understand nominal term premiums, one reason for the less pronounced non-monotonic pattern is that real term premiums do not involve inflation uncertainty. Another reason is the expected change in correlation between the short rate and macroeconomic variables at the ELB.[53]

F.1.2 Epstein-Zin Preferences

According to the top two panels of Figure F.3, the dynamics of real yields and real term premiums under EZ preferences are qualitatively similar to those under power utility. Quantitatively, the absolute size of real term premiums is larger under EZ preferences than under power utility, consistent with what we saw in nominal term premiums. The differences in the size of real term premiums between the two preference specifications spill over to the differences in real yields. Finally, real yield uncertainty and real term premium uncertainty are also amplified under EZ preferences compared to power utility, as shown in the bottom two panels.

Figure F.3: **Equilibrium Real Term Structure—Epstein-Zin Preferences—**



*Solid lines indicate results of the model with the ELB constraint, and dashed lines indicate results of the model without the ELB constraint. The solid vertical line indicates the threshold state where the ELB binds.

[53]In contrast to nominal yields, real yields becomes riskier near and at the ELB. Then the effect of increased macro uncertainty on real term premiums could reverse.

F.2 The Real Term Structure in the Quantitative Model

F.2.1 Moments at and above the ELB

We compare the moments of real yields and term premiums implied by our model with their data counterparts and their empirical estimates, respectively. Due to poor liquidity of shorter-maturity TIPS, we focus only on the 5-year and 10-year maturities.

Moments above the ELB: The average real yields for 5-year and 10-year maturities above the ELB are around 2 percent in our model and are similar to those in the data (first two columns of panel A in Table F.1). The volatility of real yields declines with maturity both in the data and in our model (first two columns of panel B). Real yield volatility is higher in our model compared to the data, but note that our model is calibrated to a sample period longer than the TIPS data sample. In particular, since the TIPS data excludes the earlier sample period of 1997 to 2003 when nominal yields were more volatile compared to the more recent sample, this discrepancy is somewhat expected. Real term premiums increase with maturity and they are positive, but smaller than nominal term premiums. Empirical estimates also indicate that real term premiums are increasing in maturity (first two columns of panel A in Table F.2). The volatility of real term premiums increases with maturity and is smaller than the volatility of nominal term premiums. They are similar to the size of the empirical estimates (first two columns of panel B).

Table F.1: **Real Yield Moments at and above the ELB —Model versus Data—**

	Above the ELB		At the ELB		Below the ELB
	Data	Model	Data	Model	Model w/o ELB
	Real Yields (Mean)				
5-year	1.67	1.97	-0.20	-0.52	-0.98
10-year	2.04	2.15	0.59	0.48	0.25
	Real Yields (Volatility)				
5-year	0.65	1.03	0.87	0.38	0.82
10-year	0.36	0.72	0.79	0.30	0.51

*This table contains summary statistics for selected real yields comparing data versus model counterparts. The sample period is 2004:Q1-2008:Q4 (starting date restricted by the availability of TIPS data) for data above the ELB, and 2009:Q1-2015:Q3 for data at the ELB.

Moments at the ELB: Real yields are on average lower at the ELB than above the ELB, both in the data and in our model. Also similarly to nominal yields, the average 5-10 year yield spread is significantly larger at the ELB both in the data and in our model (third and fourth columns of Table F.1, panel A). In contrast to nominal yields, the ELB increases average real yields (the fourth and last columns of panel B), consistent with the intuition discussed for the stylized models. While real yield volatility is higher at the ELB than above the ELB in the data, it is lower at the ELB in our model (third and fourth columns of panel B). The absolute size of real term premiums and their volatilities are lower on average at the ELB than above the ELB in our model (the fourth and last columns of panel B, Table F.2).

Table F.2: **Real Term Premium Moments at and above the ELB —Model versus Estimates—**

	Above the ELB		At the ELB		Below the ELB
	Estimates	Model	Estimates	Model	Model w/o ELB
	Real Term Premium(Mean)				
5-year	0.31	0.18	—	-0.19	-0.27
10-year	0.57	0.38	—	-0.12	-0.16
	Real Term Premium (Volatility)				
5-year	0.14	0.08	—	0.05	0.06
10-year	0.18	0.12	—	0.05	0.07

*This table contains summary statistics for selected term premiums comparing estimates versus model counterparts. The real term estimates away from the ELB are derived from the term structure model of D'Amico, Kim, and Wei (2014). The sample period is 2004:Q1-2008:Q4 for estimates above the ELB, and 2009:Q1-2015:Q3 for estimates at the ELB. All variables are in annualized percentage terms.

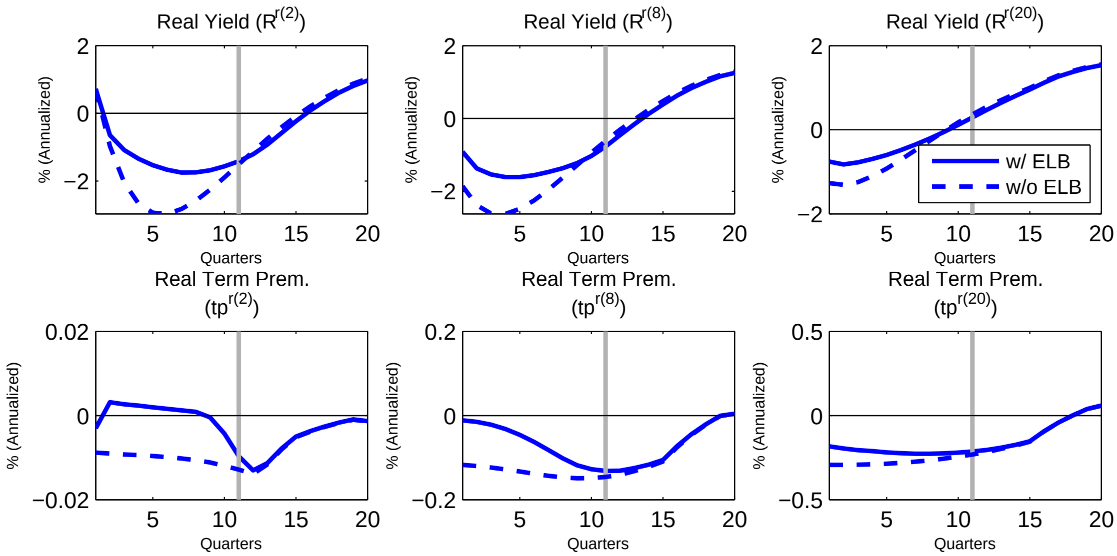
F.2.2 Dynamics

As shown in the top row of Figure F.4, real yields dip below zero for all maturities, reflecting the fact that inflation is positive during most of the simulation. The paths of real term premiums—shown in the bottom row—are qualitatively similar to those of nominal term premiums for all maturities. In particular, term premiums are compressed while the ELB constraint is binding. One key difference with nominal term premiums is that, while nominal term premiums can get stuck at zero for short maturity bonds when the policy rate is expected to remain at the ELB for long, real term premiums do not necessarily get stuck at zero.

F.3 The RW rule and the Real Term Structure

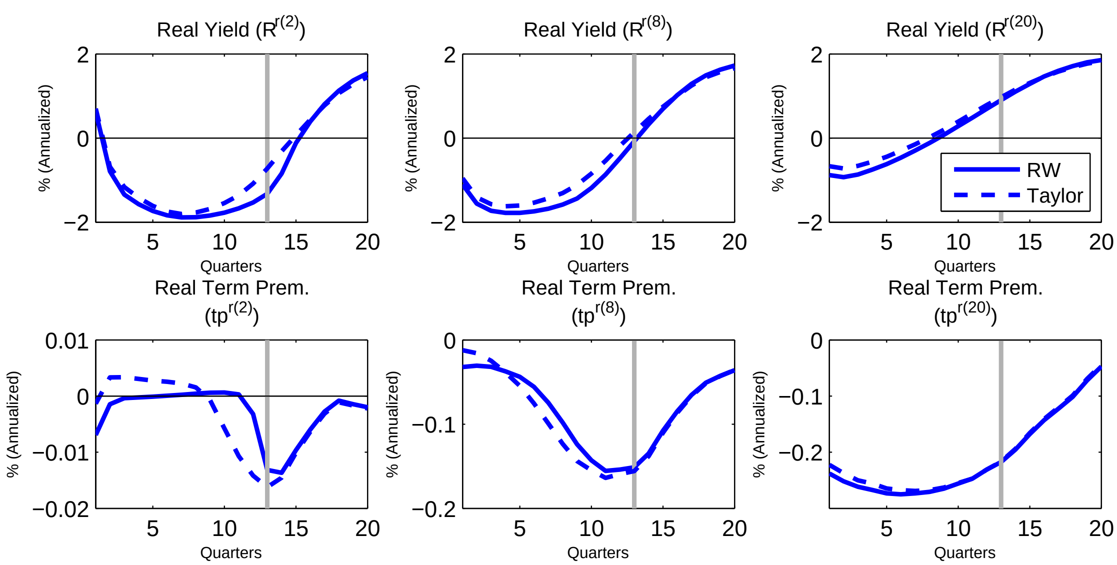
As discussed in the main body of the paper, the nominal short rate is kept at the ELB for longer, and the path of inflation is higher, under the RW rule than under the baseline rule. Thus, the paths of real yields are lower under the RW rule than under the baseline for all maturities, as seen in the top row of figure F.5. This is also true for the version of our model in which term premiums are positive on average at the ELB, as shown in the top row of figure F.6.

Figure F.4: **Impulse Responses of the Real Term Structure (to β shock)**



*We plot $median[X_{t+h} \mid \varepsilon_t^d = +\vartheta_\beta, \bar{X}]$ for a given variable X_t. The solid blue lines are the responses from the model with the ELB constraint, and the dashed blue lines are the responses from the model without the ELB constraint. The vertical grey bars indicate the timing of liftoff from the ELB.

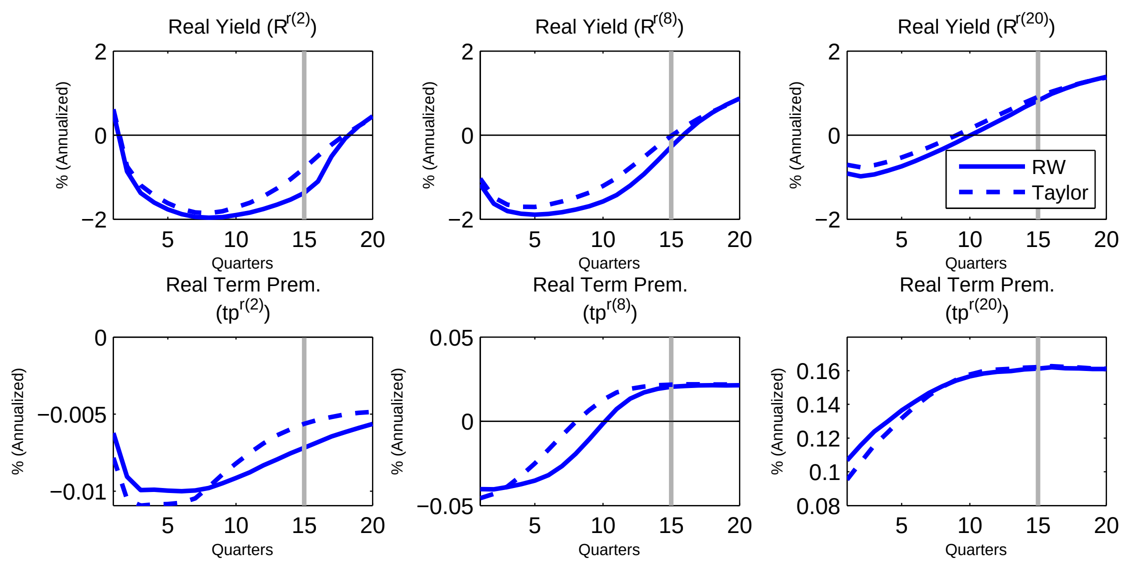
Figure F.5: **Effect of Accommodative "Forward Guidance" at the ELB**



*We plot $median[X_{t+h} \mid \varepsilon_t^d = +\vartheta_\beta, \bar{X}]$ for a given variable X_t. The solid blue lines are the responses from the model under the RW rule, and the dashed blue lines are the responses from the model under the Taylor rule with the ELB constraint. The vertical grey bars indicate the timing of liftoff from the ELB.

The effect of adopting the RW rule is intricate. As shown in the bottom-left panel of Figure F.5, the real term premium for the 2-quarter bond is lower and higher in the first and second half of the ELB episode, respectively. For the 2-year maturity, the opposite is the case. The RW rule lowers the real term premium for the 5-year bond in the first several quarters, but leaves them essentially unchanged thereafter. Interestingly, the effects of adopting the RW rule on real term premiums are opposite of the effects just described in the version of the model with positive nominal term premiums at the ELB, as seen in the bottom row of Figure F.6.

Figure F.6: **Effect of Accommodative "Forward Guidance" at the ELB —Case of Positive Nominal Term Premiums at the ELB—**



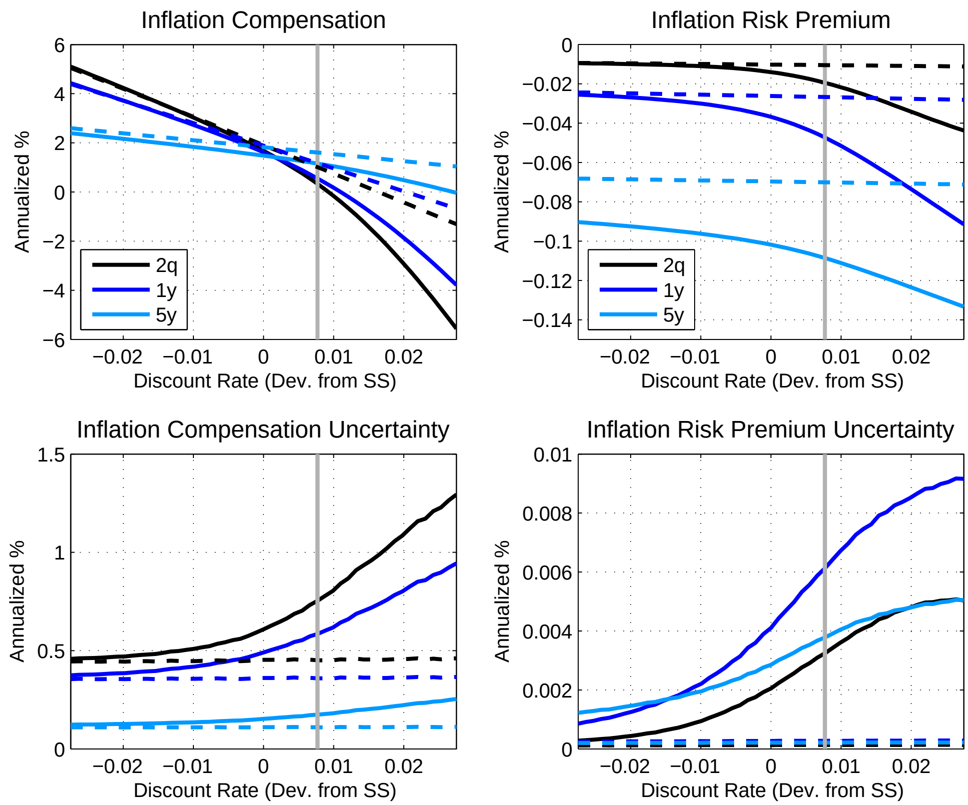
*We plot $median[X_{t+h} \mid \varepsilon_t^d = +\vartheta_\beta, \bar{X}]$ for a given variable X_t. The solid blue lines are the responses from the model under the RW rule, and the dashed blue lines are the responses from the model under the Taylor rule with the ELB constraint. The vertical grey bars indicate the timing of liftoff from the ELB.

G A Note on Inflation Compensation and Risk Premium

In this section, we briefly discuss the model implications for inflation compensation and inflation risk premium. We keep the discussion short based on our stylized model since, by definition, their dynamics are largely implied by our discussion above on the nominal and real term structures.

Inflation compensation is defined as the difference between nominal and real yields. Analogous to the decomposition of nominal and real yields, we can further decompose inflation compensation into expected inflation and inflation risk premiums. A simple and standard way to compute n-period inflation risk premiums ($irp^{(n)}$) is by taking the difference between nominal and real term premiums, i.e. $irp^{(n)} = tp^{(n)} - tp^{r(n)}$. In Figure G.1 we plot inflation compensation as well as inflation risk premiums derived from our stylized model with Epstein-Zin preferences. Note that results using power utility are qualitatively similar.

Figure G.1: **Inflation Compensation and Risk Premium —Epstein-Zin Preferences—**



*Solid lines indicate results from the model with the ELB constraint, and dashed lines indicate results from the model without the ELB constraint. The solid vertical line indicates the threshold state where the ELB binds.

The top-left panel plots inflation compensation of different maturities both from the model with (solid lines) and without (dashed lines) the ELB constraint. Similar to nominal yields (see top-left panels of Figures 2 and 5), inflation compensation is decreasing with respect to the discount rate β, the volatility of inflation compensation is decreasing with respect to maturity and the slope of the term structure of inflation compensation is countercyclical. These features are observed regardless of the ELB constraint. In contrast, inflation compensation decreases more sharply as the discount rate increases when the ELB constraint is present, and the decrease is largest for shorter maturities.

The decline in inflation compensation near and at the ELB is attributable to declines in both inflation expectations and inflation risk premiums. As discussed in Section 3, inflation decreases more at the ELB, which is reflected in the agents' inflation forecasts particularly when the economy is more likely to be in the ELB state. In addition, as shown in the top-right panel of Figure G.1 (solid lines), inflation risk premiums are also lower when the economy is subject to an ELB constraint. Specifically, in contrast to the constant (and negative) inflation risk premium under the economy without the ELB (dashed lines), inflation risk premiums under the economy with an ELB are generally lower, and decreasing with respect to β particularly strongly near and at the ELB. This result can be understood by recalling that while nominal term premiums eventually approach zero as β increases due to the compressing effect of the ELB (see the top right panels of Figures 2, 5), real term premiums are increasing with respect to β

at the ELB (see the top-right panels of Figures F.1 and F.3). Another way to understand the mechanism of an increasingly negative inflation risk premium is by noticing that inflation risk premiums are approximately the covariance between the $n-$period real stochastic discount factor ($m^r_{t,t+n}$) and $n-$period inflation ($\pi_{t,t+n}$), i.e. $irp^{(n)} \approx Cov_t(m^r_{t,t+n}, \pi_{t,t+n})$. When the economy is predominantly driven by demand shocks that cause positive comovement in marginal utility and inflation, a hypothetical asset with a return equivalent to inflation becomes increasingly valuable as uncertainty in consumption and inflation rise at the ELB. Finally, the stronger non-linearity in inflation compensation and risk premium naturally leads to an increase in uncertainty of both objects near and at the ELB, as shown in the bottom two panels of Figure G.1.

U.S. data on inflation compensation, measured from either the difference between nominal Treasury yields and TIPS yields or inflation swaps, suggests that inflation compensation has been lower on average since the U.S. economy hit the ELB compared to when it was above the ELB. Moreover, as most measures of longer-term expected inflation have been largely stable since the late 1990s including the ELB episode, a significant part of the decline in inflation compensation is likely to reflect a decline in inflation risk premiums. Although many factors could be behind these declines, our analysis suggests one potential factor of interest, which is the changing macroeconomic dynamics due to the presence of the ELB.