COMMERCIAL PROPERTY PRICE INDEXES AND THE SYSTEM OF NATIONAL ACCOUNTS

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Abstract. This paper studies the problems associated with the construction of price indexes for commercial properties that could be used in the System of National Accounts (SNA). Property price indexes are required for the stocks of commercial properties in the Balance Sheets of the country. Related service price indexes for the land and structure input components of a commercial property are required in the Production Accounts of the country if the Multifactor Productivity of the Commercial Property Industry is calculated as part of the SNA. The paper reviews existing methods for constructing an overall Commercial Property Price Index (CPPI) and concludes that most methods are biased (due to their neglect of depreciation) and more importantly, not able to provide separate land and structure subindexes. A class of hedonic regression models that is not subject to these problems is discussed.

Keywords. Commercial property price indexes; Net operating income; Discounted cash flow; System of National Accounts; Balance sheets; Methods of depreciation; Land and structure prices; Hedonic regressions; Repeat sales method

1. Introduction

This paper presents a conceptual framework for measuring prices (and the corresponding quantities or volumes) of inputs, outputs and assets of the commercial property industry of a country in the context of the requirements for the production accounts in the System of National Accounts (SNA).[1]

In Sections 2 and 3 of the paper, we outline some of the measurement problems associated with measuring the outputs and variable inputs[2] of a commercial property firm.[3] These measurement problems are not particularly troublesome.

The main measurement problems associated with the commercial property industry occur when we attempt to measure asset values associated with a property and the corresponding service flow input contributions of the land plot and the structure on the plot. The fundamental problem is that once the structure on a property has been completed, it is a fixed factor of production as is the land plot associated with the structure. Since there are two fixed factors, it is difficult to determine the separate contributions

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to production of each factor. Thus, when a commercial property is sold, it is sold at a price that is the joint value of the structure and the land at that moment. But for SNA purposes, we need to decompose that value into structure and land components. How exactly should this be done? In order to answer this question, we need a model of the determinants of property value. In Sections 4–6, we provide such a model[4] and show how it can lead to decompositions of asset values into land and structure components. The model assumes that the fundamental determinants of property value are the discounted stream of cash flows that the property is expected to generate. In order to obtain a decomposition of aggregate property value into land and structure components, an extra assumption is needed, that is, investors are able to form expectations about the future price of the land that the structure sits on.

The theoretical model of property value indicates that the pattern of structure depreciation is likely to be quite variable over time and that it is unlikely that standard structure depreciation models (declining balance, straight line or one hoss shay) will be able to capture the actual amounts of depreciation that occur in commercial property structures.[5] Thus, in Section 7, we describe the recent research by Diewert and Shimizu (2014) that allows for more arbitrary patterns of depreciation to be measured. Their method requires information on asset values for a group of commercial properties over a time period. For their particular application, they used the appraised values for a group of office buildings in Tokyo (along with a few other characteristics of the property which will be explained in Section 7). They applied a hedonic regression model to the panel data set of commercial properties and estimated depreciation rates as well as providing separate indexes for the structure and the land components of the properties.

The depreciation rates that Diewert and Shimizu estimated were for continuing properties and their estimated CPPI was constructed for this group of continuing properties. But structures on commercial properties are often demolished even though they are still earning positive cash flows. This early retirement of a productive asset is basically due to *obsolescence*, that is, new (unanticipated) uses of the property spring up over time and so better returns on the property can be earned by investors if the existing structure is torn down and replaced with an alternative more productive structure. Section 8 of the paper also draws on Diewert and Shimizu (2014) and outlines how this extra depreciation component can be estimated if information on births and deaths of commercial buildings is available.

Section 9 reviews most of the methods that have been used in order to construct practical CPPIs. Section 10 concludes.

2. The Construction of Output Price Indexes

We consider the problems associated with constructing quarterly input and output price indexes for a particular commercial property in a single location. We assume that the property has K sources of revenue and the quarter t price for product $k = 1, 2, \ldots, K$ is p_k^t and the corresponding quantity sold during the quarter is q_k^t. Before proceeding further, we need to discuss the exact meaning of the microeconomic prices and quantities if there are *multiple* transactions for say commodity n within quarter t. In this case, it is natural to interpret q_k^t as the *total* amount of commodity k sold within quarter t. In order to conserve the value of transactions, it is necessary that p_k^t be defined as a *unit value*;[6] that is, p_k^t must be equal to the value of transactions for commodity n during quarter t divided by the total quantity transacted, q_k^t. Thus define *the revenue of the commercial property in quarter t* as:

$$R^t \equiv \sum_{k=1}^{K} p_k^t q_k^t \equiv p^t \cdot q^t \tag{1}$$

where $p^t \equiv (p_1^t, \ldots, p_K^t)$ is the quarter t output price vector, $q^t \equiv (q_1^t, \ldots, q_K^t)$ is the quarter t output quantity vector and $p^t \cdot q^t$ denotes the inner product of these two vectors.

The outputs produced by an office or retail building will typically consist primarily of the rental or leasing of individual units of floor space. The total floor space leased or rented will generally be well

below the total floor space of the building since some space will be taken up by hallways, utility rooms, caretaker and managerial offices.[7] When measuring outputs, rented space is what counts but in Section 5 when valuing the cost of the services provided by the basic building structure, it is total floor space that matters. In addition to leased office and retail space, the building may make additional revenues from renting parking spaces and other miscellaneous sources of revenues.[8]

If the revenue generating property leases (for commercial space in the building) are monthly or quarterly, there is no problem in determining quarterly revenues. However, if the leases extend longer than a quarter and there is a fixed payment at the beginning of the lease for the use of leased space that covers the entire leasing period, then this total lease payment has to be *amortized* into imputed quarterly payments over the life of the lease. There are various commercial accounting methods for accomplishing this amortization and the price statistician may have no choice but to use whatever amortization method was used in the quarterly statements of the property owner.

The index number problem can be explained in the following manner. Consider the property's revenue ratio going from say quarter 0–1, R^1/R^0. Index numbers attempt to decompose a *value ratio* for the two periods under consideration into a *price change component P* times a *quantity change component Q*. Thus, we look for two functions of $4K$ variables, $P(p^0, p^1, q^0, q^1)$ and $Q(p^0, p^1, q^0, q^1)$ such that:[9]

$$p^1 \cdot q^1 / p^0 \cdot q^0 = P(p^0, p^1, q^0, q^1) Q(p^0, p^1, q^0, q^1) \qquad (2)$$

It can be seen that if the price index function $P(p^0, p^1, q^0, q^1)$ has been determined, then the quantity index $Q(p^0, p^1, q^0, q^1)$ can be residually determined using equation (2). If the functional form for $P(p^0, p^1, q^0, q^1)$ is known, then we can use (2) to determine the period 0 and 1 aggregate price *levels*, P^0 and P^1, respectively, and the period 0 and 1 aggregate quantity (or volume) *levels*, Q^0 and Q^1 respectively, as follows:

$$P^0 \equiv 1; \quad P^1 \equiv P\left(p^0, p^1, q^0, q^1\right); \quad Q^0 \equiv p^0 \cdot q^0; \quad Q^1 \equiv p^1 \cdot q^1 / P\left(p^0, p^1, q^0, q^1\right) \qquad (3)$$

Thus once the functional form for the price index $P(p^0, p^1, q^0, q^1)$ is determined (and detailed price and quantity data are available for the two quarters), aggregate output price and quantity levels for the two quarters under consideration can be determined using the definitions in (3).

There are four main approaches to the determination of the functional form for a price index $P(p^0, p^1, q^0, q^1)$ that compares the prices (and associated quantities) between two periods: fixed basket and averages of fixed basket approaches; the test or axiomatic approach; the stochastic approach and the economic approach.

These four approaches are explained in detail elsewhere.[10]

There are two main functional forms for the price index that are used by statistical agencies as ideal target indexes: the Fisher ideal price index P_F and the Törnqvist–Theil price index P_T. Before defining these two indexes, it is useful to define some other indexes that are frequently used by statistical agencies.

One of the earliest approaches to defining a price index is the *fixed basket approach*. In this approach, we are given a 'representative' basket of commodities that is defined by the positive quantity vector q. Given the price vectors for periods 0 and 1, p^0 and p^1, respectively, we can calculate the cost of purchasing this same basket in the two periods, $p^0 \cdot q$ and $p^1 \cdot q$. Then the ratio of these costs is a very reasonable indicator of pure price change over the two periods under consideration, provided that the basket vector q is 'representative'. Thus, define the *Lowe (1823) price index*, P_{Lo}, as follows:

$$P_{Lo}\left(p^0, p^1, q\right) \equiv p^1 \cdot q / p^0 \cdot q \qquad (4)$$

As time passed, economists and price statisticians demanded a bit more precision with respect to the specification of the basket vector q. There are two natural choices for the reference basket: the period

0 commodity vector q^0 or the period 1 commodity vector q^1. These two choices lead to the Laspeyres (1871) price index P_L defined by (5) and the Paasche (1874) price index P_P defined by (6):[11]

$$P_L\left(p^0, p^1, q^0, q^1\right) \equiv p^1 \cdot q^0 / p^0 \cdot q^0 = \sum_{k=1}^{K} s_k^{\,0}\left(p_k^1/p_k^0\right); \tag{5}$$

$$P_P\left(p^0, p^1, q^0, q^1\right) \equiv p^1 \cdot q^1 / p^0 \cdot q^1 = \left[\sum_{k=1}^{K} s_k^1\left(p_k^1/p_k^0\right)^{-1}\right]^{-1} \tag{6}$$

where the period t expenditure share on commodity n, s_k^t, is defined as $p_k^t q_k^t / p^t \cdot q^t$ for $k = 1, \ldots, K$ and $t = 0, 1$. Thus, the Laspeyres price index P_L can be written as a base period expenditure share weighted average of the M price ratios (or price relatives), p_k^1/p_k^0.[12] The last equation in (6) shows that the Paasche price index P_P can be written as a period 1 (or current period) expenditure share weighted *harmonic* average of the M price ratios.[13]

The problem with these index number formulae is that they are equally plausible but in general, they will give different answers. This suggests that if we require a single estimate for the price change between the two periods, then we should take some sort of evenly weighted average of the two indexes as our final estimate of price change between periods 0 and 1. An example of a symmetric average is the geometric mean, which leads to the *Fisher* (1922) *ideal index*, P_F, defined as

$$P_F\left(p^0, p^1, q^0, q^1\right) \equiv \left[P_L\left(p^0, p^1, q^0, q^1\right) P_P\left(p^0, p^1, q^0, q^1\right)\right]^{1/2} \tag{7}$$

Another useful functional form for the target price index is the Törnqvist–Theil index P_T. Theil (1967; 137) provided a strong justification for the use of this index from the perspective of the stochastic or descriptive statistics approach to index number theory. Theil's measure of overall logarithmic price change is defined as follows:

$$\ln P_T\left(p^0, p^1, q^0, q^1\right) \equiv \sum_{k=1}^{K} (1/2)\left(s_k^0 + s_k^1\right) \ln\left(p_k^1/p_k^0\right) \tag{8}$$

Exponentiating the right-hand side of (8) provides a formula for $P_T(p^0, p^1, q^0, q^1)$.[14]

The Fisher index P_F receives a strong justification as a target index from the perspectives of the basket, axiomatic and economic approaches to index number theory while the Törnqvist–Theil index P_T receives a strong justification from the perspectives of the axiomatic, stochastic and economic approaches to index number theory.[15] Thus, choosing between these two alternative indexes is difficult but fortunately, using time series data, these two index formulae give very similar results and so typically, it will not matter which of these two indexes is chosen.[16]

The above material dealt with aggregating revenue information on multiple sources of revenue from the same building. This information can be aggregated over multiple buildings in the same location and in the same industry to form an industry output price index using the two stage aggregation procedures that are commonly used in the construction of a Producer Price Index.[17] Typically, commercial buildings in a particular location are classified into three broad groups: offices, retail sales and industrial (or factories).

Buildings could be further subdivided according to the type of construction, the location of the building and other characteristics.

We conclude this section with a discussion of the problems raised by *vacancies*; that is, for some quarters, some parts of the building may be temporarily vacant and thus for these components m of the building's revenue stream for quarter t, it may be the case that p_m^t equals 0. These zero components can cause the Törnqvist–Theil index P_T to be ill-defined since the log of 0 is infinite.[18] The solution to this problem is to note that typically, the individual output quantities q_k^t are constant[19] and so we have $q_k^0 = q_k^1$

for $k = 1, 2, \ldots, K$, irrespective of whether all units are rented for the two periods or not. Thus under these conditions, the Laspeyres, Paasche and Fisher output price indexes reduce to the Lowe index, P_{Lo} $(p^0, p^1, q) \equiv p^1 \cdot q / p^0 \cdot q$, where q is the common quantity vector for the two periods under consideration. Under these conditions, letting P be either P_L, P_P or P_F, we find using definitions (3) that the price and quantity levels for quarters 0 and 1 become:

$$P^0 \equiv 1; P^1 \equiv P\left(p^0, p^1, q^0, q^1\right) = R^1/R^0 = p^1 \cdot q^1 / p^0 \cdot q^0; Q^0 \equiv p^0 \cdot q^0; Q^1 \equiv p^0 \cdot q^0 = Q^0 \quad (9)$$

Thus, the price index $P(p^0, p^1, q^0, q^1)$ collapses to the revenue ratio, R^1/R^0 if we use the Laspeyres, Paasche or Fisher index number formula. Hence our solution to the vacancy problem is simple: use the Fisher formula P_F and not the index P_T in order to form a commercial property output price index.

It should be noted that our suggested national income accounting treatment of commercial property revenues differs somewhat from the treatment used by commercial property appraisers. Appraisers impute a 'normal' rent for vacant suites in the building and add it to actual rent. They then reduce this imputed total rent by one minus a 'normal' vacancy rate and they use this adjusted rent as their estimate of Net Operating Revenues for the building in the period under consideration.[20] However, this type of imputation would typically not be made for national accounting purposes where revenues are taken to be actual period t operating revenues for the production unit, not imputed revenues. It can be seen that, typically, appraised operating income will be smoother than actual operating revenues, since vacancies are not uniform over time.

We turn our attention to the construction of (noncapital) input indexes for a commercial property.

3. The Construction of Variable Input Price Indexes

In this section, we focus on nondurable variable inputs that have well defined price and quantity components and that are used by the commercial property firm in quarters $t = 0, 1$. We assume that there are M such variable inputs.[21] Denote the quarter t unit value price of input m by p^t_{Im} and the corresponding total quantity purchased during the quarter by q^t_{Im} for $m = 1, 2, \ldots, M$.[22] Examples of these nondurable inputs include: inputs used to heat the building such as fuel oil, coal and natural gas; electricity inputs; telecommunication inputs; cleaning supplies; janitorial, maintenance and repair inputs; insurance services; security and caretaker services and managerial and legal services inputs.[23]

In the standard SNA and in Multifactor Productivity Accounts,[24] these nondurable inputs are further classified as intermediate inputs or as labour inputs. The last three classes of inputs listed above could be listed as labour inputs if the type of service rendered is provided by an employee of the commercial property firm. Alternatively, these three types of input could be classified as intermediate inputs if the type of service is contracted out.

The construction of an input price index for the M classes of input proceeds in a manner that is entirely analogous to our discussion of an output price index in the previous section. Thus, the property's *quarter t variable input cost C^t* is defined as follows:

$$C^t \equiv \sum_{m=1}^{M} p^t_{Im} q^t_{Im} \equiv p^t_I \cdot q^t_I \quad (10)$$

where $p^t_I \equiv (p^t_{I1}, \ldots, p^t_{IM})$ denotes the quarter t variable input price vector, $q^t_I \equiv (q^t_{I1}, \ldots, q^t_{IM})$ is the corresponding quarter t variable input quantity vector and $p^t_I \cdot q^t_I$ denotes the inner product of these two vectors.

Consider the property's variable cost ratio going from say quarter 0–1, C^1/C^0. As in the previous section, we decompose this *value ratio* for the two quarters under consideration into an *input price*

change component P_I times an *input quantity change component Q_I*. Thus, we look for two functions of $4M$ variables, $P_I(p^0, p^1, q^0, q^1)$ and $Q_I(p^0, p^1, q^0, q^1)$ such that:

$$p_I^1 \cdot q_I^1 / p_I^0 \cdot q_I^0 = P_I\left(p_I^0, p_I^1, q_I^0, q_I^1\right) Q_I\left(p_I^0, p_I^1, q_I^0, q_I^1\right) \tag{11}$$

It can be seen that if the price index function $P_I(p_I^0, p_I^1, q_I^0, q_I^1)$ has been determined, then the quantity index $Q_I(p_I^0, p_I^1, q_I^0, q_I^1)$ can be residually determined using equation (11). If the functional form for $P_I(p_I^0, p_I^1, q_I^0, q_I^1)$ is known, then we can use (11) to determine the quarter 0 and 1 aggregate input price *levels*, P_I^0 and P_I^1, respectively, and the quarter 0 and 1 aggregate input quantity (or volume) *levels*, Q_I^0 and Q_i^1, respectively, as follows:

$$P_I^0 \equiv 1; P_I^1 \equiv P_I\left(p_I^0, p_I^1, q_I^0, q_I^1\right); \quad Q_I^0 \equiv p_I^0 \cdot q_I^0; \quad Q_I^1 \equiv p_I^1 \cdot q_I^1 / P_I\left(p_I^0, p_I^1, q_I^0, q_I^1\right) \tag{12}$$

As in the previous section, the four major approaches to index number theory that have been considered in the literature to date suggest that the Fisher or Törnqvist–Theil functional forms defined earlier by (7) and (8) would be good choices for $P_I(p_I^0, p_I^1, q_I^0, q_I^1)$.

The problem of zero purchases of a particular input during one of the two periods under consideration needs to be addressed.[25] We cannot use the solution to this problem that was used in the previous section because quantities of variable inputs are generally not constant across periods. To explain our suggested solution to the problem of zero purchases of an input during one period, we suppose q_i^0 units of a particular input are purchased in quarter 0 at price p_m^0 but no units of the input are purchased in quarter 1. Thus, it is clear that we can set q_m^1 equal to 0 but if we set p_m^1 equal to zero as well, our preferred Fisher and Törnqvist–Theil input indexes can generate anomalous results. In order to obtain stable input indexes over time, it is best to impute a positive price for the missing price, p_m^1.

There are at least three possible choices for this imputation exercise.[26] First, carry forward the price of the previous period; that is, set p_m^1 equal to the observed price p_m° for the product m in quarter 0. Second, collect a price for the same product in quarter 1. Thus if a quantity of a particular type of cleaning fluid was purchased by the firm in quarter 0 but not purchased in period 1, a price for the same product is collected for period 1. Or third, assume that the rate of price change for product m going from quarter 0 to 1 is the same as the rate of change of an available price index for a similar product or class of products. Thus, suppose the level of a statistical agency price index for cleaning fluids is $P_C F^0$ in quarter 0 and $P_C F^1$ in quarter 1, then the imputed price for product m in quarter 1, p_m^1, is set equal to $p_m^0 [P_C F^1 / P_C F^0]$.

The first method is not recommended if the inflation rate for product m is high or very variable.[27] The second method is a preferred method but it may be very costly to obtain the missing price quote from the marketplace.[28] The third method will generally be satisfactory but of course, price indexes by product category must be available.

The above discussion has focussed on nondurable variable inputs for which (at least in principle) it is possible to obtain period by period unit value prices and the corresponding period by period quantities purchased for the commercial property under consideration. However, there are four additional periodic input costs for which there are values but no obvious breakdown into price and quantity components. These classes of *value only nondurable variable input costs* are quarterly property tax payments, quarterly business income tax payments, quarterly property insurance payments and quarterly direct and indirect charges for undertaking monetary transactions and holding bank balances.

These costs need to be decomposed into price and quantity components so that the real output and input of the Commercial Property sector can be computed for national income accounting purposes. There are several alternative approaches that countries have used to decompose property monetary and insurance payments into price and quantity components and since there is no generally accepted methodology, we will leave this decomposition problem to others to solve. However, the first two types of charges belong in the primary income accounts of the Commercial Property Industry and so we will defer the

problems associated with decomposing these charges until we have studied the problems associated with costing out the contribution of the building or structure and of the land that the structure sits on. Thus, in the following sections, we study in some detail what the determinants of property value are and how stock and flow price indexes for the building and for the property's land area can be constructed.[29]

4. The Determinants of Commercial Property Value

The SNA requires separate measures for the input contributions of a commercial property structure and the associated land plot. For the most part, this decomposition problem has been neglected in the commercial property academic literature, which has focused on the total investment return of a commercial property project; see for example Fisher *et al.* (2007). Bokhari and Geltner (2012) and Geltner *et al.* (2014; 635–644). However, decomposing property value into structure and land components is a very difficult task and before we can tackle these problems, it is necessary to first outline a theory of the determinants of the value of a commercial property, taking into account the fixity of the structure once it has been built.[30]

We outline a simplified model for an investment in a given property. We assume that a group of investors has either purchased a commercial property building at the end of period 0 (or the beginning of period 1) or has constructed a new building which is just ready for occupancy at the end of period 0.[31] We assume that the total actual cost of the structure at the beginning of period 1 is known to the investor group and is $C_S^0 > 0$, and the opportunity cost value of the land plot at the beginning of period 1 is $V_L^0 > 0$. The *total initial cost of the commercial property*, C^0, is then defined as

$$C^0 \equiv C_S^0 + V_L^0 \tag{13}$$

Time is divided up into discrete periods, $t = 0, 1, 2, \ldots$ and we assume that the *end of period t value of the land plot* is expected to be V_L^t for $t = 1, 2, \ldots$. Thus, the investors form definite expectations about the price movements of the land plot that the structure utilizes. It will be convenient to relate these expected land values to period by period *land price inflation rates* i_t; that is, we assume that the period t land prices V_L^t and land inflation rates i_t satisfy the following equations, with $1 + i_t > 0$ for all t:[32]

$$V_L^t = (1 + i_1)(1 + i_2)\ldots(1 + i_t)\,V_L^0; \qquad t = 1, 2, \ldots \tag{14}$$

We assume that the *beginning of period t cost of capital* (or interest rate) that the investors face is $r_t > 0$ for $t = 1, 2, \ldots$. Finally, we assume that the building is expected to generate net operating income (NOI; or cash flow) equal to $N^t \geq 0$,[33] which, following Peasnell (1981) and Diewert (2005; p. 485), we assume to be realized at the end of each period $t = 1, 2, \ldots$. Thus, the information set that we assume is known to the investors consists of the building cost C_S^0, the sequence of end of period land values V_L^t (or equivalently V_L^0 and the sequence of land inflation rates i_t), the sequence of one period interest rates r_t and the sequence of cash flows N^t. As will be seen shortly, it is these variables that determine the value of the property.

Using the above information set, we can define an expected discounted profit maximization problem for each choice of time period $t = 1, 2, \ldots$. Problem t assumes that the firm demolishes the structure at the end of period t, at which time the structure has no remaining value, but of course the land will have (expected) value V_L^t.[34] The resulting *expected discounted profit* (Π^t) for the investor group will then be defined as follows:[35]

$$\prod{}^t \equiv -C_S^0 - V_L^0 + \alpha_1 N^1 + \alpha_2 N^2 + \ldots + \alpha_t N^t + \alpha_t \beta_t V_L^0; \qquad t = 1, 2, \ldots \tag{15}$$

where the α_t and β_t are defined recursively as follows:

$$\alpha_1 \equiv (1 + r_1)^{-1}; \alpha_t \equiv (1 + r_t)^{-1}\alpha_{t-1} \qquad \text{for } t = 2, 3, \ldots; \tag{16}$$

$$\beta_1 \equiv (1 + i_1); \beta_t \equiv (1 + i_t)\beta_{t-1} \qquad \text{for } t = 2, 3, \ldots \qquad (17)$$

Thus, Π^t is the sum of the discounted cash flows that the property is expected to generate over time periods 1 to t, $\alpha_1 N^1 + \alpha_2 N^2 + \ldots + \alpha_t N^t$, plus the discounted expected land value of the property at the end of period t, $a_t b_t V_L^0 = a_t V_L^t = (1 + r_1)^{-1}(1 + r_2)^{-1} \ldots (1 + r_t)^{-1} V_L^t$, less the initial value of the structure at the beginning of period 1, C_S^0, less the market value of the land at the beginning of period 1, V_L^0.

We assume that the sequence of Π^t is maximized at t equal to $T \geq 1$. We assume that Π^T is nonnegative:[36]

$$\prod^T \equiv -C_S^0 - V_L^0 + \alpha_1 N^1 + \alpha_2 N^2 + \ldots + \alpha_T N^T + \alpha_T \beta_T V_L^0 \geq 0 \qquad (18)$$

Thus, T is the *endogenously determined expected length of life for the structure*. Note that the determination of the length of life of the structure is not a simple matter of determining when the building will collapse due to the effects of aging and use: it is an *economic decision*.

The above analysis shows that the decision to retire a commercial property structure is an endogenous one that is not determined exogenously by wear and tear physical deterioration of the building. Of course, building deterioration will eventually affect cash flows but the point is that demand conditions will also affect cash flows so that deterioration alone does not determine when a building will be retired. The retirement decision depends crucially on the intertemporal pattern of cash flows generated by the building and on the movements in the price of the land plot over time; structures in locations with a higher land inflation rate will tend to have a shorter life than structures in locations with a lower land inflation rate. Thus, our theory of structure retirement is somewhat different from existing theories in the real estate literature about the retirement and depreciation of a commercial property structure.[37]

In the following section, we show how the above theory of property value leads to estimates for structure depreciation.

5. Property Asset Values, User Costs and Depreciation

We assume that the optimal length of life of the structure T has been determined and that the nonnegative discounted profits constraint (18) holds.

Our task in this section is to determine the sequence of project asset values and the changes in asset value over each time period.

The sequence of expected *end of period t project asset values* A^t can be defined as follows:[38]

$$
\begin{aligned}
A^0 &\equiv \alpha_1 N^1 + \alpha_2 N^2 + \ldots + \alpha_T N^T + \alpha_T V_L{}^T; \\
A^1 &\equiv (1 + r_1)\left[\alpha_2 N^2 + \ldots + \alpha_T N^T + \alpha_T V_L{}^T\right]; \\
A^2 &\equiv (1 + r_1)(1 + r_2)\left[\alpha_3 N^3 + \ldots + \alpha_T N^T + \alpha_T V_L{}^T\right]; \\
&\ldots \\
A^{T-1} &\equiv (1 + r_1)(1 + r_2)\ldots(1 + r_{T-1})\left[\alpha_T N^T + \alpha_T V_L{}^T\right] \\
&= (1 + r_T)^{-1}\left(N^T + V_L{}^T\right); \\
A^T &\equiv V_L{}^T
\end{aligned}
\qquad (19)
$$

Thus, at the end of period t (which is equal to the beginning of period $t + 1$), the expected property asset value A^t is equal to the expected period $t + 1$ to period T cash flows, N^{t+1} to N^T, discounted appropriately to the beginning of period $t + 1$ plus the discounted to the beginning of period $t+1$ expected value of the land plot at the end of period T, $(1 + r_{t+1})^{-1}(1 + r_{t+2})^{-1} \ldots (1 + r_T)^{-1} V_L^T$.

Note that the last T equations in (19) can be rearranged to give us the following relationships between the end of period t asset values A^t and the period t cash flows N^t:

$$
\begin{aligned}
N^t &= (1 + r_t) A^{t-1} - A^t \qquad\qquad t = 1, \dots, T \\
&= r_t A^{t-1} + (A^{t-1} - A^t) \\
&= r_t A^{t-1} + \Delta^t
\end{aligned}
\tag{20}
$$

where $r_t A^{t-1}$ reflects the opportunity costs of the capital that is tied up in the project at the beginning of period t, and Δ^t is the period t expected *asset value change* for the project defined by (21):[39]

$$
\Delta^t \equiv A^{t-1} - A^t; \qquad\qquad t = 1, \dots, T
\tag{21}
$$

Thus Δ^t is simply the anticipated decline in asset value of the project from the beginning of period t to the end of period t. The Δ^t can be interpreted as *time series depreciation* for the project.[40]

It can be seen that the expressions on the right-hand side of (20) are analogous to expressions for the traditional user cost of capital; see Jorgenson (1963, 1989).[41]

Diewert (2009; p. 3) noted that measuring depreciation for a *sunk cost asset* like a commercial structure is difficult since there are no second hand asset markets for a sunk cost asset that can provide period-by-period opportunity costs in order to value the structure asset as it ages. Sales of commercial properties can provide some information but are infrequent and the sale price is for the combined land and structure. Thus, it seems difficult to obtain a sequence of objective measures of period by period depreciation or amortization amounts over the life of the building. Let $N^{t*} \geq 0$ be a *period t amortization amount* for the commercial property for $t = 1, 2, \dots, T$ where the N^{t*} satisfy the following equation:

$$
\alpha_1 N^{1*} + \alpha_2 N^{2*} + \dots + \alpha_T N^{T*} = C_S^0 + V_L^0
\tag{22}
$$

Note that N^{t*} can be interpreted as a payment made to the owners of the project at the end of period t for $t = 1, 2, \dots, T$. Equation (22) says that the initial project cost, $C_S^0 + V_L^0$, can be distributed across the T time periods before the building is demolished and the land plot sold by the series of period by period cost allocations N^{t*} where the discounted value of these cost allocations (to the beginning of period 1) is equal to the project cost. Note that the amortization schedules N^{t*} which satisfy (22) are largely arbitrary; the indeterminacy of amortization schedules for sunk cost assets was noticed by Peasnell (1981; p. 54), Schmalensee (1989; pp. 295–296) and Diewert (2009; p. 9).

In the case where $\Pi^T = 0$, it can be seen that the following intertemporal cost allocations satisfy equation (22):

$$
N^{t*} \equiv N^t \text{ for } t = 1, 2, \dots, T - 1 \text{ and } N^{T*} \equiv N^T + V_L^T
\tag{23}
$$

Thus, if $\Pi^T = 0$ and if the period t cash flow N^t is distributed back to the owners at the end of each period t and the end of period T market value of the land plot V_L^T is also distributed to the owners at the end of period T, then the present value of the resulting sequence of distributions will just be equal to the initial project cost. In the general case where optimal discounted profits Π^T are positive, the distribution pattern defined by (23) is consistent with the sequence of end of period asset values A^t defined by (19) and the depreciation amounts Δ^t defined by (21). The intertemporal allocation of project net revenues defined by (23) is preferred to any other allocation as it is useful for the property firm to value its assets at the end of each period at *market values*. As shown by Diewert (2009; pp. 9–10) and Cairns (2013; pp. 640–641), at the end of period t, the market value of the firm's assets will be A^t defined by equation t in (19) (if anticipations are realized) and thus the project depreciation schedule defined by (21) will be uniquely determined.

The Δ^t defined by equations (21) can be interpreted as *aggregate period t time series depreciation allocations* for the project as a whole. In the $\Pi^T = 0$ case, the period t cash flow N^t can be interpreted as a period t *aggregate user cost of capital* value for the property.[42] But for national income accounting

purposes, it is necessary to decompose the aggregate asset values A^t into land and structure components, and for productivity accounts to similarly decompose the aggregate user cost values N^t into land and structure components. We turn to this decomposition problem in the following section.

6. The Decomposition of Asset Values into Land and Nonland Components

The balance sheet accounts in the SNA requires a decomposition of the aggregate property value at the end of period t, A^t, into *additive* structure and land components, say V_S^t and V_L^t for $t = 0, 1, \ldots, T$. In Section 4 above, it was assumed that investors could form expectations about the future end of period value of the land plot and V_L^t was defined as the expected land price at the end of period t. Given the sequence of expected land prices V_L^t and the sequence of expected property asset values A^t, it is natural to define the *expected structure value at the end of period t* residually as follows:

$$V_S^t \equiv A^t - V_L^t; \qquad\qquad t = 0, 1, \ldots, T \qquad (24)$$

Defining the value of structures in this way will of course lead to an additive decomposition of aggregate property asset value.

The above *cash flow valuation* for the property structure at the beginning of period 1, V_S^0, will exceed the *cost-based value* of the structure at the beginning of period 1, C_S^0, if the property project makes a positive expected profit; that is, $V_S^0 > C_S^0$ if the strict inequality holds in (18) so that $\Pi^T > 0$. In order to obtain an operational depreciation model, in what follows, we will assume that (18) holds as an equality so that:

$$\prod^T = 0 \qquad (25)$$

Using (18) and (25), we have

$$
\begin{aligned}
0 &= -C_S^0 - V_L^0 + \alpha_1 N^1 + \alpha_2 N^2 + \ldots + \alpha_T N^T + \alpha_T \beta_T V_L^0 \\
&= -C_S^0 - V_L^0 + \alpha_1 N^1 + \alpha_2 N^2 + \ldots + \alpha_T N^T + \alpha_T V_L^T \qquad && \text{using (14)} \\
&= -C_S^0 - V_L^0 + A^0 && \text{using (19)} \qquad (26) \\
&= -C_S^0 - V_L^0 + V_S^0 + V_L^0 && \text{using (24)} \\
&= -C_S^0 + V_S^0
\end{aligned}
$$

which implies that $C_S^0 = V_S^0$. Thus, if the zero expected profits condition (25) holds, then at the beginning of period 1, the cost-based and expected cash flow-based valuations for the structure will coincide.

The theory of property value that has been developed in Sections 4–6 shows that depreciation (or amortization) of the structure is largely driven by the sequence of cash flows that the structure is able to generate over time. The pattern of cash flows is likely to be quite variable, depending on the location of the structure and the type of business that is associated with the structure. Thus, in the following section, we turn to a more practical approach to the determination of structure depreciation rates. In doing so, we move away from the expected cash flow approach of Diewert and Fox (2014), which assumes an independent valuation for land, to the Residual Theory of Land approach used by Diewert and Shimizu (2014); see also, for example, Geltner *et al.* (2014). This alternative approach is consistent with the usual national income accounting approach to measuring the depreciation of a structure: a sequence of depreciation rates for the asset is assumed that enables the original cost of the asset to be depreciated over time but the amounts of depreciation are expressed in terms of current period replacement cost. The practical problem with this approach is: how exactly is the sequence of depreciation rates to be determined? If current period asset values for a group of similar properties are available, either by appraisals or assessments, then these assessed values can be used as dependent variables in a system of hedonic regressions where the sequence of depreciation rates become parameters that can be estimated. The details of this alternative approach to asset valuation will be explained in the following section.

7. The Builder's Model Applied to Commercial Property Assessed Values

Suppose that we have a sample of N commercial properties of the same general type in a particular geographic area for T consecutive quarters. We assume that appraised or assessed values V_{tn} are available for each property n at the beginning of quarter t as well as other variables which will be defined below.

The *builder's model* for valuing a commercial property postulates that the value of a commercial property is the sum of two components: the value of the land which the structure sits on plus the value of the commercial structure.[43] However, the structure is valued on a replacement cost basis rather than on a cash flow basis.

Consider a developer for property n who builds a structure on a particular property that is ready for commercial use at the beginning of quarter t. The total cost of the property after the structure is completed will be equal to the floor space area of the structure, say S_{tn} m^2, times the building cost per square meter, β_t say, plus the cost of the land, which will be equal to the land cost per square meter, γ_{tn} say, times the area of the land site, L_{tn}. Thus, if property n has a new structure on it at the start of quarter t, the value of the property, V_{tn}, should be equal to the sum of the structure and land value, $\beta_t S_{tn} + \gamma_{tn} L_{tn}$.[44] Note that we assume that the building cost price β_t depends on time only and not on the location of the building. Thus, we are assuming that the structures in our sample of properties are all of the same type of construction.[45] On the other hand, the land price (per unit area) for property n at the beginning of period t, γ_{tn}, will generally depend on both the time period t and the location of the property which is indexed by n.

The above model applies to new structures, but it is likely that a similar model applies to older structures as well. In general, older structures will be worth less than newer structures due to the depreciation of the structure. Assuming that we have information on the age of the structure on property n at time t, say $A(t,n)$, and assuming a *geometric depreciation model*, a model for the asset value of property n at the beginning of quarter t is the following *basic builder's model*:[46]

$$V_{tn} = \beta_t S_{tn}(1 - \delta)^{A(t,n)} + \gamma_{tn} L_{tn} + \varepsilon_{tn}; \quad t = 1, \ldots, T; \quad n = 1, \ldots, N \qquad (27)$$

where δ is defined as the quarterly geometric depreciation rate for the structure[47] and ε_{tn} is a random error term with mean 0. Note that (27) is a nonlinear regression model. There are two problems with the model defined by (27). First, we have only T times N observations on V but there are T new structure price parameters β_t, TN land price parameters γ_{tn} and one depreciation rate parameter δ to be estimated and so there are not enough degrees of freedom to estimate all of the parameters in the above model. Second, the above model does not take into account the capital expenditures that were made in order to improve the structure after its initial construction.

In order to deal with the second problem, it is necessary to capitalize the sequence of property capital expenditures, assume a depreciation rate for these expenditures, and form a beginning of quarter t stock value for these expenditures which we denote by V_{CEtn} for property n.[48] Now subtract V_{CEtn} from total asset value V_{tn} in order to obtain a new dependent variable. Then a hedonic regression model can be used to decompose $V_{tn} - V_{CEtn}$ into structure and land components. The problem of too many parameters in the model (27) can be dealt with by applying the Country Product Dummy model[49] to the land component on the right-hand side of equations (27) above; that is, set

$$\gamma_{tn} = \alpha_t \omega_n; \quad t = 1, \ldots, T; \quad n = 1, \ldots, N \qquad (28)$$

The sequence of parameters, $\alpha_1, \alpha_2, \ldots, \alpha_T$, can be interpreted as the overall *constant quality land price index* for the group of properties while ω_n can be interpreted as a *quality adjustment factor* for the land component of property n relative to the other properties in the sample. It is obvious that not all of the

α_t and ω_n parameters can be identified so it is necessary to impose a normalization on these parameters. A natural normalization is

$$\alpha_1 = 1 \tag{29}$$

which means that the land price index will start at one in the first period of the sample.

Even though assumptions (28) and (29) have substantially reduced the number of unknown parameters to be estimated, experience has shown that it is difficult to obtain sensible estimates for the sequence of structure prices, the β_t, and the sequence of land prices, the α_t.[50] National statistical agencies routinely produce construction cost price indexes because these indexes are necessary to deflate nominal structure investments into real structure investments. Thus, we assume that there is an official construction cost index P_S^t that will adequately approximate the rate of change of structure prices for the sample of properties under consideration. This index is normalized to equal 1 in quarter 1 of our sample; that is, $P_S^1 = 1$. Set the new structure price for each quarter t, β_t, equal to a single price of structures in quarter 1, say β,[51] times the official construction cost index P_S^t. Thus, we have:

$$\beta_t = \beta P_S^t; \quad t = 1, \dots, T \tag{30}$$

Replacing V_{tn} by $V_{tn} - V_{CEtn}$ and substituting (28) and (30) into equations (27) leads to the following nonlinear regression model:

$$V_{tn} - V_{CEtn} = \beta P_S^t S_{tn} (1 - \delta)^{A(t,n)} + \alpha_t \omega_n L_{tn} + \varepsilon_{tn}; \quad t = 1, \dots, T; \quad n = 1, \dots, N \tag{31}$$

This nonlinear regression has one unknown structure price level parameter β, one unknown geometric quarterly depreciation rate δ, $T - 1$ unknown α_t (the land price series for our sample) and N unknown ω_n (which reflect the relative discount or premium in the land price for property n relative to other properties). There are *TN* degrees of freedom.

Diewert and Shimizu (2014) implemented this model for a panel data set of 50 REITs (Real Estate Investment Trusts) that had a single commercial property in the Tokyo area for 22 consecutive quarters, starting in Q1 of 2007 and ending in Q2 of 2012. The variables that were used in this paper were V_{tn}, the assessed value of property n at the beginning of quarter t; CE_{tn}, the quarterly capital expenditures made on property n during quarter t; $L_{tn} = L_n$, the area of the land plot for property n in square meters; $S_{tn} = S_n$, the total floor area of the structure for property n in m^2 and $A_{tn} = A(t,n)$, the age of the structure on property n in quarters at the beginning of quarter t. In addition, Diewert and Shimizu used a Construction Cost Price Index for Tokyo, P_S^t.[52]

Overall, there were 73 independent parameters to be estimated with 1100 degrees of freedom. Diewert and Shimizu used the nonlinear option in the econometric programming package, Shazam to estimate the nonlinear regression defined by (29) and (31); see White (2004). The R^2 between the observed variables and the predicted variables turned out to be 0.9943 and the log likelihood was -7658.84. The estimated quarterly depreciation rate turned out to be 0.00453. Thus, a structure geometric depreciation rate of about 0.45% per quarter led to a model that could explain Tokyo commercial property appraised values very well.

The age of the structures in the Diewert and Shimizu sample of Tokyo commercial office buildings ranged from about 4 years to 40 years. As we have indicated in Sections 4–6 above, it is very likely that the quarterly depreciation rates change as the structure on the property ages. Thus, Diewert and Shimizu (2014) also experimented with a hedonic model that allowed for different rates of geometric depreciation over each 10-year period. However, they found that there were not enough observations of 'young' buildings to accurately determine separate depreciation rates for the first and second age groups so they divided observations up into three groups where the change in the depreciation rates occurred at ages (in quarters) 80 and 120. They found that 550 observations fell into the interval $0 \leq A(t,n) \leq 80$, 424 observations fell into the interval $80 < A(t,n) \leq 120$ and 126 observations fell into the interval 120

$< A(t,n) \leq 160$. Now define the *real depreciation function* $g[A(t,n)]$ as a function of the structure age, $A(t,n)$, as follows:

$$g\left[A\left(t,n\right)\right] \equiv (1-\delta_1)^{A(t,n)} \text{ if } 0 \leq A\left(t,n\right) \leq 80;$$

$$\equiv (1-\delta_1)^{80}(1-\delta_2)^{[A(t,n)^{-80}]} \text{ if } 80 \quad < A\left(t,n\right) \leq 120;$$

$$\equiv (1-\delta_1)^{80}(1-\delta_2)^{40}(1-\delta_3)^{[A(t,n)^{-120}]} \text{ if } 120 \quad < A\left(t,n\right) \leq 160 \qquad (32)$$

Note that if $\delta_1 = \delta_2 = \delta_3 \equiv \delta$, then $g[A(t,n)] = (1-\delta)^{A(t,n)}$, which is the real depreciation function that was used in the standard geometric depreciation model defined by (31). Thus, the following model defined by (28) and (32) is a generalization of the usual geometric depreciation model used by national income accountants:

$$V_{tn} - V_{CEtn} = \beta P_S^t S_{tn} g\left[A\left(t,n\right)\right] + \alpha_t \omega_n L_{tn} + \varepsilon_{tn}; \quad t = 1, \ldots, T; \quad n = 1, \ldots, N \qquad (33)$$

The economic meaning of all of the new models is as follows: first the first 80 quarters of a building's life, the constant price quantity of the structure declines at the quarterly geometric rate $(1 - \delta_1)$. Then for the next 40 quarters, the quarterly geometric rate of depreciation switches to $(1 - \delta_2)$. Finally, after 120 quarters, the quarterly geometric rate of depreciation switches to $(1 - \delta_3)$. Thus, for the Diewert and Shimizu data set, the new model contains two additional depreciation parameters or 75 parameters in all.

Using the nonlinear regression program in Shazam, Diewert and Shimizu estimated the new generalized geometric depreciation model for their Tokyo data set. The R^2 between the observed variables and the predicted variable increased to 0.9946 and the log likelihood increased to -7633.63, which was a fairly large increase in log likelihood of 25.2 over the previous model for the addition of two new depreciation parameters. The estimated quarterly depreciation rates turned out to be $\delta_1 = 0.00327$, $\delta_2 = 0.00702$ and $\delta_3 = 0.03558$. Compare these rates to the single quarterly geometric depreciation rate from the previous model, which was 0.00453. Thus, the new results indicate that the quarterly depreciation rate is around 0.33% for the first 20 years of building life, increasing to 0.70% for the next 10 years and then finishing its useful life with a 3.6% per quarter depreciation rate. These rates seem to be quite reasonable.

The above material indicates how estimates of structure depreciation rates can be obtained provided that a suitable panel data set of market-based appraised values or assessed values for a number of commercial properties can be obtained.[53] Once depreciation rates have been estimated for the sample of properties, the same depreciation rates can be applied to nationwide investment data (for the same general type of commercial property) and national estimates for the stock of commercial structures can be obtained using the usual perpetual inventory method for constructing reproducible capital stocks.[54] The above econometric models will also generate estimated constant quality land price indexes for the sample of properties and these indexes can be applied to national benchmark data for the value of commercial land; see for example, EuroStat (2011) for how price indexes can be constructed from the estimated hedonic models.

Unfortunately, the depreciation rates that are estimated by the above type of model will underestimate 'true' depreciation of commercial property structures. The problem will be explained in the following section.

8. Estimating Demolition or Obsolescence Depreciation

The models that were described in the previous section are useful for national income accountants because they facilitate the accurate estimation of structure depreciation, which is required for the national accounts. The depreciation estimates that are generated by those models are estimates that apply to structures that continue in existence over the sample period. However, there is another form of structure depreciation that

is not addressed by the methodology presented in the previous section: namely the loss of residual structure value that results from the *early demolition* of the structure. This problem was noticed and addressed by Hulten and Wykoff (1981).[55] Below, we will describe the methodology suggested by Diewert and Shimizu (2014) to address this problem.[56]

Their solution to the problem of measuring the effects of the early retirement of a building requires the existence of data on the date of construction and the date of retirement of each building in the class of buildings under consideration and for the region that is in scope.[57] Komatsu *et al.* (1994) collected date of construction and date of retirement data for reinforced concrete office buildings in Japan for the reference year 1987. Thus, for each age of building s (in years), they were able to calculate the number of office buildings of age s (in years), N_s, as of January 1, 1987 along with the number of office buildings of age s, n_s, that were demolished in 1987 for ages $s = 1, 2, \ldots, 75$. Given this information, they were able to calculate the *conditional probability*, ρ_s, that a surviving structure of age s at the beginning of the year would be demolished during 1987; that is, they defined ρ_s as follows:

$$\rho_s \equiv n_s / N_s; \quad s = 1, \ldots, 75 \tag{34}$$

Under the assumption that the conditional probabilities defined by (34) have persisted through time, the *unconditional probability* π_s that a building of age s is still in existence at the beginning of the year 1987 is defined as follows:

$$\pi_0 \equiv 1; \pi_s \equiv \pi_{s-1}(1 - \rho_s); \quad s = 1, \ldots, 75 \tag{35}$$

It can be seen that the series π_s are a building counterpart to *life expectancy tables*; that is, the births and deaths of a population of buildings are used to construct the probability of building survival as a function of age instead of the probability of individual survival as a function of age.

Once the probabilities of survival π_s have been determined, then the conditional probabilities of demolition ρ_s can be determined from the π_s using equations (35) above.[58] For the Japanese data, the conditional probabilities of demolition are very small for the first 20 years or so of building life. From 20 to 42 years, these probabilities gradually increase from 1.4% to about 11% and then the probabilities fluctuate around the 10% level from age 43 to 67. Finally, after age 67, the conditional probabilities of demolition increase rapidly to end up close to unity at age 75.

Recall that the (single) geometric depreciation rate for continuing office structures in Tokyo that Diewert and Shimizu (2014) estimated for Japanese commercial office buildings was about 0.45% per quarter. Diewert and Shimizu (2014) formed a rough approximation to the possible magnitude of *demolition depreciation*[59] using the life table information provided by Komatsu, Kato and Yashiro. We will now explain how Diewert and Shimizu constructed their rough estimate of demolition depreciation.

Suppose that the *annual wear and tear geometric depreciation rate* is 1.8%[60] so that the annual δ is 0.018. Suppose further that investment in Tokyo office buildings has been constant for 75 years. Normalize the annual structure investment to equal unity in constant yen units. Finally, suppose that the Komatsu, Kato and Yashiro survival probabilities π_s apply to the hypothetical steady state investment data. Thus, after 75 years of steady investment, the index of constant yen value of the Tokyo commercial *office building stock* can be K defined as follows:

$$K \equiv \pi_0 + \pi_1(1 - \delta) + \pi_2(1 - \delta)^2 + \ldots + \pi_{75}(1 - \delta)^{75} \tag{36}$$

The corresponding index of real value of *wear and tear depreciation* Δ is defined as follows:

$$\Delta \equiv \delta\pi_0 + \delta\pi_1(1 - \delta) + \delta\pi_2(1 - \delta)^2 + \ldots + \delta\pi_{75}(1 - \delta)^{75} = \delta K \tag{37}$$

The corresponding amount of *demolition depreciation D* is defined as each component of the surviving capital stock on the right hand side of equation (36), $\pi_s(1 - \delta)^s$, multiplied by the corresponding conditional probability of demolition, ρ_s; that is, define D as follows:

$$D \equiv \rho_0\pi_0 + \rho_1\pi_1(1 - \delta) + \rho_2\pi_2(1 - \delta)^2 + \ldots + \rho_{75}\pi_{75}(1 - \delta)^{75} \qquad (38)$$

Once the surviving capital stock K, the amounts of wear and tear depreciation Δ and demolition depreciation D have been defined, the *average wear and tear depreciation and demolition depreciation rates*, δ and d, are defined as the following ratios:

$$\delta \equiv \Delta/K; d \equiv D/K \qquad (39)$$

Of course, the assumed annual wear and tear depreciation rate of 0.018 turns out to equal the average wear and tear depreciation rate defined in (39) and the average demolition depreciation rate d turns out to be equal to 0.01795. Thus, for the first depreciation model considered in Section 7 above, *it is likely that demolition depreciation is almost equal to wear and tear depreciation*. Note that the sum of the two depreciation rates is approximately 3.6% per year.[61]

The demolition depreciation rates estimated by Diewert and Shimizu are only rough approximations to actual demolition depreciation rates. The actual rates of demolition depreciation depend on actual investments in commercial property office buildings in Tokyo for the past 75 years and this information was not available. However, the above calculations indicate that accounting for premature retirements of buildings adds significantly to the wear and tear depreciation rates that are estimated using hedonic regressions on continuing buildings. Thus, it is important that national statistical agencies construct a data base for building births and retirements so that depreciation rates for buildings that are not retired can be adjusted to reflect the loss of building asset value that is due to premature retirement.

The analysis presented in this section does not invalidate the hedonic regression approach for constructing constant quality price indexes for commercial properties, since price indexes compare like to like and thus apply only to continuing structures.[62] As a byproduct of the hedonic regression approach outlined in Section 7, statisticians can form estimates of wear and tear depreciation for buildings that remain in use. The analysis in this section simply warns the reader that wear and tear depreciation is not the entire story;[63] there is also a loss of asset value that results from the early retirement of a building that needs to be taken into account when constructing national income accounting estimates of depreciation.

There is a need to determine exactly how obsolescence depreciation should be treated in the SNA. Treatment 1 would simply add the obsolescence depreciation to deterioration depreciation for the period when the asset is prematurely retired. Both forms of depreciation would appear in the Consumption of Fixed Capital accounts. Treatment 2 would simply add expected demolition depreciation to regular deterioration depreciation, which spreads out the demolition depreciation out over the average useful life of the asset. Hulten and Wykoff (1981) follow the second treatment of demolition depreciation. Thus, following their example, the 1.8% demolition depreciation rate would simply be added to the 1.8% deterioration geometric depreciation rate, leading to an overall 3.6% geometric depreciation rate. The problem with the second treatment is that while it will be approximately correct for the entire economy, it will distort the 'truth' for each producing unit in the economy. It is also possible that national income accountants could decide to place demolition depreciation into the Other Changes in Assets account.

The method for obtaining an overall commercial property price index, building depreciation rates and subindexes for commercial structures and land explained in Section 7 relied on information on appraisal values for commercial properties. There are other methods that can be used for form overall commercial property price indexes. In the following section, we will review these methods and comment on their strengths and weaknesses.

9. Alternative Methods for Constructing Commercial Property Price Indexes

There are four general classes of methods that have been suggested in the literature that can be used to form commercial property price indexes: Capitalization Methods,[64] Sales Transaction-Based Methods,[65] Appraisal-Based Methods[66] and Stock Market-Based Methods.[67] We will briefly discuss each class of methods.

9.1 *Capitalization or Ratio Methods*

In order to introduce this class of methods, recall the first equation in equations (19) in Section 5 which defined the asset value of the commercial property A^0 at the end of period 0 (or the beginning of period 1) in terms of a discounted stream of expected cash flows (N^t for period t) and an end of period T expected value of the property. Suppose that instead of ending our analysis of the property's cash flow at the end of period T, we assumed that the cash flows persisted into the indefinite future. Suppose further that the sequence of cash flows is expected to grow at the constant geometric rate g so that $N^t = (1+g)^{t-1} N^1$ and suppose further that the one period discount rates r^t are all equal to a constant r for all i = 1, 2, ... Then the beginning of period 1 asset value for the property, V^1, should be equal to the following expression where $\gamma \equiv (1 + g)/(1 + r)$:[68]

$$
\begin{aligned}
V^1 &= (1 + r)^{-1} N^1 [1 + \gamma + \gamma^2 + \ldots] \\
&= (1 + r)^{-1} N^1 / (1 - \gamma) \\
&= N^1 / (r - g)
\end{aligned}
\tag{40}
$$

Thus, if estimates for r and g can be formed and the cash flow generated by the property can be observed in each period, equation (40) can be used to form an asset value for the property. Equation (40) is known as the *Gordon Growth Model* in the finance and commercial property literature.[69]

The period t *capitalization rate or cap rate* for the property under consideration, k^t, is defined as the ratio of period t asset value V^t to the cash flow generated by the property during period t, N^t:[70]

$$
k^t \equiv V^t / N^t
\tag{41}
$$

Under the assumptions of the Gordon Growth Model, k^t will be constant over time; that is, k^t will be equal $1/(r - g)$ for all periods t.

An *imperfect commercial property price index*[71] P_{IN}^t for the property under consideration going from period $t - 1$ to t is simply the ratio of property values for the two periods; that is, define

$$
P_{IN}^t = V^t / V^{t-1} \equiv N^t / N^{t-1}
\tag{42}
$$

where the second equality follows if the Gordon Growth Model defined by (40) is valid with r and g constant over time or, more generally, if the capitalization rates k^t defined by (41) are constant over time. Thus, under these conditions, a commercial property price index for a single property or a group of properties can be constructed by calculating (one plus) the rate of cash flow growth for the single property or for the group of properties.

A problem with using the cash flow ratio, N^t / N^{t-1}, as a property price index is that cash flows can be very volatile due to fluctuations in vacancy rates, intermediate input expenditures and in capital expenditures.

Thus, the price index defined by N^t / N^{t-1} will generally fluctuate too much to be useful for national income accounting purposes if commercial property price indexes are needed by disaggregated industries and by regions within the country.[72]

The last fluctuation problem can be mitigated by interpreting N^t as NOI for period t (rather than cash flow for period t which subtracts very variable capital expenditures made in period t from period t NOI) and the second fluctuation problem can be mitigated by assuming that period t asset value V^t is a multiple

of period t revenue R^t for the property (or group of properties). Thus, define the *revenue cap rate for period t* as κ^t:

$$\kappa^t \equiv V^t / R^t \tag{43}$$

Now assume that the revenue cap rates κ^t are constant across time and under this assumption, an alternative *imperfect commercial property price index* for period t, P^t_{IR}, is defined as the revenue ratio, R^t/R^{t-1}:

$$P^t_{IR} = V^t / V^{t-1} \equiv R^t / R^{t-1} \tag{44}$$

The revenue-based imperfect price index P^t_{IR} defined by (44) will generally fluctuate much less than the cash flow-based imperfect price index P^t_{IN} defined by (42).[73] However, the assumption that revenue cap rates (or cash flow cap rates) remain constant over time is unlikely to hold empirically, thus casting doubt on the accuracy of the resulting property indexes.

The *strengths of the ratio approach* to the construction of a commercial property price index are that the methodology is relatively simple and easy to explain to the public, and the resulting indexes can be constructed by disaggregated sector (on a quarterly basis) using information on NOI or revenues by sector that are available to national statistical agencies.

The *weaknesses of the ratio approach* to the construction of a commercial property price index are: (1) the resulting indexes will have a downward bias due to their neglect of net depreciation. (2) The indexes may fluctuate more than is warranted, particularly when indexes are required by detailed industrial sector or by region. (3) The indexes are based on the assumption of constant cap rates over time; if this assumption is not satisfied, then the resulting indexes will be biased. (4) The indexes cannot be decomposed into separate land and structure subindexes.

9.2 *Sales Transaction-Based Methods*

It is generally agreed that the best measure of property value in period t is the *sale price of the property* if it sold in period t.[74] Commercial property price indexes that are based on the sales of commercial properties are *sales transactions-based* indexes.

The *advantages and disadvantages* of sales transaction-based indexes were discussed in detail in the context of the sales of residential properties in the *Residential Property Price Handbook*; see Eurostat (2011). The following transactions-based methods were discussed there: Stratification or Mix Adjustment Methods (Chapter 4); Hedonic Regression Methods (Chapter 5); Repeat Sales Methods (Chapter 6); and Hedonic Regression Methods that lead to a Land and Structure Decomposition of Property Value (Chapter 8).

The discussions about the above methods in the *Residential Property Price Handbook* are valid in the present Commercial Property context but there are two additional factors that make the construction of a CPPI much more difficult than a RPPI if a transactions-based method is used. First, commercial property sales are much less frequent than residential house sales, and second commercial property characteristics are much more diverse and complex than the main characteristics that explain the value of residential properties. Both of these differences mean that a transaction-based CPPI will be much less accurate than a corresponding RPPI.

We will conclude this section by offering a very brief outline of the three main transactions-based methods for constructing a CPPI.

9.2.1 *Stratification Methods*

The simplest method that could be used to construct a CPPI would be to simply track the change in the average (mean or median) price of commercial properties in scope that were sold during a period. However, mean (and median) indexes confound changes in prices with changes in the quality of the properties because the mix of heterogeneous properties sold in one period could be quite different from the mix of properties sold in another period.

The stratification method works as follows. Commercial properties are classified according to their main price determining characteristics. Thus, the land area of the property, the structure floor space area, the age of the structure, the type of construction of the structure, the location or neighbourhood of the property and the property's industry would be important examples of price determining property characteristics. Each of these characteristics is decomposed into a finite number of nonoverlapping cells. Suppose there were three cells in each of the above six classifications. The overall number of cells in the classification is 3^6 which is 729 cells. Then each property sale is classified into 1 of these 729 cells. The average selling price within each cell can be used as a constant quality price for that type of property. Regular index number theory can then be used to aggregate up the average prices by cell into an overall index.

The *main advantages* of the stratification method are that the method adjusts for compositional change of the properties sold, and that it is relatively easy to explain to users.

The *main disadvantages* of the stratification method are: (1) It cannot deal adequately with depreciation of the structure unless age of the structure is a stratification variable. (2) It cannot deal adequately with structures that have undergone major repairs or renovations (unless renovations are a stratification variable). (3) It requires information on property characteristics so that sales transactions can be allocated to the correct strata. (4) If the classification scheme is very coarse, compositional changes will affect the indexes, that is, there may be some unit value bias in the indexes. (5) It does not lead to a decomposition of property value into land and structure components. (6) Due to paucity of commercial property transactions and the multiplicity of relevant commercial property characteristics, the method is basically unworkable: too many cells will be empty and index number theory breaks down under these conditions.

9.2.2 *Hedonic Regression Methods*

Hedonic regression models use the selling price of a property as the dependent variable in a regression model with continuous quantities of characteristics (or dummy variables for characteristics groupings) as independent variables. The models described in Section 7 above are examples of hedonic regression models, except that appraised values were used as the dependent variables instead of market sales values.[75]

The advantages and disadvantages of the hedonic regression approach to the construction of property price indexes were extensively discussed in Chapters 5 and 8 of Eurostat (2011). We summarize the advantages and disadvantages of hedonic methods below.

The *main advantages* of the hedonic method are as follows: (1) if the list of available property characteristics is sufficiently detailed, the method adjusts for both sample mix changes and quality changes of the individual properties. (2) The method is probably the most efficient method for making use of the available data on market sales of commercial properties. (3) The method can be used to decompose the overall price index into land and structures components.

The *main disadvantages* of the hedonic regression approach using property sales as the dependent variable are: (1) the method is data intensive since it requires data on all important price determining property characteristics and so it is relatively difficult to implement. (2) The method may not lead to reasonable results due to multicollinearity problems. (3) Different choices will be made about the set of characteristics that are entered into the regression, the functional form for the model, the stochastic

specification, and possible transformations of the dependent variable, which could lead to varying estimates of overall price change. (4) The general idea of the hedonic method is easily understood but some of the technicalities may not be easy to explain to users. (5) Sparseness of observations for sales of commercial properties and the multiplicity of price determining characteristics means that the method may not always lead to 'reasonable' indexes.

9.2.3 *The Repeat Sales Method*

The repeat sales method was initially proposed by Bailey *et al.* (1963). The only information required to estimate a standard repeat sales regression equation is the sales price of the commercial property, sales date and address of properties that have sold more than once during the sample period. Thus, this method requires no information on property and structure characteristics except for the location of the properties (so that repeat sales of the same property can be ascertained). Hence, this method is much less data intensive than hedonic methods. The repeat sales method is described in some detail in Chapter 6 of Eurostat (2011) and this discussion will not be repeated here.[76]

The *main advantages* of the repeat sales method are as follows: (1) the repeat sales method in its basic form needs no characteristics other than address of the commercial properties that are transacted more than once over the sample period. Source data may be available from administrative records such as those from the Land Registry. (2) Standard repeat sales regressions are easy to run and the resulting price indexes easy to construct. (3) The repeat sales method is a matched model type of method[77] without any imputations. Locational factors that affect the value of the property are basically held constant[78] and thus it is not necessary to collect information on location characteristics.[79] (4) The results are essentially reproducible provided that the treatment of outliers and possible corrections for heteroskedasticity and sample selectivity bias are clearly described.

The *main disadvantages* of the repeat sales method are as follows: (1) the method is inefficient in the sense that it does not use all of the available transaction prices; it uses only information on units that have sold more than once during the sample period. (2) The method does not generate a pure price index since it ignores depreciation of the structure and capital expenditures made on the structure. The repeat sales index will generally have a downward bias since net depreciation (equal to gross depreciation less offsetting capital expenditures) will generally be positive. (3) The set of properties that transact more than once during the sample period may not be representative for the population of properties under consideration, leading to a sample selectivity bias. (4) The method cannot provide separate price indexes for land and for structures. (5) Each additional period adds new data to the sample of properties and new estimates of property inflation are obtained for the entire sample period. Call this Approach 1 to index construction. In order to avoid perpetual revision of the overall index that Approach 1 entails, one could use the newly computed price level for the last sample period relative to the previous period's price level to update the existing index, leading to an Approach 2 index. The two types of index may not agree very closely, leading to problems in deciding what is the 'truth'.[80] (6) The repeat sales method cannot be used if indexes are required for fine classifications (e.g. by type of property or by location or by both) of the type of property sold. This problem is much bigger for commercial properties than for residential properties due to the relative infrequency of commercial property sales.[81]

9.3 *Appraisal-Based Methods*

The discussion in Section 7 above illustrated how the appraisal method can be used to construct an overall CPPI and its land and structure components. The method can also be used to develop estimates for deterioration depreciation but additional information on births and retirements of buildings is required in order to form estimates for demolition depreciation, which will not be picked up by the hedonic regression.

How exactly are appraised values (or assessed property values for taxation purposes) formed? Basically, a mixture of methods are used, including ratio methods, estimation of expected discounted cash flows, comparable property comparisons,[82] and formal or informal hedonic regression methods.[83]

It can be seen that appraised values are necessarily only estimates of market values (prices at which commercial properties are transacted) and hence will have a range of uncertainty around them. Commercial property appraisals generally have the following two characteristics;[84] appraised values tend to be smoother than market values and they tend to lag behind the turning points in market values.

Existing commercial property price indexes based on appraisals simply compare the appraised value of a group of properties with the corresponding appraised value of the same group of properties in a base period. These indexes are not pure price indexes since they do not adjust for depreciation and capital expenditures and thus it is likely that they have a downward bias (compared to a hedonic price index) due to net depreciation of the structure over time.[85]

Are appraised or assessed values reliable enough to be used for national statistical purposes? Appraisers do have an incentive to 'get things right' since when a sale of an appraised property is actually sold, the sale price can be compared with the assessed value and if the discrepancy is too great, the appraisers reputation will be adversely affected which in turn may lead to reduced work for the appraiser in the future. Assuming that appraised values are supposed to approximate market values, official assessments of property value for tax purposes should be approximately unbiased on average since the tax authority will want to make the assessed values as large as possible to maximize tax revenue while the property owners will want assessed values to be as low as possible in order to minimize their property tax bills. Thus, there will be incentives for the assessments to be approximately in line with market values.[86] Thus, if commercial property price indexes are required at a relatively fine degree of disaggregation (say by detailed industry and by region or location), then working with market-based assessed values is likely to lead to the most accurate indexes.

If appraisals are only made infrequently, then the appraised value information for the base period can be combined with information on sales of commercial properties to form a type of modified repeat sales method. This method is called the Sale Price Appraisal Ratio method (SPAR method) and it is described fully in Chapter 7 of Eurostat (2011). This method is used in practice by several countries but it suffers from the same bias problem as the Repeat Sales method and the direct comparison of assessed values method: these methods will tend to have a downward bias due to their neglect of net depreciation. The *advantages and disadvantages of the SPAR method* for constructing property price indexes in the context of residential housing were discussed in Chapter 7 of Eurostat (2011) and this discussion is also relevant in the present context.

The *advantages of the builder's model* hedonic regression approach for constructing commercial property price indexes that was used by Diewert and Shimizu (2014) and explained in Section 7 above are: (1) It generates relatively smooth price indexes. (2) It generates not only an overall commercial property price index but it also generates a separate component land price index. (It requires an exogenous construction cost index which is used to provide a structure component price index). (3) It generates evidence-based estimates of deterioration depreciation rates for commercial property structures. (4) It does not have a known bias. All other methods, with the exception of the hedonic regression method applied to property sales data, are likely to be biased because of their net depreciation bias. (5) It can generate property price indexes at a very disaggregated level if market-based assessed values for taxation purposes are available and made on a frequent basis. (6) The informational requirements are intermediate between the large requirements for the hedonic regression approach applied to property sales and the small requirements for the repeat sales approach. It requires information on assessed or appraised values of the commercial property, the floor area and the age of the structure, capital expenditures on the property and an exogenous construction price index that is applicable for the structure on the property.

The *disadvantages of the method* are: (1) Although the informational requirements are less than the requirements for a hedonic regression model, they are still more than the requirements for the repeat sales method. (2) The assumed form for the structure depreciation function may not be 'correct' and the depreciation estimates generated by the model may not be accurate. (3) The assessed value information may not be accurate. In particular, it is likely that the assessed values lag behind the corresponding 'true' market values, leading to indexes which miss the turning points in market value-based indexes.[87] (4) The assumed depreciation rate for capital expenditures may not be correct. (5) Total property value may not be additive in its land and structure components.

9.4 *Stock Market-Based Methods*

A Real Estate Investment Trust (REIT) is a type of company (similar to a corporation) that must engage in commercial real estate investments. The following quotation explains their origin in the United States and their nature:[88]

'The REIT investment vehicle was created by Congress in 1960 through legislation called the Real Estate Investment Trust Act, which authorized a real estate ownership structure with tax treatment similar to that of mutual funds: a pass-through entity that distributes most of its earnings and capital gains. The idea was to do for commercial real estate investment what mutual funds had done for stocks; to provide small investors ("retail investors") a means to invest in a diversified portfolio of many individual assets without requiring a huge fortune to do so'. Geltner *et al.* (2014; p. 574).

Since 1960, REITs have spread to many countries. Since REITs trade on national stock markets, up to date market values for the equity portion of REIT asset value is available on virtually a daily basis. From annual and quarterly reports on the activities of a REIT to shareholders, one can determine the debt position of the REIT and then add this to the equity market value to get a good approximation to the market value of the properties owned by the REIT. If there are no additions or subtractions to the list of properties owned by the REIT, then a daily estimate of the market value of the REIT's commercial properties can be obtained and a daily asset value type of price index can be formed. Geltner *et al.* (2014; 655) provide a chart of such a stock market-based price index for US REITs over the period 1999–2011 and compare this index with a repeat sales index and an appraisal-based index for the US commercial property market. As might be expected, the stock market-based index is considerably more volatile than the other indexes but it has the advantage of showing turning points in the aggregate commercial property market before the other indexes indicate that cyclical turning points have occurred. The same chart also shows that at times, the stock market-based index is rather far removed from the other indexes.[89] Day to day volatility of a stock market-based index could be removed by averaging or smoothing the daily data but quarterly and annual volatility of the index makes stock market-based indexes largely useless for national income accounting purposes.

The *advantages of a stock market-based method* for constructing a commercial property price index are that it has relatively low informational requirements,[90] it generates indexes which indicate turning points sooner than alternative indexes, and it can easily be explained to the public.

The *disadvantages* of the method are: (1) it leads to very volatile indexes; that is, while a stock market-based index will indicate broad cyclical turning points, large quarterly fluctuations will make the index largely useless for national income accounting purposes. (2) The index is an *asset value index* and as such it does not take depreciation of the structures into account and thus will generally have a downward bias. (3) The index cannot be decomposed into structure and land components without additional information. (4) It will usually be very difficult to construct a family of regional and industry subindexes using the stock market approach since the largest REITs will generally hold properties in different regions and for different lines of business. Smaller REITs that may have a narrower focus may not trade very frequently and the resulting price indexes will be even more volatile than national indexes.

10. Conclusion

This paper has provided an overview of measurement problems that national income accountants and price statisticians face in attempting to construct the production accounts for the commercial property sector in a county. The measurement problems encountered in measuring sectoral revenues, intermediate input and labour expenses are standard ones but the measurement problems in accounting for structure and land inputs are substantial. Thus, Sections 4–9 discuss the measurement problems associated with measuring the price of land and structures and the value of their contributions to production in the commercial property sector of an economy.

We highlight some important practical implications of our analysis. First, all of the methods for forming aggregate Commercial Property Price indexes that were discussed in Section 9 above can be used to form rough approximations to a 'true' aggregate CPPI but most of the methods are biased (due to their neglect of net depreciation of the structure on a commercial property) and, more importantly, cannot be used to obtain a breakdown of property value into structure and land components.

The two methods which can provide unbiased estimates for a CPPI and also provide a decomposition of property value into land and structure components are the *asset value hedonic regression models* described in Section 7 and the (builder's model) *property sales hedonic regression models* described in Section 9.2.2.

The property sales hedonic regression approach to the construction of property price indexes works reasonably well in the residential housing context but is unlikely to work well in the commercial property context due to the infrequency of property sales and the heterogeneity of commercial properties, which implies that many characteristics will have to enter the regression as independent variables in order to obtain a satisfactory model. The scarcity of commercial property sales means that this approach is unlikely to work well when disaggregated (by type of commercial property and by region) indexes are required.

Finally, at this point in time, it appears that that the builder's model applied to assessed or appraised values is the most promising approach. In order to obtain indexes at a disaggregated level, it will be necessary to use the assessed values of properties that local government authorities make for property tax purposes. Thus, it will be useful for the national statistics agency to cooperate with the local taxation authorities to ensure that assessed values reflect market values and that assessments are carried out on a rolling year basis so that the assessed values are not out of date.

Notes

1. See the UN, Eurostat, IMF, OECD and World Bank (2009) for the latest international version of the SNA.
2. Variable inputs comprise intermediate and labour inputs.
3. Once prices and quantities for outputs and variable inputs for a single production unit have been determined, standard index number theory can be used to aggregate up to the industry level.
4. This model draws on the work of Diewert and Fox (2014).
5. Dixon *et al.* (1999) and Crosby *et al.* (2012) provide extensive reviews of the depreciation literature as it applies to commercial properties.

6. The early index number theorists Walsh (1901; p. 96), Fisher (1922; p. 318) and Davies (1924; p. 96) all suggested unit values as the prices that should be inserted into an index number formula. This advice is followed in the *Producer Price Index Manual: Theory and Practice* with the proviso that the unit value be a narrowly defined one; see the IMF/ILO/OECD/UNECE/Eurostat/World Bank (2004).

7. However, an industrial building is typically leased to a single business and in this case, total floor space would coincide with rented floor space.

8. For example, a building may derive revenues from a transmission tower on the top of the building.

9. If $K = 1$, then we define $P(p_1^0, p_1^1, q_1^0, q_1^1) \equiv p_1^1/p_1^0$ and $Q(p_1^0, p_1^1, q_1^0, q_1^1) \equiv q_1^1/q_1^0$, the single price ratio and the single quantity ratio respectively. In the case of a general $K > 1$, we think of $P(p_1^0, p_1^1, q_1^0, q_1^1)$ as being a weighted average of the price ratios $p_1^1/p_1^0, p_2^1/p_2^0, \ldots, p_K^1/p_K^0$. Thus, we interpret $P(p_1^0, p_1^1, q_1^0, q_1^1)$ as an aggregate price ratio, P^1/P^0, where P^t is the aggregate price level for period t for $t = 0,1$.

10. See the *Producer Price Index Manual*, IMF/ILO/OECD/UNECE/Eurostat/The World Bank (2004).

11. Note that $P_L(p^0, p^1, q^0, q^1)$ does not actually depend on q^1 and $P_P(p^0, p^1, q^0, q^1)$ does not actually depend on q^0.

12. This result is due to Walsh (1901; pp. 428, 539).

13. This expenditure share and price ratio representation of the Paasche index is described by Walsh (1901; 428) and derived explicitly by Fisher (1911; p. 365).

14. The U.S. Bureau of Labor Statistics uses this index number formula as its target Consumer Price Index at higher levels of aggregation.

15. See the ILO (2004).

16. Diewert (1978; p. 888) showed that $P_T(p^0, p^1, q^0, q^1)$ approximates P_F to the second order around an equal price and quantity point. However, Diewert's results assumed that all prices and quantities were positive.

17. See the IMF/ILO/OECD/UNECE/Eurostat/The World Bank (2004).

18. The second alternative formulae for the Laspeyres and Paasche indexes defined by (5) and (6) can also become ill-defined if some prices are zero.

19. If renovations occur while the unit is vacant, then there will be a quality adjustment problem.

20. See Baum and Crosby (2008; p. 67) and Chapter 11 in Geltner *et al.* (2014).

21. Using national income accounting terminology, the variables inputs in this section refer to intermediate inputs and labour inputs. In the Commercial Property literature, these inputs are aggregated into Operating Expenses; see Geltner *et al.* (2014; p. 236).

22. We have added the subscript I to the prices and quantities in order to distinguish input prices and quantities from the output prices and quantities considered in the previous section.

23. Another possible expense that could be included here are property taxes; see Geltner *et al.* (2014; p. 236). However, property taxes are best regarded as being part of the user cost of structure and land services.

24. For explanations on how Multifactor Productivity Accounts can be constructed, see Jorgenson and Griliches (1967), Christensen and Jorgenson (1973), Diewert (1980, 1992) and Schreyer (2001, 2009).

25. If the input m is not purchased in *both* quarters 0 and 1, then this input can simply be omitted in the list of inputs and normal index number theory is applied to the remaining commodities.

26. Additional imputation methods are considered in Diewert (1980; pp. 498–501) and in Feenstra and Diewert (2001).

27. In periods of high or moderate inflation, the carry forward method of imputation will tend to understate input cost inflation over the periods where the input is not purchased but then the index will jump up when the input is finally repurchased.

28. We are essentially assuming that the price statistician has access to building level data on revenues and costs, where the costs are listed by transaction. In many cases, only quarterly accounting data will be available and costs will generally be available for only a few product categories. Average prices by category will usually not be available and so separate price indexes will have to be constructed as separate exercises.

29. There is one additional cost category associated with a commercial property that needs to be taken into account and this category is period by period capital expenditures on the property. We will deal with these expenditures in Section 7.

30. The material in this section and the subsequent one is drawn from Diewert and Fox (2014). A revised version of this working paper differs somewhat from what is described here, as it explicitly takes into account demolition costs and the consideration of goodwill as an asset.

31. The length of each period will typically be a quarter if our analysis is applied empirically because, usually, information on building cash flows will only be available on a quarterly (or annual) basis.

32. We are assuming that investors have perfect foresight about price movements of the land plot, a strong assumption that can be relaxed but with complications beyond the scope of the current paper.

33. Cash Flow in the commercial property literature is defined as Net Operating Income less Capital Expenditures; see Geltner *et al.* (2014; p. 231). In our equations (3), N^t should be interpreted as cash flow for period t.

34. Our analysis of the determinants of property value follows the conventional commercial property valuation literature, as explained by Geltner *et al.* (2014; p. 204) for example, but instead of considering just a single problem for a fixed horizon defined by (3), we consider the entire sequence of intertemporal profit maximization problems defined by (3).

35. Our analysis utilizes the intertemporal production plan methodology that was pioneered by Hicks (1946).

36. Our analysis largely follows that of Cairns (2013; p. 639) who noted that unless the inequality in (18) is satisfied, investors will not participate in the project: 'This participation constraint provides that the cash flows of the project allow investors to recover their sunk investment as a stream of quasi-rents or user costs'.

37. Baum (1991; p. 59) and Dixon *et al.* (1999; p. 162) distinguished *physical deterioration* and *obsolescence* of the structure as the primary causes of depreciation (decline in the value of the building over time). In our approach, it is increases in the price of land along with falls in cash flows that drives obsolescence. Dixon *et al.* (1999; pp. 168–170) also noted that *rental decline* (i.e. falls in net operating income as the building ages) contributed to building depreciation and of course, this effect is also part of our approach. Crosby *et al.* (2012) investigate the rental decline phenomenon for UK commercial properties and they also take into account post construction capital expenditures on the properties. When depreciation rates for commercial properties are reported in the real estate literature, they are generally reported as fraction of property value (which includes the value of the land plot). Thus, these reported property depreciation rates will understate depreciation rates on the structure by itself.

38. These equations implicitly assume that at the end of each period t, the period t cash flow N^t is distributed to the owners of the property.

39. By rearranging equation (20), it can be seen that $\Delta^t = N^t - r_t A^{t-1}$ so that fluctuations in the cash flows N^t will drive fluctuations in the Δ^t since the variations in r_t and A^{t-1} as t varies will generally be small.

40. The term *time series depreciation* is due to Hill (2000) but the concept dates back to Hotelling (1925; p. 341). We note that Δ^t incorporates both the effects of wear and tear depreciation and anticipated revaluation; see Hill (2000; p. 6), Hill and Hill (2003; p. 617), Diewert (2009; p. 9) and Cairns (2013; p. 640).

41. See also Diewert (2005; p. 485, 2009; p. 8). Baumol *et al.* (1982; p. 384) identify $r_t A^{t-1} + \Delta^t$ as the period *t payment to capital*; see also Cairns (2013; p. 640).

42. If $\Pi^T > 0$, then the N^t are not equal to user *cost* allocations since they will contain a pure profit component.

43. The material in this section is based on Diewert and Shimizu (2014). The builder's model has been applied to residential property sales by Eurostat (2011), Diewert *et al.* (2011, 2014) and Diewert and Shimizu (2013) except that straight line or piecewise linear depreciation was used as the depreciation model for the residential structures whereas in Diewert and Shimizu (2014), geometric depreciation models were used. Geometric depreciation models have the advantage that the implied structure asset values that the models generate always remain positive whereas piecewise linear depreciation models can generate negative asset values.

44. Note that V_{tn} is now the beginning of period *t* value for property *n*, which would equal the end of period $t - 1$ asset value A^{t-1} defined in previous sections.

45. In practice, commercial properties are typically very heterogeneous. See Diewert and Shimizu (2013) on how to include more structure characteristics in the builder's model.

46. Note that depreciation applies only to the structure part of asset value. Typical hedonic regression models that use the time dummy approach and age as an explanatory variable estimate depreciation as a proportion of entire asset value instead of just the structure portion of asset value. This type of model will be biased unless the ratio of structure value to land value is constant. For a recent example of this type of model, see Bokhari and Geltner (2014).

47. Note that in their nonlinear regression model, Diewert and Shimizu (2014) replaced $1 - \delta$ by e^{φ}, where φ is an unknown parameter to be estimated. However, the φ estimate can readily be converted into a δ estimate.

48. Diewert and Shimizu (2014) used a quarterly geometric depreciation rate of 10% for capital expenditures. They attempted to estimate this depreciation rate but had only limited success in determining an accurate estimate for this rate.

49. See Summers (1973) who introduced the method. Diewert and Shimizu (2013) used a similar approach to account for unobserved structure characteristics.

50. The problem is a multicollinearity problem: structure size is frequently correlated with lot size. See Diewert *et al.* (2011, 2014) for an explanation of the problem in the context of constructing residential house price indexes using hedonic regressions.

51. This parameter determines the overall quality of the structures for the group of properties.

52. This index was constructed by the Construction Price Research Association which is now an independent agency but prior to 2012 was part of the Ministry of Land, Infrastructure, Transport and Tourism (MLIT), a ministry of the Government of Japan. The quarterly values were constructed from the Monthly Commercial Construction Cost index for Tokyo for reinforced concrete buildings.

53. The same type of builder's model can be applied using a sample of commercial property *sales*; see Diewert and Shimizu (2013) for an example of this type of hedonic model applied to residential property sales. However, the problems with applying this type of model to commercial property sales are twofold: (i) commercial properties do not sell at the same frequency as residential properties and (ii) commercial properties are much more heterogeneous than residential properties so the estimated depreciation rates may not turn out to be sensible.

54. See section 4 in Diewert and Shimizu (2014) for a worked example showing how a commercial property price index can be constructed using the methodology described in this section.

55. 'Any analysis based only on survivors will therefore tend to overstate both the value and productivity of estimated capital stocks'. Hulten and Wykoff (1981; p. 377). Wear and tear depreciation is often called *deterioration* depreciation and *demolition* or *early retirement depreciation* is sometimes called *obsolescence* depreciation. Crosby *et al.* (2012; p. 230) distinguish the two types of depreciation

and in addition, they provide a comprehensive survey of the depreciation literature as it applies to commercial properties. Note that if we use the expected discounted cash flow approach to the valuation of a commercial property, deterioration depreciation is simply the loss of cash flow generated by the building during the accounting period, adjusted for a normal return to capital invested.

56. There is also the problem of how new properties should be taken into account in the construction of price indexes. Although we do not explicitly address this problem here, the usual approach is to construct chained indexes to take into account the changing stock.

57. Usually, land registry offices and/or municipal authorities issue building permits for the construction of new buildings and demolishment permits for the tearing down of buildings. It may be difficult to classify buildings into the desired economic categories.

58. Define $\rho_0 \equiv 0$.

59. Another term for demolition depreciation is *obsolescence depreciation*. Recall the analysis in Section 4 above where a model for the expected length of life of the structure was developed. The model depended on the sequence of expected cash flows, the sequence of expected interest rates and the sequence of expected land prices. If any of these expectations are not realized, then it may become optimal to retire the building prematurely; that is, before the original expected life has been reached. In particular, technical progress may lead to new types of structure that are able to generate much higher cash flows than the existing building and this will lead to an increase in land prices which in turn will lead to an unanticipated early retirement of the building.

60. This is four times the estimated quarterly depreciation rate of 0.45%.

61. Our method for adjusting wear and tear depreciation rates for the early retirement of assets is similar to the method suggested by Hulten and Wykoff. The main difference between our suggested method and their method is that we use a building life table to form estimates of building survivor probabilities whereas Hulten and Wykoff used somewhat arbitrary assumptions to form their estimates of survivor probabilities.

62. The worked example of how a hedonic regression model on appraised values for continuing commercial properties can be used to generate a CPPI for the group of properties that is developed in section 4 of Diewert and Shimizu (2014) also developed capital stock estimates for both structures and land for the group of properties. These capital stock estimates did not take into account demolition depreciation (since the sample of properties was a continuing sample. However, when developing national estimates for the stock of structures on commercial properties, demolition depreciation must be taken into account.

63. What we have labelled as wear and tear depreciation could be better described as *anticipated amortization of the structure* rather than wear and tear depreciation. Once a structure is built, it becomes a fixed asset which cannot be transferred to alternative uses (like a truck or machine). Thus, amortization of the cost of the structure should be roughly proportional to the cash flows that the building generates over its expected lifetime; see equations (22) and (23) of Section 5 and the related discussion. The pattern of cash flows generated by a commercial property can be quite volatile but market-based assessed values should be able to forecast these cash flows to some degree. However, technical progress, obsolescence or unanticipated market developments can cause the building to be demolished before it is fully amortized. See Diewert and Fox (2014) for a more complete discussion of the fixity problem.

64. Methods of this type are discussed in some detail in Baum and Crosby (2008; p. 27) and Geltner *et al.* (2014; pp. 585–591) and are labelled as *discounted cash flow models* or *capitalization of income approaches*.

65. For a fairly comprehensive review of the alternative sales-based methods for constructing property price indexes in the context of residential properties, see Eurostat (2011). For comparisons of the hedonic regression and the repeat sales methodologies, see Diewert *et al.* (2009), Diewert (2010, 2011), Shimizu *et al.* (2010) and Deng *et al.* (2013). See Devaney and Diaz (2010) and Bokhari and

Geltner (2014) for applications of the hedonic regression methodology in the commercial property context.

66. Appraisers frequently base their estimates for the asset value of a commercial property on sales of similar properties during the same time period: 'Real estate valuation is founded primarily on the use of comparable sales information'. Andrew Baum and Neil Crosby (2008; p. 17). This valuation method can be considered as a type of informal hedonic regression model.

67. Papers that use this methodology include Fisher *et al.* (1994), Geltner (1997), Geltner *et al.* (2010), Bokhari and Geltner (2012) and Shimizu *et al.* (2013).

68. We assume that $g < r$ so that the infinite series in (28) converges.

69. In the finance literature, N^1 is interpreted as the period 1 dividend or payout to investors. Geltner *et al.* (2014; p. 585) note that while the Gordon Growth Model did not originate with Gordon, it is most often attributed to Gordon and Shapiro (1956). Geltner *et al.* (2014; pp. 585–591) develop a series of generalizations of the Gordon Growth Model.

70. Geltner *et al.* (2014; p. 17) define the cap rate k^t as in definition (29) above but N^t is interpreted as period t net operating income rather than as period t cash flow. Recall that cash flow subtracts capital expenditures from net operating income.

71. The index is imperfect because it is not a pure price index that compares like with like; capital expenditures and depreciation make the structure part of the property asset different as the structure ages. Since depreciation of the structure will eventually be bigger than capital expenditures on the structure, we expect that the imperfect index defined by (30) will have a downward bias. Diewert and Shimizu (2014) called this type of index an *asset value price index*.

72. However, at a country wide aggregate level, the fluctuations in countrywide cash flow may not be a problem.

73. The revenue-based indexes will still fluctuate due to short term variations in vacancy rates. The problem is that these revenue fluctuations will probably be larger than the corresponding fluctuations in market asset values since forward looking asset values will average over short-term fluctuations in vacancy rates.

74. There are complications in this definition of property value if (i) the reported sale price is not the true sales price (as can happen when the buyer and seller try to avoid property transactions taxes); (ii) the agreed upon sale price takes place at a time that is different from the time when payment is received and (iii) if there are transactions costs associated with the transfer of title for the property. These are standard difficulties that national income accountants are familiar with and so in the interests of brevity, we will ignore these complications and assume that the sale price of a commercial property is unambiguous and pertains to a given period t.

75. However, the hedonic regression models presented in Section 7, which used appraised values for commercial properties as the dependent variable, are generally not suitable when appraised values are replaced by market sales values. When using appraised values as the dependent variable, the panel data nature of the regression model means that there is no need to account for the characteristics of the commercial property except for the age of the structure, capital expenditures on the structure and the floor space area of the structure. When we do not have panel data, it is necessary to take into account additional characteristics of the structure as well as locational characteristics. Examples of the hedonic regression approach applied to sales of residential properties that lead to land and structure decompositions of property value can be found in Eurostat (2011), Diewert *et al.* (2011, 2014) and Diewert and Shimizu (2013).

76. For a discussion of the repeat sales method in the commercial property context, see Geltner *et al.* (2014; pp. 657–662).

77. The method is not quite a matched model since depreciation and capital expenditures change the nature of the structure between the sales of the property.

78. Some price determining locational characteristics can change between sales of a property; for example, a transportation improvement that improves access to the property could have taken place between sales or a neighbouring polluting factory may have been closed and so on. However, these effects will generally be small.

79. Many characteristics of the structure will be constant between sales of the property such as the type of building construction, the floor space area of the structure, etc. However, as noted above, some structure characteristics will change between sales due to depreciation and capital expenditures.

80. This may not be a big problem in the context of commercial property price indexes; some empirical evidence on this topic is required.

81. Geltner *et al.* (2014; pp. 660–662) provide charts of some commercial property repeat sales price indexes for the United States over the past 15 or so years. It can be seen from these charts as the indexes become more specific, the indexes tend to become more wiggly, indicating that noise in the estimation procedure is increasing relative to the trends in property prices.

82. The appraised or assessed value of a particular property is valued at the sale price of a comparable property that sold in the period under consideration.

83. See Geltner *et al.* (2014; pp. 634–656) for an excellent summary of how appraisals are made in practice.

84. See the chart and surrounding discussion in Geltner *et al.* (2014; p. 655) for an illustration of these points.

85. Diewert and Shimizu (2014) in their study of Tokyo commercial office buildings called this type of index an *asset value index*. They found that the asset value index generated by their sample of commercial properties had a downward bias of about 0.4–0.7 percentage points per year compared to corresponding 'true' price indexes.

86. Assessed values may be slightly too low on average in order to minimize appeals by property owners. However, market-based assessments cannot drift too low since every time a property is sold, the selling price becomes the basis for the current period assessed value. This will influence the assessed values of 'comparable' properties in the right direction.

87. See Shimizu and Nishimura (2006) on biases in appraisal information.

88. For additional information on REITs, see Chapter 23 in Geltner *et al.* (2014).

89. The same type of phenomenon occurs in ordinary stock markets: the stock market-based index of aggregate asset value is much more volatile than the corresponding replacement-based index of asset value and the two measures of asset values can differ considerably over the course of a business cycle; see the literature on Tobin's (1969) Q.

90. Some care must be taken to account for the debt position of the REIT, to account for sales and acquisitions of properties and to ensure that there are no international properties but the required information is usually available in the quarterly and annual reports of the REIT.

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