Does the New Keynesian Model Have A Uniqueness Problem?

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Disclaimer: The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Board, the FOMC, or anyone else associated with the Federal Reserve System.
Introduction

• NK models have been enormously influential in terms of their policy implications.

• Models’ implications for fiscal policy are particularly striking when ZLB is binding.

• Key results:
  – When ZLB binds, output fall is potentially very large.
  – The multiplier is larger when the ZLB binds than when it doesn’t.
  – The more binding is the ZLB the larger is the drop in output and the larger is the multiplier.

• These results generated using linearized version of NK model, e.g. EW, CER.
Non-uniqueness and policy

- Non-linear NK models have multiple equilibria.

- Policy prescriptions can vary a lot across equilibria (Mertens and Ravn, Braun et. al. (2012), Cochrane (2015).
  - At some ZLB equilibria, multiplier is small or even negative.
  - At other ZLB equilibria, mutliplier very large.

- So, in principle non-uniqueness of equilibria poses an enormous challenge for policy analysis in NK models.
Is non-uniqueness a substantive problem?

• Yes, if there’s no compelling way to select among different equilibria that give different answers to critical policy questions.

• Our argument starts from presumption that the assumption of RE obviously wrong.

• But it can be a useful modeling strategy for thinking about a world where RE isn’t literally true.

“... the model described above ’assumes’ that agents know a great deal about the structure of the economy and perform some non-routine computations. It is in order to ask, then: will an economy with agents armed with ‘sensible’ rules-of-thumb, revising these rules from time to time so as to claim observed rents, tend as time passes to behave as described...” Lucas (1978)
Selecting among equilibria

• Suppose agents make a ‘small’ error in forming expectations about variables relative to their values in a particular REE.
  – Does economy converge to that REE?
  – If yes, the RE equilibrium is stable-under-learning.

• In our view, learnability is a necessary condition for an REE to be empirically interesting.

• Non-learnable REE equilibria are best viewed as mathematical curiosities.
Apply learnability criterion to standard fully non-linear NK model with Calvo pricing frictions.

Unlike linearized NK models, ZLB REE can’t be characterized by a set of numbers.
  – There’s an endogenous state variable (past price dispersion), so ZLB REE is a set of \textit{functions}.
  – Must think about how agents learn about these functions.
Key Results...

- There’s multiple REE, including sunspot equilibria (Mertens and Ravn).
  - When we consider fundamental shocks that trigger ZLB episodes, we find two minimum state variable ZLB equilibria.
  - These equilibria converge to different inflation rates if the ZLB episode lasts forever.

- Impact of government consumption can be very different in the different ZLB equilibria.
  - For example, there are ZLB REE in which multiplier is negative.
Key Results...

- There exists a unique interior ZLB REE that’s stable-under-learning.
  - That REE that converges to a relatively low ZLB deflation rate.

- Controversial predictions of linearized NK model about fiscal policy in the ZLB, are satisfied at unique learnable ZLB REE.

- We conclude Calvo model doesn’t have a substantive uniqueness problem, at least for analysis of fiscal policy.
What about the Rotemberg model?

- Many authors used non-linear versions of the Rotemberg model to proxy for Calvo model.
  - Much easier to work with, no endogenous state variables in ZLB.
  - ZLB REE is a set of numbers (not functions).

- Linearized versions of these models are the same.

- But non-linear versions of the two models are potentially very different.
The Rotemberg model...

- Some properties of non-linear Rotemberg model are very sensitive to how you formulate adjustment costs for prices.

- Number of ZLB REE and their stability properties depend on whether and exactly how you scale adjustment costs for growth.

- But there always exists a unique ZLB REE that’s stable-under-learning.

- At that equilibrium, impact of fiscal policy in the ZLB are same as those implied by log-linear NK model.
By-product: Linear Approximations

- Use non-linear Calvo model to assess robustness of log-linear approximations.

- Log-linear approximations work reasonably well for analysis of ZLB and fiscal policy.

- Evidence that quality of linear approximations is poor rests on examples where output deviates by more than 15 percent from its steady state.
Conclude talk with remarks about neo-Fisherian view of monetary policy.

- To achieve a high inflation rate, the monetary should target a high nominal interest rate.

Theoretical foundation for that view collapses in the face of stability-under-learning criterion.

Non-linear flexible price (BSGU) and NK models (Calvo, Rotemberg) have unique RE equilibrium that’s stable-under-learning.

- First result eliminates BSGU based arguments for neo Fisherian view
- Second result eliminates Cochrane arguments based on NK model.
Standard NK model

• Household maximizes

$$E_0 \sum_{t=0}^{\infty} d_t \left[ \log(C_t) - \frac{\chi}{2} h_t^2 \right]$$

$$d_t = \prod_{j=0}^{t} \left( \frac{1}{1 + r_{j-1}} \right)$$

• Discount factor $r_t$ can take on two values: ‘normal value’ $r$ and $r^l$, where $r^l < 0$.
  
  – If $r_t = r^l$, it stays at that value with prob $p$.
  – Once you switch back to $r$, you stay there forever.

$$P_tC_t + B_t \leq (1 + R_{t-1}) B_{t-1} + W_t h_t + \Pi_t.$$
Model

- Final homogeneous good, $Y_t$, produced by competitive and identical firms:

$$Y_t = \left[ \int_0^1 (Y_{j,t})^{\frac{\epsilon}{\epsilon-1}} dj \right]^{\frac{\epsilon-1}{\epsilon}}, \ \epsilon > 1.$$

- Input $j$ produced by firm $j$ using technology $Y_{j,t} = h_{j,t}$.
  - competitive in factor markets
  - monopolist in product market.
Model

- Monopolist $j$ choosing $\tilde{P}_t$ to maximize

$$E_t \sum_{k=0}^{\infty} \beta^k \lambda_{t+k} ( (1 + \nu) \tilde{P}_t - P_{t+k} s_{t+k} ) Y_{j,t+k}$$

$\nu$ is a subsidy that removes steady state distortions owing to monopoly power.

- Monopolist $j$ sets price, $P_{j,t}$, subject to demand curve for its good and Calvo sticky price friction

$$P_{j,t} = \begin{cases} P_{j,t-1} & \text{with probability } \theta \\ \tilde{P}_t & \text{with probability } 1 - \theta. \end{cases}$$
Model

- Aggregate output
  \[ Y_t = p_t^* h_t \]

- \( p_t^* \) is a measure of price dispersion
  \[ p_t^* = \left( 1 - \theta \right) \left[ \frac{1 - \theta \pi_t^\varepsilon - 1}{1 - \theta} \right]^{\frac{-\varepsilon}{1 - \varepsilon}} + \theta \pi_t^\varepsilon (p_{t-1}^*)^{-1} \].

- Aggregate resource constraint
  \[ C_t + G_t \leq Y_t. \]

- Monetary policy rule
  \[ R_t = \max \{ 1, 1 + r + \alpha (\pi_t - 1) \} \]

  - Max operator reflects ZLB and \( \alpha > 1 + r. \)
Solving the model

- Can reduce equilibrium conditions to four non-linear equations.

- There’s an endogenous state variable, $p_{t-1}^*$, and an exogenous state variable, $r_t$.

- So a solution to the model is a set of functions which satisfy these conditions.

- Stage 1: solve for the equilibrium functions that obtain when $r_t = r$, i.e. after the economy has exited the ZLB.

$$Y(p_{t-1}^*), \pi(p_{t-1}^*), F(p_{t-1}^*), p^*(p_{t-1}^*)$$

- Stage 2: solve for equilibrium functions that obtain when $r_t$ is equal to $r_\ell$

$$Y_\ell(p_{t-1}^*), \pi_\ell(p_{t-1}^*), F_\ell(p_{t-1}^*) \text{ and } p^*_\ell(p_{t-1}^*)$$
ZLB REE Steady State

- Consider limit as $t \to \infty$ when the economy stays in the ZLB.
- $p_t^*$ converges to a number, $\hat{p}$, for any interior equilibrium.
- System of equations collapses to a system of equations in four unknowns,
\[
\pi_\ell(\hat{p}), \ Y_\ell(\hat{p}), \ p_\ell^*(\hat{p}), \text{and} \ F_\ell(\hat{p}).
\]
Solving for steady state ZLB

- Equations defining an interior steady-state ZLB equilibrium collapse into one equation one unknown
  \[ f(\pi_\ell) = 0. \]

- In a slight abuse of notation we drop explicit dependence of \( \pi_\ell \) on \( \hat{\rho} \).

- A necessary condition for ZLB REE equilibrium to be unique
  - There’s a unique solution to this equation.
Parameterizing the model

- Benchmark values:
  \[ \varepsilon = 7.0, \ \beta = 0.99, \ \alpha = 2.0, \ p = 0.75, \]
  \[ r^L = -0.02/4, \ \theta = 0.85, \ \eta_g = 0.2. \]

- Steady state output is normalized to 1 by setting \( \chi = 1.25 \).

- Sensitivity analysis in appendix.
ZLB Steady States

• The function $f(\pi_\ell)$ has inverted U shape so there’s either two interior steady-state ZLB equilibria or none.

• Benchmark case: two steady-state ZLB REE equilibria
  – ‘high’ and ‘low’ inflation.

• Number of minimum state variable ZLB REE equilibria coincides with number of steady state ZLB REE equilibria.
  – A numerical result, not a theorem.
\[ G_{\ell} = G \]

\[ G_{\ell} = 1.05 \times G \]
Dynamic Response Functions

• Unlike Rotemberg, ZLB REE isn’t a number because of endogenous state variable, $p_{t-1}^*$.  

• Consider dynamic response of $\pi_t$ and $C_t$ to $r_t$ shock when economy converges to high and low $\pi$ steady-state ZLB REE.  

• Refer to these paths as: high and low inflation ZLB REE.
Dynamic Response Functions

![Graphs showing dynamic response functions for inflation and consumption. The graphs display the relationship between the period in ZLB and the corresponding inflation or consumption values for high and low inflation scenarios.]
Dynamic Response Functions

• Along high $\pi$ ZLB REE path,
  – Quarterly $\pi$ and $C$ initially drop by 1.5 and $-6.35$ percentage points, respectively.
  – After about 5 quarters $\pi$ and $C$ declines stabilize at $-1.3$ and $-6.3$ percentage points.

• Along low $\pi$ ZLB REE path, quarterly $\pi$ and $C$ initially drop by $-7.25$ and $-23.5$ percentage points.
  – After about 5 quarters $\pi$ and $C$ declines stabilize at $-6.0$ and $-23.3$ percentage points.
The Multiplier

- $G^\ell = 1.05 \times G^h$, i.e. when economy is in ZLB, $G$ rises by 1 percentage of steady state output.

- Multiplier:
  $$\frac{G^\ell}{(C^\ell(p^*_{t-1}) + G^\ell)} \frac{\Delta (C^\ell(p^*_{t-1}) + G^\ell)}{\Delta G^\ell}.$$

- If economy is in high $\pi$ (low $\pi$) ZLB REE for low value of $G$, it’s in high $\pi$ (low $\pi$) ZLB REE for low value of $G$.
  - A non-trivial assumption.
The Multiplier

High-Inflation ZLB REE

Multiplier

Period in ZLB

Low-Inflation ZLB REE

Multiplier

Period in ZLB
Comparisons

• Multiplier in high-$\pi$ ZLB REE is large, exceeding two over the time period displayed.

• Multiplier is *negative* in low-$\pi$ ZLB REE.

• To understand this result, note that an increase in $G^\ell$ shifts $f(\pi^\ell)$ upwards.
  – So effect of increase in $G$ depends on which equilibrium we focus on.

• Dramatic illustration of basic result in Mertens and Raven (2011) where multiplier in one REE is a lot smaller than in the other REE.
$f(\pi_\ell) = G_{\ell} = G$

$G_{\ell} = 1.05 \times G$
Comparison to linearized model

Impact period of shock

- Responses of linearized model similar to those in high-$\pi$ ZLB REE.
- Properties of low-$\pi$ ZLB REE are very different.

<table>
<thead>
<tr>
<th>Model</th>
<th>Output</th>
<th>Inflation</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>-2.18</td>
<td>-0.0066</td>
<td>1.63</td>
</tr>
<tr>
<td>Nonlinear, high-inflation</td>
<td>-2.84</td>
<td>-0.0093</td>
<td>2.24</td>
</tr>
<tr>
<td>Nonlinear, low-inflation</td>
<td>-17.87</td>
<td>-0.0734</td>
<td>-0.35</td>
</tr>
</tbody>
</table>
Comparison to linearized model

- Basic qualitative results reported in CER using log-linear approximation hold up when we focus on high-$\pi$ ZLB REE.

- Multiplier can be much bigger than 1 when ZLB binds.

- When duration of ZLB increases or degree of flexibility of prices increases,
  - Severity of output collapse and multiplier are larger.

- One interesting difference:
  - For parameter values that imply linear multipliers explode, REE ceases to exist in non-linear model.
Stability-under-learning

- In Calvo model, firms choose \( P_{j,t} \), based in part on value of \( P_t \).
- But, \( P_t \) is a function of firms’ collective price decisions.
- Firms can’t ‘know’ \( P_t \) when they choose their own price, in sense of actually observing it.
- Standard assumption: firms form a ‘belief’ about \( P_t \) when they make their decision.
- In REE that belief is correct.
Stability-under-learning

- If firms don’t have rational expectations, it’s not natural to assume they see $P_t$ when they choose their prices.

- But if they don’t see $P_t$, they also don’t see $C_t$.

- Must attribute to firms views about equilibrium functions for current and future aggregate $\pi$ and $C$. 
Stability-under-learning

- $x^e_f(p^*_{t-1}, t - 1)$: firm’s belief, formed using information up to time $t - 1$, about equilibrium function for $x_\ell$.

- To make time $t$ decision, firms must forecast values of future variables as $p_t^*$ evolves.
  - FONC’s involve objects like $x^e_f(p^*_{t-1}, t - 1)$ and $x^e_f(p^*_{t+j}, t - 1)$ for $j \geq 0$.
  - So firms must have views about the entire function.

- All these functions have time $t - 1$ as argument
  - Reflects our assumption that firms think they’re in stationary environment.
Stability-under-learning

- Firms’ beliefs evolve according to

\[ x^e_f(p_t^*, t) = \omega x^e_f(p_{t-1}^*, t-1) + (1 - \omega)x^e_f(p_{t-1}^*, t-1). \]

- For \( \omega > 0 \), this formulation embodies the heroic assumption that agents know time \( t-1 \) equilibrium function for \( x_{\ell} \).
  - Paper reports sensitivity analysis to simpler rules.

- In Rotemberg model, there’s no state variables in ZLB.
  - Replace above rule with assumption that agents’ expectations evolve according to simple constant gain algorithm about the values of variables.
Stability-under-learning in the NK model

- When households make their time $t$ consumption decisions, firms’ actions have already determined $\pi_t$.

- So households can compute the time $t$ equilibrium function for $\pi$ (again heroic!).

- $\pi_{e,h}^* (p_{t-1}^*, t)$ : households’ belief, at time $t$, about equilibrium function for $\pi_{e,h}$.

- Given new information, households beliefs evolve according to

$$\pi_{e,h}^* (p_t^*, t + 1) = \omega \pi_{e,h} (p_{t-1}^*, t) + (1 - \omega) \pi_{e,h}^* (p_{t-1}^*, t).$$
A learning ZLB equilibrium

• Assume agents know REE functions when economy isn’t in ZLB.

• A sequence of functions for all of endogenous variables that satisfy
  – Resource constraint,
  – Monetary policy rule,
  – Household and firm optimality conditions for all $t$,
  – given initial set of beliefs $\pi_{\ell}^{e,f}(\cdot,0)$, $C_{\ell}^{e,f}(\cdot,0)$, and $\pi_{\ell}^{e,h}(\cdot,0)$ that evolve according to above rules.
Stability-under-learning

- A ZLB REE is *stable-under learning* if a learning equilibrium with initial beliefs close to, but not equal to, the REE functions, converges back to the ZLB REE equilibrium.

- If an economy stays in ZLB forever, it will converge to a steady state ZLB REE.

- Learning equilibrium must also approach steady state ZLB REE if initial ZLB REE is stable-under-learning.

- Allows us to eliminate *all* of ZLB REE that lead to low-$\pi$ steady state ZLB REE.
  - They’re not stable-under-learning.
Stability-under-learning

• Consider a firm that believes that
  – Steady state inflation rate is $\pi^e_f$.
  – Economy is in steady state corresponding to that rate of inflation.

• The belief $\pi^e_f$ isn’t an REE belief so the steady state associated with it (including $p^*_t$) isn’t a steady state ZLB REE.

• So $f(\pi^e_f)$ isn’t equal to zero.
Stability-under-learning

- There's an equivalence between a belief $\pi_{\ell}^{ef}$ and a value of $\tilde{p}_{\ell}^{ef}$ that will be chosen by firms who can update their price.

$$\tilde{p}_{\ell}^{ef} = \frac{\tilde{P}_t}{P_{t-1}}$$

- So we use function $f(\pi_{\ell}^{ef})$ to define a new function

$$\tilde{f}(\tilde{p}_\ell^e)$$

that must be equal to zero at a steady state ZLB REE.

- Write firms’ FONC for $\tilde{p}_t = \tilde{P}_t/P_{t-1}$ can be written, after imposing all of equilibrium conditions, as

$$\tilde{F} \left( \tilde{p}_t, \tilde{p}_{\ell}^{ef} \right) = 0.$$
Stability-under-learning

- Define the best-response function

\[ \tilde{p}_t = g(\tilde{p}_e^f). \]

- This function has the property that,

\[ \tilde{F} \left( g(\tilde{p}_e^f), \tilde{p}_e^f \right) = 0. \]

- In a steady state RE ZLB equilibrium

\[ \tilde{p}_t = \tilde{p}_e^f. \]
Stability-under-learning

- Following figure plots typical firm’s best response function, i.e. $\tilde{p}_t$ as a function of $\tilde{p}_e^{e,f}$.

- Steady state ZLB REE equilibria correspond to the two points where best response function intersects 45 degree line.

- Given any belief, $\tilde{p}_e^{e,f}$, between RE steady state beliefs, best response $g(\tilde{p}_e^{e,f})$ is greater than $\tilde{p}_e^{e,f}$.

- Follows that realized $\pi$ will exceed beliefs about $\pi$.

- So learning equilibrium will move towards high-$\pi$ ZLB REE steady state.
Best Response Function

• Now consider any belief, $\tilde{p}_{\ell}^{ef}$, that exceeds high-$\pi$ REE ZLB steady state.
  
  – Best response function $g(\tilde{p}_{\ell}^{ef})$ is less than $\tilde{p}_{\ell}^{ef}$.
  – So realized $\pi$ will be lower than beliefs about $\pi$.
  – Learning equilibrium will move towards high-$\pi$ ZLB REE steady state.

• Finally, consider any belief, $\tilde{p}_{\ell}^{ef}$, that’s less than low-$\pi$ ZLB REE ZLB steady state.
  
  – Here best response function $g(\tilde{p}_{\ell}^{ef})$ is less than $\tilde{p}_{\ell}^{ef}$.
  – So realized $\pi$ will be lower than beliefs about inflation.
  – Learning equilibrium will move away from the low-$\pi$ ZLB REE steady state.
Stability-under-learning

- Previous discussion focused on limiting point of ZLB REE.

- To be stable-under-learning, functions defining a learning equilibrium must converge point-wise to functions defining a ZLB REE for every possible for $p_t^*$, including steady state value of $p^*_\ell$.

- Just showed that any ZLB REE that converges to low-$\pi$ steady state ZLB REE doesn’t satisfy this condition.

- So those equilibria aren’t stable-under-learning.
Stability-under-learning

- Previous discussion doesn’t establish that a ZLB REE that converges to high-\(\pi\) steady state REE ZLB is stable-under-learning.

- Our solution algorithm parameterizes ZLB REE functions with a finite number of parameters, \(z_t\).

- Learning algorithm defines a mapping from current values of those parameters to next period’s values:

\[
  z_{t+1} = s(z_t).
\]

- Define

\[
  S(\tilde{z}) = \left[ \frac{ds_i(z)}{dz_j} \right] |_{\tilde{z}},
\]

for all \(i, j < N\) where \(N\) is number of parameters.
### Stability-under-learning

- Evaluate $S$ for the parameters of the high-$\pi$ REE.

- Max eigenvalue is less than one in absolute value.
  - So, locally, functions in neighborhood of high-$\pi$ ZLB REE will converge to those REE functions in a learning equilibrium.

- Repeat analysis for parameters of the low-$\pi$ ZLB REE.
  - Maximum eigenvalue is greater than one in absolute value.
  - So, locally, functions in neighborhood of low-$\pi$ ZLB REE, equilibrium will diverge from those REE functions in a learning equilibrium.
Learning equilibria

Dynamic Paths

• At $t = -1$ economy is in high-$\pi$ ZLB REE steady state where $p^*_1 = p_\ell$.

• At time 0,

\[
x^e_f(p^*_{-1}, -1) = x_\ell(p^*_{-1}) + \bar{x}_\ell, \quad \bar{x}_\ell > 0.
\]

• Agents think that if ZLB ends, economy will be in REE that converges to high-$\pi$ steady state.
  
  – Also assume parameter $\omega = 1$ (tomorrow will be like today).

• $\bar{x}_\ell = (-.02, -.01, 0, 0.01, 0.02)$. 
Learning equilibria

Red line: REE value in high inflation ZLB REE

Inflation

Period In ZLB

Inflation

Period In ZLB
Learning equilibria

- Regardless of value of $\bar{x}_l$, $\pi$ converges to high-$\pi$ ZLB REE.

- Establishes that learning equilibrium converges to equilibrium function defining an REE when evaluated at the $p_{l}^*$.

- Second panel: begin from the low-$\pi$ ZLB REE.

- Inflation diverges from that equilibrium in the learning equilibrium.
Learning equilibria

- Till now we’ve assumed that agents believe that once ZLB is over, economy will go to REE that converges to the high-\( \pi \) steady state.

- Redo analysis assuming agents think that post-ZLB, economy will go to REE that converges to low-\( \pi \) steady state.

- All of qualitative results hold, e.g. REE that converge to low \( \pi \) steady state are not stable-under-learning.
Fiscal policy in learning equilibria

- Initially assume that agents think that when ZLB episode is over, economy goes to REE that converges to high-$\pi$ steady state.

- Economy begins in high-$\pi$ steady state.

- At time $0$, $p^*_1 = 1$, $r$ falls to $r_\ell$.

- Firms obey learning laws discussed above.
Fiscal policy in learning equilibria

- Consumption
- Inflation
- Multiplier

Period In ZLB

Learning Equilibrium
RE Equilibrium
Fiscal policy in learning equilibria

- $C$ and and $\pi$ converge to high $\pi$ ZLB REE from above.

- Reason that they initially take on higher values is that initial expectations about higher future $\pi$ and $C$ spur demand now.

- As expectations adjust downward with realized $\pi$ and $C$, they push $C$ and $\pi$ down further.

- Multiplier starts out low because ZLB isn’t binding in first few periods.

- Once ZLB starts to bind, the multiplier quickly rises above 1.
Alternative experiment

• After $r_t$ shock, firms and household have beliefs near the low-$\pi$ ZLB REE.

• $C$ and $\pi$ converge to high $\pi$ ZLB REE from below.

• Reason that they initially take on lower values is that expectations about low future inflation and consumption depress demand in the present.
**Alternative experiment**

- Multiplier starts out around 1 and then rises after that.

- Multiplier rises because fiscal expansion helps quickly move expectations toward those associated with the high-$\pi$ ZLB REE.
  - Without change in $G$, expectations remain close to low-inflation ZLB REE for some time.

- After many periods, the multiplier eventually approaches the high-inflation steady state ZLB REE value.
Mertens and Ravn (2015)

- Report multiplier is *small* when they analyze a learning equilibrium near the low inflation steady state RE ZLB equilibrium.

- This result is very different than ours - we just argued that the relevant multiplier is *large*.

- Why?
When we calculate the multiplier we initially consider an economy in which agents initial expectations about inflation differ by $\epsilon_\pi$ from the low $\pi$ steady state ZLB REE.

We then consider a separate economy with shocks that set $r$ to $r_\ell$ and a shock to $G$.

Expectations start in the same place for the two economies.

We then use difference in output between the two economies to calculate the multiplier.
Mertens and Ravn (2015)

- In their experiment when $G$ increases, the rate of inflation in the steady state of the ZLB REE falls by $\epsilon'_\pi$.

- When they raise $G$ in the learning equilibrium, they also decrease agents’ expectations about inflation by $\epsilon'_\pi$.
  - In and of itself this fall in inflation reduces output in the ZLB.

- Next figure displays multiplier if we adopt their assumption.

- We obtain a *negative* multiplier that persists for roughly 10 years.

- Change in expectations is quantitatively much more important than the increase in $G$. 
Rotemberg model

- Scaling term of price-adjustment costs can have large effect on properties of the equilibria that we find that aren’t stable-under-learning.

\[ \Phi_{t+k} \left( \frac{P_{j,t+k}}{P_{j,t+k-1}} - 1 \right)^2 \]

- But there’s always a unique stable under learning equilibrium in our examples.

<table>
<thead>
<tr>
<th>Adj. Cost</th>
<th>Stable Equilibrium</th>
<th>Unstable Equilibrium</th>
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</thead>
<tbody>
<tr>
<td>( \Phi_t = \frac{\phi}{2} )</td>
<td>1.56</td>
<td>0.98</td>
</tr>
<tr>
<td>( \Phi_t = \frac{\phi}{2} )  ( (C_t + G_t) )</td>
<td>1.70</td>
<td>0.36</td>
</tr>
<tr>
<td>( \Phi_t = \frac{\phi}{2} Y_t )</td>
<td>1.65</td>
<td>1.07</td>
</tr>
</tbody>
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Concluding Remarks

- Non-uniqueness of equilibria in NK models does not pose a substantive challenge to key conclusions about the efficacy of fiscal policy in ZLB episodes.

- A close derivative of our analysis is that the ‘neo-Fisherian’ views of monetary policy based on NK models of flexible price model like BSGU are not empirically relevant.