Inflation as a Long-run Optimal Policy in an Open Economy

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Fact 1: Trend inflation (in many countries)
Fact 2: Price rigidity: X-country, X-good diffs
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- Fact 2: Price rigidity: X-country, X-good diffs
- Question: Long-run implications w/ trade
- Model: Trend Calvo + Ricardo (-Viner) trade
  - No previous papers
Inflation → Production → Terms of trade

- Fact 1: Trend inflation (in many countries)
- Fact 2: Price rigidity: X-country, X-good diffs
- Question: Long-run implications w/ trade
- Model: Trend Calvo + Ricardo (-Viner) trade
  - No previous papers
  - Inflation affects trade pattern
    - Breaking the dichotomy
  - Welfare enhancing inflation
    - Contrast to closed or small-open models
    - Effect of the terms of trade
Model environment

- $t = 0, 1, \ldots, \infty$, analyze stationary state
- Home & foreign (w/ * if necessary)
- Stand-in HH in each country
- Final goods $i$, costless-tradeable
- Continuum $\nu \in (0, 1)$ of non-tradeable intermediate goods for each $i$
- No intl asset trade & balanced trade
- Money-less economy
Model environment: CRS (Ricardian trade)
Model environment: DRS (Ricardo-Viner trade)
Home & the rest of the world

- foreign: the rest of the world
- Symmetry except for
  - Size: \(N, N^*\)
  - Exog. tech of int. good: \(\theta_{it}, \theta_{it}^*\)
  - Prob. of price change: \(\omega_i, \omega_i^*\)
  - Tax/subsidy rates: \(\tau_{Lt}, \tau_{Lt}^*, \tau_{it}, \tau_{it}^*\)
  - Inflation rate: \(\Pi, \Pi^*\)
- Not consider strategic situation
  - \(\Pi\): treated as a policy parameter
  - \(\Pi^*\): exogenously fixed
Households

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t u \left( \left( \sum_i \alpha_i c_{it}^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho}{\rho-1}}, l_t \right),$$

s.t. $$\sum_i P_{it} c_{it} \cdot \frac{P_t}{P_t} + b_t + \tau_{Lt} = \frac{1 + i_{t-1}}{\Pi_t} b_{t-1} + w_t l_t + f_t,$$

where $$u(c, l) = c^\psi (1 - l)^{1-\psi}.$$

- $$b_t$$: intra-country nominal bond
- $$\tau_{Lt}$$: lump-sum tax
- $$i_t$$: nominal int. rate
- $$\Pi_t = P_t/P_{t-1}$$: gross inf. rate
- $$f_t$$: firms’ real prfts
Final goods firms

\[
\max P_{it}y_{it} - \int_0^1 P_{it}(v)y_{it}(v)dv,
\]

s.t. \[ y_{it} = \left( \int_0^1 y_{it}(v)\frac{\eta-1}{\eta} dv \right)^{\frac{\eta}{\eta-1}}. \]
Intermediate good firms

- Each int. firm produces differentiated product
  - Index \( \nu \)
- Facing the demand curve
- Using labor
  - RTS: \( \gamma \in (0, 1] \)
- Input subsidy \( \tau_{it} \in [0, 1) \)
- \( \theta_{it} \): productivity, common w/ i industry
- Can update price w/ prob. \( 1 - \omega_i \)
- Intertemporal problem of prfts max
  - discounting: \( \Lambda_{tt+j} \equiv \beta^j \frac{u_{ct+j}}{u_{ct}} \)
Intermediate good firms

\[
\max_{P_{it}(v), \{l_{it+j}(v), y_{it+j}(v)\}} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{tt+j} \omega_{i,j} \\
\times \left[ \frac{P_{it}(v)}{P_{t+j}} y_{it+j}(v) - (1 - \tau_{it+j}) w_{t+j} l_{it+j}(v) \right],
\]

s.t.

\[
y_{it+j}(v) = \theta_{it+j} l_{it+j}(v)^\gamma,
\]

\[
y_{it+j}(v) = \left( \frac{P_{it+j}}{P_{it}(v)} \right)^\eta y_{it+j}.
\]
Industry-price

- Those who adjust price: pick $\tilde{P}_{it}$

\[
\left( \frac{\tilde{P}_{it}}{P_t} \right)^{1-\eta + \frac{\eta}{\gamma}} = \frac{\eta}{\eta - 1}
\]

\[
E_t \sum_{j=0}^{\infty} \Lambda_{tt+j} \omega_i^j \left( \frac{P_{it+j}}{P_{it}} \right)^{\frac{\eta}{\gamma}} \left( 1 - \tau_{it+j} \right)^{\frac{\omega_{t+j}}{\gamma}} \left( \frac{y_{it+j}}{\theta_{it+j}} \right)^{\frac{1}{\gamma}}
\]

\[
\times E_t \sum_{j=0}^{\infty} \Lambda_{tt+j} \omega_i^j \frac{P_{it+j}}{P_{t+j}} \left( \frac{P_{it+j}}{P_{it}} \right)^{\eta-1} y_{it+j}
\]
www Industry-price

- Industry-price, law of motion
  \[ P_{it} = \left( \omega_i P_{it-1}^{1-\eta} + (1 - \omega_i) \tilde{P}_{it}^{1-\eta} \right)^{\frac{1}{1-\eta}} \]

- LoM of \( p_{it} \equiv \frac{P_{it}}{P_t} \) determined
Price dispersion leads to resource costs

\[ y_{it} \left( \frac{P_{it}}{P_{it}(\nu)} \right)^{\eta} = y_{it}(\nu) = \theta_{it}l_{it}(\nu)^{\gamma} \]

Integration over \( \nu \)

\[ y_{it} \left( \int_{0}^{1} \left( \frac{P_{it}}{\tilde{P}_{it}} \right)^{\frac{\eta}{\gamma}} \, d\nu \right)^{\gamma} = \theta_{it} \left( \int_{0}^{1} l_{it}(\nu) \, d\nu \right)^{\gamma}, \]

\[ \equiv s_{it}^{\gamma} = l_{it} \]

i.e.,

\[ y_{it} = \frac{\theta_{it}}{s_{it}^{\gamma}} l_{it}^{\gamma}. \]
Stationary state

Consider $\theta_{it} = \theta_i$, $\Pi_t = \Pi$

- Agg. & industry-level variables: constant

$$y_i = \frac{\theta_i}{s_i^\gamma} l_i^\gamma, \quad p_i = (1 - \tau_i) v_i s_i^\gamma \frac{w}{\theta_i} l_i^{1-\gamma}$$

where

$$s_i = \frac{1 - \omega_i}{1 - \omega_i \Pi_i \eta^{-1}} \left( \frac{1 - \omega_i \Pi_i \eta^{-1}}{1 - \omega_i} \right)^{-\frac{1}{\eta-1}}$$

$$v_i = \frac{\eta}{\eta - 1} \frac{1 - \beta \omega_i \Pi_i \eta^{-1}}{1 - \beta \omega_i \Pi_i \eta^{-1}} \frac{1 - \omega_i \Pi_i \eta^{-1}}{1 - \omega_i \Pi_i \eta^{-1}}.$$
• $\Pi > 1, \omega > 0 \rightarrow \text{markup gradually ↓}$
• Price re-setter picks larger markup
• Avg markup ($v_i$), gradually ↓
• Given price dispersion, output dispersion
• Ex ante symmetric, ex post not
• Allocation inefficiency, $s_i$
- $s = 1$ (when $\Pi = 1$ or $\omega = 0$)
- Asymmetrically increasing as $\Pi$ deviates
- Decreasing in $\Pi$ around $\Pi = 1$
- Often abstracted (thru tax adjustment)
Relative price (against annual $\Pi$)

- Min: slightly larger than $\Pi = 1$
- Small impact of $\nu \rightarrow$ Assume $(1 - \tau_i)\nu_i = 1$
CRS, single-good autarky

- CRS \((\gamma = 1)\), single-good, autarky
- Welfare

\[
u \propto \left(\frac{\theta}{s}\right)^\psi
\]

- \(s = 1\) (by \(\Pi = 1\)) achieves max
- Result not much affected by cost of hitting zero-lower bound (Schmitt-Grohé & Uribe 11, Coibion et al. 12)
CRS, two-good autarky

- CRS, two-good autarky
- Welfare
  - Max at $\Pi = 1$
- Relative price

\[
\frac{p_A}{\theta_A} = \frac{s_A}{\theta_B} \quad \frac{s_B}{\theta_B}
\]

- Relative price depends on $\Pi$, $\omega_i$, ... thru $s$ (& $v$)
Labor market

\[ \sum_{i} l_i = l \]

Bond market

\[ b_i = 0 \]

Costless trade goods market for \( i = A, B \)

\[ Nc_i + N^*c_i^* = Ny_i + N^*y_i^* \]
Two-good model, trade pattern (Prop. 1)

- CRS → Ricardian trade
- Home exports good $A$ if

$$
\frac{s_A}{\theta_A} \frac{s_B}{\theta_B} = \frac{p_A}{p_B} < \frac{p_A^*}{p_B^*} = \frac{s_A^*}{\theta_A^*} \frac{s_B^*}{\theta_B^*}
$$

- Inf. rate affects trade pattern
  - thru $s$ (& $v$)
  - Eqm rel. price = home’s terms of trade (TOT)
Prop 2: $\text{TOT} \uparrow$ by $\Pi$’s deviation from 1
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\[ \frac{p_A}{p_B} \]

\[ \frac{(1-\tau_A^*)v_A^* s_A^* \theta_B^*}{(1-\tau_B^*)v_B^* s_B^* \theta_A^*} \]

\[ \frac{(1-\tau_A)v_A s_A \theta_B}{(1-\tau_B)v_B s_B \theta_A} \]

\[ N \frac{\theta_A}{s_A} \]

\[ N \frac{\theta_A}{s_A} + N^* \frac{\theta_A^*}{s_A^*} \]

\[ Ny_A + N^* y_A^* \]
Welfare under trade

- Home’s welfare

\[ u = \left( \frac{\psi}{1 - \psi} \right)^{\psi-1} \left( \frac{\theta_A}{s_A} \right)^{\psi} \times \left( \alpha_A^\rho + \alpha_B \alpha_A^{\rho-1} \left( \frac{\theta^*_B s_A N^*}{\theta_A s_B^* N} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\psi}{\rho-1}} \]
Prop 3: Welfare enhancing inflation

\[
\frac{\partial u}{\partial \Pi} = -\Phi_0 \Phi_1 \frac{1}{s_A} \frac{\partial s_A}{\partial \Pi}
\]

where $\Phi_0 > 0$ (constant) and

\[
\Phi_1 = 1 - \frac{1 - \rho \alpha_B}{\rho} \frac{\alpha_A}{\frac{1}{\rho}} \left( \frac{\theta_B^{*} s_A N^{*}}{\theta_A s_B^{*} N} \right)^{\frac{\rho - 1}{\rho}}
\]

- If $\Phi_1 > 0$, $\Pi = 1$: maximizer
- If $\Phi_1 < 0$, $\Pi = 1$: (local) minimizer
Welfare enhancing inflation

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Welfare enhancing inflation

![Graph showing the relationship between Good B and Good A, with Consumer and Production points, PPF, and Budget line.](image-url)
Welfare enhancing inflation

![Graph showing consumption, production, PPF, and budget lines.]

- **Consumption**: The point on the curve where the consumer's choice lies.
- **Consumption’**: The new consumption point with inflation.
- **PPF**: Production Possibility Frontier.
- **PPF’**: Inflation shifts the PPF outwards.
- **Budget line**: The line represents the budget constraint.
- **Budget line’**: Shifted due to inflation.
- **Production**: The point on the x-axis representing production.
- **Production’**: The new production point with inflation.

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Mechanism known as immiserizing growth

- Deviation of $\Pi$ from 1: effective TFP ↓
  (a) Production capacity ↓
  (b) Exports ↓ $\rightarrow$ Export price ↑ $\rightarrow$ TOT ↑
- (Reverse version) known as “immiserizing growth” (Johnson 55; Bhagwati, 58)
- Closed & Small-open: only (a)
- Condition on $\Phi_1$: making (b) stronger
  - A key parameter: $\rho$ (subst. b/w $A$ & $B$)
Comparison: small-open vs. two-country

![Graph showing comparison between small-open and two-country models](image-url)
DRS case: Ricardo-Viner trade model

- DRS ($\gamma < 1$)
  - Both countries produce both goods
  - Ricardo-Viner (also known as specific factors) trade model
  - Consider home exports $A$, imports $B$
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- Prop 4: Unique eqm exists under costless trade
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- DRS ($\gamma < 1$)
  - Both countries produce both goods
  - Ricardo-Viner (also known as specific factors) trade model
  - Consider home exports $A$, imports $B$
- Prop 4: Unique eqm exists under costless trade
- Prop 5: TOT ↑ when $\Pi$’s deviation from 1 if

$$\left| \frac{\partial s_A^\gamma \Pi}{\partial \Pi s_A^\gamma} \right| \geq \left| \frac{\partial s_B^\gamma \Pi}{\partial \Pi s_B^\gamma} \right|$$

- Exporting industry responds more to $\Pi$ (i.e., more sticky)
Prop 6: Welfare formula

\[
\frac{du}{d\Pi} u \propto \frac{p_B(y_B - c_B)}{c} \frac{dp}{d\Pi} \frac{\Pi}{p} - \sum_{i=A,B} \frac{l_i \partial s_i^\gamma}{l} \frac{\partial \Pi}{\partial s_i^\gamma} \frac{\Pi}{s_i^\gamma}
\]

- \(du/d\Pi \geq 0\)
- \(dp/d\Pi\) depends on various parameters
- Closed or small-open: only 2nd term
  - \(\Pi = 1\) always optimal
Welfare enhancing inflation
Welfare enhancing inflation

Good B

Consumption

Consumption’

Production

Production’

Budget line

Budget line’

PPF

PPF’

Good A
Back of the envelope calculation

Assume $s_A = s_B$

$$
\frac{du \Pi}{d\Pi u} \propto \frac{IM}{GDP} \times \frac{dp \Pi}{d\Pi p} - \frac{\partial s^\gamma \Pi}{\partial \Pi s^\gamma} = 0.11 \times 0.63 - 0.57
$$

- $\frac{IM}{GDP} = 0.11$ US data
- $\frac{dp \Pi}{d\Pi p} = 0.63$ regression using US time series
- $\frac{ds^\gamma \Pi}{d\Pi s^\gamma} = 0.57$ calculated by setting $\gamma = 0.95$, $\eta = 10$, $\omega = 0.753$, $\Pi^{1/12} = 1.03$
Why important?: positive optimal inf.

- Long-run optimality of zero-inf. in closed economy
  - Eliminating price dispersion cost (King & Wolman 99, Schmitt-Grohé & Uribe 11, Coibion et al. 12)
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- Sticky-price two country models
  - Short-run: nominal exchange rate, terms of trade (e.g., Benigno 04, Corsetti et al. 11, Bergin & Corsetti, 16)
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  ★ Provide long-run optimal in open-economy thru the terms of trade change
Why important?: digging up TFP

- Ricardian models: empirically good
  - Trade pattern & X-country prosperity
    (Eaton & Kortum 02, Alvarez & Lucas 07, Waugh 10, Costinot et al. 12)
  - TFP (= labor productivity): the driver
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- The source of sector-level TFP (LP)?
  - Trade context (Matsuyama 05, Ishise 16)
  - Amplification thru allocation (e.g., Hsieh & Klenow 09) given exogenously hetero TFP
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- TFP (≡ labor productivity): the driver
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★ Provide a story of sector-level TFP
★ Breaking the classical dichotomy
Summary & future works

- Trend-Calvo & Ricardo (-Viner) trade
- Inf. rate affects trade pattern
- TOT attenuates the welfare loss
- Optimal inflation rate
  - $\Pi = 1$ may not be optimal under some parameters
- More empirical assessment
  - $\Pi$’s role of determining trade pattern
  - TOT on the long-run welfare
Model structure (c.f., Obstfeld-Rogoff)
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Law of motion of the price dispersion

\[ s_{it} = (\omega_i)^0 (1 - \omega_i) \left( \frac{P_{it}}{\tilde{P}_{it}} \right)^\eta \]

\[ + (\omega_i)^1 (1 - \omega_i) \left( \frac{P_{it}}{\tilde{P}_{it-1}} \right)^\eta \]

\[ + (\omega_i)^2 (1 - \omega_i) \left( \frac{P_{it}}{\tilde{P}_{it-2}} \right)^\eta + \ldots \]

\[ = (1 - \omega_i) \left( \frac{P_{it}}{\tilde{P}_{it}} \right)^\eta + \omega_i \left( \frac{P_{it}}{P_{it-1}} \right)^\eta s_{it-1} \]