Fiscal policy and debt management with incomplete markets

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Classical question in macro public finance

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  What is the "right" level of public debt?
  How quickly debt should be repaid?
  How much debt and taxes should be used to respond to agg shock?
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- Renewed interest in the aftermath of 2008 crisis
- Concerns that current debt levels are "too high" for rich countries...
- And "too low" (negative) for China, Norway,...
• A theory of optimal public debt management
  • Ramsey planner with distortionary taxation and incomplete markets
• Contribution: develop quadratic approximations that characterize moments of the invariant distribution in closed form
• Derive explicit formulas ("sufficient statistics") for the moments of the invariant distribution
This paper

- Most of the focus:
  - mean ("target") debt level
  - speed of reversion to the target
  - variance of debt in the invariant distribution

- Key insight: optimal debt minimizes risk for the gov’t

- Other questions that our framework addresses
  - what is the optimal composition of portfolio of gov’t debts?
  - how should gov’t debt respond to shocks?
  - how should government set taxes, transfers, tax rates over the cycle?
Results

• Main formulas:

\[
\text{target debt} = - \frac{\text{cov (returns, deficit)}}{\text{var (returns)}}
\]

\[
\text{speed of convergence} = \frac{1}{1 + \beta^2 \text{var (returns)}}
\]

• Here:
  • returns: MU-adjusted returns on gov’t portfolio of debts/assets
  • deficit: MU-adjusted present value of primary deficits

• Sufficient statistics: can be easily computed given observed data
Optimal debt level keeping maturity constant:

- target debt level: -7% of GDP
- speed of mean reversion: 250 years (half life)
- std. deviation: 0.26

Tax rates are persistent and smooth

Taxes and debt have similar volatility in the data but are less persistent
Related literature

   - any debt level is optimal, all fiscal hedging through (equivalent of) Arrow securities
   - hard to see how to achieve that with real world instruments

2. Incomplete markets: Barro, Bohn, Faraglia-Marcet-Scott, Lustig-Sleet-Yeltekin
   - mostly numerical, often for models with counterfactual returns
   - analytics (Barro): any debt level is optimal

   - can get their results in the limit, knife-edge cases

4. Portfolio theory: Markowitz, Merton, ...
   - GE, benevolence, interaction of portfolio decisions with taxation

5. Nominal debt, possibility of default
   - have not studied, but our approach should work there too
The simplest model

- Continuum of identical agents with preferences
  \[ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ c_t - \frac{1}{1 + \gamma} l_t^{1+\gamma} \right] \]

- No capital + exogenous gov't expenditures
  \[ c_t + g_t = l_t \]

- Gov't can use proportional tax \(\tau_t\) and trade with agents one-period security (in zero net supply) at price \(q_t\) with stochastic payoff \(p_t\)
  \[ g_t + p_t B_{t-1} = \tau_t l_t + q_t B_t \]

- iid shocks for \((g_t, p_t)\), \(B_t\) is in a compact set
- Let \(B_t \equiv q_t B_t\), \(R_t \equiv p_t / q_{t-1}\)
Characterization

Lemma

\[ \{ c_t, l_t, R_t, B_t, \tau_t \}_{t=0}^\infty \text{ is a competitive equilibrium if and only if} \]

\[ \{ l_t, B_t \}_{t=0}^\infty \text{ satisfies} \]

\[
\underbrace{l_t - l_t^{1+\gamma} + B_t}_{= \tau_t l_t} = R_t B_{t-1} + g_t
\]

- Easier to express hours as a function of tax revenues \( Z \)

\[
Z \equiv l(Z) - l(Z)^{1+\gamma}
\]

\[
\Psi(Z) = \frac{1}{1 + \gamma} l(Z)^{1+\gamma}
\]

- Consumption is a residual

\[
c_t = (1 + \gamma) \Psi(Z_t) + R_t B_{t-1} - B_t
\]
Ramsey problem in recursive form

- Bellman equation (state $s = (g, p)$):

\[
V(B) = \max_{\{Z(s), B'(s)\}} \mathbb{E} \left[ R B - B' + \gamma \Psi(Z) + \beta V(B') \right]
\]

subject to

\[
Z(s) + B'(s) = R(s) B + g(s) \quad \text{for all } s
\]

- Policy functions $\tilde{B}(B, s), \tilde{Z}(B, s), \tilde{\tau}(B, s)$ induce optimum

$\{\tilde{B}_t, \tilde{Z}_t, \tilde{\tau}_t\}_t$
Optimal policy

- **Monotonicity**: $\tilde{B}, \tilde{Z}, \tilde{\tau}$ are increasing in $E$
- **Distortion smoothing**: 
  $$V' (\tilde{B}_t) = \mathbb{E}_t V' (\tilde{B}_{t+1}) + \beta \text{cov}_t (R_{t+1}, V' (\tilde{B}_{t+1}))$$
- **Uniqueness**: $\tilde{B}_t$ converges to a unique invariant distribution
Optimal policy

- Our goal: characterize properties of the invariant distribution
- Amount of risk depends on debt level:
  \[ E(B, s) = R(s)B + g(s) \]
- Let \( B^* \) be the debt level that minimizes \( \text{var}(E(B, \cdot)) \):
  \[ B^* \equiv -\frac{\text{cov}(R, g)}{\text{var}(R)} \]
- Let \( Z^* \) be the level of tax revenues that satisfies budget constraint in expectation
  \[ Z^* \equiv \bar{g} + \frac{1 - \beta}{\beta}B^* \]
Special case: p and g are perfectly correlated

- If $\text{corr}(p, g) = \pm 1$ then $E(B^*, s)$ is independent of $s$
  - risk is completely eliminated if $B_t = B^*$
- Monotonicity of policy rules:
  \[
  B < B^* \implies \text{cov}(R(\cdot), V'(\tilde{B}(B, \cdot))) > 0 \\
  B = B^* \implies \text{cov}(R(\cdot), V'(\tilde{B}(B, \cdot))) = 0 \\
  B > B^* \implies \text{cov}(R(\cdot), V'(\tilde{B}(B, \cdot))) < 0
  \]
- Euler equation and Martingale convergence theorem imply
  \[
  \tilde{B}_t \to B^*, \  \tilde{Z}_t \to Z^*, \ \text{var}(\tilde{\tau}_t) \to 0
  \]
Imperfect hedging

• If shocks are imperfectly correlated, complete elimination of risk is impossible, invariant distribution of \( \{ \tilde{B}_t, \tilde{Z}_t \} \) is not degenerate

• Our approach: take quadratic approximation of \( \tilde{B} (B, s) \) around \( B \) as variance of shocks goes to zero

• Simple linear policy rules

\[
\tilde{B} (s, B) = B + \beta [g (s) - \bar{g}] + \beta \left[ R (s) - \beta^{-1} \right] \\
\quad - \beta^2 \text{var} (R) B - \beta^2 \text{cov} (R, g) + O \left( \| s \|^3, (1 - \beta) \| s \|^2 \right)
\]
Main result: moments of invariant distribution

**Proposition**: the mean, variance and mean reversion of \( \{ \tilde{B}_t, \tilde{Z}_t \} \) satisfy, up to order \( O(||s||, (1 - \beta)) \):

- The mean of the invariant distribution
  \[
  \mathbb{E}\tilde{B}_t = B^*, \quad \mathbb{E}\tilde{Z}_t = Z^*
  \]

- Speed of mean reversion
  \[
  \frac{\mathbb{E}_{t-1} (\tilde{B}_t - B^*)}{\tilde{B}_{t-1} - B^*} = \frac{\mathbb{E}_{t-1} (\tilde{Z}_t - Z^*)}{\tilde{Z}_{t-1} - Z^*} = \frac{1}{1 + \beta^2 \text{var} (R)}
  \]

- Variance of the invariant distribution
  \[
  \text{var} (\tilde{B}_t) = \frac{\text{var} (\mathbb{E} (B^*))}{\text{var} (R)}
  \]
  \[
  \text{var} (\tilde{Z}_t) = 0
  \]
Intuition

- Back to Euler equation:

\[
\text{cov} \left( R_{t+1}, V' (\tilde{B}_{t+1}) \right) \propto \text{cov} \left( R_{t+1}, E_{t+1} \right) + O \left( \| s \|^3 \right) \\
\propto \frac{\partial}{\partial B} \text{var} \left( R_{t+1}, E_{t+1} (B, \cdot) \right) + O \left( \| s \|^3 \right)
\]

- \( \text{var} \left( R_{t+1}, E_{t+1} (B, \cdot) \right) \) is minimized at \( B = B^* \):

\[
B < B^* \implies \text{cov} \left( R_{t+1}, E_{t+1} (B, \cdot) \right) > 0 \\
B = B^* \implies \text{cov} \left( R_{t+1}, E_{t+1} (B, \cdot) \right) = 0 \\
B > B^* \implies \text{cov} \left( R_{t+1}, E_{t+1} (B, \cdot) \right) < 0
\]

The optimal policy is to revert to risk-minimizing position.
Main insights

- Target debt level: minimizes risk
  - target level is positive if $\text{cov}(R, g) < 0$
  - target level is negative (accumulate assets) if $\text{cov}(R, g) > 0$
- Speed of mean reversion is determined by $\text{var}(R)$
  - $\text{var}(R) = 0$ implies debt is random walk as in Barro (1979)
- The less hedging $B^*$ offers, the bigger the variance of the invariant distribution is
- For $\beta$ close to one, $\text{var}(\tilde{Z}_t)$ and $\text{var}(\tilde{\tau}_t)$ is close to 0 $\implies$ all adjustment to shock is done via debt
Reliability of approximations

Figure 1: Using the quadratic approximation (red line) and a more accurate global approximator (black line), the top, middle, and bottom panels plot smoothed kernel densities (left side) and decision rules (right side) associated with values of $\sigma = 0.001, 0.02,$ and $0.04$. The right panel displays policies $\tilde{H}(s, H) - H$ for states $s$ that attain the extreme values for $\{g(s)\}$ and $\{p(s)\}$.
Extensions

- Richer asset structure
- Persistence, other shocks
- Risk aversion
Extension 1: richer market structure

- Suppose there are $K$ assets with arbitrary payoffs, duration
  - note that fixed portfolio weights are isomorphic to one security
- Notation: $\mathbf{R} = \begin{bmatrix} R^1, \ldots, R^K \end{bmatrix}$; $\mathbb{C} [\mathbf{R}, \mathbf{R}]$ and $\mathbb{C} [\mathbf{R}, \mathbf{g}]$ are covariances matrices
  - assume that $\mathbb{C} [\mathbf{R}, \mathbf{R}]$ is non-singular
- Risk-minimizing total debt level and portfolio are

$$
(B^*, \mathbf{B}^*) \equiv \arg \min_{B=\mathbf{1}^T \mathbf{B}} \operatorname{var} \left( \sum R^k B^k + g \right)
$$

$$
= \left( -\mathbf{1}^T \mathbb{C} [\mathbf{R}, \mathbf{R}]^{-1} \mathbb{C} [\mathbf{R}, \mathbf{g}], \mathbb{C} [\mathbf{R}, \mathbf{R}]^{-1} \mathbb{C} [\mathbf{R}, \mathbf{g}] \right)
$$
Optimal portfolio with active debt management

- Mean debt level:
  \[ E(\tilde{B}_t) = B^* \]

- Mean reversion:
  \[
  \frac{\mathbb{E}_{t-1}(\tilde{B}_t - B^*)}{(\tilde{B}_{t-1} - B^*)} = \frac{\beta^{-2} \mathbf{1}^T C [R, R]^{-1} \mathbf{1}}{1 + \beta^{-2} \mathbf{1}^T C [R, R]^{-1} \mathbf{1}}
  \]

- Optimal portfolio:
  \[
  B_t = B^* + \frac{C [R, R]^{-1} \mathbf{1}}{\mathbf{1}^T C [R, R]^{-1} \mathbf{1}} \left( \tilde{B}_t + \mathbf{1}^T C [R, R]^{-1} C [R, g] \right)
  \]
Some insights

- Optimal portfolio chosen to **minimize risk**
  - unlike Merton’s investor’s, no risk-return trade-off
  - gov’t benevolent + general equilibrium implies that not optimal to chase returns for gov’t

- Speed of mean reversion is slower with more asset: can hedge risks better when $B_t \neq B^*$

- Higher debt $B_t \implies$ higher weight of securities with small $\text{var} \left( R^k \right)$
Suppose that shocks are first order Markov + TFP shocks $\theta$ + discount factor shocks.

For any random variable $x$ let

$$PV(x; s) = \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t x_t \bigg| s_0 = s \right].$$
Optimal policy with persistent shocks

- Optimal debt satisfies

\[ V'_t (\tilde{B}_t) = \mathbb{E}_t V'_{t+1} (\tilde{B}_{t+1}) + \beta \text{cov}_t (R_{t+1}, V'_{t+1} (\tilde{B}_{t+1})) \]

- Our quadratic approximations imply that in invariant distribution

\[ \mathbb{E} \tilde{B}_t = \frac{\text{cov} (R, \text{PV} (g)) - \bar{g} \text{cov} \left( R, \text{PV} \left( \theta \frac{1+\gamma}{\gamma} \right) \right)}{\text{var} (R)} \]

mean reversion:

\[ \frac{1}{1 + \beta^2 \text{var} (R)} \]
Intuition: risk minimization

- Planner wants to minimize fluctuations in $\tau_t$
- Primary deficit, holding $\tau$ constant is

$$X_\tau \equiv g - \theta \frac{1+\gamma}{\gamma} Z_\tau = g - \theta \frac{1+\gamma}{\gamma} \tau (1 - \tau)^{\frac{1}{\gamma}}$$

- Mean level of debt $B$ and $\tau$ related through budget constraint:

$$\frac{1 - \beta}{\beta} B = \bar{g} - \tau (1 - \tau)^{\frac{1}{\gamma}} \mathbb{E} \theta^{\frac{1+\gamma}{\gamma}}$$

- The mean of invariant distribution is risk-minimizing debt:

$$B^* \equiv \arg\min_B \text{var} \left( RB + PV \left( X_{\tau(B)} \right) \right)$$

- Effect from $\tau (B)$ is second order:

$$B^* \approx -\frac{\text{cov} \left( R, X_{\tau(B)} \right)}{\text{var} \left( R \right)} \text{ for any } B$$
Extension 3: Risk aversion

- Same environment as extension 1 but utility is

\[
\frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}
\]

- New implementability constraint

\[
U_{c,t}B_t + U_{c,t} \left[ l_t + \frac{U_{l,t}}{U_{c,t}} l_t - g_t \right] = \frac{p_t U_{c,t}}{\beta E_{t-1} p_t U_{c,t}} U_{c,t-1}B_{t-1}
\]
Effective debt and return

- Define
  - effective debt: \( B_t = U_{c,t}B_t \)
  - effective return: \( R_t = \frac{p_t U_{c,t}}{\beta E_{t-1} p_t U_{c,t}} \)
  - effective primary deficit: \( \chi_t = U_{c,t} \chi_t \)
- All can be written as functions of \( c_t \)
Recursive problem

- Bellman equation

\[ V(\mathcal{B}, s) = \max_{\{c(s), \mathcal{X}'(s)\}} \mathbb{E} \left[ U \left( c(s), \frac{c(s) + g(s)}{\theta(s)} \right) + \beta V(\mathcal{B}, s) \right] \]

subject to

\[ \mathcal{B}'(s) = \mathcal{R}(s) \mathcal{B} + \mathcal{X}(s) \text{ for all } s \]

- Similar to recursive formulation in quasi-linear case, same optimality condition for effective debt:

\[ V_t(\tilde{\mathcal{B}}_t) = \mathbb{E}_t V'_{t+1}(\tilde{\mathcal{B}}_{t+1}) + \beta \text{cov}_t (\mathcal{R}_{t+1}, V'_{t+1}(\tilde{\mathcal{B}}_{t+1})) \]
Risk-minimizing effective debt

- Planner wants to minimize fluctuations in $\tau$
- The risk-minimizing effective debt is

$$\tilde{B}^* = -\frac{\text{cov} (R, PV (X))}{\text{var} (R)}$$

- Terms on the r.h.s. are endogenous but, up to the second order, do not depend on $\tau$
- Can be easily computed without doing dynamic programing
- Risk-free $R \implies R$ is when $X$ is high $\implies$ optimal to hold negative quantity of risk-free debt
- Easy to generalize to $K$ asset
Quantitative exercise

- Apply our analysis to the U.S. economy
- Since formulas are approximation, also evaluate how well they do
Model specification

- Preferences
  \[ \ln c \equiv \frac{1}{3} l^3 \]

- 1 asset, return are matched to returns of the U.S. gov't portfolio

- 3 shock process:

  \( \ln \theta_t = \rho_\theta \theta_{t-1} + \sigma_\theta \epsilon_{\theta,t} \)

  \( \ln g_t = \ln \bar{g} + \chi_g \epsilon_{\theta,t} + \sigma_g \epsilon_{g,t} \)

  \( \ln p_t = \chi_p \epsilon_{\theta,t} + \sigma_p \epsilon_{p,t} \)
Target statistics:
- dynamics of GDP
- dynamics of returns to U.S. gov’t portfolio

Returns computed from budget constraint:

\[(q_t + p_t) B_{t-1} = X_t + q_t B_t\]

\[\implies R_t = \frac{\text{market value of debt}_t + \text{primary deficit}_t}{\text{market value of debt}_{t-1}}\]

GDP and returns are endogenous, depend on tax policy. We estimate

\[\tau_t = (1 - \rho_\tau) \tau_{t-1} + \rho_\tau \bar{\tau} + \rho_Y \ln Y_t + \rho_{Y-} \ln Y_{t-1}\]
## Model fit

<table>
<thead>
<tr>
<th>Param</th>
<th>Value</th>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Log Output</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\theta}$</td>
<td>0.020</td>
<td>std. dev</td>
<td>1.7%</td>
<td>1.70%</td>
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<tr>
<td>$\rho_{\theta}$</td>
<td>0.160</td>
<td>auto corr</td>
<td>0.28</td>
<td>0.28</td>
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<tr>
<td></td>
<td></td>
<td>Returns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>0.05</td>
<td>std. dev</td>
<td>5.1%</td>
<td>5.02%</td>
</tr>
<tr>
<td>$\chi_p$</td>
<td>0.650</td>
<td>corr with $\log Y_t$</td>
<td>-0.06</td>
<td>-0.08</td>
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<td></td>
<td></td>
<td>$G/Y$</td>
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</tr>
<tr>
<td>$\bar{g}$</td>
<td>0.230</td>
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<td>23%</td>
<td>23%</td>
</tr>
<tr>
<td>$\sigma_g$</td>
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<td>std. dev</td>
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<td>4.7%</td>
</tr>
<tr>
<td>$\chi_g$</td>
<td>-0.150</td>
<td>corr with $\log Y_t$</td>
<td>-0.42</td>
<td>-0.41</td>
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</table>
Optimal policy: computed and analytical

Correlation of returns and output is close to 0:
- correlation with effective returns is negative
- accumulate assets

Variability of effective returns is quite low, provides bad hedge:
- slow convergence to the mean
- large variance of debt

<table>
<thead>
<tr>
<th>Effective debt: $X_t$</th>
<th>Using simulation</th>
<th>Using formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.07</td>
<td>-0.06</td>
</tr>
<tr>
<td>Half life (years)</td>
<td>250</td>
<td>257</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>0.26</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Table 4: Ergodic moments and comparison with formula
Simple back of envelope

- Run VAR

\[
\begin{pmatrix}
X_t \\
Y_t
\end{pmatrix} = A \begin{pmatrix}
X_{t-1} \\
Y_{t-1}
\end{pmatrix} + \varepsilon_t
\]

- Let

\[
\begin{pmatrix}
\alpha X \\
\alpha Y
\end{pmatrix} = \left( I - \beta^{-1} A \right)^{-1} \begin{pmatrix}
1 \\
0
\end{pmatrix}
\]

- Then

\[
PV_t (X') = \alpha X X_t + \alpha Y Y_t
\]

- Risk minimizing effective debt

\[
B^* = - \frac{\text{cov} (R_t, PV_t (X'))}{\text{var} (R_t)} = - \frac{\alpha Y \text{cov} (R_t, Y_t) + \alpha X \text{cov} (R_t, X_t)}{\text{cov} (R_t)}
\]

- Applying to the U.S. data

\[
B^* = -0.08
\]
Comparison to the U.S. policy

<table>
<thead>
<tr>
<th>Moments</th>
<th>Benchmark</th>
<th>Comparison to U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Simulated</td>
</tr>
<tr>
<td>Tax Rate</td>
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<tr>
<td>std. dev</td>
<td>0.2%</td>
<td>0.2%</td>
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<tr>
<td>auto corr</td>
<td>0.97</td>
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<tr>
<td>Log Debt</td>
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<tr>
<td>std. dev</td>
<td>10%</td>
<td>1.8%</td>
</tr>
<tr>
<td>auto corr</td>
<td>0.95</td>
<td>0.31</td>
</tr>
</tbody>
</table>

- Similar orders of magnitude
- Debt in the U.S. too smooth, reverts to the mean too quickly
Conclusion

- Portfolio theory for government assets
  - general equilibrium effects
  - benevolence
- Easily extend to other countries
  - open economy and accumulating foreign debt (e.g. China)
  - investing in stocks (e.g. Norway, sovereign funds)