The Home Market Effect and Patterns of Trade Between Rich and Poor Countries

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Introduction

- Empirically rich (poor) countries tend to export high (low) income elastic products

- Standard trade models assume *homothetic preferences* to focus on the supply side determinants of the patterns of trade

- Just adding *nonhomothetic preferences* in the standard models would, *ceteris paribus*, make rich countries *import* high income elastic goods

- *Virtually all* models of trade with nonhomothetic preferences *assume* that the rich have CA in high income elastic goods.
  
  
  ✓ **Factor endowment**: Markusen (1986), Caron-Fally-Markusen (2014)

These models suggest that the rich export high income elastic goods *despite* they demand relatively more of them.

- Here, we explain *why* the rich have CA in high income elastic goods based on *Home Market Effect*, suggesting that the rich export high income elastic goods *because* they demand relatively more of them.
Home Market Effect (HME): Krugman’s (1980) example

- Two Dixit-Stiglitz monopolistic competitive sectors, $\alpha$ & $\beta$, with iceberg trade costs
- One factor of production (labor)
- Two countries of equal size, A & B, mirror-images of each other
  - A is a nation of $\alpha$-lovers; with the minority of $\beta$-lovers.
  - B is a nation of $\beta$-lovers, with the minority of $\alpha$-lovers.

In equilibrium,
- **Under autarky**, proportionately large share of firms in A operates in sector $\alpha$.
- **Under trade**, disproportionately large share of firms in A operates in sector $\alpha$.
- A becomes a net-exporter in $\alpha$; B a net exporter in $\beta$.

**Key Insight:** With scale economies and positive but finite trade costs, a relatively larger domestic market is a source of comparative advantage.

**Notes:** In Krugman (1980),
- Demand composition differs across countries due to *exogenous variations in taste*
- The mirror image setup obscures crucial factors of HME. Also restricts comparative static exercises
This Paper: Krugman-type HME model with demand composition difference due to nonhomothetic preferences. Also dispenses with the mirror-images setup.

- Continuum of Dixit-Stiglitz monopolistic competitive sectors with iceberg trade costs
- Two countries; may differ only in per capita labor endowment and population size.
- Preferences across sectors: Implicitly Additively Separable Nonhomothetic CES
  - Sectors indexed such that their income elasticity is increasing in the index.
  - The Rich has relatively larger domestic market than the Poor in the higher indexed

Under Trade Equilibrium, HME implies
- The Rich’s share of firms are disproportionately larger in higher-indexed sectors
- The Rich run trade surpluses (deficits) in higher (lower)-indexed sectors.

Comparative Statics: Due to endogenous demand compositions, uniform productivity improvement and a trade cost reduction cause
- Product cycles: The Rich switches from a net exporter to a net importer in the middle
- Welfare gaps to widen (narrow), when different sectors produce substitutes (complements)
- When two countries differ in size, a trade cost reduction has additional effects due to the ToT change; Leapfrogging and Reversal of the patterns of trade
Explicitly vs. Implicitly Additive Separability: Hanoch (1975)

**Explicit Additivity:** \[ u = \int_{0}^{1} f_s(c_s)ds; \quad \text{CES if} \quad u = \int_{0}^{1} \omega_s(c_s)^{1-1/\eta} ds \]

Pigou’s Law:  
Income Elasticity of Good s = constant  
Price Elasticity of Good s

Two Problems:  
i) Empirically false (Deaton 1974 and others)  
ii) Conceptually impossible to disentangle the effects of income elasticity differences from those of price elasticity differences

**Implicit Additivity:** \[ \int_{0}^{1} f_s(u,c_s)ds = 1; \quad \text{CES if} \quad \int_{0}^{1} \omega_s(u)(c_s)^{1-1/\eta} ds = 1 \]

i) Price elasticities & income elasticities can be separate parameters.  
ii) *Nonhomothetic CES* if \( \frac{\partial \log \omega_s(u)}{\partial u} \) varies with s. When we can index s to make it monotone increasing in s, \( \frac{\partial^2 \log \omega_s(u)}{\partial s \partial u} > 0, \) *log-supermodularity*
Fajgelbaum-Grossman-Helpman (2011); FGH

- A monopolistic competitive sector producing indivisible products with trade costs, with two segments, H&L, across which products are *vertically* differentiated.
- A competitive outside sector producing the divisible numeraire to pin down the ToT
- Each household consumes one unit of a particular product from either H or L.
  o A *discrete choice model* a la McFadden, a *nested-logit demand structure*
  o The rich consumers more likely to choose an H-product if marginal utility of the numeraire is higher when combined with an H-product
- The Rich (Poor) becomes a net-exporter of high-quality H (low-quality L) products.

FGH focuses on specialization along the quality dimension within a single industry. Our model focuses on specialization across a broader range of industries.

**Some Advantages of Our Framework**
- A minimum departure from the standard HME models
- Parsimonious and yet flexible
  o Comparative statics with any number of sectors and the ToT effect
  o Income elasticities are separate parameters from price elasticities
  o Different sectors may produce substitutes, as in Flam-Helpman (1987), Stokey (1991), and FGH (2011), or complements, as in Matsuyama (2000)
Organization of the Paper

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Home Market Effect with Nonhomothetic Preferences
One Nontradeable Factor (Labor)

Two Countries: \( (j \text{ or } k = 1 \text{ or } 2) \)

\( N^j \) identical households with labor endowment \( h^j \), supplied inelastically at \( w^j \).

- \( w^j h^j = E^j \): Household Income (and Expenditure)
- \( L^j = h^j N^j \): Total Labor Supply in \( j \)

\( N^j \) and \( h^j \) are the only possible sources of heterogeneity across the two countries.

Tradeable Goods:

- A continuum of monopolistically competitive sectors, \( s \in [0,1] \),
- Each sector produces a continuum of tradable differentiated goods, \( \nu \in \Omega^j_s = \Omega^1_s + \Omega^2_s \),

\( \Omega^j_s \): Disjoint sets of differentiated goods in sector \( s \) produced in country \( j \) in equilibrium
Household Preferences: Two-Tier structure

Lower-level, usual Dixit-Stiglitz aggregator (Homothetic within each sector)

\[
\tilde{C}_s^k \equiv \left[ \int_{\Omega_s} (c_s^k(v))^{-\frac{1}{\sigma}} \, dv \right]^\frac{\sigma}{\sigma-1}; \quad \sigma > 1, \quad s \in [0,1]
\]

Upper-level, \( \tilde{U}^k = U(\tilde{C}_s^k, s \in [0,1]) \), implicitly given by

\[
\int_0^1 (\beta_s)^{\frac{1}{\eta}} (\tilde{U}^k)^{\frac{\epsilon(s)-\eta}{\eta}} (\tilde{C}_s^k)^{\frac{\eta-1}{\eta}} \, ds \equiv 1; \quad \beta_s > 0 \text{ and } \sigma > \eta \neq 1
\]

- \((\epsilon(s) - \eta)/(1 - \eta) > 0\) for global monotonicity & quasi-concavity
- \(\int_0^1 \epsilon(s) \, ds = 1\), without loss of generality.
- If \(\epsilon(s) = 1\) for all \(s \in [0,1]\), standard homothetic CES
- If \(\epsilon(s) \neq 1\), nonhomothetic. Index sectors so that \(\epsilon(s)\) is increasing in \(s \in [0,1]\). Then,

\[
\omega(s, \tilde{U}^k) \equiv (\beta_s)^{\frac{1}{\eta}} (\tilde{U}^k)^{\frac{\epsilon(s)-\eta}{\eta}} \text{ is log-supermodular in } s \text{ and } \tilde{U}^k.
\]
**Lemma 1:** For a positive value function, \( \hat{g}(\bullet; x) : [0,1] \rightarrow \mathbb{R}_+ \), with a parameter \( x \), define

\[
g(s; x) \equiv \frac{\hat{g}(s; x)}{\int_0^1 \hat{g}(t; x) dt} \text{ (a density function)} \quad \text{and} \quad G(s; x) \equiv \int_0^s g(t; x) dt = \frac{\int_0^s \hat{g}(t; x) dt}{\int_0^1 \hat{g}(t; x) dt} \text{ (its cumulative distribution function)}.
\]

If \( \hat{g}(s; x) \) is **log-supermodular** in \( s \) and \( x \), i.e. \( \frac{\partial^2 \log \hat{g}(s; x)}{\partial s \partial x} > 0 \),

i) \( \frac{g(s; x)}{g(s; x')} \) is decreasing in \( s \) for \( x < x' \); **Monotone Likelihood Ratio (MLR)**

ii) \( G(s; x) > G(s; x') \) for \( x < x' \). **First-Order Stochastic Dominance (FSD)**

The happier households put more weights on the higher-indexed goods.
Household Maximization: Two-Stage Budgeting

1\textsuperscript{st} Stage (Lower-level) Problem: Chooses $c^k_s(\nu)$ for $\nu \in \Omega_s$ to:

$$\text{Max } \tilde{C}_s^k \equiv \left[ \int_{\Omega_s} \left(c^k_s(\nu)\right)^{1-\frac{1}{\sigma}} d\nu \right]^{\frac{\sigma}{\sigma-1}}, \text{ subject to } \int_{\Omega_s} p^k_s(\nu)c^k_s(\nu)d\nu \leq E^k_s,$$

$p^k_s(\nu)$ & $c^k_s(\nu)$: the unit consumer price and consumption of variety $\nu \in \Omega_s$;

$E^k_s$: Expenditure allocated to sector-s, taken as given.

Solution:

$$c^k_s(\nu) = \left(\frac{p^k_s(\nu)}{P^k_s}\right)^{-\sigma} C^k_s = \frac{(p^k_s(\nu))^{-\sigma}}{(P^k_s)^{1-\sigma}} E^k_s, \text{ where } P^k_s \equiv \left[ \int_{\Omega_s} (p^k_s(\nu))^{1-\sigma} d\nu \right]^{\frac{1}{1-\sigma}}$$

$C^k_s$: the maximized value of $\tilde{C}_s^k$, satisfying $E^k_s = P^k_s C^k_s$. 

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2\textsuperscript{nd} stage (Upper Level) Problem: Choose $E_s^k = P_s^k C_s^k$ to:

$$\text{Max } \tilde{U}^k \text{, subject to } \int_0^1 \left( \frac{1}{\eta} \left( \tilde{U}^k \right) \right)^{\varepsilon(s)-\eta} \left( C_s^k \right)^{\eta-1} ds \equiv 1 \text{ and } \int_0^1 P_s^k C_s^k ds = \int_0^1 E_s^k ds \leq E^k.$$  

Solution: the share of sector-s in k’s expenditure, $m_s^k$

$$m_s^k \equiv \frac{E_s^k}{E^k} \equiv \frac{P_s^k C_s^k}{E^k} = \frac{\beta_s \left( U^k \right)^{\varepsilon(s)-\eta} \left( P_s^k \right)^{1-\eta}}{\int_0^1 \beta_i \left( U^k \right)^{\varepsilon(i)-\eta} \left( P_t^k \right)^{1-\eta} dt},$$

where $U^k$ is the maximized value of $\tilde{U}^k$, given implicitly by:

$$\left( E^k \right)^{1-\eta} \equiv \int_0^1 \beta_s \left( U^k \right)^{\varepsilon(s)-\eta} \left( P_s^k \right)^{1-\eta} ds. \quad (U^k \text{ is strictly increasing in } E^k.)$$

Notes:

- $\partial \log(m_s^k / m_s^{k'}) / \partial \log(U^k) = \varepsilon(s) - \varepsilon(s')$. Higher-indexed more income elastic; Income elasticity differences are constant across different per capita income levels.

- $\beta_s \left( U^k \right)^{\varepsilon(s)-\eta} \left( P_s^k \right)^{1-\eta}$ is log-supermodular in $s$ and $U^k$. From Lemma 1, for fixed prices, a higher $E^k$ (and $U^k$) shifts the expenditure share towards higher-indexed.
The Rest of the model: Deliberately kept the same with Krugman (1980).

Iceberg Trade Costs: Only $1/\tau < 1$ fraction of exports survives shipping, reducing the export revenue to its fraction, $\rho \equiv (\tau)^{1-\sigma} < 1$

CES Demand for each good: $D_s(\nu) = A_s^j (p_s^j(\nu))^{-\sigma}$, $\nu \in \Omega_s^j$, where

$$A_s^j = b_s^j + \rho b_s^k \quad (k \neq j): \text{ Aggregate demand shifter for the producers in } j \text{ in } s$$

$$b_s^k = \beta_s \left( E_s^k \right)^\eta \left( U_s^k \right)^{\varepsilon(s)-\eta} N_s^k \left( p_s^k \right)^{\sigma-\eta}; \quad k's \text{ demand shifter for sector } s$$

Standard CES demand curve, but $U^k$ affects $b_s^k$ and hence $A_s^j$ differently across $s$.

Constant Mark-Up: $\psi_s$ units of labor to produce one unit of each variety in sector-$s$

$$p_s^j(\nu) = \frac{w_s^j \psi_s}{1 - 1/\sigma} \equiv p_s^j \quad \text{for } \nu \in \Omega_s^j$$

Free Entry (Zero-Profit) Condition: $\phi_s$ units of labor per variety to set up in sector-$s$.

Labor Market Equilibrium: $\int_0^1 f_s^j ds = 1$, $f_s^j$: sectoral share in employment (and value-added) and, if appropriately normalized, in the measure of firms (and varieties).
**Autarky Equilibrium** ($\rho = 0$):

**Standard-of-Living:**

\[
U^k_0 = u(x^k_0)
\]

where

\[
x^k_0 \equiv (h^k)^\sigma N^k = (h^k)^{\sigma - 1} L^k
\]

where \( u(x) \) is defined implicitly by

\[
(x)^{1-\eta} \equiv \int_0^1 \left( \beta_s (u(x))^{(\varepsilon(s) - \eta)} \right)^{\frac{\sigma - 1}{\sigma - \eta}} ds.
\]

- \( U^k_0 = u(x^k_0) \) is increasing both in \( h^k \) and in \( N^k \). **Aggregate increasing returns**
- Even if \( h^1 > h^2 \), \( U^1_0 < U^2_0 \) holds when \( L^1 / L^2 < (h^1 / h^2)^{1-\sigma} < 1 \).

**Market Size (and Firm) Distributions:**

\[
f^k_s = m^k_s = \frac{\left( \beta_s (u(x^k_0))^{(\varepsilon(s) - \eta)} \right)^{\frac{\sigma - 1}{\sigma - \eta}}}{\int_0^1 \left( \beta_t (u(x^k_0))^{(\varepsilon(t) - \eta)} \right)^{\frac{\sigma - 1}{\sigma - \eta}} dt}
\]

**Notes:**

- In autarky, firms (and labor) are distributed proportionately with market sizes.
- \( \left( \beta_s (u(x^k_0))^{(\varepsilon(s) - \eta)} \right)^{\frac{\sigma - 1}{\sigma - \eta}} \) is log-supermodular in \( s \) and \( x^k_0 \). From Lemma 1,

With a higher \( x^k_0 \equiv (h^k)^\sigma N^k \), the household becomes happier and spends relatively more on higher-indexed goods in equilibrium.
• Compare \( m_s^k = \frac{\left( \beta_s (u(x_0^k))^{(\varepsilon(s)-\eta)} \right)^{\sigma-1}}{\int_0^1 \left( \beta_i (u(x_0^k))^{(\varepsilon(t)-\eta)} \right)^{\sigma-1}} \) \( dt \) & \( m_s^k = \frac{\beta_s \left( U^k \right)^{(\varepsilon(s)-\eta)} \left( P_s^k \right)^{1-\eta}}{\int_0^1 \beta_i (U^k)^{(\varepsilon(t)-\eta)} \left( P_s^k \right)^{1-\eta}} \) dt and notice \( \frac{\sigma - 1}{\sigma - \eta} > 1 \) iff \( \eta > 1 \).

Given price indices, \( U \uparrow \) shifts the expenditure toward the higher-indexed.

In equilibrium, this causes entries (exits) and hence more (less) varieties in the higher (lower)-indexed sectors, reducing the effective relative prices of higher-indexed goods, which amplifies (moderates) the shift if \( \eta > (<) 1 \).

\[ \frac{d \log u(\lambda x)}{d \log \lambda} = \frac{\lambda xu'(\lambda x)}{u(\lambda x)} = \zeta(\lambda x) \] is increasing (decreasing) in \( x \), if \( \eta > (<) 1 \). Hence,

i) If \( \eta < 1 \), gains from a percentage increase in \( x \) is lower at a higher \( x \).

ii) If \( \eta > 1 \), gains from a percentage increase in \( x \) is higher at a higher \( x \).
Trade Equilibrium and Patterns of Trade
Figure 1: (Factor) Terms of Trade Determination

\[ \frac{L^1}{L^2} = \Lambda(\omega; \rho) \equiv (\omega)^{2\sigma} \frac{1 - \rho(\omega)^{-\sigma}}{1 - \rho(\omega)^{-\sigma}}, \text{ where } \omega \equiv \frac{w^1}{w^2}. \]

\[ (\rho)^{-1/\sigma} \]

\[ (\rho)^{1/\sigma} \]

\[ \omega \equiv \frac{w^1}{w^2} \]

\[ \lambda \equiv \frac{L^1}{L^2} \]

- The factor price lower in the smaller economy (Aggregate increasing returns)
- Globalization (\( \tau \downarrow \) or \( \rho \uparrow \)) reduces the smaller country’s disadvantage and hence the factor price differences.
Standard-of-Living: summarized by a single index, $x^k_\rho$

$$U^1_\rho = u(x^1_\rho), \text{ where } x^1_\rho \equiv \frac{(1-\rho^2)x_0^1}{1-\rho(\omega)^{-\sigma}} > x_0^1; \quad U^2_\rho = u(x^2_\rho), \text{ where } x^2_\rho \equiv \frac{(1-\rho^2)x_0^2}{1-\rho(\omega)^{-\sigma}} > x_0^2$$

$u(x)$, defined as before. 

Gains from trade

Market Size Distributions: $m^k_s = \frac{\left(\beta_s (u(x^k_\rho))^{(\varepsilon(s)-\eta)}\right)^{\frac{\sigma-1}{\sigma-\eta}}}{\left(x^k_\rho\right)^{\frac{1-\eta}{\sigma-\eta}}} = \frac{\left(\beta_s (u(x^k_\rho))^{(\varepsilon(s)-\eta)}\right)^{\frac{\sigma-1}{\sigma-\eta}}}{\int_0^1 \left(\beta_s (u(x^k_\rho))^{(\varepsilon(i)-\eta)}\right)^{\frac{\sigma-1}{\sigma-\eta}} dt}$

$\left(\beta_s (u(x^k_\rho))^{(\varepsilon(s)-\eta)}\right)^{\frac{\sigma-1}{\sigma-\eta}}$ is log-supermodular in $s$ & $x^k_\rho$. From Lemma 1, if $u(x^1_\rho) < u(x^2_\rho)$

i) MLR: 

$$\frac{m^1_s}{m^2_s} = \left(\frac{x^1_\rho}{x^2_\rho}\right)^{\frac{\eta-1}{\sigma-\eta}} \left(\frac{u(x^1_\rho)}{u(x^2_\rho)}\right)^{\frac{\sigma-1}{\sigma-\eta}}$$

is strictly decreasing in $s$:

ii) FSD: 

$$\int_0^1 m^1_s dt > \int_0^1 m^2_s dt$$

The Rich (Poor) has relatively larger domestic markets in higher(lower)-indexed sectors.
Firm Distributions: \[ f_s^1 = \frac{m_s^1 - \rho(\omega)^{-\sigma} m_s^2}{1 - \rho(\omega)^{-\sigma}}; \quad f_s^2 = \frac{m_s^2 - \rho(\omega)^{\sigma} m_s^1}{1 - \rho(\omega)^{\sigma}} \]

HME; \[ \frac{f_s^1}{f_s^2} > \frac{m_s^1}{m_s^2} > 1; \quad \frac{f_s^1}{f_s^2} = \frac{m_s^1}{m_s^2} = 1; \quad \text{or} \quad \frac{f_s^1}{f_s^2} < \frac{m_s^1}{m_s^2} < 1. \]

Sectoral Trade Balances: From \[ NX_s^1 = -NX_s^2 \equiv V_s^1 \rho b_s^2 (w^1)^{1-\sigma} - V_s^2 \rho b_s^1 (w^2)^{1-\sigma}, \]

\[ NX_s^1 = -NX_s^2 = \frac{\rho w^2 L^2}{(\omega)^{-\sigma}} - \rho (m_s^1 - m_s^2) = \frac{\rho w^1 L^1}{(\omega)^{\sigma}} - \rho (m_s^1 - m_s^2) \propto (m_s^1 - m_s^2). \]

Determined by the difference in the Demand Composition, not in the Market Size.

\[ U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2) \rightarrow m_s^1 / m_s^2 \text{ is strictly decreasing in } s \rightarrow \]

a unique cutoff sector, \( s_c \in (0,1) \), such that

\[ NX_s^1 = -NX_s^2 > 0 \text{ for } s < s_c; \quad NX_s^1 = -NX_s^2 < 0 \text{ for } s > s_c. \]
Figure 2: Home Market Effect and Patterns of Sectoral Trade Balances:

For $U^1_\rho = u(x^1_\rho) < U^2_\rho = u(x^2_\rho)$

The Rich (Poor) runs surpluses in the higher-(lower-) indexed sectors, which produce with higher (lower) income elastic goods.
Figure 3: Ranking the Countries

Red Curve: $U^1_0 < U^2_0$ below, $U^1_0 > U^2_0$ above

Black Curve: $U^1_\rho < U^2_\rho$ below, $U^1_\rho > U^2_\rho$ above
Comparative Statics
**Uniform Productivity Improvement:** \((\partial \log(h^1) = \partial \log(h^2) = \partial \log(h) > 0)\)

\(h^1 / h^2, L^1 / L^2, \omega = w^1 / w^2, x^1_0 / x^2_0, x^1_\rho / x^2_\rho\) all unchanged, with \(\partial \log(x^1_\rho) = \partial \log(x^2_\rho) = \sigma \partial \log(h) > 0\).

- Both \(U^1_\rho = u(x^1_\rho)\) and \(U^2_\rho = u(x^2_\rho)\) go up. Since \(\left(\beta_s (u(x^k_\rho))^{(e(s) - \eta)}\right)^{\frac{\sigma - 1}{\sigma - \eta}}\) is log-supermodular in \(s\) and \(x^k_\rho\), from Lemma 1, the market size distributions shift toward higher-indexed sectors in both countries, in the sense of MLR and FSD.

- \(\text{sgn} \frac{\partial \log(U^1_\rho / U^2_\rho)}{\partial \log(h)} = \text{sgn}(\eta - 1) \text{sgn}(x^1_\rho - x^2_\rho)\), from Lemma 2.

Welfare gaps widen (narrow) if sectors produce substitutes (complements).

- \(\text{sgn} \frac{\partial \log(m^1_s / m^2_s)}{\partial \log(h)} = \text{sgn}(x^2_\rho - x^1_\rho) \rightarrow s_e \text{ goes up.}\)
Figure 4: Product Cycles Due to Uniform Productivity Improvement

- As everyone becomes more productive, they shift their spending towards the higher-indexed.
- The relative weights of the sectors in which the Rich runs surpluses go up.
- To keep the overall trade account between the two countries in balance, the Rich’s trade account in each sector must deteriorate.
- The Rich switches from being the net-exporter to the net-importer in middle sectors.
Globalization, a higher $\rho = (\tau)^{1-\sigma}$, when two countries are equal in size: $L^1 = L^2 = L$

$$\omega = 1 \rightarrow x^k_\rho = (1 + \rho)x^k_0 = (1 + \rho)(h^k)^\sigma N^k = (1 + \rho)(h^k)^{\sigma-1} L$$

The relative factor price fixed at $\omega = 1$ and independent of $\rho$. No ToT change
- The country with higher per capita labor endowment is richer.
- A higher $1 + \rho$ is isomorphic to a uniform increase in $h^k$.

Figure 4: Product Cycles Due to Globalization
Globalization, a higher $\rho = (\tau)^{1-\sigma}$, when two countries are unequal in size:

**Leapfrogging and Reversal of the Patterns of Trade**

For $h^1 / h^2 > 1$ and below the Red curve,

$U^1_\rho < U^2_\rho$ at a low $\rho$,
Closer to autarky, Country 1 is poorer due to its disadvantage of being smaller, running surpluses in lower-indexed.

$U^1_\rho > U^2_\rho$ at a high $\rho$,
Globalization leads to a factor price convergence, which makes the smaller but smarter 1 richer, running surpluses in higher-indexed.

**Figure 5**
HME with Exogenous Taste Variations: A Comparison
An Extension of Krugman (1980):

Keep the same structure, except the upper-level preferences are homothetic CES,

\[
U^k \equiv \left[ \int_0^1 (\beta_s^k)^{\eta} \left( \tilde{C}_s^k \right)^{1-\eta} \, ds \right]^{\frac{\eta}{\eta-1}}, \quad \text{normalized to } \int_0^1 (\beta_s^k)^{\sigma-\eta} \, ds = 1
\]

with different weights \(\beta_s^k\), and \(\beta_s^1 / \beta_s^2\) strictly decreasing in \(s\).

Then,

**Standard-of-living:** \(U_{\rho}^k = \left( x_{\rho}^k \right)^{\frac{1}{\sigma-1}}\)

**Market Size Distribution:** \(m_s^k = \left( \beta_s^k \right)^{\frac{\sigma-1}{\sigma-\eta}} \) \(\Rightarrow m_s^1 / m_s^2 = \left( \beta_s^1 / \beta_s^2 \right)^{\frac{\sigma-1}{\sigma-\eta}}\) strictly decreasing in \(s\).

Otherwise, the same
**Figure 2**

Notes:
- $m_s^1 / m_s^2$ depends solely on the exogenous preferences parameters. Independent of $\rho$ and $h^k$. Effects on $s_c$ in the previous model are entirely due to nonhomotheticity.
- Uniform productivity growth cannot change the welfare gap.
- Leapfrogging can occur; Reversal of Patterns of Trade cannot.
- Krugman (1980), a special case with $\eta = 1$, $L^1 = L^2$, and $\beta_s^1 / \beta_s^2 = \gamma > 1$ for $0 \leq s < 1/2$; $\beta_s^1 / \beta_s^2 = 1/\gamma < 1$ for $1/2 < s \leq 1$. 

Adding An Outside Goods Sector

The same structure as before, except

**Homogeneous Good (Numeraire):** competitive, CRS (1-to-1), zero trade cost

**Household Preferences:** Three-Tier structure

\[ \tilde{C}^k_s \equiv \left[ \int_{\Omega} \left( c^k_s(v) \right)^{1-\frac{1}{\sigma}} d\nu \right]^{\frac{\sigma}{\sigma-1}} ; \sigma > 1, \quad s \in [0,1] \]

**Lower-level**, \( \tilde{C}^k_s \)

\[ \tilde{C}^k_s \]

**Middle-level**, \( \int_0^1 (\beta_s)^{\eta} (\tilde{U}^k_s)^{\eta} (\tilde{C}^k_s)^{\eta-1} ds \equiv 1 ; \beta_s > 0 \text{ and } \sigma > \eta \neq 1 \)

**Upper-level**, \( \tilde{W}^k = (1-\alpha) \log \tilde{C}^k_o + \alpha \log(\tilde{U}^k) \)

\( \tilde{C}^k_o \): Household consumption of the numeraire

\( \alpha \): (Fixed) expenditure share of differentiated goods
With a sufficiently small $\alpha$, both countries produce the numeraire.

- $L^j - \int_0^1 V^j_s ds > 0$; a positive employment in the numeraire sector.
- $w^j = 1$; (Factor) Terms of Trade uniquely pinned down and independent of $\rho$.
- Each household earns $h^k$ and spends $E^k = \alpha h^k$ on differentiated goods.

The Equilibrium Conditions would be the same otherwise.

**Autarky Equilibrium**

**Standard-of-Living:** $W^k_0 = (1 - \alpha) \log((1 - \alpha)h^k) + \alpha \log(u(x^k_0))$,

with $x^k_0 \equiv (\alpha h^k)^\sigma N^k = \alpha (\alpha h^k)^{\sigma - 1} L^k$

**Market Size Distributions:** $m^k_s = \frac{\left(\beta_s \left(u(x^k_0)\right)^{(\varepsilon(s)-\eta)}\right)^{\frac{\sigma - 1}{\sigma - \eta}}}{\int_0^1 \left(\beta_t \left(u(x^k_0)\right)^{(\varepsilon(t)-\eta)}\right)^{\frac{\sigma - 1}{\sigma - \eta}} dt}$
**Trade Equilibrium:**

**Standard-of-Living:** \( W_k^\rho = (1 - \alpha) \log((1 - \alpha)h^k) + \alpha \log(u(x^k_\rho)) \),

where \( x^k_\rho \equiv (1 + \rho)(\alpha h^k)^\sigma N^k = (1 + \rho)x^k_0 \)

**Market Size Distributions:** \( m^k_s = \frac{\left( \beta_s \left( u(x^k_\rho) \right)^{(e(s) - \eta)} \right)^{\frac{\sigma - 1}{\sigma - \eta}} \int_{0}^{1} \left( \beta_t \left( u(x^k_\rho) \right)^{(e(t) - \eta)} \right)^{\frac{\sigma - 1}{\sigma - \eta}} dt \)}{\left( \beta_t \left( u(x^k_\rho) \right)^{(e(t) - \eta)} \right)^{\frac{\sigma - 1}{\sigma - \eta}} dt} \)

**Firms Distributions:**

From \( V^1_s = \frac{m^1_s(\alpha L^1) - \rho m^2_s(\alpha L^2)}{1 - \rho} > 0; \quad V^2_s = \frac{m^2_s(\alpha L^2) - \rho m^2_s(\alpha L^2)}{1 - \rho} > 0, \)

\( f^1_s = \frac{m^1_s L^1 - \rho m^2_s L^2}{L^1 - \rho L^2} > 0; \quad f^2_s = \frac{m^2_s L^2 - \rho m^1_s L^1}{L^2 - \rho L^1} > 0 \quad \text{for} \quad \rho < \frac{m^1_s L^1}{m^2_s L^2} < \frac{1}{\rho}. \)
Sectoral Trade Balances:

\[ NX_s^1 = -NX_s^2 \equiv V_s^1 \rho b_s^2 - V_s^2 \rho b_s^1 = \frac{\rho}{1 + \rho} (V_s^1 - V_s^2) = \frac{\alpha \rho}{1 - \rho} (m_s L^1 - m_s L^2) \propto (m_s L^1 - m_s L^2) \]

What matters is the cross-country difference in the market size in each sector itself.

Trade Balances in Differ. Goods Sectors:

\[ \int_0^1 NX_s^1 ds = -\int_0^1 NX_s^2 ds = \frac{\alpha \rho}{1 - \rho} (L^1 - L^2) \]

Instead of having a higher factor price, the larger country runs an overall surplus in the differentiated goods sectors, with a deficit in the outside good sector.

Factor Price Equalization Condition; \( \alpha < \text{Min} \left\{ \frac{(1 - \rho)L^1}{L^1 - \rho L^2}, \frac{(1 - \rho)L^2}{L^2 - \rho L^1} \right\} \)
Patterns of Trade: Home Market Effect

- $m_s^1 / m_s^2$ is strictly decreasing in $s$, for $x_0^1 < x_0^2 \Leftrightarrow L^1 / L^2 < (h^1 / h^2)^{1-\sigma}$

- When $L^1$ and $L^2$ are not too different, a unique cutoff sector, $s_c \in (0,1)$ such that

$$NX_s^1 = -NX_s^2 = \frac{\alpha\rho L}{1 - \rho} (m_s^1 L^1 - m_s^2 L^2) > 0 \text{ for } s < s_c; < 0 \text{ for } s > s_c.$$

Comparative Statics: With a uniform productivity improvement and globalization,

- $m_s^k$ shifts towards the higher-indexed in the sense of MLR and FSD.
- $\text{sgn} \frac{\partial \log(U^1_\rho / U^2_\rho)}{\partial \log(h)} = \text{sgn} \frac{\partial \log(U^1_\rho / U^2_\rho)}{\partial \log(1 + \rho)}$

  $$= \text{sgn}(\eta - 1) \text{sgn}(x^1_\rho - x^2_\rho).$$

- $\text{sgn} \frac{\partial \log(m_s^1 / m_s^2)}{\partial \log(h)} = \text{sgn} \frac{\partial \log(m_s^1 / m_s^2)}{\partial \log(1 + \rho)}$

  $$= \text{sgn}(x^2_\rho - x^1_\rho) \Rightarrow s_c \in (0,1) \text{ moves up.}$$

Rich’s Sectoral Trade Balances switch from Surpluses to Deficits
In Summary:

- With the ToT pinned down by the numeraire good, a higher $\rho$ does not change ToT change, even when the country sizes are different.
- With no ToT change, the effect of a higher $\rho$ is isomorphic to the effects of uniform productivity improvement (an equi-proportional increase in $h^k$), as in the $L^1 = L^2$ case of the previous model.
- With no ToT change, Leapfrogging and A Reversal of Patterns of Trade cannot occur.

Two Caveats: Unlike in the $L^1 = L^2$ case of the previous model, $L^1 \neq L^2$ generates the possibility:

- $U^1_\rho < U^2_\rho \iff L^1 / L^2 < (h^1 / h^2)^{1-\sigma}$ may occur, even if $h^1 > h^2$.
- If $L^1$ and $L^2$ are too different, the larger country may run a surplus in all $s$. 
Concluding Remarks
• Empirically, goods differ widely in their income elasticities; rich (poor) countries tend to export goods with high (low) income elasticities.

• We aim to explain why the rich (poor) have CA in high (low) income elastic goods with two ingredients, *Nonhomothetic Preferences & Home Market Effect*

• Simple intuition
  ✓ Demand composition of the Rich (Poor) more skewed towards high (low) income elastic goods
  ✓ With scale economies and positive but finite trade costs, such cross-country differences in the demand composition become a source of comparative advantage.

• No previous studies capture this intuition in a setup flexible and yet tractable enough to allow for a variety of comparative static exercises, because GE models with *imperfect competition, scale economies, positive but finite trade costs, and nonhomotheticity* would be intractable
  ✓ *Explicitly additively separable nonhomothetic preferences*, such as Stone-Geary or CRIE, are too restrictive and too intractable

• *Implicitly additively separable nonhomothetic preferences* enables us to overcome this difficulty