Learning, Confidence, and Business Cycles

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Parsimonious mechanism for business cycle dynamics

- Propose: Endogenous idiosyncratic uncertainty
  - firms learn about own profitability prospects

- Behaves *as if* linear RBC model with endogenously determined

  1. Countercyclical labor wedge and spreads (from excess returns)
  2. Co-movement from demand shocks
  3. Amplification, propagation and hump-shaped dynamics

Parsimonious mechanism for business cycle dynamics

- Propose: Endogenous idiosyncratic uncertainty
  - firms learn about own profitability prospects

- Do not require additional shocks or rigidities such as

  1. Wedge shocks (countercyclical labor wedge and spreads)
  2. Nominal rigidities (co-movement)
  3. Habit, adjustment cost (internal propagation)

Countercyclical endogenous idiosyncratic uncertainty

Firms face Knightian uncertainty about own profitability

1. Learning through production: lower scale $\rightarrow$ more uncertainty
2. Uncertainty affects input choice: more uncertainty $\rightarrow$ lower scale

Feedback arises from any shock that moves activity

Countercyclical idiosyncratic uncertainty shows up

- As countercyclical wedges: labor and asset prices move 'too much' compared to what econometrician measures
  - rationalize 'excess volatility'
- In linear decision rules at firm level
- In the cross-sectional average through aggregation

Model: Preferences

Representative household: recursive multiple priors utility

$$U_t(C; s^t) = \ln C_t - \varphi \frac{H_t^{1+\eta}}{1+\eta} + \beta \min_{p \in \mathcal{P}_t(s^t)} E^p[U_{t+1}(C; s^t, s_{t+1})]$$

- $\mathcal{P}_t(s^t)$: one-stead-ahead set of probability distributions
- Larger set $\mathcal{P}_t(s^t) \rightarrow$ less confidence

Production

- Firms: continuum, indexed by $l \in [0,1]$, perfectly competitive

$$Y_{l,t} = A_t\{z_{l,t}K_{l,t-1}^{\alpha}H_{l,t}^{1-\alpha} + \nu_{l,t}\}$$

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$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{A,t}, \qquad \epsilon_{A,t} \sim N(0, \sigma_A^2)$$

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- Idiosyncratic additive shock, $\quad \nu_{l,t} \sim N(0, \sigma_\nu^2)$

Information

$$Y_{l,t} = A_t\{z_{l,t} K_{l,t-1}^{\alpha} H_{l,t}^{1-\alpha} + \nu_{l,t}\}$$

- $z_{l,t}$ and $\nu_{l,t}$ unobservable to agents $\rightarrow$ learning
- Non-invertibility problem: path of output and input not fully revealing about the unobservable shocks
- Interpretations of additive shock
  1. Aggregation of production units with common and idiosyncratic shocks
  2. Sale is signal on unobservable persistent demand shock

Details

Heterogeneous-firm RBC model

- Firms: choose $\{K_{l,t}, H_{l,t}, I_{l,t}\}$ to maximize

$$E_0^* \sum_{t=0}^{\infty} M_0^t D_{l,t}$$

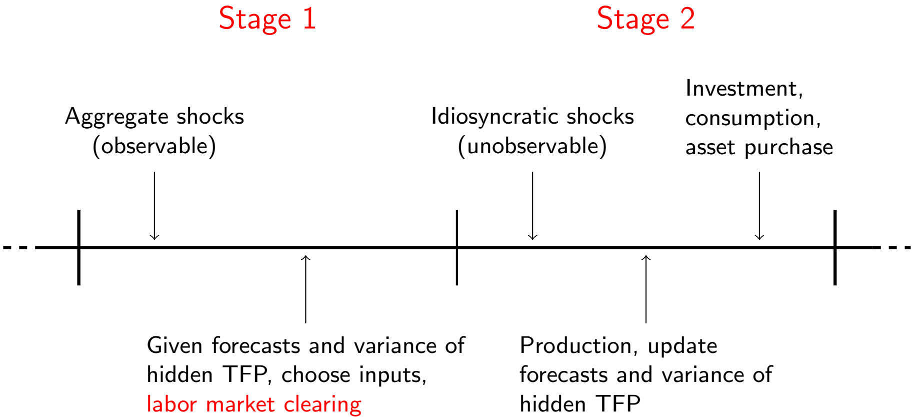
  - M_0^t : prices of contingent claims, under worst case probabilities

$$D_{l,t} = Y_{l,t} - W_t H_{l,t} - I_{l,t}$$

- Resource constraint:

$$Y_t = C_t + I_t + G_t$$
$$\ln G_t = (1-\rho_g)G + \rho_g \ln G_{t-1} + \epsilon_{g,t}, \qquad \epsilon_{g,t} \sim N(0, \sigma_g^2)$$

Timeline of events within a period



RCE

Learning and ambiguity about idiosyncratic productivity

- Estimate $z_{l,t}$ from observables: linear + Gaussian → Kalman filter

$$Observation : Y_{l,t}/A_t = K_{l,t-1}^{\alpha} H_{l,t}^{1-\alpha} z_{l,t} + \nu_{l,t}$$

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- Low production input $K_{l,t-1}^{\alpha} H_{l,t}^{1-\alpha}$ $\rightarrow$ high Mean Square Error $\Sigma_{l,t|t}$
- Not confident in the Kalman filter estimate: set of distributions

$$E_t z_{l,t+1} = (1-\rho_z)\bar{z} + \rho_z \tilde{z}_{l,t|t} + \mu_{l,t}; \qquad \mu_{l,t} \in [-a_{l,t}, a_{l,t}]$$

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- Confidence lower when estimation uncertainty is higher

$$-a_{l,t} = -\eta_a \sqrt{\Sigma_{l,t|t}}$$

  - Distributions "close" to filter estimate (relative entropy distance)

Ambiguity and the law of large numbers

- Each firm's expected $z_{l,t+1}$ under worst-case probability

$$E_t^* z_{l,t+1} = (1-\rho_z)\bar{z} + \rho_z \tilde{z}_{l,t|t} \underbrace{-a_{l,t}}_{=-\eta_a\sqrt{\Sigma_{l,t|t}}}$$

  - Household acts as if conditional mean of each $z_{l,t+1}$ is lower
  - First-order effect of uncertainty

- Cross-sectional average given by a set

$$\left[\bar{z} - \int a_{l,t} dl, \bar{z} + \int a_{l,t} dl\right]$$

  - Epstein & Schneider (2003): formal treatment of LLN with ambiguity

Linearized solution

1. Filtering problem is linear $\rightarrow$ analytic law of motion for $\Sigma_{l,t|t}$
   - Inputs have first-order effect on the level of posterior variance
2. First-order feedback from uncertainty to decision rules through $-a_{l,t}$
3. In turn, linear decision rules $\rightarrow$ easy aggregation
   - Cross-sectional mean: sufficient statistic for tracking distributions

Implication: comovement and countercyclical labor wedge

- Standard model

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  Low confidence $\rightarrow$ C low $\rightarrow$ standard effect is H high
  $\rightarrow$ choose H *as if* productivity low $\rightarrow$ H low

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- Labor wedge: implicitly define labor tax

$$\varphi H_t^{\eta} = (1-\tau_t)\lambda_t MPL_t \quad \Rightarrow \quad \frac{E_t^{*}[\lambda_t MPL_t]}{\lambda_t MPL_t} = 1-\tau_t$$

Low confidence $\rightarrow$ econometrician rationalizes 'surprisingly low' H by high labor tax

Implication: countercyclical ex-post excess return

- Euler conditions for capital and risk-free assets

$$\lambda_t = \beta E_t^*[\lambda_{t+1} R_{t+1}^K]$$
$$\lambda_t = \beta E_t^*[\lambda_{t+1} R_t]$$

$\rightarrow$ under linearization, $E_t^* R_{t+1}^K - R_t = 0$

- Pricing based on worst case $\neq$ econometrician's DGP

- During low confidence times, demand for capital 'surprisingly low'
$\rightarrow$ ex-post excess return $R_{t+1}^K - R_t$ high

- Implication extends to defaultable corporate bonds
$\rightarrow$ countercyclical excess bond premia (Gilchrist & Zakrajsek 2012)

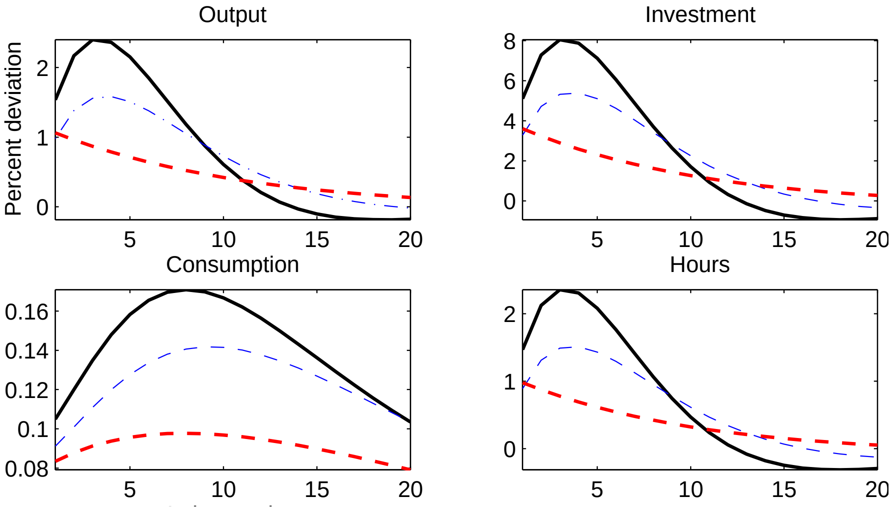
Calibration

Magnitude of feedback loop determined by

1. Variability of inputs
   - Inverse Frisch elasticity $\eta = 0$
   - Capital utilization
2. Size and variability of posterior variance
   - Idiosyncratic TFP shock $\rho_z = 0.5, \sigma_z = 0.4$
     - establishment-level data (Bloom et al. 2014, Kehrig 2015)
   - SS posterior variance $\Sigma = 0.1$
     - estimated posterior variance of firm-specific shocks (David et al. 2015)
3. Size of entropy constraint
   - Reasonable theoretical upper bound $\eta_a = 2$ (Ilut & Schneider 2014)
   - Empirical: firm-level capital return forecasts across analysts
     - Set $\eta_a = 0.4$ to get average dispersion of 39% (vs 43% in Senga 2014)

Parameters

IRF to aggregate TFP shock



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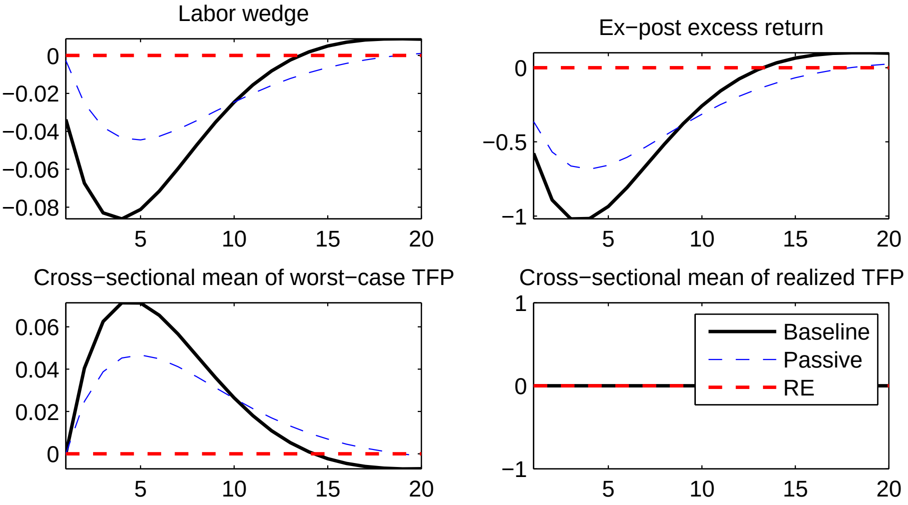
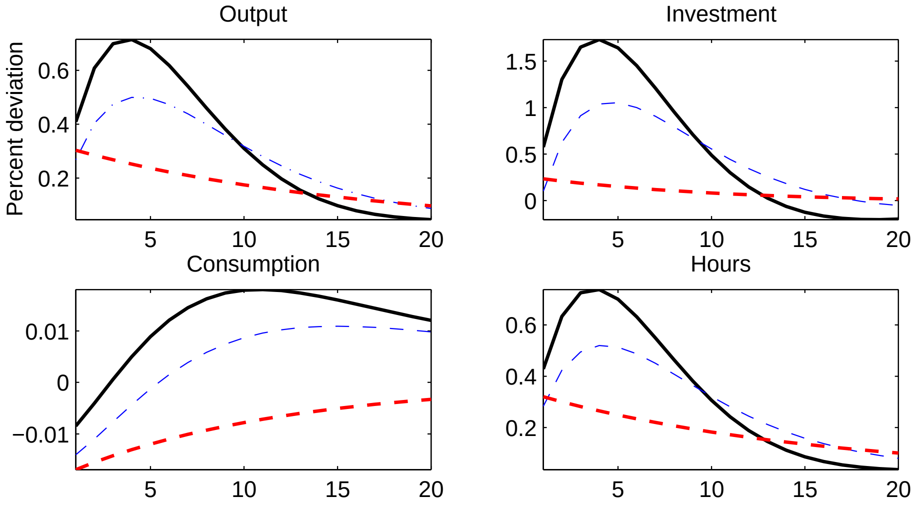


IRF to government spending shock



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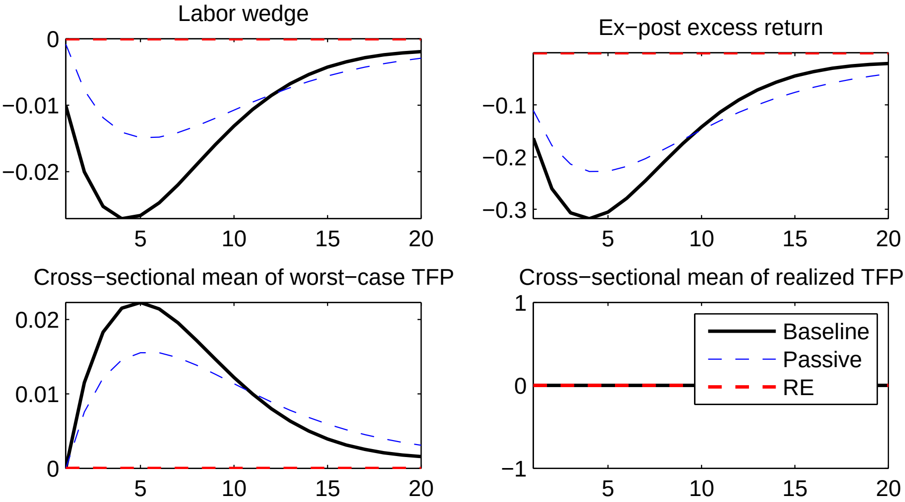

Bayesian estimation on US aggregate data

- Linearization ⇒ estimation using standard Kalman filter
- Quantitative model with additional rigidities (CEE, 2005)
  - real: habit formation, investment adjustment costs
  - nominal: sticky prices and wages
- Shocks: TFP, G, mon. policy and 'financial wedge' shock

$$\Delta_t^k \simeq E_t^* R_{t+1}^k - R_t$$

- US Data: $Y_t, H_t, I_t, C_t, \pi_t, R_t, Spread_t$ (on BAA corporate bond)

$$\begin{aligned} Spread_t &\equiv R_t^k - R_{t-1} \\ &= \underbrace{\left(E_{t-1}^* R_t^k - R_{t-1}\right)}_{\text{wedge shock}} + \underbrace{\left(R_t^k - E_{t-1}^* R_t^k\right)}_{\text{endogenous uncertainty}} \end{aligned}$$

  - estimate both flex and sticky price versions
  - stochastic singularity ⇒ iid measurement error

Results

1. Endogenous uncertainty: parsimonious friction $\Rightarrow$ reduce other rigidities

Model	η_a	Pr(price Δ)	Pr(wage Δ)	Inv. adj. cost	Habit
RE	0	0.24	0.04	0.3	0.62
Baseline	1.3	0.44	0.98	0.06	0.47

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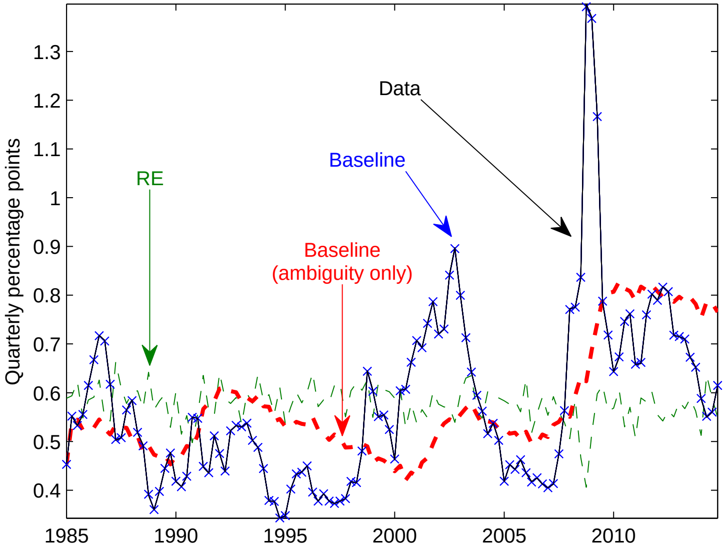
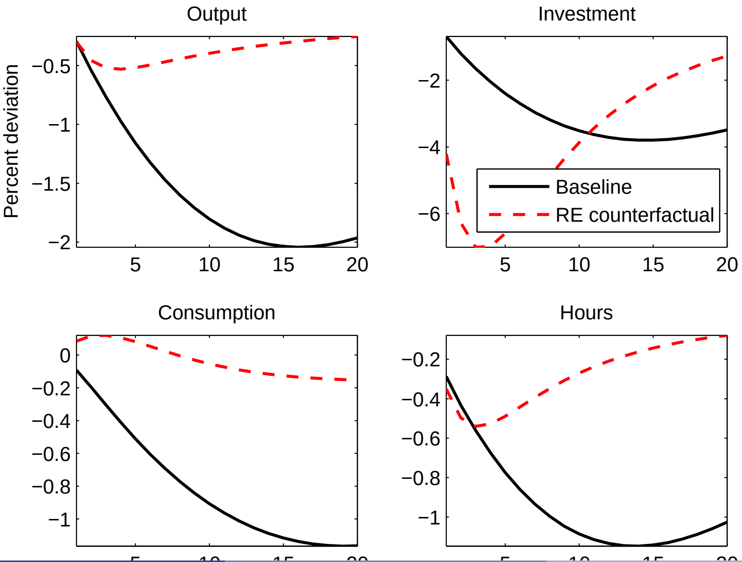
   - marginal data density is higher (both flex and sticky price versions)
   - under RE: observed spread is mostly just measurement error
   - but well fitted under model with endogenous uncertainty

Spread: data vs. models



Endogenous uncertainty: countercyclical spread ⇒ bus. cycle comovement



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3. Variance decomposition: financial shock more important with learning

Model (sticky price)	Y	H	I	C	π	R
RE	0.15	0.23	0.12	0.22	0.88	0.90
Baseline	0.73	0.81	0.76	0.61	0.88	0.84

Policy implication of endogenous uncertainty

- Endogenous uncertainty $\Rightarrow$ Policy matters
- Policy experiment:
  - modify Taylor rule to include adjustment to credit spread ϕ_{spread}
  - lower output growth variation: from stabilizing endogenous uncertainty

	Std. of output growth	
ϕ_{spread}	Baseline	Fixed uncertainty
0	0.60	0.60
-0.5	0.59	0.60
-1.0	0.57	0.60
-1.5	0.52	0.63

Conclusion

- Heterogeneous-firm business cycle model where firms face Knightian uncertainty about their own profitability
- Feedback loop between uncertainty and economic activity produces
  - Countercyclical labor wedge and ex-post excess return on capital
  - Co-movement in response to non-TFP shocks
  - Strong internal propagation with amplified and hump-shaped dynamics
- Estimation: inference on rigidities and shocks
- Policy implications

Interpreting the additive shock ($\nu_{l,t}$)

1. At the aggregate level, observationally equivalent to model where firms face unobservable demand shock
   - Each unit of good l : provides sum of good specific and idiosyncratic quality

$$\widetilde{Y}_{l,t} = \sum_{j=1}^{Y_{l,t}} (z_{l,t} + \tilde{\nu}_{l,j,t})$$

   - where units produced $Y_{l,t} = K_{l,t-1}^{\alpha} H_{l,t}^{1-\alpha}$
   - Noisy signal about persistent quality $z_{l,t}$: procyclical precision

$$\widetilde{Y}_{l,t}/Y_{l,t} = z_{l,t} + \nu_{l,t},\ \nu_{l,t} \sim N\left(0, \frac{\sigma_{\tilde{\nu}}^2}{Y_{l,t}}\right)$$

   - demand is a function of estimate of quality $z_{l,t}$

2. Aggregation of production units with common and idio shocks

Return

Kalman filter

- Estimate

$$\tilde{z}_{I,t|t} = \tilde{z}_{I,t|t-1} + Gain_{I,t}(Y_{I,t}/A_t - \tilde{z}_{I,t|t-1}F_{I,t})$$

- Kalman gain

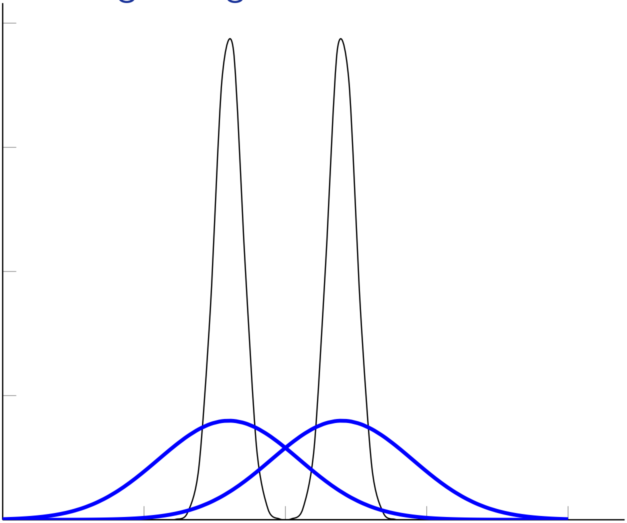
$$Gain_{I,t} = \left[\frac{F_{I,t}^2\Sigma_{I,t|t-1}}{F_{I,t}^2\Sigma_{I,t|t-1} + \sigma_{\nu,t}^2}\right]F_{I,t}^{-1}$$

- Mean square error

$$\begin{aligned}\Sigma_{I,t|t} &= (1 - Gain_{I,t}F_{I,t})\Sigma_{I,t|t-1} \\ &= \frac{\sigma_{\nu,t}^2\Sigma_{I,t|t-1}}{F_{I,t}^2\Sigma_{I,t|t-1} + \sigma_{\nu,t}^2}\end{aligned}$$

Return

Illustration: distinguishing distributions



Return

Relative entropy distance

Agents consider the conditional means $\mu^*_{l,t+1}$ that are sufficiently close to the long run average of zero in the sense of relative entropy:

$$\frac{(\mu^*_{l,t+1})^2}{2\rho_z^2\Sigma_{l,t|t}} \leq \frac{1}{2}\eta_a^2$$

- LHS: relative entropy between two normal distributions that share the same variance $\rho_z^2\Sigma_{l,t|t}$ but have different means ($\mu^*_{l,t+1}$ and zero)

Return

Linearized solution

1. Filtering problem is linear $\rightarrow$ analytic law of motion for $\Sigma_{l,t|t}$
   - Inputs have first-order effect on the level of posterior variance

$$\hat{\Sigma}_{l,t-1|t-1} = \varepsilon_{\Sigma,\Sigma}\hat{\Sigma}_{l,t-2|t-2} - \varepsilon_{\Sigma,F}\hat{F}_{l,t-1}, \tag{1}$$

2. First-order feedback from uncertainty to decision rules through $-a_{l,t}$

$$E_t^*\hat{z}_{l,t} = \varepsilon_{z,z}\hat{\hat{z}}_{l,t-1|t-1} - \varepsilon_{z,\Sigma}\hat{\Sigma}_{l,t-1|t-1}, \tag{2}$$

3. In turn, linear decision rules $\rightarrow$ easy aggregation
   - Cross-sectional mean: sufficient statistic for tracking distributions

$$E_t^*\hat{z}_t = \underbrace{\varepsilon_{z,z}\hat{\hat{z}}_{t-1|t-1}}_{=0} - \varepsilon_{z,\Sigma}\varepsilon_{\Sigma,\Sigma}\hat{\Sigma}_{t-2|t-2} + \varepsilon_{z,\Sigma}\varepsilon_{\Sigma,F}\hat{F}_{t-1} \tag{3}$$

where $\hat{x}_t \equiv \int \hat{x}_{l,t}dl$

Return

Recursive competitive equilibrium

- Household's problem at stage 1 : *hats* RVs resolved at stage 2

$$V_1^h(\overrightarrow{\theta}_l, B; \xi_1, X) = \max_H \left\{ -\varphi \frac{H^{1+\eta}}{1+\eta} + E^*[V_2^h(\hat{m}; \hat{\xi}_2, X)] \right\}$$
$$\text{s.t.} \quad \hat{m} = WH + RB + \int (\hat{D}_l + \hat{P}_l)\theta_l dl - G \tag{4}$$

- Household's problem at stage 2:

$$V_2^h(m; \xi_2, X) = \max_{C, \overrightarrow{\theta_l}', B'} \left[\ln C + \beta \int V_1^h \left(\overrightarrow{\theta_l}', B'; \xi_1', X' \right) dF(X'|X) \right]$$
$$\text{s.t. } m \geq C + B' + \int P_l \theta_l' dl; \quad \xi_1' = \Gamma(\xi_2, X) \tag{5}$$

Recursive competitive equilibrium

- Firm l's problem at stage 1

$$v_1^f(\tilde{z}_l, \Sigma_l, K_l; \xi_1, X) = \max_{H_l} E^*[v_2^f(\hat{\tilde{z}}_l', \Sigma_l', K_l; \hat{\xi}_2, X)]$$
$$\text{s.t.} \quad \text{Updating rules of Kalman filter} \tag{6}$$

- Firm l's problem at stage 2: $v_2^f(\tilde{z}_l', \Sigma_l', K_l; \xi_2, X)$ equals

$$\max_{I_l} \left[\lambda \left(Y_l - WH_l - I_l \right) + \beta \int v_1^f \left(\tilde{z}_l', \Sigma_l', K_l; \xi_1', X' \right) dF(X'|X) \right]$$
$$\text{s.t. } K_l' = (1-\delta)K_l + I_l; \ \xi_1' = \Gamma(\xi_2, X) \tag{7}$$

Return

Parameters

γ	Labor augmenting tech growth	1.004
α	Capital share	0.3
β	Discount factor	0.99
η	Inverse Frisch elasticity	0
δ_0	SS depreciation	0.025
δ_2/δ_1	Convexity of depreciation	0.15
η_a	Size of entropy constraint	0.4
$\bar{\Sigma}$	SS posterior variance	0.1
	(Kalman gain)	0.47
$\bar{g}$	SS share of gov spending	0.2
ρ_z	Idiosyncratic TFP	0.5
σ_z	Idiosyncratic TFP	0.4
ρ_A	Aggregate TFP	0.95
ρ_g	Government spending	0.95
ρ_σ	Firm-level dispersion	0.85

Return

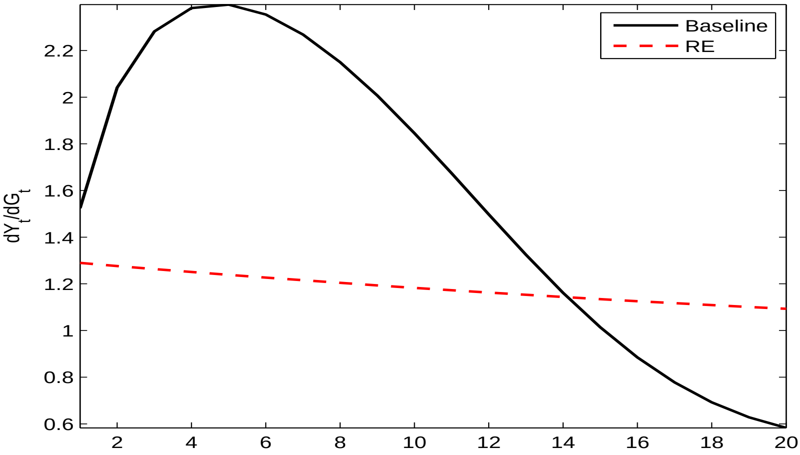
HP-filtered moments (TFP shock only)

	Data	Our model	RE
$\sigma(y)$	1.11	1.11	0.49
$\sigma(c)/\sigma(y)$	0.72	0.11	0.17
$\sigma(i)/\sigma(y)$	3.57	2.95	3.23
$\sigma(h)/\sigma(y)$	1.64	1.02	0.86
$\sigma(c, y)$	0.86	0.72	0.85
$\sigma(i, y)$	0.92	0.99	0.99
$\sigma(h, y)$	0.88	0.99	0.99
$\sigma(y, \tau_l)$	-0.83	-0.95	0
$\sigma(h, \tau_l)$	-0.97	-0.95	0
$\sigma(y_t, y_{t-1})$	0.89	0.87	0.66
$\sigma(h_t, h_{t-1})$	0.95	0.88	0.66
$\sigma(\Delta y_t, \Delta y_{t-1})$	0.39	0.44	-0.06
$\sigma(\Delta h_t, \Delta h_{t-1})$	0.71	0.52	-0.06

Note: We choose the st. dev of aggregate TFP shock so that the output st. dev in the model matches the data.

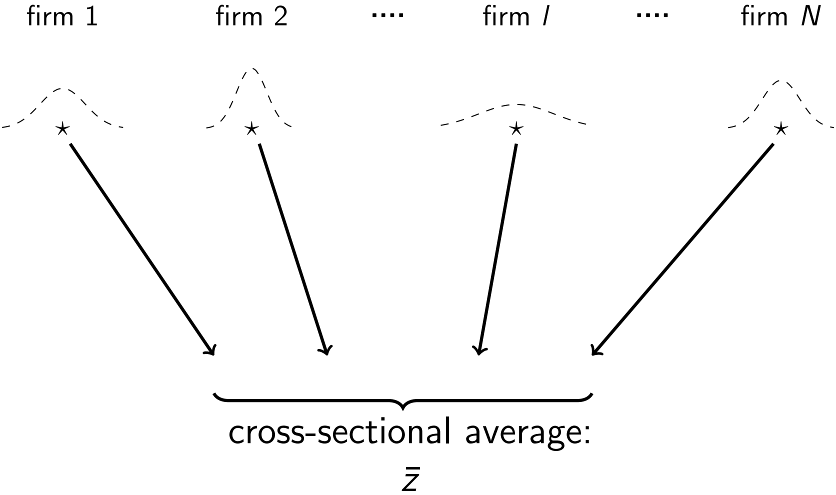
Return

Government spending multiplier



Return

Law of large numbers for risky random variables



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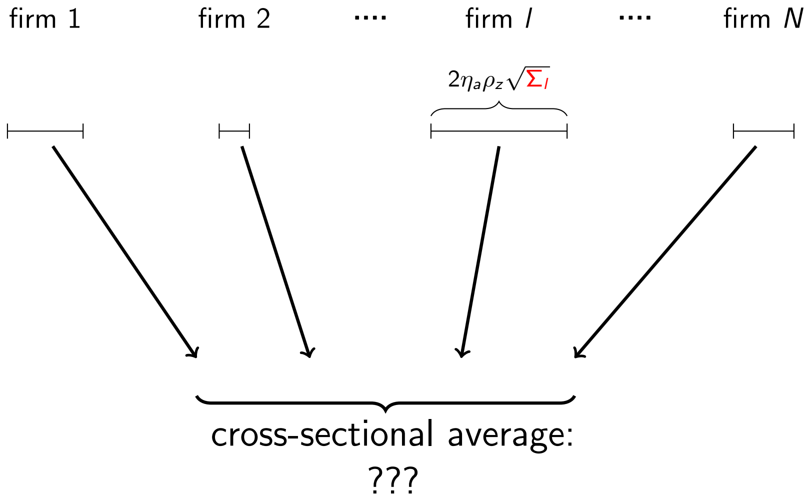

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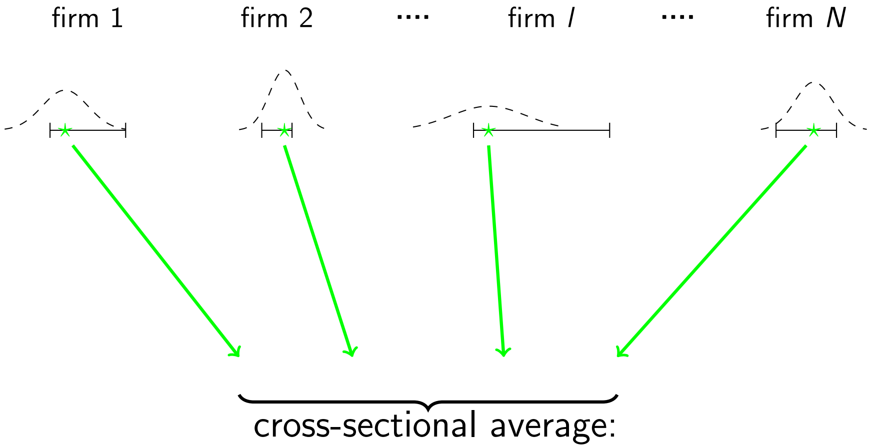
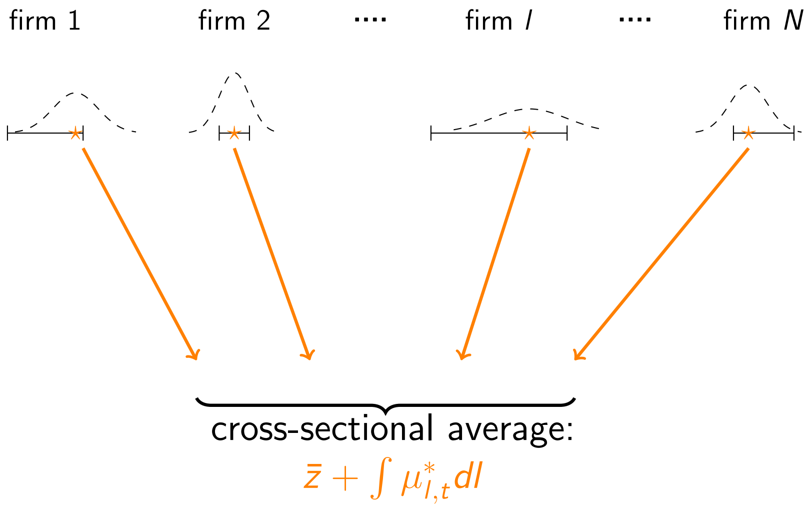
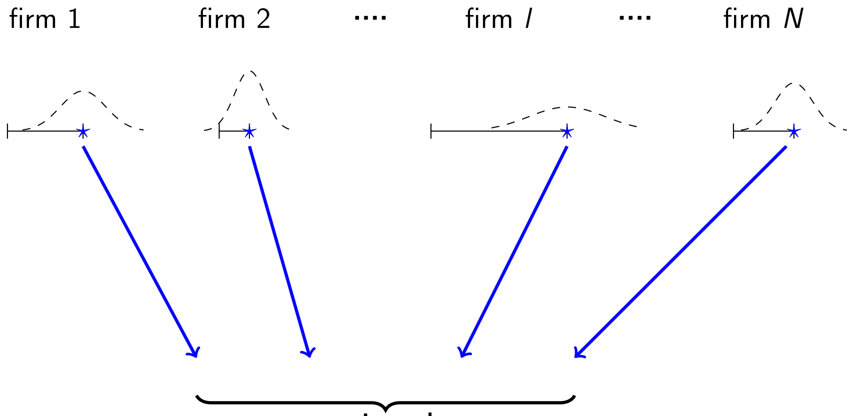


$\bar{z} + \int \mu^*_{l,t} dl$

Law of large numbers for ambiguous random variables



Law of large numbers for ambiguous random variables



cross-sectional average:

$$\left[\bar{z} - \int a_{l,t} dl,\ \bar{z} + \int a_{l,t} dl\right]$$

Law of large numbers for ambiguous random variables