A theory of nonperforming loans and debt restructuring

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Nonperforming loans

A loan is classified as nonperforming when payments of interest and/or principal are past due by 90 days or more.
Non performing loans in Euro area and Japan


Kobayashi and Nakajima

Nonperforming loans and debt restructuring
Non performing loans in some European countries

• Nonperforming loans can be a significant source of distortion.
• Our theory is related to but different from debt overhang.
  • Having nonperforming loans is different from just having a lot of debt.
• What is special about nonperforming loans?
  • When loans are nonperforming, the contractual value of debt is different from the present discounted value of repayments.
  • In other words, the value of debt is no longer a “state variable.”
- Borrowing constraint arises because the borrower may default at any time.
- There exists a maximum amount of debt that the borrower can repay.

What happens if the amount of debt exceeds the repayable amount?
- This may happen, for instance, if the borrower’s productivity declines, or if the value of the collateral asset falls.

The lender has two options:
- rewrite the contract and reduce the amount of debt (debt restructuring);
- retain the right to the original amount of debt (non-performing loans).
If the bank reduces the debt, the levels of lending and output converge to their first-best levels in a finite period of time.

- This is a kind of debt overhang, but inefficiency only lasts temporarily.

If the bank chooses not to do so, the loans become nonperforming.

- The PDV of repayments is lower than the contractual value of debt.
- The equilibrium level of output is permanently lower than the first-best level.

The value obtained by the bank is higher when the debt is restructured (reduced to a repayable amount).

- Why would the bank choose not to do that?

If the reduction of debt involves bargaining, the agreement may not be reached instantly, and debt restructuring could be delayed.

- We apply the model of Abreu and Gul (2000) to illustrate this point.
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a deterministic version of Albuquerque and Hopenhayn (2004).

A bank lends to a firm.

$r =$ common discount rate.

$D_0 =$ initial debt of the firm.

$b_t =$ repayment from the firm to the bank:

$$\dot{D}_t = rD_t - b_t.$$  

$k_t =$ short-term loans (working capital) that the firm borrows from the bank:

$F(k_t) =$ output produced using $k_t$.

$x_t =$ dividends to the owners of the firm:

$$x_t = F(k_t) - rk_t - b_t.$$  

Limited liability:

$$x_t \geq 0.$$  

Enforcement constraint

- $V_t = \text{value to the firm's owners:}$

$$V_t = \int_t^\infty e^{-r(s-t)} x_s \, ds.$$  

- The firm can choose to default at any time $t$, after receiving working capital $k_t$.
  - $G(k_t) = \text{the value of the outside opportunity of the firm.}$
  - The bank would receive none when the firm defaults.

- Enforcement constraint:

$$V_t \geq G(k_t).$$

- The liquidation value of the firm is assumed to be zero.
Plans

- At each time $t$, the contract between the bank and the firm specifies $(D_t, r)$.
- Then, given $(D_t, r)$, the bank offers a plan $\{k_{t+s}^t, b_{t+s}^t, x_{t+s}^t\}_{s \in \mathbb{R}_+}$ to the firm:
  - $k_{t+s}^t = \text{working capital provided at time } t + s$;
  - $b_{t+s}^t = \text{repayment at } t + s$;
  - $x_{t+s}^t = F(k_{t+s}^t) - rk_{t+s}^t - b_{t+s}^t$.
- The associated values for the bank and the firm are:
  
  $$D_{t+s}^t = \int_{t+s}^{\infty} e^{-r(u-t-s)} b_u^t \, du,$$
  $$V_{t+s}^t = \int_{t+s}^{\infty} e^{-r(u-t-s)} x_u^t \, du.$$
- In equilibrium, the bank’s offers must be time consistent, i.e.,
  
  $$k_s^t = k_s^{t'}, \quad b_s^t = b_s^{t'}, \quad x_s^t = x_s^{t'}, \quad \text{for all } t < t' \leq s \in \mathbb{R}_+.$$
Feasible plans

- A plan offered at time $t$ is **feasible** if the limited liability and enforcement conditions are satisfied for all $s \geq 0$:

$$0 \leq x_{t+s}, \quad \text{and} \quad G(k_{t+s}^t) \leq V_{t+s}^t.$$ 

- $\Gamma = \text{the set of all feasible plans}.$
- $\Gamma(D) = \text{the set of all feasible plans such that the value to the bank is } D:$

$$D = \int_0^\infty e^{-rt}b_t \, dt$$

- $D_t$ is the state variable in this model.
  - We shall consider the “best” Markov plans under different circumstances.
First-best level of production

- $k^* = \text{the first-best level of production:}

  \[ F'(k^*) = r. \]

  Associated with $k^*$, define:

  \[
  V^* = G(k^*), \\
  x^* = rV^*, \\
  b^* = F(k^*) - rk^* - x^*, \\
  D^* = \frac{b^*}{r}.
  \]

  If $D_0 \leq D^*$, the first-best plan with $k_t^0 = k^*$ for all $t$ is feasible.
Efficient plans

- Given $D \in \mathbb{R}_+$, the (constrained) efficient plan is a plan that solves

$$\max_{\{k_t, b_t, x_t\} \in \mathbb{R}_+} \int_0^\infty e^{-rt} x_t \, dt$$

- The efficient plans are expressed using the value of debt as a state variable:
  - There exists a maximum value of debt, $D_{\text{max}}$, which can be repaid by the firm.
  - $V_t = V_e(D_t)$, where $V_e : [0, D_{\text{max}}] \to \mathbb{R}_+$ is a strictly decreasing function.
  - $k_t, x_t$, and $b_t$ are given as

$$k_e(D_t) = \begin{cases} 
G^{-1}[V_e(D_t)], & \text{for } D_t > D^*, \\
k^*, & \text{for } D_t \leq D^*,
\end{cases}$$

$$x_e(D_t) = \begin{cases} 
0, & \text{for } D_t > D^*, \\
rV_e(D_t), & \text{for } D_t \leq D^*,
\end{cases}$$

$$b_e(D_t) = F[k_e(D_t)] - rk_e(D_t) - x_e(D_t),$$
Dynamics of the efficient plans

- If $D_0 \leq D^*$, the first-best is attained in the efficient plan:

$$k_{e,t} = k^*, \text{ for all } t \geq 0.$$ 

- For $D_0 \in (D^*, D_{\text{max}}]$, the level of production is inefficiently low initially (debt overhang), but converges to the first-best level in finite time.

  - Let

$$\bar{t} \equiv \frac{1}{r} \ln \left( \frac{V^*}{V_e(D_0)} \right).$$

Then

$$V_{e,t} = \begin{cases} 
  e^{rt} V_e(D_0), & \text{for } t < \bar{t}, \\
  V^*, & \text{for } t \geq \bar{t},
\end{cases}$$

$$k_{e,t} = \begin{cases} 
  G^{-1}(V_{e,t}), & \text{for } t < \bar{t}, \\
  k^*, & \text{for } t \geq \bar{t},
\end{cases}$$
Markov perfect equilibrium

- At each point in time $t$, given contract $(D_t, r)$, the bank offers a plan $\{k_{t+s}^t, b_{t+s}^t, x_{t+s}^t\}_{s \in \mathbb{R}_+}$ to the firm subject to the constraint:

$$\int_0^\infty e^{-r(s-t)} b_{t+s}^t \, ds, \leq D_t, \quad \text{and} \quad x_{t+s}^t \geq 0.$$ 

Then, given this offer, the firm decides whether or not to default.

- The efficient plan $\{k_e(D), x_e(D), b_e(D), V_e(D)\}$ is attained as a Markov perfect equilibrium with the following strategies:

1. at each time $t$, the bank offers $\{k_{t+s}^t, b_{t+s}^t, x_{t+s}^t\}_{s \in \mathbb{R}_+}$ such that $k_{t+s}^t = k_e(D_{t+s})$, $b_{t+s}^t = b_e(D_{t+s})$, and $x_{t+s}^t = x_e(D_{t+s})$, where $D_{t+s}$ is the solution to $\dot{D}_{t+s} = r D_{t+s} - b_e(D_{t+s})$ with initial value $D_t$;

2. given $(D_t, b_t^t, k_t^t, x_t^t)$, the firm defaults if either (i) $V_e(D) < G(k_t^t)$, or (ii) $x_t^t < x_e(D_t)$. 
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To analyze non-performing loans, suppose that there is an unexpected shock in period 0 so that

\[ D_0 > D_{\text{max}}. \]

Two options for the bank:
1. rewrite the contract to reduce the amount of debt to \( D_{\text{max}} \);
2. retain the right to \( D_0 \) with understanding the firm is never able to repay it.

If the debt is reduced to \( D_{\text{max}} \),
- then the efficient plan discussed in the previous section can be implemented.
- Nonperforming loans would not arise.

If the bank keeps the right to \( D_0 > D_{\text{max}} \),
- the PDV of future repayments to the bank would be less than the contractual value of the firm’s debt.
- The loan becomes nonperforming.
Contractual values of debt

- $D_0^c = \text{contractual value of debt in period 0.}$
- If the firm repays $\{b_t\}_{t \in \mathbb{R}^+}$, then the contractual value of debt evolves as
  \[ D_t^c = e^{rt} D_0 - \int_0^t e^{r(t-s)} b_s \, ds. \]
- $d_t(\{b_{t+j}\}_{j \in \mathbb{R}^+}) = \text{PDV of repayments} \ \{b_{t+j}\} \ \text{after} \ t$:
  \[ d_t(\{b_{t+j}\}_{j \in \mathbb{R}^+}) = \int_0^\infty e^{-rj} b_{t+j} \, dj. \]
- If $D_0^c > D_{\text{max}}$, then for any feasible repayment plan $\{b_t\}_{t \in \mathbb{R}^+}$,
  \[ D_t^c > d_t(\{b_{t+j}\}_{j \in \mathbb{R}^+}). \]

Thus, the bank also suffers from an enforcement problem.
Debt is no longer a state variable

- For $D_0^c > D_{\text{max}}$, $\Gamma(D_0^c) = \emptyset$.
- The bank can make an offer with the PDV of repayments less than $D_0^c$.
  - Thus, the set of feasible plans that the bank with $D_0^c$ can offer is
    \[ \overline{\Gamma}(D_0^c) \equiv \bigcup_{D \leq D_0^c} \Gamma(D). \]
- The set of feasible plans for the bank is independent of the value of initial debt if $D_0^c > D_{\text{max}}$:
  \[ \overline{\Gamma}(D_0^c) = \Gamma(D_{\text{max}}) = \Gamma, \quad \forall D_0^c > D_{\text{max}}. \]
- In other words, the value of debt is no longer a state variable.
Markov plans

- With $D_0^c > D_{\text{max}}$, there is no state variables.
  - Markov plans are constant plans.
- Let $\Gamma = \{ (k_t, b_t, x_t) \in \Gamma \mid (k_t, b_t, x_t) = (k, b, x), \ \forall t \in \mathbb{R}_+ \}$.
- The highest value the bank can obtain with Markov plans is:

$$\max_{\{k_t, b_t, x_t \} \in \Gamma} \int_0^\infty e^{-rt} b_t \, dt.$$ 

- The solution to this problem is given by $\{k_{\text{npl}}, b_{\text{npl}}, x_{\text{npl}}, D_{\text{npl}}, V_{\text{npl}}\}$, where $k_{\text{npl}}$ is the solution to:

$$F'(k_{\text{npl}}) = r + rG'(k_{\text{npl}}),$$

and $b_{\text{npl}} = F(k_{\text{npl}}) - rk_{\text{npl}} - rG(k_{\text{npl}})$, $x_{\text{npl}} = F(k_{\text{npl}}) - rk_{\text{npl}} - b_{\text{npl}}$, $D_{\text{npl}} = \frac{b_{\text{npl}}}{r}$, $V_{\text{npl}} = \frac{x_{\text{npl}}}{r}$. 
Markov Perfect Equilibrium

This can be obtained as a Markov Perfect Equilibrium with the following strategies:

1. at each time $t$, the bank offers $\{k_{t+s}, b_{t+s}, x_{t+s}\}_{s \in \mathbb{R}_+}$ such that $k_{t+s} = k_{npl}$, $b_{t+s} = b_{npl}$, and $x_{t+s} = x_{npl}$ for all $t$ and $s$;

2. given the offer $\{k_{t+s}, b_{t+s}, x_{t+s}\}_{s \in \mathbb{R}_+}$ from the bank, the firm defaults if either (i) $G(k_t^t) > V_{npl}$ or (ii) $x_t^t < x_{npl}$. 

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Nonperforming loans and debt restructuring
Persistence of inefficiency

- Inefficiency lasts permanently:

\[ k_t = k_{npl} < k^*. \]

- Note that \( D_{max} > D_{npl} \), i.e., the value to the bank is higher when debt is restructured.
  - Then why would the bank choose not to restructure debt?
  - If debt restructuring is costly, and the cost exceeds \( D_{max} - D_{npl} \), then the bank would choose to hold nonperforming loans.

- But even without such costs, if debt restructuring involves bargaining, then there can be an inefficient delay in reaching an agreement.
Inefficient delays in bargaining

  - The unique SPE is efficient (the agreement is reached immediately).
- Inefficient delay may occur with asymmetric information:
  - Abreu and Gul (2000), Feinberg and Skrzypacz (2005), Fuchs and Skrzypacz (2010), etc.
- Here we apply the model of Abreu and Gul (2000).
  - Debt restructuring is inefficiently delayed, and loans become nonperforming.
Abreu and Gul (2000)

- Two agents bargain over their shares of a pie.
- Each agent may be either “rational” or “irrational.”
  - An irrational type is identified by a fixed offer and acceptance rule.
- Independence-from-procedures result:
  - Regardless of the details of the bargaining protocol, the equilibrium distribution of outcomes in discrete-time bargaining games converge to the same limit.
  - This limit corresponds to the (unique) equilibrium in the continuous-time bargaining game with a war of attrition structure.
    - The rational type of each agent pretends to be their irrational type.
    - Their strategy is described by a distribution over the time to concede.
- The equilibrium exhibits inefficient delay.
  - As the probability of irrationality goes to zero, delay and inefficiency disappear.
Two banks

- Continue to consider the case where $D_0^c > D_{\text{max}}$.
- Bank $i$ holds a share $\omega^i \in (0, 1)$ of $D_0^c$.
  - Before debt restructuring, if the firm repays $\hat{b}_t$, then bank $i$ receives $\omega^i \hat{b}_t$.
- Two banks bargain over their shares of the value of the debt after it is reduced to $D_{\text{max}}$.
- Simplifying assumptions:
  - When the two banks bargain over their shares of $D_{\text{max}}$, they take as given the repayments $\{\hat{b}_t\}$ that the firm makes before debt restructuring.
  - On the other hand, the repayments before debt restructuring, $\{\hat{b}_t\}$, are determined to maximize the joint surplus of the banks taking as given the equilibrium in the bargaining game.
Bargaining between the two banks

- The irrational type of bank $i$ is identified by a number $\alpha^i \in (0, 1)$.
  - It always demands $\alpha^i D_{\text{max}}$ and would accept the offer from the other bank if and only if its share is greater than or equal to $\alpha^i$.
  - $z^i =$ initial probability that bank $i$ is irrational.
- Each bank’s strategy is described by a cdf function $\Phi^i(t)$, i.e., the probability that lender $i$ concedes to the other lender by time $t$ (inclusive).
- In equilibrium, there exists a time $T^0 > 0$ such that
  - $\Phi^i(t)$ is continuous for all $t > 0$ and $i = 1, 2$;
  - $\Phi^i(t)$ is constant for $t \geq T^0$ and $i = 1, 2$;
  - $\Phi^i(t)$ is strictly increasing for $t \in [0, T^0)$ and $i = 1, 2$. 
Given \( \{ b_t \} \) and \( \Phi^j(t) \), the expected value of bank \( i \) when it concedes to bank \( j \) at time \( t \) is:

\[
u^i_t = \int_{s=0}^{t} \left\{ \int_{w=0}^{s} e^{-rw} \omega^i \hat{b}_w \, dw + e^{-rs} \alpha^i D_{\max} \right\} d\Phi^j(s)
\]

\[+ [1 - \Phi^j(t)] \left\{ \int_{s=0}^{t} e^{-rs} \omega^i \hat{b}_s \, ds + e^{-rt}(1 - \alpha^j) D_{\max} \right\} \]

Using the condition that \( \frac{d\nu^i_t}{dt} = 0 \) for \( t \in (0, T^0) \), the equilibrium is given by:

\[
\Phi^j(t) = \begin{cases} 
1 - c^j \exp \left( - \int_0^t \lambda^j(s) \, ds \right), & t < T^0, \\
1 - z^j, & t \geq T^0.
\end{cases}
\]

where

\[
\lambda^j(t) \equiv \frac{(1 - \alpha^j)r - \beta^i(t)}{\alpha^1 + \alpha^2 - 1},
\]

with \( \beta^i(t) \equiv \frac{\omega^i \hat{b}_t}{D_{\max}} \).
(c^1, c^2, T^0) is determined as follows:

\[ T^0 \equiv \min( T^1, T^2), \]

where \( T^i \) is defined implicitly by

\[ 1 - \exp \left( \int_0^{T^i} \lambda^i(s) \, ds \right) = 1 - z^i, \]

and \( c^i \) is determined by

\[ 1 - c^i \exp \left( \int_0^{T^0} \lambda^i(s) \, ds \right) = 1 - z^i. \]
Define $\Phi(t) = \text{the probability that either one of the two lenders concede by time } t$:

$$\Phi(t) = \begin{cases} 
1 - c \exp \left( - \int_0^t \lambda(s) \, ds \right), & t < T^0, \\
1 - z, & t \geq T^0,
\end{cases}$$

where $c \equiv c^1c^2$, $\lambda(s) \equiv \lambda^1(s) + \lambda^2(s)$, and $z = z^1z^2$.

- Given $\Phi$ and $T^0$, we consider a Markov plan that maximizes the joint value of the two banks.
- Because of $T^0$, time $t$ is a payoff-relevant state variable in this problem.
Let \( \{ \hat{k}_t, \hat{b}_t, \hat{x}_t \} \) is the plan before the debt reduction.

Conditional on the event that debt restructuring has not been done by \( t \), the values to the banks and the firm are:

\[
\hat{D}_t = \int_t^{T_0} \left\{ \int_t^s e^{-r(w-t)} \hat{b}_w \, dw + e^{-r(s-t)} D_{\max} \right\} \frac{d\Phi(s)}{1 - \Phi(t)} \\
+ \frac{1 - \Phi(T_0)}{1 - \Phi(t)} e^{-r(T_0 - t)} D_{\text{npl}},
\]

\[
\hat{V}_t = \int_t^{T_0} \left\{ \int_t^s e^{-r(w-t)} \hat{x}_w \, dw + e^{-r(s-t)} V_{\min} \right\} \frac{d\Phi(s)}{1 - \Phi(t)} \\
+ \frac{1 - \Phi(T_0)}{1 - \Phi(t)} e^{-r(T_0 - t)} V_{\text{npl}}.
\]

Given \( \Phi \) and \( T_0 \), let \( \hat{\Gamma}(t) \) be the set of all plans \( \{ \hat{k}_t, \hat{b}_t, \hat{x}_t \}_{t \in [0, T_0)} \) such that

\[
\hat{V}_t \geq G(\hat{k}_t), \quad \text{and} \quad \hat{x}_t = F(\hat{k}_t) - r \hat{k}_t - \hat{b}_t \geq 0,
\]

where \( \hat{V}_t \) is given as above.
The Markov plan that maximizes the joint surplus of the two banks is:

$$\max_{\{\hat{k}_t, \hat{b}_t, \hat{x}_t\} \in \hat{\Gamma}(0)} \hat{D}_0.$$ 

In this solution, $\hat{x}_t = 0$ for all $t < T^0$ and

$$\hat{V}_t = V_{\min} \int_t^{T^0} \lambda(s) \exp \left( - \int_s^T [\lambda(w) + r] \, dw \right) \, ds$$

$$+ V_{\text{npl}} \exp \left( - \int_t^{T^0} [\lambda(s) + r] \, ds \right).$$

Given $\hat{V}_t$ for $t < T^0$,

$$\hat{k}_t = G^{-1}(\hat{V}_t), \quad \hat{b}_t = F(k_t) - r\hat{k}_t.$$ 

The equilibrium as a whole is given by $(\{\hat{b}_t, \hat{k}_t, \hat{x}_t\}_{t \in [0, T^0]}, \Phi)$ that jointly solves these two sets of the conditions.
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Nonperforming loans in Japan (relative to GDP)

Notes: Outstanding loans are measured as a fraction of GDP. Sources: Financial Services Agency, The Japanese Government, Status of Non-Performing Loans.

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Interpretation of Japan’s lost decades

- Evidence on evergreening and “zombie firms” in Japan:
  - Peek and Rosengren (2005), Caballero, Hoshi, and Kashyap (2008), etc.
  - Fukuda and Nakamura (2011): Most firms which are identified as zombies by Caballero, Hoshi and Kashyap (2008) did recover substantially in the 2000s.

- In the 1990s, nonperforming loans piled up and evergreening was widespread.
  - It created zombie firms as discussed by Caballero, Hoshi, and Kashyap (2008).

- In the 2000s, the bankruptcy and reorganization procedures were reformed.
  - The Civil Rehabilitation Law was enacted in 2000 and the Alternative Dispute Resolution Law followed in 2004.
  - Outstanding debt decreased rapidly, and most zombie firms recovered as shown by Fukuda and Nakamura (2011).
Summary: Debt restructuring

- Suppose that $D_0^c > D_{\text{max}}$.
- Debt restructuring:
  - The bank reduces $D_0^c$ to $D_{\text{max}}$.
- The contractual value of debt is used as a state variable.
- The efficient plan is the solution to
  \[
  \max_{\{k_t, b_t, x_t\} \in \mathbb{R}^+ \in \Gamma(D_{\text{max}})} \int_0^\infty e^{-rt} x_t \, dt
  \]
  which has a Markovian form: \(\{k_e(D), x_e(D), b_e(D)\}\).
- Inefficiency (debt overhang) only lasts temporarily.
  - The first best allocation is attained in a finite period of time.
Summary: Nonperforming loans

- Suppose that the bank does not reduce $D_0^c$.
  - The loan becomes nonperforming.
  - The PDV of repayments is less than the value of debt.
- The contractual value of debt is no longer a state variable.
  - The set of feasible plans does not depend on the value of debt on the contract.
- Markov plans are constant plans.
  - Let $\bar{\Gamma}$ = the set of constant feasible plans.
- The most profitable Markov plan for the bank is the solution to
  \[
  \max_{\{k_t, b_t, x_t\}_{t \in \mathbb{R}_+} \in \bar{\Gamma}} \int_0^\infty e^{-rt} b_t \, dt
  \]
  which is given as $\{k_{npl}, x_{npl}, b_{npl}\}$.
- Inefficiency lasts permanently.
- Note, however, that $D_{\text{max}} > D_{npl}$.
Summary: Bargaining with two banks

- Apply Abreu and Gul’s model of bargaining with asymmetric information.
  - Each bank may be irrational with a fixed offer and acceptance rule.
- The equilibrium in the bargaining game between the two banks exhibits an inefficient delay.
  - Debt restructuring is not done immediately, and the loan becomes nonperforming.
- The time $t$ becomes a state variable.
  - $\hat{\Gamma}(t) =$ the set of all feasible plans after $t$ (before debt restructuring).
- The Markov plan that maximizes the joint surplus for the banks is the solution to:
  \[
  \max_{\{\hat{k}_t, \hat{b}_t, \hat{x}_t\} \in \hat{\Gamma}(0)} \hat{D}_0,
  \]
  which has a Markovian form: $\{\hat{k}(t), \hat{x}(t), \hat{b}(t)\}$. 