Leverage and Productivity

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Abstract

Financial frictions can reduce aggregate productivity, in particular when firms with high productivity cannot borrow against their earnings. This paper investigates the quantitative importance of this form of borrowing constraint using a large panel of firms in Japan. The firms are young and unlisted, precisely the firms for which credit frictions are expected to be the most severe. In this data, I find that firm leverage (asset-to-equity ratio) and firm output-to-capital ratios rise with firm productivity, both over time in a firm and across firms of the same age and cohort. I use these facts in indirect inference to estimate a standard general equilibrium model where financial frictions arise from the limited pledgeability of earnings and assets. In this model more financially constrained firms have higher output-to-capital ratios. The model matches the two facts the best when firms can pledge the equivalent of over half of their one-year-ahead earnings and one-fifth of their assets. Compared to the common assumption that firms can pledge only assets, aggregate productivity loss due to financing frictions is one-third smaller when earnings are also pledgeable to the degree seen in Japan.

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Introduction

To what extent are young and unlisted firms borrowing constrained? The answer to this question matters for understanding the impact of financial frictions on aggregate productivity. Recent quantitative studies of aggregate productivity loss due to financial frictions found that frictions at the entry margin is key to generating large aggregate productivity losses. Also, the answer may be informative as an upper bound on the overall severity of financing frictions because one would expect young and unlisted firms to be affected the most by financing frictions. They have not yet accumulated retained earnings and have not been able to tap equity markets. Furthermore, many countries have programs that give financing to young firms because it is believed that they are important for aggregate growth but are financially constrained[1].

One important source of financial frictions derives from firms not being able to credibly commit to fully repay loans out of their future earnings or assets. As a result, the borrowing capacity of a firm may be smaller than the financing needed for the firm to produce at its optimal scale, which could happen for a firm with low internal funds relative to its productivity. When many firms are borrowing constrained in this way, aggregate productivity can be significantly smaller than in an economy where firms can fully commit to repay. In this paper, we investigate the extent of this form of friction for a large panel of young and unlisted firms from Japan[2]. In particular, we study whether the borrowing constraint is due to low plegeability of earnings or low plegeability of assets. We also demonstrate why this distinction matters for aggregate productivity loss due to financial frictions.

More specifically, we use a standard general equilibrium model of aggre-

[1]For example, expansion of financing to young firms features prominently in the current Japanese government's growth strategy (See "Japan Revitalization Plan" Cabinet Office 2014) and the SBIR grants in the U.S. give funds to young firms to stimulate innovation.

[2]We use Japanese data because it has a measure of inside fund that can be used for robustness checks. Moreover, policymakers in Japan have often attributed Japan's long stagnation, in part, to the stunted growth of young firms due to financial frictions (See "Japan Revitalization Plan" Cabinet Office (2014)).

gate productivity loss due to financial frictions where the extent of the frictions depends on the share of assets and one-period-ahead earnings that a lender can recover when its client-firm defaults. We depart from standard inference approaches by allowing the share pledgeable to differ for assets and earnings. When earnings are not pledgeable, the borrowing capacity of a firm is a constant fraction of its inside fund and does not vary with the firm's productivity. On the other hand, when earnings are pledgeable, more productive firms have higher earnings and hence can borrow more than less productive firms even if they all have the same inside funds[3].

We estimate these two shares by indirect inference with our micro-data. More precisely, we choose the shares so that the empirical regression coefficients of leverage[4] and output-to-capital ratio on productivity and inside fund matches as close as possible with that obtained from the same regression ran on data simulated from the model. Here, leverage is defined as total asset over equity and output-to-capital ratio is a proxy for the marginal product of capital. We choose these empirical targets for the following reason. The model has two forces governing how leverage vary with firm productivity conditional on firm's inside fund. In one case, firms are unconstrained and more productive firms have higher leverage because their optimal production scale is larger. In the second case, firms are constrained and more productive firms have higher leverage because they have higher borrowing capacity. The second force is turned on only when earnings are pledgeable. Hence we can identify the share of earnings that is pledgeable by looking at how leverage varies with productivity for constrained firms, who have higher output-to-capital ratio in the model.

[3]More productive firms having higher borrowing capacity is also a feature of microfoundations where more productive firms have more to lose if they default. See, for example, Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Buera and Shin (2013) and Arellano et al. (2012).

[4]We measure leverage by the ratio of total assets to inside equity, which corresponds to the equity multiplier measure of leverage, as in Berk and DeMarzo (2013). A more commonly used measure of leverage in empirical corporate finance is the debt-to-asset ratio, because the focus of these studies is the choice between equity and debt. We do not distinguish between debt and outside equity because the firms in our dataset are young, unlisted, and owner-managed.

In the extreme when earnings are not pledgeable at all, leverage and output-to-capital ratio cannot *simultaneously* rise with productivity after controlling for inside funds.

In the data, we find *both* leverage and output-to-capital ratio rise strongly with firm productivity after controlling for inside equity (see Figure 1 and 2). Leverage increases by 1%, on average, for every 1% increase in firm productivity, while the output-to-capital ratio rises by 0.7%, on average, for every 1% increase in firm productivity. This pattern of leverage and output-to-capital ratio both rising with firm productivity holds within a firm over time and across firms of the same cohort, age and detailed industry group under various empirical specifications. It is also robust to several alternative empirical specifications such as using an alternative measure of inside fund and capital. The model matches these empirical elasticities the best when firms can pledge half of their one-year-ahead profits and a fifth of their assets. At these parameter values the aggregate productivity loss due to financial frictions is 11%[5].

We explore the aggregate implications of our findings by comparing aggregate productivity loss due to financial frictions under our benchmark estimation with the loss when we estimate the model restricting the share of earnings that can be pledged to be zero[6]. This restriction appears in many papers quantifying aggregate productivity loss due to financial frictions. We find aggregate productivity loss is approximately 14% in the restricted parameterization. That is, the common assumption that firms cannot borrow against earnings results

[5]Aggregate productivity loss is defined as the difference between the first-best productivity and the model productivity as a percentage of the model productivity. Here, the first best is achieved when all firms are unconstrained. It is the same in both models.

[6]This is an interesting exercise because there does not appear to be consensus on the modeling the microfoundation of financial frictions in macroeconomic models. Some papers such as Moll (2014), Midrigan and Xu (2014), and Buera and Shin (2013) assume that borrowing capacity does not depend on firm productivity. Other papers such as Buera et al. (2011), Buera et al. (2014), Arellano et al. (2012), Bah and Fang (2014) assume borrowing capacity increases with firm productivity. More broadly, in the literature on firm dynamics and aggregate fluctuations, the assumption tend to be that firm borrowing capacity rises with firms productivity or expected returns, e.g. Bernanke and Gertler (1989), Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004). Our empirical evidence is more consistent with the assumption of borrowing capacity rising with firm productivity.

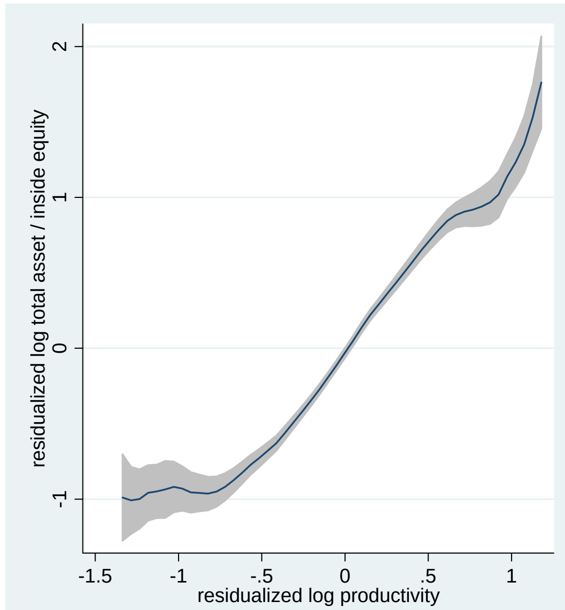

Figure 1: Firm leverage rises with firm productivity

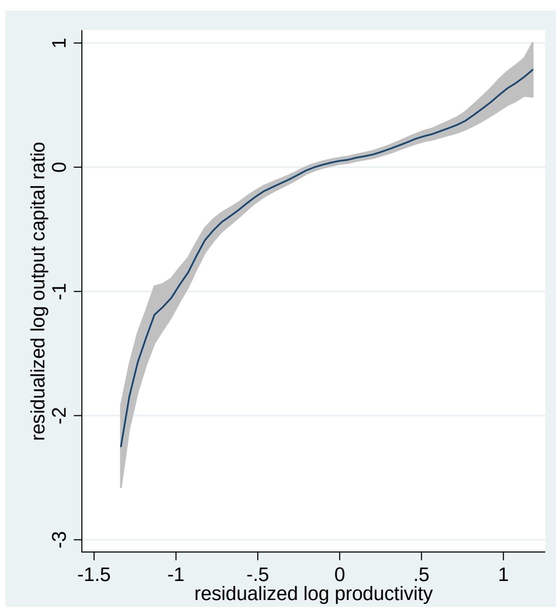

Figure 2: Firm output-capital ratio rises with firm productivity

in an overstatement of aggregate productivity loss by 30% relative to the loss that is consistent with our empirical findings. Hence our empirical findings suggest that assuming borrowing capacity does not depend on productivity is not innocuous: it can lead to quantitatively significant overstatement of aggregate productivity loss due to financial frictions.

Our paper contributes to the empirical literature on young firm financing. Studies that use the Survey of Small Business Finance by the U.S. Federal Reserve Board, such as Bates (1990), Åstebro and Bernhardt (2003), Ang et al. (2010), and Cole (2013), find that the relationship between firm leverage and firm size or firm return-on-assets varies by the year of the survey. Cassar (2004) documents a positive relationship between startup leverage and total assets, while finding a negative relationship between leverage and the share of noncurrent assets, a measure of liquidity for Australia. Brav (2009) finds that leverage is negatively related to return-on-assets and positively related to firm size and sales growth. Arellano et al. (2012) use Amadeus data to study cross country differences in the relationship between firm size and leverage. They find that the size-leverage relationship depends on the level of financial development in a country.

Our paper differs from previous empirical literature by documenting the relationship between firm leverage, firm output-to-capital ratios and firm total factor productivity (TFP) for young unlisted firms. Many of the aforementioned empirical papers use size to proxy for productivity. However, empirically distinguishing between productivity and size is crucial for the study of financial frictions. It is precisely the presence of small but productive firms that is thought to create big inefficiencies due to financial frictions. For example, Hsieh and Klenow (2009) and Foster et al. (2008) find large firms are not necessarily highly productive firms. Also, in our dataset, size and measured productivity are far from being perfectly correlated.

There are numerous studies of productivity and leverage for listed firms. A common finding is that smaller firms or less well performing firms have higher leverage, where leverage is measured as the debt to equity ratio (see Frank and Goyal (2008) for a survey). This appears to be driven by better firms having better access to *outside* equity[7] In contrast, we find a positive correlation between productivity and leverage. This is likely due to the firms in our dataset not having access to the outside equity market markets[8]. Higher productivity evidently allows firms in our sample to borrow more and have higher ratios of debt to *inside* equity. Nonetheless, our point that better firms have better access to financing is consistent with existing findings of larger firms having lower borrowing costs or better access to credit markets, e.g., Gilchrist et al. (2013), Hennessy and Whited (2007).

Our paper is also related to the large literature testing for financial frictions in corporate finance with seminal papers such as Fazzari et al. (1987), Kaplan and Zingales (1997) and Whited (1992). Recent studies in development economics use exogenous variations in policy and weather (e.g. Banerjee and Duflo (2014) and De Mel et al. (2008)). Our approach is similar to Whited (1992) in that we use a structural model to infer which firms are financially constrained.

Finally, our paper is related to the recent literature quantifying aggregate productivity losses due to financial frictions such as Moll (2014), Midrigan and Xu (2014), and Buera et al. (2011). We contribute to this literature by documenting rising leverage capacity with productivity and showing its importance for quantifying aggregate productivity losses due to financial frictions. More broadly, the empirical evidence in our paper is useful for distinguishing between various microfoundations of financial frictions in macroeconomic models. By documenting how external financing and output-to-capital ratio varies at the firm-level, we complement studies such as Beck et al. (2000) that con-

[7]For example, for listed firms in Japan, Pushner (1995) finds that leverage (debt-to-asset ratio) is negatively correlated with TFP because active institutional shareholders reduce the cost of outside equity financing while disciplining managers, leading to both a lower share of debt and higher productivity.

[8]This is likely to be due to the high fixed costs associated with accessing stock and bond markets. See Russ and Valderrama (2009) and Begenau and Salomao (2014) for evidence and theory

struct aggregate measures of external financing.

The paper is organized as follows. Section 1. describes our dataset and empirical findings. Section 2. presents a standard model of aggregate TFP loss due to financial frictions. Section 3. estimates the model by indirect inference while Section 4. conducts counterfactual exercises. Section 5. concludes. Details such as robustness checks are provided in the Appendix.

1. Firm leverage and firm productivity in the data

In this section we describe the data source and document the key empirical patterns.

1.1. Data

Our firm-level data comes from TSR, Japan's largest credit rating agency. This data is known for its coverage and rich information of small private firms that is not found in other datasets for Japan. For example, it used by the Japanese government whitepapers such as the White Paper on Small and Medium Enterprises. We observe the balance sheets of these firms from 2004 to 2013 as well as information on their incorporation date, legal status, detailed industry classification, listing status etc.

TSR is a credit rating agency so the unit of observation in our data is a unit that requires independent credit rating. This is suitable for our study of firm financing. Table 1 shows a breakdown of units that formed between 2001 - 2014 by Japan's legal forms observed in 2014 when the data was downloaded. We have some sole-proprietorships but the sample size is very small relative to the total number of sole-proprietors in the economy and a proxy for their inside equity is not readily available. Hence we use only the top two largest categories ("Corporations" and " Limited liability company") for our analysis.

We measure the age of a firm by the difference between the year of observa-

Legal form	Share of firms
Corporation	67.0%
Limited liability company	25.4%
Sole proprietorship	4.7%
Unknown and others	2.9%

Table 1: Breakdown of firms incorporated 2001-2014 in the TSR-Orbis database by legal form at 2014

tion and the year of the reported incorporation date[9]. Table 2 gives a breakdown of firms with known incorporation date by incorpoation year (only for odd years to save space). We use only the sample that incorporated between 2001 and 2013 because we observe firms from 2004 to 2013. The cutoff at 2001 ensures we observe firms when they are young and *shihonkin* or shareholders fund are good proxies for the inside equity of the founders. For the same reason, we drop firms that were ever listed.

We check the coverage of our dataset by comparing the number of firms in our final dataset with the Japanese Census[10]. The Census (or Establishment and Enterprise Survey before 2009) reports the number of establishments by opening year, legal form, single versus multi-unit, branch versus main, *shihonkin* bin and employment bin etc. We define a firm in the Census as an establishment whose legal form is company (*kaisha*) and is either a single unit or a main branch. The unit of observation in the Census is an establishment with continuous economic activities at a physical location under a single management. The opening year is not the creation year or incorporation year of the firm but the year when operation began at the location under the current management. Also, it is the number of establishments operating at the time

[9]A few firms are observed before their incorporation date probably due to a change of legal form. We drop these observations from the data used for the analysis.
[10]Details of these datasets are provided in the appendix.

incorporation year	share of firms
before 2001	63.9%
2001	2.3%
2003	2.4%
2005	2.8%
2007	2.8%
2009	2.7%
2011	3.0%
2013	3.2%

Table 2: Breakdown of firms in the TSR-Orbis database by incorporation year

of the Census. For example, the firm count for the 2011 cohort is the number of single or main establishments that began operating at the surveyed location in 2011 and is operating on the Census survey date Feb 1st, 2012. It is likely that incorporation took place before operation started and some incorporated firms may not reached the operation stage or survived to the time of the Census survey.

For the available census years, Table 3 displays the Census count of non-agricultural companies that opened in that year as well as the TSR-Orbis count of companies incorporated in that year with all variables observed at some point between 2004-2014. It shows that the sample size in TSR is close to 30% of the Census counts.

Figure 3 and 4 compares the *shihonkin* and worker distribution in our final sample and the 2006 Census[11] for newly incorporated companies[12]. It shows that our dataset slightly selects on larger entrants in terms of their *shihonkin*

[11]the Census is for all establishments because publicized data does not report a breakdown by single, branch and main establishments.

[12]As the Census count of newly incorporated companies include those incorporated in 2005, we use the 2005 and 2006 cohorts distributions in 2006 from TSR-Orbis to make the comparison.

incorp year	TSR-Orbis	Census[1]
2001	8,995	35,114
2006	9,826	28,946
2011,2012	9,405	21,312

[1] single unit or main companies establishments

Table 3: Company counts. TSR-Orbis, Census

level but more than half of our observations are small firms (with *shihonkin* below 100K dollars or below 10 employees). In terms of workforce, we actually select on smaller firms. This could be because the Census includes owners and non-paid family members while TSR-Orbis does not. For our numerical exercise in Section 4., this selection issue may not be a problem. The economic significance of our findings increases with the standard deviation of firm productivity and the elasticity with which leverage capacity increases with productivity. Our data selects on larger entrants who are likely to have better access to financing. Hence the selection issue is likely to lead to an understatement of productivity variation and the elasticity of leverage with respect to productivity. So we believe the results of our numerical exercise are not inflated by data selection.

Our dataset is close to being representative along the employment and *shihonkin* margin. Ideally, we also want to show our dataset is representative of productivity, leverage and output-to-capital ratio. Unfortunately, the Census does not cover revenue and asset until 2012. For 2012, it does not publicize the revenue and capital distribution by incorporation year so we do not have a way of checking the representativeness of our data on these important dimensions.

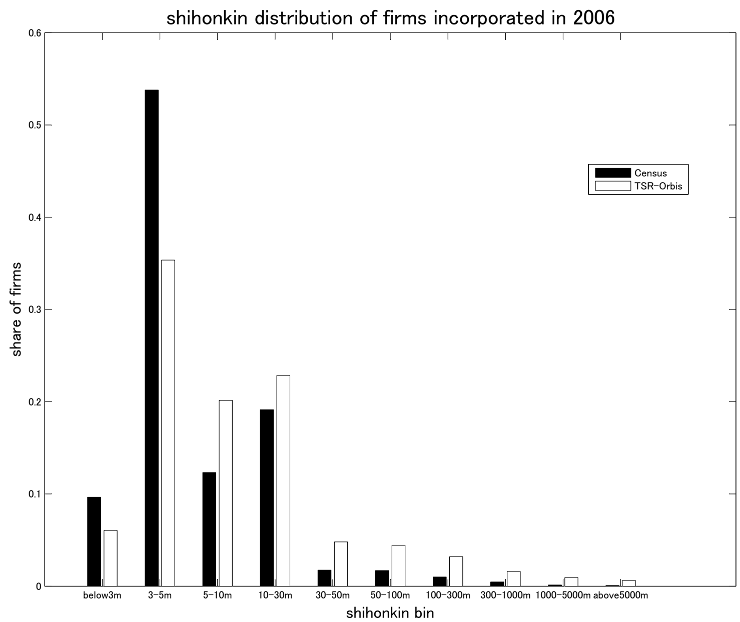

Figure 3: *shihonkin* distribution, Orbis-tsr versus Census

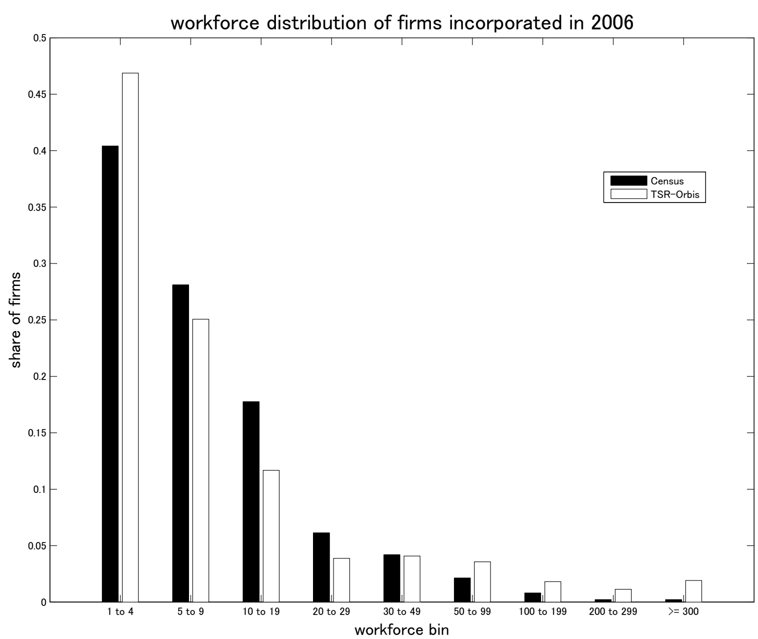

Figure 4: Workforce distribution, Orbis-tsr versus Census

1.2. Empirical findings

This section documents empirical relationships between firm borrowing and firm productivity using balance sheet information from TSR[13]. Table 4 displays the data variables we use in the benchmark regressions. We do not have material costs in the data so we use industry level material shares to impute a value added measure from the operating revenue of the firm. The industry material shares come from JIP Database 2013 created by RIETI, Japanese government METI's main research branch. It has 108 sectors which we match to the TSR-Orbis database using the official JIP-ISIC concordance table. We use the average material share over 2000-2010. While there are likely to be intra-industry differences in in factor shares, using industry shares reduces the dispersion in measured productivity due to firm level measurment error in cost shares. Syverson (2004) for example uses both industry averages and and plant-level shares in his benchmark results.

Variable	Notation	Data item
capital	k	book value of capital stock (total asset)
value added	y	operating revenue $\times$ (1 - factor share of materials)
inside equity	a	*shihonkin* or shareholders fund
labor	l	number of employees

Table 4: Map of variables used in the empirical analysis to balance sheet items

We calculate firms productivity using a decreasing-returns-to-scale Cobb-Douglas production function which will be later embedded in our general equilibrium model. More specifically, productivity z of a firms is measured by

$$\ln z = \ln y - \eta\alpha \ln k - \eta(1-\alpha)\ln l$$

[13]Definitions of variables can be found at Orbis glossary. We also provide a table with the definition of the key variables in the Appendix.

where the scale parameter $\eta = 0.85$ is taken from Midrigan and Xu (2014) and capital intensity α is calculated from the JIP database in the same manner as the material shares.

The key to our empirical strategy is controlling for a, inside equity. To this end, we use an item in Japanese firms' balance sheet called *shihonkin* as the proxy for inside equity. *shihonkin* is also called stated/share/legal/paid-in capital. The institution of *shihonkin* existed in the U.S. until the late 1950's and is still prevalent in many European countries and China. In Japan, the legal definition of *shihonkin* is the "the amount of properties contributed by persons who become shareholders at the incorporation or share issued" (Article 445 of the Companies Act). In practice, it is seen as the founder's own stake in the firm at the time of incorporation (e.g. see Ministry of Economy, Trade and Industry SME Agency advisory website). In the appendix of the paper, we layout institutional and empirical justifications for using *shihonkin*. We also report regression results using shareholders fund instead to proxy for inside equity.

Figure 5 and 6 display the main empirical patterns we will use to discipline the model. These are local linear regressions of log leverage $\ln \frac{k}{a}$ and log output-to capital ratio $\ln \frac{y}{k}$ on industry fixed effects at the NAICS 6-digit level, log productivity and log inside equity. That is, the regression equations are

$$\ln \frac{y_i}{k_i} = \text{Industry FE} + \theta_1 \ln z_i + \theta_2 \ln a_i$$

$$\ln \frac{k_i}{a_i} = \text{Industry FE} + \nu_1 \ln z_i + \nu_2 \ln a_i$$

We run this regression within cohort-year. The figures display the result for the 2006 cohort in 2011, when they are five years old. We find that both firm leverage and output-to-capital ratio rise strongly with firm productivity. Further more, the pattern appears linear. We obtain similar results for other cohort-years (see Figures 9 to 12 in the Appendix).

We also run an OLS regression of the above form. Column (1) in Table 5

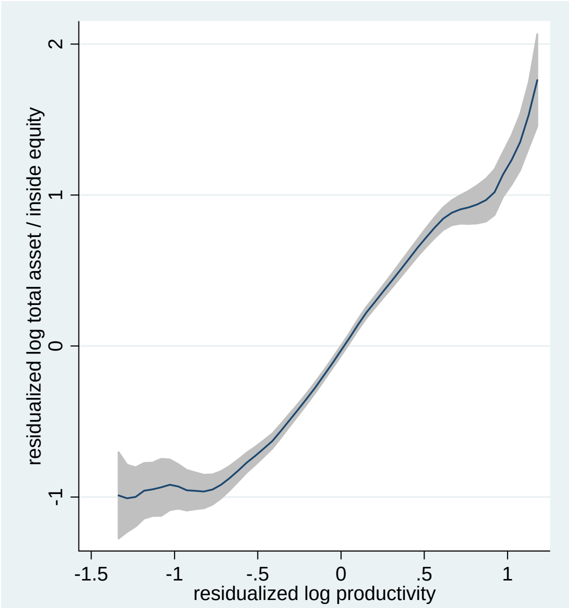

Figure 5: Firm leverage rises with firm productivity

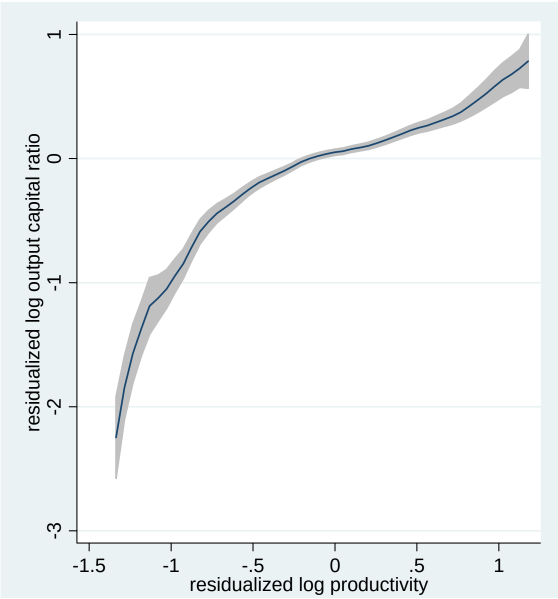

Figure 6: Firm output-capital ratio rises with firm productivity

shows the coefficient on log productivity for each dependent variable. It shows that leverage rises close to one-for-one with productivity while the elasticity of output-to-capital ratio with respect to productivity is approximately 0.7.

Columns (2) to (6) display robustness checks for the regression results. Column (2) displays the regression coefficients on log productivity after including the quadratic term $(\ln z)^2$ in the regression. This is to control for rising leverage and output-to-capital due to a combination of constrained and unconstrained firms. The coefficients of interest do not change much. This is not surprising given that in our non-parameteric regression, the relationship appears linear.

In the third column, we use shareholders fund (total asset - total debt) instead of *shihonkin* to proxy for inside equity. In TSR-Orbis, total shareholders fund is the sum of *shihonkin* and all other shareholders funds not linked to *shihonkin* such as reserve capital, undistributed profit, include also minority interests if any. This is to address the concern that firms may have other unobservable inside stake that positively correlate with firm productivity. For example, suppose the true inside equity held by the firm is $a^* = \hat{a} + \epsilon$ where $\hat{a}$ is our proxy *shihonkin*. Then our regression equation for leverage on productivity becomes

$$\ln \frac{k}{\hat{a}} = \nu_0 + \nu_1 \ln z + \ln \left(1 + \frac{\epsilon}{a}\right)$$

So if more productive firms have more unobserved inside equity, we would infer a positive on productivity in the leverage regression even if the true ν_1 is zero. We find that using this alternative measure of inside fund does not change our benchmark regression results.[14]

[14]Of course, the shareholders fund may not capture all of the unobserved inside equity. For example, Berger and Udell (1998) and Robb and Robinson (2014) find that U.S. young and startups firms have significant personal loan guarantees and collaterals such as home equity from owners that do not appear on the balance sheet. Anecdotes suggest this is also true in Japan. Hence an alternative hypothesis is that leverage capacity is constant with respect to productivity but the off balance sheet inside equity is positively correlated with productivity. We do not have data to form a strong test against this case. However, it is likely that as a firm expands with age, the off balance sheet equity share of total equity declines (e.g. Berger and Udell (1998) find older firms accumulate retained earnings). This means that if the alternative hypothesis is true, when we use shareholders fund as the proxy for inside equity, we should expect to see a higher estimate of ν for younger firms. In contrast, as shown in Figure 16, the

The fourth column uses fixed assets instead of total assets to proxy for capital. The coefficient on productivity in the leverage regression drops from 1.125 to 0.346 while the coefficient on output-to-capital ratio doubles. We do not use this as our benchmark results because the firms in our data are small young firms that do not have a lot of financial assets unrelated to production. Working capital and trade credits make up the bulk of non-fixed asset. These items should be counted towards firm borrowing used in their main production.

In column (5) we run our regression with firm fixed effects to control for unobserved cross-section variation in firms. We find that even within firms, one percent increase in productivity is associated with 0.5% increase in leverage and slightly over 1% increase in the output-to-capital ratio.

Finally, one may be concerned that the positive relationship between leverage and output-to-capital ratio with productivity is driven by measurement error in capital and output that affects both measured productivity and the dependent variables. Assuming the employment is well measured and is uncorrelated with the measurement error in capital and output but is correlated with firm productivity, we apply 2SLS using labor as an instrument. We find that while the elasticity of leverage with respect to productivity drops to 0.2 while the elasticity of output-to-capital ratio with respect to productivity rises to 3.2.

1.3. Capital share variation

In the DRTS model with common leverage, all else constant, firms with higher capital share has lower marginal revenue product of capital. The inference of productivity depends on the assumed capital share. Hence our regression results could be due to correlation between capital share variation and infered firm productivity. We check for this alternative explanation by running our regressions on data simulated from a DRTS model with a common leverage. We randomly draw productivty z from $LN(0, 1)$ and capital share α from $U(a, b)$

estimated ν is smaller for age 1 firms[15]. So it is unlikely that the alternative hypothesis is driving the positive correlation between leverage and productivity.

Dep. Var	(1) linear	(2) quad.	(3) diff a	(4) diff k	(5) firm FE	(6) 2SLS
leverage	1.125	1.120	0.973	0.346	0.489	0.207
	(0.039)	(0.050)	(0.040)	(0.051)	(0.034)	(0.048)
output-	0.690	0.598	0.751	1.316	1.126	3.205
capital	(0.028)	(0.035)	(0.032)	(0.039)	(0.034)	(0.684)
N	5872	5872	5872	5870	21962	5872

NAICS 6-digit industry FE. Control for log inside fund. 2SLS use employment to instrument for productivity. 2006 cohort. Age 5 except for (5). Similar results for other year-cohort.

Table 5: Regression coefficient on log productivity

with $z \perp a$. We then calculate $y, l, k, \ln \frac{y}{k}$ using the DRTS model assuming firms are not constrained and infer productivity assuming capital share is $\hat{\alpha} = 0.33$ by

$$\ln \hat{z} = \frac{1}{\hat{\alpha}} \left[\frac{\ln y}{\eta} - \hat{\alpha} \ln k - (1 - \hat{\alpha}) \ln l \right].$$

Regressing the simulated $\ln \frac{y}{k}$ against $\ln \hat{z}$ shows that $\ln \frac{y}{k}$ is not positively correlated with $\ln \hat{z}$. This suggests that capital share variation that is uncorrelated with productivity does not explain our regression results. We also conducted the exercise for capital share variation that is correlated with z. We find that while correlation between capital share and productivity can generate leverage and capital revenue product both rising with productivity, the model implies high revenue firms have lower capital share. This is contrary to the common prior that larger firms are more capital intensive. Hence, we rule out capital share variation as an alternative hypothesis.

1.4. Adjustment cost of capital

In our empirical strategy, we assume higher capital revenue product firms are more constrained. An alternative explanation is that these firms are not financially constrained but face higher adjustment cost of capital. However, as shown in Figure 7, growth rate of shareholders fund between t and $t+1$ increases with log output-to-capital ratio at t for each cohort at age 2[16]. The regression we ran is

$$\ln \frac{sf_{t+1}}{sf_t} = \theta_0 + \theta_1 \ln \frac{y_t}{k_t} + \theta_2 \ln \frac{z_{t+1}}{z_t} + \theta_3 \ln a_t$$

which controls for productivity growth rate and current period internal funds.

In the model in the next section, this pattern is consistent with higher output-capital ratio indicating more financially constrained. Firms with higher output-to-capital ratio are firms that are more productive relative to their borrowing capacity. Their propensity to save (savings as a share of wealth) is higher because these firms earn an excess return. So if higher output-to-capital ratio is driven mostly by higher productivity, we should see firms with higher output-to-capital ratio today having higher growth rate in *shihonkin* or shareholders fund. A pure adjustment cost theory without a distinction between internal and external funds can not explain why firms with higher output-to-capital ratio accumulate internal funds more quickly.

2. Model

In this section we layout a commonly used model of aggregate productivity loss due to financial frictions to illustrate the implications of our findings. We parameterize the model in two ways. In one version, we use indirect inference to estimate the model using the regression coefficients in our empirical analysis. In another version we take the common approach in the literature

[16]We use age 2 because firms are still young but there are more observations than age 1. We also did the same regression for other years and found similar results.

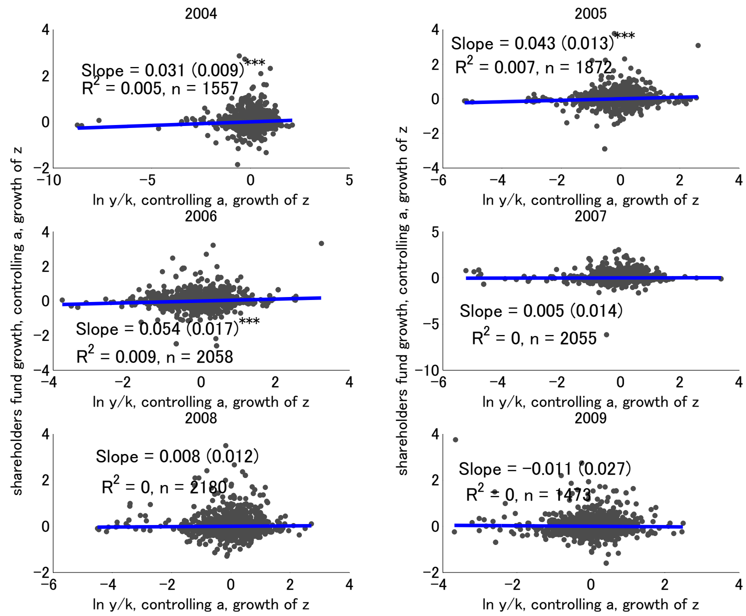

Figure 7: Growth rate of shareholders fund and $\frac{y}{k}$

and assume firms borrowing capacity does not vary with firm productivity. We choose ϕ_k to match the average leverage in the first version to make the two versions comparable. We compare TFP loss relative to the first best in the two scenarios and find that assuming a constant leverage capacity overstates the TFP loss by almost 30% relative to the benchmark parameterization that fits the regression coefficients.

2.1. Model

Consider an economy populated with a continuum of infinitely lived entrepreneurs born with wealth a_0 and productivity z_0 drawn from distribution $G(a, z)$. Each entrepreneur's productivity post birth is governed by an AR(1) process with autocorrelation parameter ρ and iid normal innovation shock with mean μ_e and standard deviation σ_e^2. That is

$$\ln z' = \rho \ln z + \epsilon, \quad \epsilon \overset{iid}{\sim} N(\mu_e, \sigma_e^2)$$

The economy also has L measure of hand-to-mouth workers each supplying one unit of labor. Entrepreneurs can save and can choose between inactive versus active. Inactive entrepreneurs rent capital to the active entrepreneurs.

In each period, entrepreneurs have access to a Cobb-Douglas production technology with capital share α and decreasing-returns-to-scale η[17]

$$y = f(z, k, l) = z(k^{\alpha} l^{1-\alpha})^{\eta}$$

When $\eta = 1$, firms have constant-returns-to-scale technology. We assume each entrepreneur can operate only one business in order for the distribution of firms to be well defined. This serves the same purpose as assuming a fixed cost of production.

The entrepreneur decides capital and labor inputs after seeing her productivity to maximize her profit.

$$\pi(a, z) := \max_{k,l} f(z, k, l) - (r + \delta)k - wl, \quad k \leq \bar{k}(a, z)$$

$\bar{k}(a, z)$ denotes the maximum amount of capital the entrepreneur can raise. We call $\bar{k}(a, z)/a$ the leverage capacity of the entrepreneur[18].

The entrepreneur's networth consists of her current period profit and her saving $a(1 + r)$. She decides her consumption and savings policy functions to maximize her lifetime utility. The entrepreneur's problem can be written recursively as

$$V(a, z) = \max_{a',c} u(c) + \beta\, \mathbb{E}\left[V(a', z')|z\right]$$

subject to

$$c + a' \leq a(1 + r) + \pi(a, z)$$

[17]We can also model firms as having constant-returns-to-scale technology but face CES demand.

[18]Here we abstract from other dimensions of entrepreneur's problem such as heterogeneity in preferences that may lead to an overstatement of the cost of financial frictions. For example, Hurst and Pugsley (2011) find small businesses choosing to stay small due to owner's preferences and Strebulaev and Yang (2013) find family firms tend to use zero debt.

Let $\underline{z}$ denotes the productivity level of the least productive firm among the active firms. When $\eta < 1$, all firms are active so $\underline{z}$ is lower support of the exogenous productivity distribution. When $\eta = 1$, some firms with productivity level above the lower support may be inactive. There are two markets that need to clear at the equilibrium. First, capital market clearing requires aggregate capital demand equating total wealth in the economy

$$K := \int_{a,z\geq\underline{z}} k(a,z)dG(a,z) = \int_{a,z} adG(a,z) =: A.$$

This condition can be equivalently stated as requiring aggregate leverage to be equal to the wealth-weighted average leverage of active entrepreneurs

$$\frac{A}{A^e} = \int_{a,z\geq\underline{z}} \lambda(z)\frac{adG(a,z)}{A^e}, \quad A^e = \int_{a,z\geq\underline{z}} adG(a,z).$$

The labor market clearing condition is

$$L = \int_{a,z\geq\underline{z}} \left(\frac{\pi}{\alpha}\right)^{\frac{1}{1-\alpha}} z\lambda(z)adG(a,z).$$

Furthermore, at the equilibrium, the evolution of wealth and productivity distribution must be consistent with the law of motion of firm productivity and firm policy functions. That is

$$G(a',z') = \int_{a,z} prob(z'|z)1\{a' = savings(a,z)\}dG(a,z)$$

Equilibrium definition: A stationary competitive equilibrium consists of labor demand $l(a,z)$, capital demand $k(a,z)$, productivity cutoff level $\underline{z}$, savings policy, interest rate and wage, wealth and productivity distribution $G(a,z)$ such that

1. given prices, $l(a,z)$, $k(a,z)$, $\underline{z}$ and savings policy solve the entrepreneur's problem

2. capital market and labor market clear

3. $G(a, z)$ is consistent with the savings policy and the law of motion of z

In this economy, aggregate output is given by

$$Y = \int_a \int_z y(a, z) dG(a, z)$$

and we define aggregate productivity, TFP, as

$$Z := \frac{Y}{(K^\alpha L^{1-\alpha})^\eta}$$

The variation of aggregate TFP with exogenous parameters governing leverage capacity $\bar{k}(a, z)/a$ is interpreted as the impact of financial frictions on aggregate productivity.

2.2. Model of borrowing capacity

A common[19] assumption used in the literature that quantifies aggregate TFP loss due to financial frictions is that leverage capacity is constant, i.e.,

$$k \leq \lambda a$$

Here we use a parsimonous model of borrowing capacity from Buera et al. (2011) where financial frictions arise from limited enforcement of contracts. If the entrepreneur defaults, she can keep $1 - \phi_y$ fraction of revenue and $1 - \phi_k$ fraction of depreciated capital but lose all her wealth $a(1 + r)$. It is assumed that entrepreneurs can use the financial market after one period without further penalties[20]. Borrowing capacity is the maximum capital $\bar{k}(a, z)$ that satisfies the

[19] E.g. Kiyotaki and Moore (1997), Midrigan and Xu (2014), Moll (2014), Buera and Shin (2013).

[20] An alternative way to introduce a relationship between borrowing capacity and productivity is through the persistence of productivity shock and prolonged autarky, e.g., as in Arellano et al. (2012). Also, we are assuming that firms do not have access to equity or bond markets. This assumption is likely to be innocuous for our data sample because the firms in our data are

following incentive compatibility constraint so that there is no default in the equilibrium

$$\phi_y \max_l \{f(z,k,l) - wl\} + (1+r)a \geq (r+\delta)k + (1-\phi_k)(1-\delta)k$$

When $\phi_y = 0$, the borrowing limit reduces to the constant one when $\lambda = \frac{1+r}{R+(1-\phi_k)(1-\delta)}$. Another common approach assumes $\phi_y = \phi_k$ such as Buera et al (2011, 2014). In both approaches, ϕ_k is chosen to match some measure of aggregate external financing such as the aggregate debt-to-GDP ratio.

One potential shortcoming of the common approach is that it may overstate the aggregate productivity loss due to financial frictions. In this model, financial frictions reduce TFP by introducing misallocation of capital where entrepreneurs who are more productive relative to their wealth can not raise sufficient external funds to meet their profit maximizing capital demand. They can not raise sufficient funds either because they cannot pledge all of their wealth or all of their revenue. However, if entrepreneurs with low wealth are able to pledge their current and future revenue, they are in essence borrowing against their productivity which ameliorates the extent of misallocation due to financial frictions.

To illustrate this idea, let us assume $\eta = 1$, entrepreneurs have log utility and the borrowing capacity can be written as $\bar{k}(a,z) = \lambda(z)a$. As shown in Moll (2014), these assumptions yield an analytical expression of aggregate productivity where TFP is just the capital-share weighted average of all active entrepreneur's productivity. We can further decompose that expression for TFP into the following terms

$$TFP^{\frac{1}{\alpha}} = Cov_\omega[z^{\frac{1}{\alpha}}, \lambda(z)|z \geq \underline{z}](1 - D/K) + \mathbb{E}_\omega\left[z^{\frac{1}{\alpha}}|z \geq \underline{z}\right]$$

where $\omega(z)$ denotes the wealth share of entrepreneurs with productivity z, $\underline{z}$ is

small. There are high fixed costs for accessing the bond and equity markets. Small firms may optimally choose to use only bank financing. See Russ and Valderrama (2009).

the minimum productivity of active entrepreneurs and D/K is the aggregate debt-to-capital ratio. The $\omega(z)$ reflects the effect of wealth accumulation by productivity types on the TFP and the $\underline{z}$ term reflects competition for capital that changes the break even productivity. Holding these general equilibrium forces fixed, as in the case of IID productivity shocks and calibrating to D/K in the data, higher covariance between leverage capacity and productivity has a direct positive impact on TFP. That is, two model economies with the same aggregate financing (D/K) can have different aggregate productivity from differences in the covariance between borrowing capacity and productivity z. Assuming firm leverage capacity is constant with respect to firm productivity and calibrating the model to aggregate external financing leads to lower model TFP. Since aggregate TFP loss due to financial frictions is often measured as the difference between the model TFP and the first best TFP, which is unaffected by parameters governing finanical frictions, assuming a constant leverage capacity ultimately implies an overstatement of the TFP loss due to financial frictions.

3. Parameterization

In this section we first describe how we solve the model given parameters. Then, we layout our strategy for choosing the parameters. In short, we use indirect inference to choose ϕ_k and ϕ_y that best match the empirical relationship between firm leverage, output-to-capital ratio and firm productivity. We calibrate the remaining parameters to firm level data and common values used in the literature.

3.1. Solving the model

Given parameters, we find the stationary equilibrium by first computing all equilibrium objects given guesses of the interest rate and the wage. This involves solving the entrepreneur's dynamic programming problem and calculating the stationary joint distribution of asset and productivity that is consistent with the entrepreneur's optimal savings decision. We then apply bisection methods to find the pair of prices that satisfies the capital and labor market clearing conditions (See Appendix for details of computation methods).

The entrepreneur's production decision is straightforward to derive. All entrepreneurs choose to produce because there are no fixed costs and the marginal return to producing is infinite at zero production. Conditional on producing, the entrepreneur's factor demand can be derived using standard first-order conditions. Given a particular capital input level, the entrepreneur chooses labor so that the marginal product of labor equals the marginal cost of labor w. This yields a profit function that is concave in capital. Then, the entrepreneur chooses her capital level subject to the borrowing constraint. Without the borrowing constraint, the entrepreneur chooses the capital level which equate the marginal increase in profit with the marginal cost of capital, $r + \delta$. The optimal level is higher for more productive entrepreneurs because they have higher marginal returns to capital. More specifically, the unconstrained optimal capital demand increases with productivity with an elasticity of $\frac{1}{1-\eta}$ and at this optimal level, the marginal product of capital equals $r + \delta$:

$$k^u(a,z) \propto z^{\frac{1}{1-\eta}} \qquad \eta\alpha\frac{y^u(a,z)}{k^u(a,z)} = r + \delta.$$

The entrepreneur can choose the unconstrained optimal capital level only if it is below her borrowing capacity $\bar{k}(a,z)$. If it exceeds her maximum borrowing capacity, then she will hit her borrowing limit because, below this level, profit is monotonically increasing in capital. That is, the entrepreneur's optimal capital demand is the smaller of $k^u(a,z)$ and $\bar{k}(a,z)$. When the entrepreneur is constrained, her marginal product of capital exceeds the marginal cost of capital. The size of the gap, or the excess return to capital, is higher for entrepreneurs with higher productivity relative to their maximum borrowing limit. That is, the constrained entrepreneurs have capital demand and output-to-capital ratio

$$k^c(a,z) = \bar{k}(a,z) \qquad \eta\alpha\frac{y^c(a,z)}{k^c(a,z)} = r + \delta + \mu(a,z) \propto \left(\frac{z}{\bar{k}(a,z)^{1-\eta}}\right)^{\frac{1}{1-(1-\alpha)\eta}}$$

In what follows, we say an entrepreneur is more financially constrained than another when her excess return $\mu(a,z)$ is higher.

Solving the entrepreneur's savings decision is less straightforward as there are no analytical solutions to the dynamic programming problem. We solve the problem using fitted value function iteration with linear interpolation. We choose this method because it has been shown to be a contraction mapping for a general class of income processes including the one in the current model (see Stachurski (2008)). Having a globally convergent method that works over a wide range of parameter space is essential for our estimation procedure. To implement fitted value iteration, we need to discretize the state space of productivity and asset. For assets, we choose a grid that assigns more points to the lower end of assets where there is more curvature in the value function. For productivity, we use the Rouwenhorst method which Kopecky and Suen (2010) found to be significantly more accurate than Tauchen's method for calculating aggregates. Given the entrepreneur's policy function, we need to find the stationary joint distribution of asset and productivity that is consistent with the entrepreneur's savings decisions. There are several methods for calculating the distribution from given policy functions. We choose the forward iteration technique proposed by Young (2010) because it has been shown to be effective for solving similar models (see Haan et al. (2010)). After finding the stationary equilibrium, we can calculate the aggregate demand and supply of labor and capital. We repeat the above procedures for different guesses of interest rate and wage rate until we find a pair of prices where labor and capital demand equals supply.

3.2. Parameterization

We parameterize the model by calibrating ϕ_k, ϕ_y and σ to the firm level data and setting the remaining parameters to values commonly used in the literature. Table 6 display the values we used in our benchmark calibration. Following Midrigan and Xu (2014), we set the scale parameter of production, η, to be 0.85 and the capital intensity parameter, α, to be 0.33. Moll (2014) which shows that aggregate productivity loss due to financial frictions at the steady state decreases with the persistence of the productivity shocks. We choose a highly persistent process that is consistent with the literature. Finally, we choose σ to match the 90/10 ratio of log productivity in our firm level data.

Parameter	Description	Value	Source
η	returns-to-scale	0.85	Midrigan and Xu (2014)
α	capital intensity	0.33	Midrigan and Xu (2014)
ρ	productivity persistence	0.95	Moll (2014)
σ	productivity dispersion	0.627	90/10 ratio of productivity

Table 6: Pre-set parameters

We use indirect inference (Smith (1993) and Gourieroux et al. (1993)) to choose ϕ_k, ϕ_y. The data objects we choose to match are the regression coefficients of leverage, output-to-capital on moments of productivity and inside equity. Namely

$$\ln \frac{y}{k} = \beta_0 + \beta_1 \ln z + \beta_2 \ln z^2 + \beta_3 \ln a$$

$$\ln \frac{k}{a} = \theta_0 + \theta_1 \ln z + \theta_2 \ln z^2 + \theta_3 \ln a$$

We choose these data objects because they speak directly to the financial constraint and borrowing capacity in the model. Table 7 illustrates why the regression coefficients are informative of ϕ_y and ϕ_k. When ϕ_k is very high, many firms are unconstrained and capital rises with productivity after controlling for inside

	Leverage	Output-capital
high ϕ_k	+	0
low ϕ_k, low ϕ_y	0	+
low ϕ_k, high ϕ_y	+	+

Table 7: Identification

fund due to the increase in capital demand with productivity. On the other hand, since many firms are unconstrained, output-to-capital ratio is flat with respect to productivity. When ϕ_k is low but ϕ_y is zero, most firms are financially constrained and leverage does not vary with productivity after controlling for inside equity. However, the output-to-capital ratio rise strongly with productivity because the firms with high productivity relative to their inside equity are more constrained. In order to match both rising leverage and output-to-capital ratio with firm productivity, the model needs a positive ϕ_y and low ϕ_k.

We choose ϕ_y and ϕ_k to minimize the distance between regression coefficients from the model simulated data and that from the actual data where distance is defined using weighting matrix Ω. That is, the parameter estimates are defined by

$$[\hat{\phi}_y, \hat{\phi}_k] := \arg\min_{\phi_y,\phi_k} \quad ([\beta,\theta]-[\hat{\beta},\hat{\theta}])\Omega([\beta,\theta]-[\hat{\beta},\hat{\theta}])^T$$

Here, β and θ denote the vector of regression coefficients $[\beta_1,\beta_2,\beta_3]$ and $[\theta_1,\theta_2,\theta_3]$ from the simulated data. The value of these depends on parameters ϕ_y and ϕ_k. The hatted versions denote the regression coefficients from the empirical data. For the choice of Ω, we use equal weighting (Ω equals the identity matrix) as the benchmark and report the results with OLS robust standard error variance -covariance matrix ($\Omega := [X'X]^{-1}\hat{\epsilon}\hat{\epsilon}'[X'X]^{-1}$) as a robustness check. The latter is the efficient weighting matrix if the linear model is correctly specified[21].

[21]The regression we run on the actual data also controls for industry fixed effects which is not in the simulated regressions. We construct the OLS weighting matrix by first carrying out partial

We find the optimal values $[\hat{\phi}_y, \hat{\phi}_k]$ in the following way. First, we recover the coefficients from the actual data. Let n denote the sample size. Then, for a given pair of ϕ_k and ϕ_y, we solve for the stationary equilibrium using numerical methods described in the previous subsections. We then simulate Hn samples of $\{a_i, z_i, k_i, y_i\}$ from the stationary equilibrium. To simulate one sample, we following Hn households for T periods, keeping only the last period. Ergodic properties of the model ensure that when T is large, simulation from the last period well approximates simulation from the stationary distribution (Braun et al. (2012)). We set T to 500. We then run the two regressions on the simulated data. As shown in Gourieroux et al. (1993), this is asymptotically equivalent to simulating H samples of histories of size n. We then calculate the distance from the empirical coefficients by the aforementioned criterion. We repeat this exercise for the combinations of ϕ_y and ϕ_k over $[0, 1]$ with 0.1 increments. To avoid the cluttering problem in simulated estimation methods, we use the same draws of z for every pair of $[\phi_y, \phi_k]$.

For the equal weighting scheme, the optimal parameters are $\hat{\phi}_k = 0.2$, $\hat{\phi}_y = 0.5$ which yields an aggregate debt-to-capital ratio of $D/K = 0.27$ which corresponds to a leverage equal to 1.35. For the OLS weighting scheme, the resulting values are $\hat{\phi}_k = 0.2$, $\hat{\phi}_y = 0.6$ which yields an aggregate debt-to-capital ratio of $D/K = 0.284$. The difference between the two results is due to the OLS weighting scheme putting more weight on the curvature parameters[22]. We also parameterize the model using the two common approaches in the literature: one where we set $\phi_y = 0$ and another we set $\phi_y = \phi_k$. In both approaches, to make them comparable to our benchmark parameterization, we choose ϕ_k

regression to remove the industry fixed effects from the data regression. More specifically, we regress the dependent variable and each independent variable on industry dummies and use the residuals as dependent and independent variables. The resulting coefficients are numerically identical to running a direct regression with the industry dummies. We apply the OLS robust standard error weighting matrix formula to the regression results with the residualized variables to construct the weighting matrix for in the robustness check.

[22]One interesting observation is the low plegeability of asset. This could be due a large share of assets held by firms in our data are working capital, which may be difficult for lenders to recover. For example, for the 2006 cohort at age 5, the average share of total assets that is fixed assets is 27% while the median share is only 17%.

to match the D/K ratio in our benchmark parameterization. Table 8 displays the chosen values in the three approaches and as well as resulting the model and data coefficents at these values. The specification that performs the worst is when $\phi_y = 0$. Imposing the restriction $\phi_k = \phi_y$ brings the model closer to our empirical findings. This suggests that when only aggregate moments are available, it is better to use the $\phi_k = \phi_y$ model instead of the $\phi_y = 0$ model. Overall, the model fails to match the lack of curvature in the data. This suggests that alternative models of borrowing capacity may be needed.

4. Counterfactuals

4.1. TFP loss relative to the first best

The first best aggregate productivity in the model is given by

$$Z^{fb} := \left[\,\mathbb{E}_z\, z^{\frac{1}{1-\eta}}\right]^{1-\eta}$$

where the expectation is taken with respect to the stationary distribution of productivity $LN(\mu, \sigma^2)$. It is characterized by the non-dependence on the wealth distribution. We compute TFP loss due to financial frictions by the gap between the model TFP and the first best TFP as a percentage of the model TFP

$$\text{loss} := \frac{Z^{fb} - Z}{Z}.$$

Table 10 displays the aggregate productivity loss for each parameterization approach. It shows that compared to the benchmark parameterization, assuming leverage capacity is constant leads to almost 30% smaller productivity loss due to financial frictions. The difference is smaller when compared to the case of assuming $\phi_y = \phi_k$. These results suggest that it is quantitatively important to model borrowing capacity rising with firm productivity.

Regression	benchmark $\phi_y = 0.5$, $\phi_k = 0.2$	$\phi_y = 0$ $\phi_y = 0$, $\phi_k = 0.3$	$\phi_y = \phi_k$ $\phi_y = 0.3$, $\phi_k = 0.3$	Data (OLS s.e.)
Dep var = $\ln \frac{k}{a}$				
$\ln z$	2.297	1.710	2.060	1.120
				(0.050)
$\ln a$	-0.351	-0.318	-0.317	-0.512
				(0.018)
$(\ln z)^2$	-0.852	-1.253	-1.021	-0.002
				(0.013)
Dep var =$\ln \frac{y}{k}$				
$\ln z$	1.523	1.727	1.605	0.598
				(0.035)
$\ln a$	-0.226	-0.238	-0.238	-0.206
				(0.010)
$(\ln z)^2$	0.297	0.437	0.356	-0.029
				(0.008)
D/K	0.266	0.236	0.303	
Distance	0.516	0.574	0.520	
TFP loss	10.7%	14.6%	11.4%	

Table 8: Quality of fit of estimated key parameters versus common approaches in the literature, equal weighting

Regression	benchmark $\phi_y = 0.6$, $\phi_k = 0.2$	$\phi_y = 0$ $\phi_y = 0$, $\phi_k = 0.4$	$\phi_y = \phi_k$ $\phi_y = 0.3$, $\phi_k = 0.3$	Data (OLS s.e.)
Dep var = $\ln \frac{k}{a}$				
$\ln z$	2.489	1.823	2.060	1.120
				(0.050)
$\ln a$	-0.379	-0.341	-0.317	-0.512
				(0.018)
$(\ln z)^2$	-0.789	-1.323	-1.021	-0.002
				(0.013)
Dep var =$\ln \frac{y}{k}$				
$\ln z$	1.456	1.688	1.605	0.598
				(0.035)
$\ln a$	-0.275	-0.230	-0.238	-0.206
				(0.010)
$(\ln z)^2$	-0.216	0.461	0.356	-0.029
				(0.008)
D/K	0.266	0.316	0.303	
Distance	0.368	0.683	0.474	
TFP loss	10.1%	13.7%	11.4%	

Table 9: Quality of fit of estimated key parameters versus common approaches in the literature, OLS weighting

$\phi_k = 0.2$, $\phi_y = 0.5$	$\phi_y = 0$, $\phi_k = 0.3$	$\phi_y = \phi_k = 0.3$
10.7%	14.6 %	11.4%

Table 10: TFP loss relative to the first best, equal weighting

$\phi_k = 0.2$, $\phi_y = 0.6$	$\phi_y = 0$, $\phi_k = 0.4$	$\phi_y = \phi_k = 0.3$
10.1%	13.7%	11.4%

Table 11: TFP loss relative to the first best, OLS weighting

5. Conclusion

Using private firm data from Japan we documented that both leverage and output-to-capital ratio increase with measured productivity in a way that is inconsistent with the decreasing returns to scale model with a common leverage capacity that is often used in quantitative studies of the impact of financial frictions on TFP. We showed that allowing firms to pledge current and future revenue is more consistent with these empirical patterns and that ignoring this heterogeneity in leverage capacity can lead to an economically significant overstatement of the loss in TFP due to financial frications. We leave documenting these facts for other countries and studying the implications of our findings for the impact of financial frictions on endogenous growth and business cycles for future research.

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A Data

A1. Definition of terms from Orbis

Can also be found at Orbis glossary.

Table 12: Glossary of terms from TSR-Orbis

Balance Sheet			
BvD Code	Label	Formula	Definition

FIAS	Fixed Assets	IFAS + TFAS + OFAS	Total amount (after depreciation) of non current assets (Intangible assets + Tangible assets + Other fixed assets).
CUAS	Current Assets	STOK + DEBT + OCAS	Total amount of current assets (Stocks + Debtors + Other current assets).
TOAS	Total Assets	FIAS + CUAS	Total assets (Fixed assets + Current assets)
SHFD	Shareholders Funds	CAPI + OSFD	Total equity (Capital + Other shareholders funds)
CAPI	Capital		Issued Share capital (Authorized capital).
OSFD	Other Shareholders Funds		All Shareholders funds not linked with the Issued capital such as Reserve capital, Undistributed profit, include also Minority interests if any.
EMPL	Number of Employees		Total number of employees included in the company's payroll
Profit & Loss Account			
OPRE	Operating revenue (Turnover)		Total operating revenues (Net sales + Other operating revenues+ Stock variations). The figures do not include VAT. Local differences may occur regarding excises taxes and similar obligatory payments for specific market of tobacco and alcoholic beverage industries.

A2. *shihonkin* as a proxy for founders' stake

In this section, we provide instituitional as well as empirical justifications for using *shihonkin* to proxy for founders equity or inside equity. More details on *shihonkin* can be found in Li and Satoshi (2014).

A2.1. Institutional support

shihonkin, also called stated/share/legal/paid-in capital, is legally defined as the amount of properties contributed by persons who become shareholders at the incorporation or share issued (Article 445 of the Companies Act). While a straight reading of the law says that half of a firms initial equity is *shihonkin*, firms appear to have more flexibility in setting the level of *shihonkin*. For example, J-NET 21, a government website providing advice to entrepreneurs setting up small businesses, advices founders that not all of the initial financing needs to be put down as *shihonkin* because the initial financing can also be entered as a loan from the company head. It observes that founders tend to register more *shinhonkin* if the business requires fixed investment and operating finances while founders who only wants to incorporate would put in less. Another article on the same website also says that *shihonkin* is "a measure of trust". *shihonkin* is seen as the entrepreneurs "own skin" in the firm and is the minimum recoverable amount for creditors. Furthermore, the Ministry of Economy, Trade and Industry SME Agency website on changes in the corporate law in 2006 advice new firms to decide *shihonkin* based on financing needs and does not even mention the need to register at least half of their initial financing as *shihonkin*[23].

shihonkin is registered at the Ministry of Justice at the time of incorporation. To incorporate a firm, the founder must provide a *shihonkin* level and show evidences that deposits/physical production inputs of the declared amount is put into the firm. In the case of cash, evidence is a special bank deposit certificate. In the case of physical assets such as buildings or land, the firm needs to receive evaluation by an approved third party. Once the evidences are approved, the Ministry of Justice discloses the registered *shihonkin* in its public registry that can be accessed by anyone at the municiple registry. Changes to the *shihonkin* are updated at the Ministry of Justice. So *shihonkin* is not just a number the firm writes down but actually reflects the value of contribution by its founders.

Registering *shihonkin* carries at least two costs. First, the firms pay 0.7% tax on the amount it registers[25]. For example, if a firm registers ¥1000 at time of incorporation and then increases that by ¥1000 after incorporation, it pays a total of ¥14 in tax. Second, the level

[23]Existing companies have less flexibility in determining the fraction of new share issuance that goes into *shihonkin* Articles 199 to 213 defines the process for existing firms to increase equity through share subscription. In case new stock is issued, the subscription price needs to be calculated "fairly" so not to be disadvantagous to existing shareholders. Also, when the actual price paid for the shares are less than the market value of the shares at the time of issuance, the buyer may need to pay a gift tax[24]. Hence, while the actual contribution can differ from the issuance value due to differences in market valuation and firms issuance value, firms do not appear to have a large margin of control over the total value of the equity subject to *shihonkin* registration. Nonetheless, there are firms that put more than half of the new issuance as *shihonkin*. The justification for this regulation is that general shareholders are external financiers. They are more like creditors than owners of the firms and need protection from managers diluting the value of the firm. However as our focus is young firms, we do not delve more into this aspect of *shihonkin*.

[25]National Tax Agency tax schedule (in Japanese): https://www.nta.go.jp/taxanswer/inshi/7191.htm

of registered *shihonkin* sets a lower bound on the networth needed to pay dividend[26]. For example if a firm has ¥3 million in *shihonkin*, it could not pay dividend to its shareholders unless its networth (total asset - debt - *shihonkin* - capital reserves). While debt and capital reserves can be adjusted easily, reduction of *shihonkin* is extremely difficult. It first requires over 2/3 (kabushiki) and 3/4 (yugen) approval rate at a meeting with over 50% of shareholders with voting rights attending. Then the firm needs to undergo a debtholder protection procedure which involves an announcement on media and government's official gazette, individually contacting each debtholder known to the firm and negotiating with disapproving debtholders. The proposed reduction only becomes effective when no debtholder vetos. Only about 0.05% of firms each year try to reduce *shihonkin*[27].

A2.2. Empirical support

Here we provide some empirical support for using *shihonkin* to proxy for inside equity. Table 13 lays out a drastic reform of the minimum *shihonkin* requirement for incorporation that occurred over 2003-2006. Pre-2003, incorporation of stock-issuing limited liability firms required at least ¥10 million *shihonkin* and incorporation of non stock-issuing limited liability firms required ¥3 million of *shihonkin*. In 2003, new firms were given a five year exemption period where they can incorporate with *shihonkin* above ¥1 but must build it up to the required minimum within 5 years and cannot issue dividends until they have done so. In 2006, the *shihonkin* requirement was removed completely. In effect, limited liability firms can be incorporated with no *shihonkin*. However, stock issuing limited liability firms can not issue dividend unless they have ¥3 million of net asset (asset - debt). According to the Household Consumption Survey by the Japanese government, ¥3 mil is approximately 17% of the average household financial assets (deposits + insurance + shares) in Japan in 2006 and 43% for households with 30-39 years old household heads. [28]

Figure 8 display the *shihonkin* distribution for new limited liability firms in Japan by Census years. The top three panels are for year before the reform. It shows that tight bunching at the pre-reform requirements indicating that the requirements were enforced and they were binding for many firms. The last two panels in Figure 8 are after the reform. It shows a drastic smoothing of the distribution. This is consistent with the fact that registering positive

[26]Article 446 of the Companies Act, Article 290 of the Commercial Code and Article 46 of the Yugenkaisha Act

[27]We arrive at this estimate by counting the number of*shihonkin* reduction announcements on the government's official gazette and dividing the number of announcement by the total number firms in the Census

[28]In 2003, in order to increase startups, the Japanese government allowed new companies to incorporate with a minimum of one yen share-capital and granted them 5 years to build up to the ¥3 million or ¥10 million level. These new companies could not issue dividends until they meet the required minimums but they were able to receive limited liability for the the first five years without meeting the requirements. In 2006, the Japanese government further relaxed the capital regulation by removing the five year cap, essentially allowing limited liability firms to incorporate permanently with one yen *shihonkin*.

	for incorporation	for paying dividend
1991/4/1	¥3 mil for yugen corps (Y-corps) ¥10 mil for kabushiki corps (K-corps)	
2003/2/1	only ¥1 at registration 5 yrs to meet ¥3 mil or ¥10 mil requirement	same as before
2006/5/1	only ¥1	¥3 mil for K-corps

Table 13: Reform of minimum *shihonkin* requirement

amounts of *shihonkin* is costly.

What is puzzling is that despite the costs of registering a positive amount of *shihonkin*, entrepreneurs *choose* to register *shihonkin* above the statutory requirements after the 2006 reform. In Figure 8, although bunching at the old requirements disappeared after the 2006 reform, the majority of new firms registered positive *shihonkin*, with 70% registering over ¥3 million. This observation holds even for firms that started between 2009 and 2012, suggesting that the "excess *shihonkin*" phenomenon is structural rather than due to transitions. This "excess *shihonkin*" is consistent with the advices on Japanese government agency websites that regards *shihonkin* as the founders' own stake in the firm. The cost of registering more *shihonkin* above the required level is met by the benefit of being able to borrow more.

B Empirical findings

We reported empirical patterns for the 2006 cohort at age 5. Figures 9 and 10 shows the regression of output-to-capital ratio for other cohorts at age 5 and figures 11 and 12 display the results for age 1. They show that our findings are not driven by cohort and age factors.

B1. Regression results using shareholders fund

Figures 13, 14, 15 and 16 display the our main OLS regressions but using shareholders fund instead of *shihonkin* to measure inside equity. Our empirical findings are robust to using this alternative proxy for inside equity.

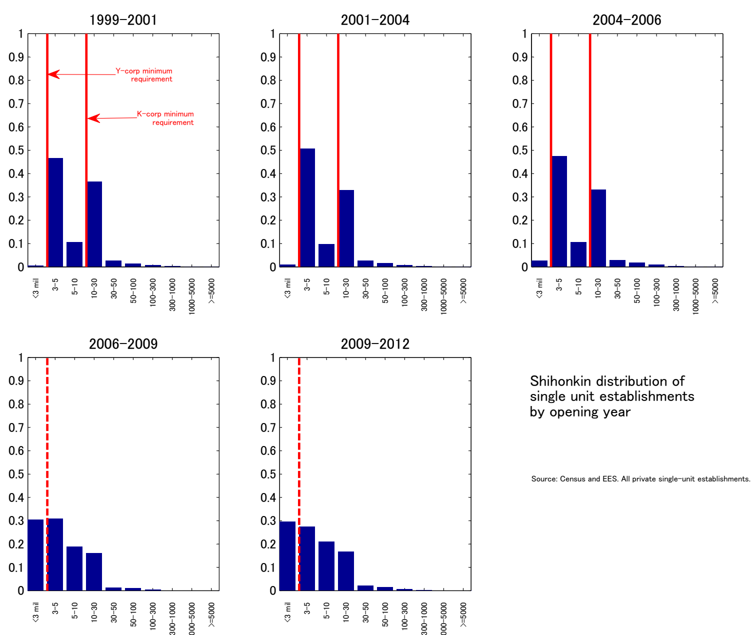

Figure 8: *shihonkin* distribution for new single-unit establishments. Source: Japan Census

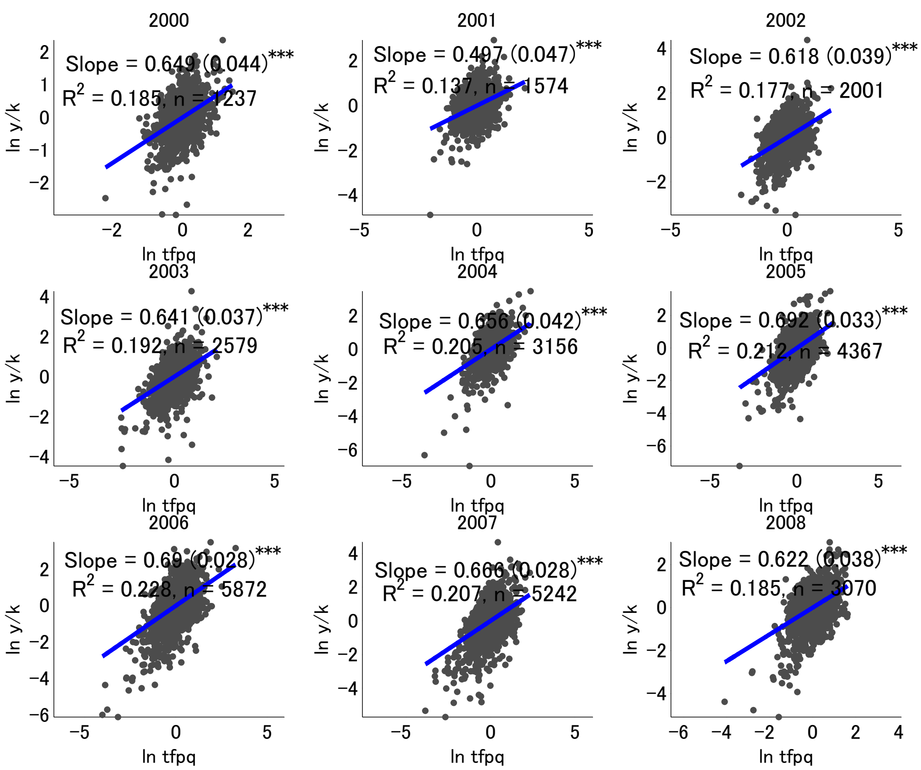

Figure 9: Output-capital ratio increases with productivity, age 5

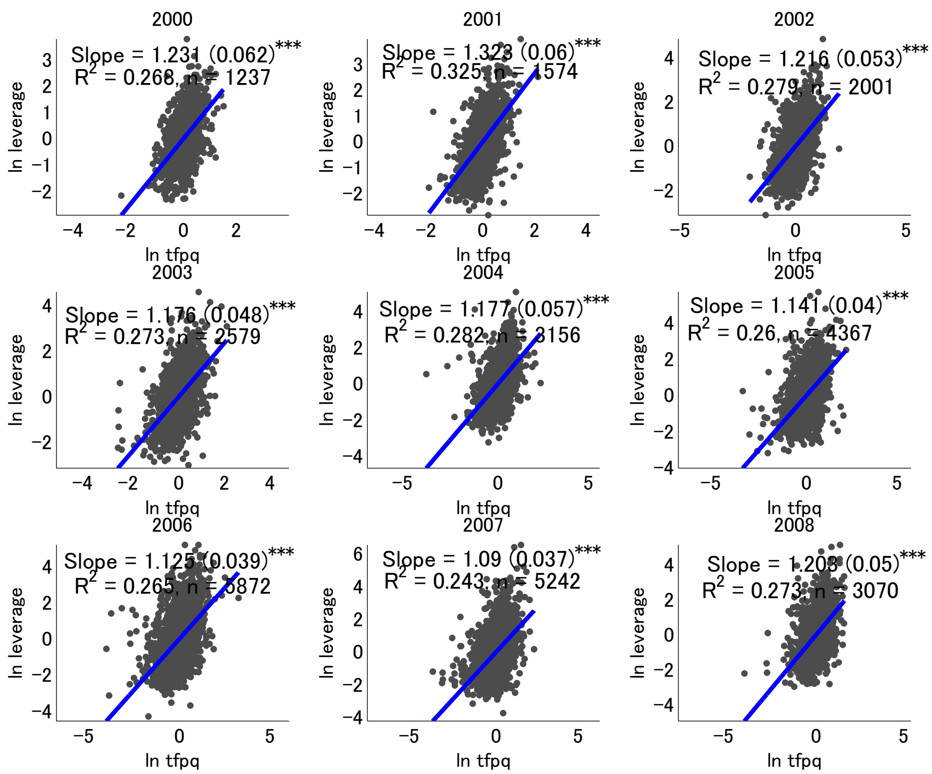

Figure 10: Leverage increases with productivity, age 5

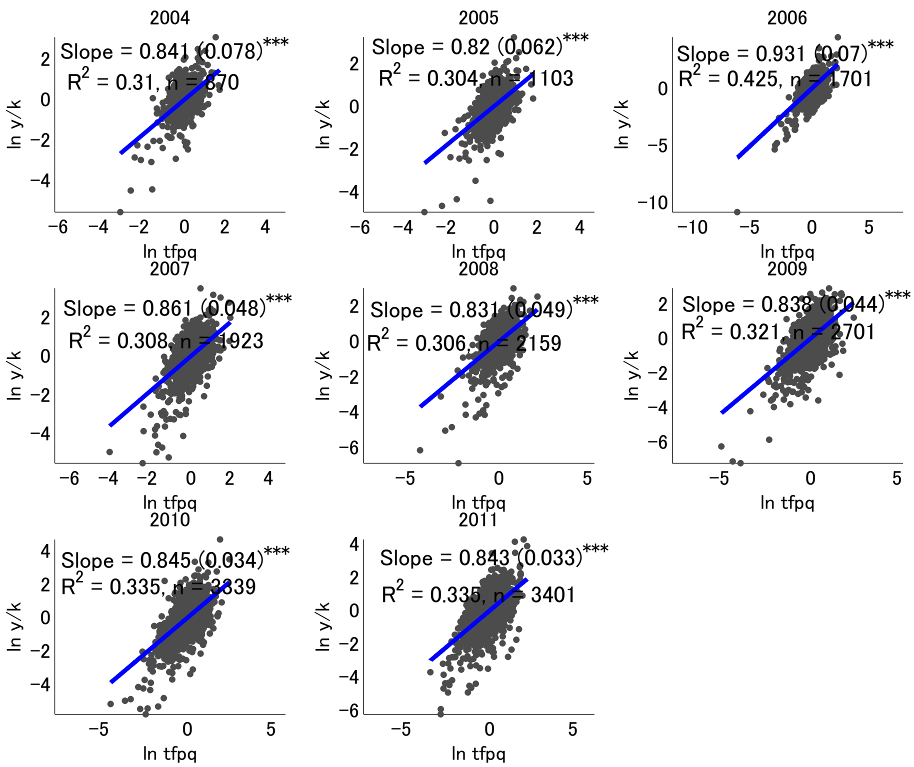

Figure 11: Output-capital ratio increases with productivity, age 1

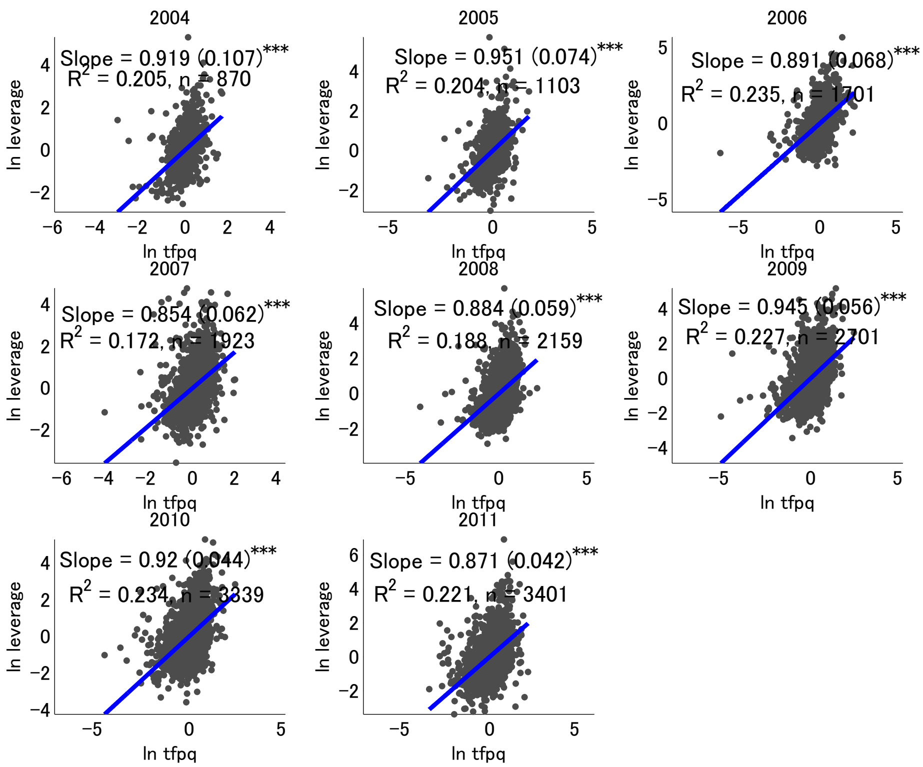

Figure 12: Leverage increases with productivity, age 1

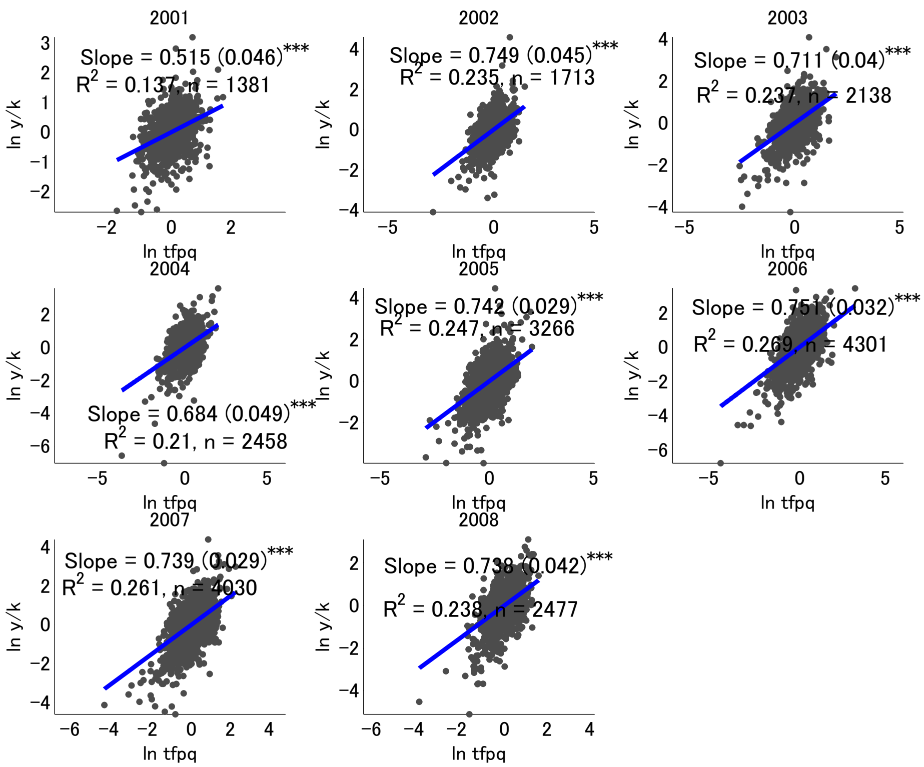

Figure 13: Output-capital ratio vs productivity using shareholders fund

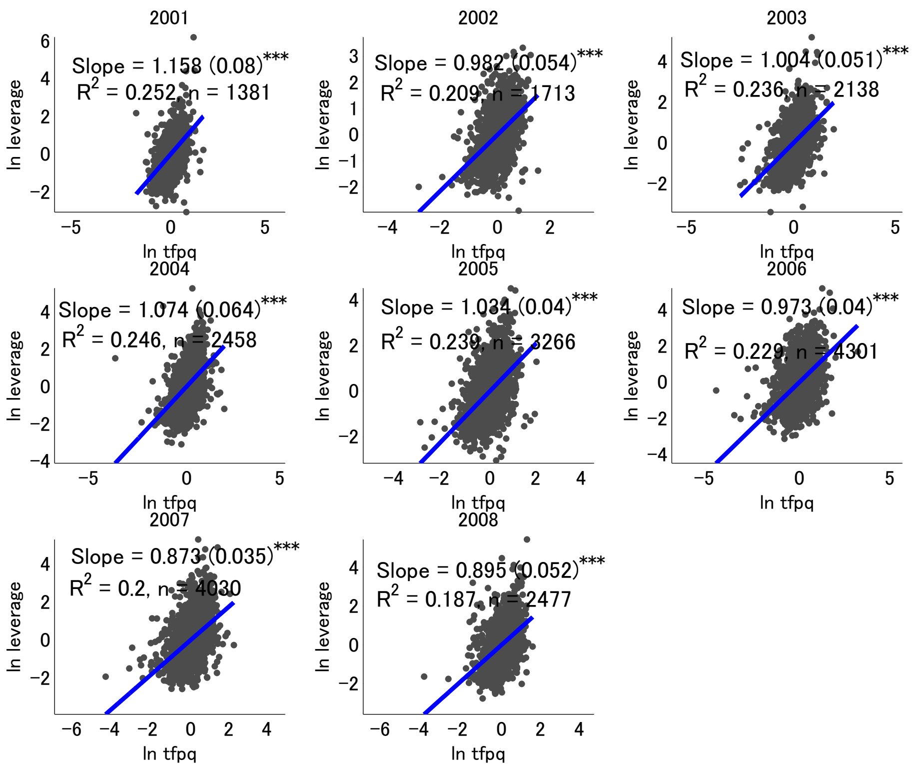

Figure 14: Leverage vs productivity, using shareholders fund

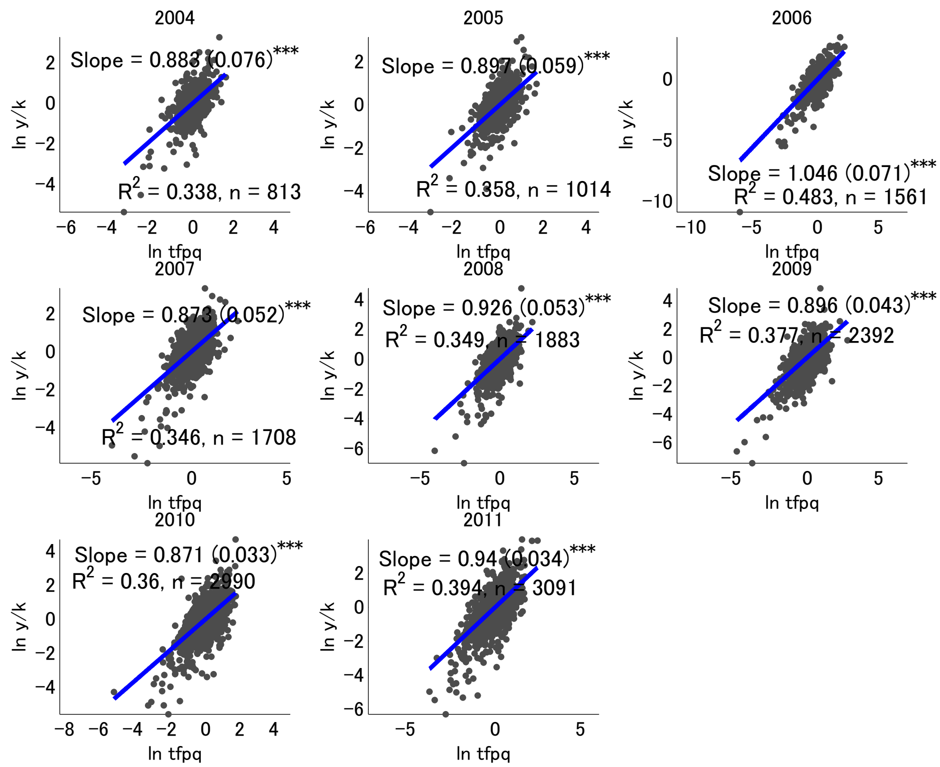

Figure 15: Output-capital ratio vs productivity, using shareholders fund, age 1

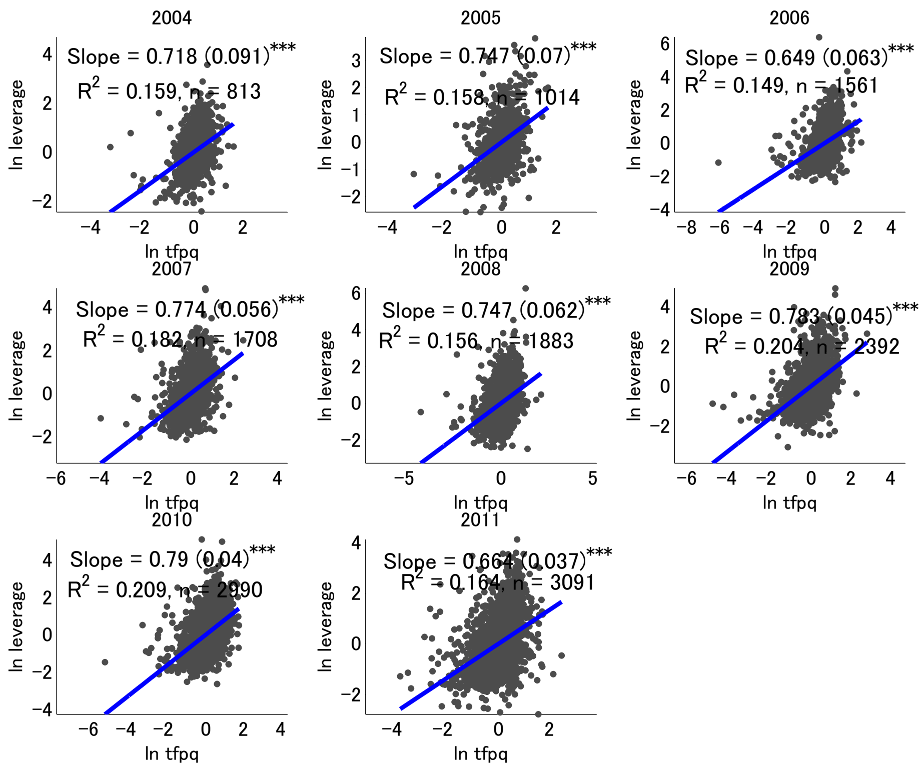

Figure 16: Leverage vs productivity, using shareholders fund, age 1