Time-Varying Wage Risk, Incomplete Markets, and Business Cycles

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How do changes in idiosyncratic labor income risk affect aggregate fluctuations?

In particular, how is labor market dynamics affected?
Growing interests in fluctuations in uncertainty
  - uncertainty shocks by Bloom 2009
  - various measures of uncertainty rose in the recent financial crisis

Cyclical variation in idiosyncratic labor earnings risk
  - Storesletten, Telmer, and Yaron 2004, Heathcote, Perri, and Violante 2010
  - previous DSGE analyses typically omit labor supply decisions
    $\implies$ little is known about the impact on labor market dynamics
What this paper does

- Augment DSGE model widely used for labor market analyses
  - idiosyncratic wage/productivity risk
  - incomplete asset markets
  - indivisible labor
  - aggregate shocks
    - TFP shocks
    - uncertainty shocks (fluctuations in idiosyncratic wage risk)
- Analyze the impact of uncertainty shocks on business cycles through stochastic simulation
  - infer the size of uncertainty shocks using individual wage data
Main findings

- Uncertainty shocks move key statistics closer to data
  - $\text{corr}(H, Y/H)$ decreases from 0.83 to –0.40
  - $\sigma_{\text{wedge}}$ increases from 17% of data to 90%

- Aggregation bias (composition effect)
  - Impacts of uncertainty shocks on employment differ across productivity groups
Related literature

- Varying uninsured idiosyncratic earnings risk
  - exogenous earnings or divisible labor

- Uninsured Idiosyncratic wage risk under indivisible labor
  - constant risk
Outline

- Model
  - Parameter values and steady state
  - Business cycle results
  - Conclusion
Individuals

- Momentary utility: $u(c, h)$
  - $c$: consumption
  - $h$: labor hours, $h \in \{\bar{h}, 0\}$

- Time-varying uninsured idiosyncratic wage risk
  - **Idiosyncratic wage risk**: person-specific labor productivity $x$
    $$\ln x' = \rho_x \ln x + \varepsilon'_x, \varepsilon'_x \sim N(0, \sigma_{\varepsilon_x}^2)$$
  - **Uninsured**: single asset $k$ (physical capital), $k \geq k$ ($k \leq 0$)
  - **Time-varying**: $\sigma_{\varepsilon_x}$ is a Markov chain
Beginning-of-period value

\[ V(k, x; z, \sigma_{\varepsilon_x}, \mu) = \max \{ V^E(k, x; z, \sigma_{\varepsilon_x}, \mu), V^N(k, x; z, \sigma_{\varepsilon_x}, \mu) \} \]

- **Value functions**
  - \( V \): beginning-of-period value
  - \( V^E \): employment value
  - \( V^N \): nonemployment value

- **State variable**
  - \( k \): individual asset holding
  - \( x \): idiosyncratic productivity
  - \( z \): aggregate TFP, AR(1) process
  - \( \sigma_{\varepsilon_x} \): idiosyncratic wage risk, learned one period in advance
  - \( \mu \): individual distribution over \( k \) and \( x \), \( \mu' = \Gamma(z, \sigma_{\varepsilon_x}, \mu) \)
Value of employment

\[ V^E(k, x; z, \sigma_{\epsilon_x}, \mu) = \max_{c, k'} \{ u(c, \bar{h}) + \beta E[V(k', x'; z', \sigma_{\epsilon_x}', \mu')] | x, z, \sigma_{\epsilon_x}, \mu] \} \]

s.t. \( c + k' = w(z, \sigma_{\epsilon_x}, \mu) x \bar{h} + [1 + r(z, \sigma_{\epsilon_x}, \mu)] k \)
\[ k' \geq k \]
\[ c \geq 0 \]

Law of motion for \( x, z, \sigma_{\epsilon_x}, \) and \( \mu \)
Value of nonemployment

\[ V^N(k, x; z, \sigma_{\epsilon_x}, \mu) = \max_{c, k'} \{ u(c, 0) \}
\]

\[ + \beta E[V(k', x'; z', \sigma'_{\epsilon_x}, \mu') | x, z, \sigma_{\epsilon_x}, \mu] \}
\]

s.t. \( c + k' = [1 + r(z, \sigma_{\epsilon_x}, \mu)]k \)

\( k' \geq k \)

\( c \geq 0 \)

Law of motion for \( x, z, \sigma_{\epsilon_x}, \) and \( \mu \)
Employment choice

\[ h = \begin{cases} 
\bar{h} & \text{if } V^E \geq V^N \\
0 & \text{otherwise} 
\end{cases} \]
Representative firm

- Produce the good $Y$
- Rent capital $K$ and labor $L$ from individuals
- Production function
  - $Y = zF(K, L)$
  - constant returns to scale in $K$ and $L$
- Maximize static profits
  $$\begin{align*}
  r(z, \sigma \epsilon_x, \mu) &= (1 - \alpha)zF_K(K, L) - \delta \\
  w(z, \sigma \epsilon_x, \mu) &= \alpha zF_L(K, L)
  \end{align*}$$
- $\delta$: capital depreciation rate
Equilibrium

A recursive competitive equilibrium consists of a set of functions,

\[(w, r, V^E, V^N, V, c, k', h, K, L, \Gamma)\],

that satisfy the following conditions:

- Individual optimization
- Firm optimization
- Market clearing (labor, capital, and good)
  - Labor: \( L = \int x h(k, x; z, \sigma_{\epsilon_x}, \mu) \mu([dk \times dx]) \)
  - Hours: \( H = \int h(k, x; z, \sigma_{\epsilon_x}, \mu) \mu([dk \times dx]) \)
- Law of motion for the distribution across individuals is consistent with individuals’ behavior and the underlying stochastic processes
Outline

- Model
- Parameter values and steady state
- Business cycle results
- Conclusion
Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>0.9829</td>
</tr>
<tr>
<td>(B)</td>
<td>1.0203</td>
</tr>
<tr>
<td>(\bar{h})</td>
<td>1/3</td>
</tr>
<tr>
<td>(k)</td>
<td>–2.0</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.64</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.025</td>
</tr>
<tr>
<td>(\rho_z)</td>
<td>0.950</td>
</tr>
<tr>
<td>(\sigma_{\varepsilon_z})</td>
<td>0.007</td>
</tr>
</tbody>
</table>

\[
u(c, h) = \begin{cases} 
\ln c - B & \text{when work} \\
\ln c & \text{when not work}
\end{cases}
\]

\[
y = zF(K, L) = zK^{1-\alpha}L^\alpha
\]

\[
\ln z' = \rho_z \ln z + \varepsilon_z', \quad \varepsilon_z' \sim \mathcal{N}(0, \sigma_{\varepsilon_z}^2)
\]
Parameters on idiosyncratic productivity

\[
\ln x' = \rho_x \ln x + \varepsilon'_x, \quad \varepsilon'_x \sim N(0, \sigma^2_{\varepsilon_x})
\]

- \(\sigma_{\varepsilon_x}\) is a 3-state Markov chain
  - \((1 + \lambda)\bar{\sigma}_{\varepsilon_x}, \bar{\sigma}_{\varepsilon_x}, (1 - \lambda)\bar{\sigma}_{\varepsilon_x}\)
  - remain unchanged with prob \(\rho_{\sigma_{\varepsilon_x}}\), transition to each of the other states with \((1 - \rho_{\sigma_{\varepsilon_x}})/2\), independent of \(z\)

- Parameters

\[
\rho_x, \quad \bar{\sigma}_{\varepsilon_x}, \quad \lambda, \quad \rho_{\sigma_{\varepsilon_x}}
\]
Moments compared between PSID and model

\[ \ln x_{i,t} = \rho_x \ln x_{i,t-1} + \varepsilon_{i,x,t}, \ varepsilon_{i,x,t} \sim N(0, \sigma^2_{\varepsilon_x,t}) \]

\[ \ln w_{i,t} = \ln x_{i,t} + \ln w_t \]

\[ \ln w_{i,t} = \rho_x \ln w_{i,t-1} + (\ln w_t - \rho_x \ln w_{t-1}) + \varepsilon_{i,x,t}, \ varepsilon_{i,x,t} \sim N(0, \sigma^2_{\varepsilon_x,t}) \]

1. Pooled estimation

\[ \Rightarrow \hat{\rho}_x, \hat{\sigma}_{\varepsilon_x} \]

- PSID, Model: OLS

2. Year-by-year estimation

\[ \Rightarrow \text{std}(\hat{\sigma}_{\varepsilon_x,t}), \ corr(\hat{\sigma}_{\varepsilon_x,t}, \hat{\sigma}_{\varepsilon_x,t-1}) \]

- PSID: OLS, controlled OLS, Heckman-type estimation
- Model: OLS
Estimated idiosyncratic wage risk in PSID

![Graph](image)
Cyclical component of estimated wage risk

\[
\text{std}(\hat{\sigma}_{\epsilon_x,t}) = 0.032, \quad 0.035, \quad 0.039 \\
\text{corr}(\hat{\sigma}_{\epsilon_x,t}, \hat{\sigma}_{\epsilon_x,t-1}) = 0.185, \quad 0.236, \quad 0.056
\]
Moments and parameter values

<table>
<thead>
<tr>
<th>Moments (annual)</th>
<th>U.S.</th>
<th>Varying risk</th>
<th>Constant risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\rho}_x$</td>
<td>0.854</td>
<td>0.855</td>
<td>0.855</td>
</tr>
<tr>
<td>$\hat{\sigma}_{\epsilon_x}$</td>
<td>0.282</td>
<td>0.283</td>
<td>0.279</td>
</tr>
<tr>
<td>$\text{std}(\hat{\sigma}_{\epsilon_x,t})$</td>
<td>0.032</td>
<td>0.032</td>
<td>0.008</td>
</tr>
<tr>
<td>$\text{corr}(\hat{\sigma}<em>{\epsilon_x,t}, \hat{\sigma}</em>{\epsilon_x,t-1})$</td>
<td>0.185</td>
<td>0.158</td>
<td>–0.240</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters (quarterly)</th>
<th>Varying risk</th>
<th>Constant risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_x$</td>
<td>–</td>
<td>0.930</td>
</tr>
<tr>
<td>$\bar{\sigma}_{\epsilon_x}$</td>
<td>–</td>
<td>0.223</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>–</td>
<td>0.090</td>
</tr>
<tr>
<td>$\rho_{\sigma_{\epsilon_x}}$</td>
<td>–</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Steady state

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini labor income</td>
<td>0.60~0.63</td>
<td>0.60</td>
</tr>
<tr>
<td>Gini wealth</td>
<td>0.78</td>
<td>0.69</td>
</tr>
<tr>
<td>corr(labor income, wealth)</td>
<td>0.23</td>
<td>0.30</td>
</tr>
</tbody>
</table>
Outline

- Model
- Parameter values and steady state
- **Business cycle results**
- Conclusion
## Business cycle statistics

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Constant risk</th>
<th>Varying risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_Y$</td>
<td>1.69</td>
<td>1.37</td>
<td>1.43</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>0.54</td>
<td>0.32</td>
<td>0.33</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>2.85</td>
<td>3.10</td>
<td>3.15</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>1.00</td>
<td>0.57</td>
<td>0.81</td>
</tr>
<tr>
<td>$\sigma_{Y/H}$</td>
<td>0.63</td>
<td>0.48</td>
<td>1.00</td>
</tr>
<tr>
<td>$\text{corr}(Y, C)$</td>
<td>0.78</td>
<td>0.90</td>
<td>0.86</td>
</tr>
<tr>
<td>$\text{corr}(Y, I)$</td>
<td>0.80</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$\text{corr}(Y, H)$</td>
<td>0.80</td>
<td>0.96</td>
<td>0.41</td>
</tr>
<tr>
<td>$\text{corr}(Y, Y/H)$</td>
<td>0.31</td>
<td>0.95</td>
<td>0.67</td>
</tr>
<tr>
<td>$\text{corr}(H, Y/H)$</td>
<td><strong>-0.32</strong></td>
<td><strong>0.83</strong></td>
<td><strong>-0.40</strong></td>
</tr>
</tbody>
</table>
Sensitivity analysis

\[ \text{corr}(H, Y/H) \]

\( \lambda \)
Sensitivity analysis

\[ \text{corr}(H, \frac{Y}{H}) \]

\[ \rho_{\sigma x} \]

\[
\begin{array}{c}
0.2 & 0.4 & 0.6 & 0.8 & -0.5 & -0.4 & -0.3 & -0.2 & -0.1 & 0
\end{array}
\]

\[
\begin{array}{c}
0 & -0.1 & -0.2 & -0.3 & -0.4 & -0.5
\end{array}
\]
One-period increase in idiosyncratic wage risk

Wage risk $\sigma_{\varepsilon x}$

Output $Y$

Hours worked $H$

Labor productivity $Y/H$

Horizontal axis – period
Vertical axis – percent deviation
Underlying two effects

- Uncertainty effect (period 0)
  - uncertainty about future wages rises $\Rightarrow H \uparrow, Y/H \downarrow$

- Distribution effect (period 1)
  - the productivity-wealth distribution shifts $\Rightarrow H \downarrow, Y/H \uparrow$
Uncertainty effect
Uncertainty effect

<table>
<thead>
<tr>
<th>Employment</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>low productivity</td>
<td>H</td>
</tr>
<tr>
<td>high productivity</td>
<td>Y/H</td>
</tr>
</tbody>
</table>

Legend:
- Density
- Not Work
- Work
- Productivity Inx
- Wealth In(k+2)
Uncertainty effect (period 0)
  - uncertainty about future wages rises $\implies H \uparrow, Y/H \downarrow$

Distribution effect (period 1)
  - the productivity distribution shifts $\implies H \downarrow, Y/H \uparrow$
Distribution effect

![Distribution effect graph](image)

Productivity $\ln x$

Density

Period 0

Period 1
Distribution effect

<table>
<thead>
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<tbody>
<tr>
<td>low productivity</td>
<td>high productivity</td>
</tr>
<tr>
<td>↓↓</td>
<td>↑</td>
</tr>
<tr>
<td></td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>↓</td>
</tr>
<tr>
<td></td>
<td>Y/H</td>
</tr>
<tr>
<td></td>
<td>↑</td>
</tr>
</tbody>
</table>
Uncertainty versus distribution effects

- Psych risk: only uncertainty effect, individuals receive signals for changes in $\sigma_{\varepsilon_x}$, but those changes in $\sigma_{\varepsilon_x}$ never materialize (Bachmann and Bayer 2013)

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</thead>
<tbody>
<tr>
<td>$\sigma_Y$</td>
<td>1.69</td>
<td>1.37</td>
<td>1.43</td>
<td>1.37</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>0.54</td>
<td>0.32</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>2.85</td>
<td>3.10</td>
<td>3.15</td>
<td>3.10</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>1.00</td>
<td>0.57</td>
<td>0.81</td>
<td>0.61</td>
</tr>
<tr>
<td>$\sigma_{Y/H}$</td>
<td>0.63</td>
<td>0.48</td>
<td>1.00</td>
<td>0.52</td>
</tr>
<tr>
<td>corr($Y, C$)</td>
<td>0.78</td>
<td>0.90</td>
<td>0.86</td>
<td>0.89</td>
</tr>
<tr>
<td>corr($Y, I$)</td>
<td>0.80</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
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<td>0.31</td>
<td>0.95</td>
<td>0.67</td>
<td>0.87</td>
</tr>
<tr>
<td>corr($H, Y/H$)</td>
<td>$-0.32$</td>
<td>$0.83$</td>
<td>$-0.40$</td>
<td>$0.58$</td>
</tr>
</tbody>
</table>
Implication for the labor wedge

- Labor wedge is calculated by

\[
\ln \text{wedge} = \ln MPL - \ln MRS = \ln \frac{Y}{H} - \ln BRH^{1/\gamma}C
\]

\[
U(C, H) = \ln C - \frac{BRH^{1+1/\gamma}}{1 + 1/\gamma}, \gamma = 1.5
\]

<table>
<thead>
<tr>
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<th>Varying risk</th>
<th>Psych risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{\text{wedge}})</td>
<td>1.40</td>
<td>0.23</td>
<td>1.26</td>
<td>0.38</td>
</tr>
<tr>
<td>(\text{corr}(H, \text{wedge}))</td>
<td>-0.94</td>
<td>-0.96</td>
<td>-0.84</td>
<td>-0.83</td>
</tr>
</tbody>
</table>

- Fluctuations in the labor wedge arise from those in the deviation of \(w\) and \(MRS\) (Karabarbounis 2014)
Countercyclical risk

- Introduce negative comovement of $\sigma_{\epsilon_x}$ with $z$
  - $z > (1 + 0.017) \bar{z} \implies \sigma_{\epsilon_x} = (1 - \lambda) \bar{\sigma}_{\epsilon_x}$
  - $z < (1 - 0.017) \bar{z} \implies \sigma_{\epsilon_x} = (1 + \lambda) \bar{\sigma}_{\epsilon_x}$
  - otherwise, $\sigma_{\epsilon_x} = \bar{\sigma}_{\epsilon_x}$
  - $\rho_{\sigma_{\epsilon_x}}$ is implied by $z$’s persistence, $\lambda$ is unchanged

- Recalibrate varying risk model to match the volatility and persistence of $\sigma_{\epsilon_x}$ in the countercyclical risk model
  - $\lambda = 0.058$, $\rho_{\sigma_{\epsilon_x}} = 0.925$
## Countercyclical risk

<table>
<thead>
<tr>
<th>Countercyclical risk</th>
<th>Recalibrated varying risk</th>
<th>Constant risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_Y$</td>
<td>1.32</td>
<td>1.38</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>0.37</td>
<td>0.33</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>2.98</td>
<td>3.11</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>0.56</td>
<td>0.68</td>
</tr>
<tr>
<td>$\sigma_Y/H$</td>
<td>0.61</td>
<td>0.74</td>
</tr>
<tr>
<td>$\sigma_{\text{wedge}}$</td>
<td><strong>0.60</strong></td>
<td><strong>0.83</strong></td>
</tr>
<tr>
<td>$\text{corr}(Y, C)$</td>
<td>0.91</td>
<td>0.88</td>
</tr>
<tr>
<td>$\text{corr}(Y, I)$</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$\text{corr}(Y, H)$</td>
<td>0.84</td>
<td>0.68</td>
</tr>
<tr>
<td>$\text{corr}(Y, Y/H)$</td>
<td>0.87</td>
<td>0.73</td>
</tr>
<tr>
<td>$\text{corr}(H, Y/H)$</td>
<td><strong>0.46</strong></td>
<td><strong>0.00</strong></td>
</tr>
<tr>
<td>$\text{corr}(H, \text{wedge})$</td>
<td>$-0.67$</td>
<td>$-0.77$</td>
</tr>
</tbody>
</table>
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Conclusion

- Examine how time-varying idiosyncratic wage risk affects aggregate fluctuations in the heterogenous-agent model commonly used for labor market analyses
- Including uncertainty shocks improves the model’s performance concerning labor market dynamics
- Future work
  - uncertainty on asset income, endogenous uncertainty
  - other shocks than aggregate TFP and uncertainty shocks
  - home production, family labor supply, and so on
Increase in wage uncertainty during recessions

TFP $z$

Wage risk $\sigma_{\varepsilon x}$

Hours $H$

Labor productivity $Y/H$
U.S. hours and labor productivity