The Optimal Degree of Discretion in Monetary Policy in a New Keynesian Model with Private Information

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Introduction
Rule vs. discretion is a recurrent theme in macroeconomics.
Emphasis is on the importance of rules.
Monetary policy is often delegated to an independent group of professionals with some flexibility and restrictions.
  - Dual mandate, inflation targeting
Q: What is the optimal degree of discretion (flexibility) in monetary policy?
Use a mechanism design approach:

- Benevolent government sets up an independent central bank at time 0.
- It designs and credibly imposes rules (a mechanism) on CB.

Optimal mechanism may grant some discretion to CB, as

- CB is benevolent but is unable to commit, and
- CB has superior (private) information that is useful in improving social welfare.
New Keynesian Model

- Canonical New Keynesian model
- Inflation and output gap must satisfy the New Keynesian Phillips curve.
- Consistent with various costly price adjustment specifications
- Introduces a specific form of time-inconsistency problem.
  - New Keynesian Phillips curve is forward-looking
    → A promise to make future policy history-dependent can improve current outcome through expected future inflation.
  - CB in the future is tempted to renege such promise.
- The stronger the desire to renege is, the tighter the gov’t must constrain CB.
Several properties of the optimal mechanism:

1. Optimal mechanism is dynamic.
   - State variable = previous period’s inflation promise
   - No need to keep track of CB’s continuation utility

2. “Degree of discretion” varies endogenously over time and is negatively linked to the “severity of time-inconsistency problem”.

3. Private information limits history-dependence.

4. No-discretion is not a long-run outcome.

5. History-dependent inflation targeting rule can do as good as the optimal direct mechanism.
Literature

- Monetary policy in New Keynesian models
  - e.g. Woodford (1999): no private info. & no gain from discretion.

- Monetary policy with private information

- Dynamic contract
  - Green (1987), Atkeson and Lucas (1992), etc.: iid private information

- Delegation
The Set-up
This presentation uses a two-period model: $t = 0, 1$.
(See the paper for an infinite horizon model.)
Exogenous shock

- The central bank (CB) privately observes the “state of the economy,” $\theta_t$.
- $\theta_t$ is drawn from an interval $\Theta = [\theta, \bar{\theta}]$ in an IID fashion over time.
- $p(\theta) > 0$ is the probability density function;
- $E[\theta] = 0$ WLOG.
- No other shocks.
Endogenous Variables

- The gov’t chooses an **allocation** (mechanism) that is “feasible” and “incentive compatible.”
- An allocation specifies:
  1. inflation rate in both periods: \( \pi_0, \pi_1 \)
  2. output gap in period 0: \( x_0 \), and
  3. “inflation promise” in period 0: \( \pi^e_0 \),

  as functions of shock history: \( \pi_0(\theta_0), x_0(\theta_0), \pi^e_0(\theta_0), \pi_1(\theta_0, \theta_1) \).
- No output gap in period 1, just for simplicity.
An allocation is **feasible** from $\pi_{-1}$ if

- **New Keynesian PC:** 
  \[
  \pi_0(\theta_0) = \kappa x_0(\theta_0) + \beta \pi_0^e(\theta_0), \quad \forall \theta_0
  \]
- “Promise-keeping” in pd. 1: 
  \[
  \pi_0^e(\theta_0) = E[\pi_1(\theta_0, \theta_1)|\theta_0], \quad \forall \theta_0
  \]
- “Promise-keeping” in pd. 0: 
  \[
  \pi_{-1}^e = E[\pi_0(\theta_0)]
  \]

- NKPC requires an allocation be consistent with price setters’ incentive.
- Last condition is there just to make the whole problem recursive.
Quadratic Social Welfare

The gov’t maximizes a quadratic social welfare:

$$E \left[ - (\pi_0(\theta_0) - \theta_0)^2 - bx_0(\theta_0)^2 - \beta E \left[ (\pi_1(\theta_0, \theta_1) - \theta_1)^2 | \theta_0 \right] \right].$$

$\theta$ is the desirable inflation rate that varies over time e.g. redistribution effects through inflation.

(Social welfare used in the paper:

$$E \left[ \sum_{t=0}^{\infty} \beta^t R(\pi_t(\theta_t), x_t(\theta_t), \theta_t) \right].$$

$R$ is slightly more general than quadratic.)
CB’s objective = SWF (i.e. he is benevolent)

An allocation is incentive compatible if and only if

\[-(\pi_1(\theta_0, \theta_1) - \theta_1)^2 \geq -(\pi_1(\theta_0, \theta') - \theta_1)^2, \quad \forall \theta_0, \theta_1, \theta',\]

and

\[-(\pi_0(\theta_0) - \theta_0)^2 - bx_0(\theta_0)^2 - \beta E \left[(\pi_1(\theta_0, \theta_1) - \theta_1)^2 | \theta_0\right] \\
\geq -(\pi_0(\theta') - \theta_0)^2 - bx_0(\theta')^2 - \beta E \left[(\pi_1(\theta', \theta_1) - \theta_1)^2 | \theta_0\right], \quad \forall \theta_0, \theta'\]
Mechanism Design Problem

\[ \overline{W}_{-1}(\pi^e_{-1}) := \sup E \left[ - (\pi_0(\theta_0) - \theta_0)^2 - bx_0(\theta_0)^2 - \beta E \left[ (\pi_1(\theta_0, \theta_1) - \theta_1)^2 \right] | \theta_0 \right] \]

subject to feasibility from \( \pi^e_{-1} \) and incentive-compatibility.
CB’s inability to commit

- CB is not required to report in a way that the promised expected inflation is indeed delivered.
- It is the government’s job to incentivize CB to deliver the promised inflation.
- **Lack of commitment power** for CB.
In the following...

We consider

- the full-information benchmark, and
- two private-information cases
  1. only $\theta_1$ is private;
  2. both $\theta_0$ and $\theta_1$ are private,

...to understand how the private information imposes restrictions on the second-best allocation.
Full-information Problem

\[
W^{FI}(\pi^e_{-1}) = \sup E \left[ - (\pi_0(\theta_0) - \theta_0)^2 - bx_0(\theta_0)^2 - \beta E \left[ (\pi_1(\theta_0, \theta_1) - \theta_1)^2 | \theta_0 \right] \right].
\]

subject to

\[
\begin{align*}
\pi_0(\theta_0) &= \kappa x_0(\theta_0) + \beta \pi^e_0(\theta_0), \quad \forall \theta_0 \\
\pi^e_0(\theta_0) &= E[\pi_1(\theta_0, \theta_1) | \theta_0], \quad \forall \theta_0 \\
\pi^e_{-1} &= E[\pi_0(\theta_0)],
\end{align*}
\]

Feasibility

- Inflation promise \( \pi^e_0 \) serves as the state variable.
Full-information Problem: Period 1

Period 1 problem:

\[
\max_\pi - E \left[ (\pi(\theta_1) - \theta_1)^2 \right] \quad \text{s.t.} \quad \pi^e_0 = E[\pi(\theta_1)].
\]

- **Solution** \( \pi_1(\theta_1, \pi^e_0) = \theta_1 + \pi^e_0 \)
  - (FONC: \( \pi_1(\theta_1, \pi^e_0) = \theta_1 - \mu \) and \( E[\pi_1(\theta_1, \pi^e_0)] = -\mu \))
- \( \pi_1 \) increasing in \( \theta_1 \) and \( \pi^e_0 \).
- Period-1 social welfare following the promise \( \pi^e_0 \) is
  
  \[ -(\pi^e_0)^2. \]
Full-information Problem: Period 0

\[
\begin{align*}
\sup E \left[ - (\pi_0(\theta_0) - \theta_0)^2 - bx_0(\theta_0)^2 - \beta \pi_0^e(\theta_0)^2 \right].
\end{align*}
\]

subject to

\[
\begin{align*}
\text{NKPC} & \quad \pi_0(\theta_0) = \kappa x_0(\theta_0) + \beta \pi_0^e(\theta_0), \quad \forall \theta_0 \\
\text{“Promise keeping”} & \quad \pi_{-1}^e = E[\pi_0(\theta_0)].
\end{align*}
\]

- \pi_0 \text{ increasing in } \theta_0 \text{ and } \pi_{-1}^e.
- \(x_0, \pi_0^e, \pi_1\) is “efficient” given \(\pi_0\) in that

\[
\begin{align*}
\text{Welfare from } (x_0, \pi_1) \text{ for } \theta_0 &= \max_{(x, \pi^e): \pi_0(\theta_0) = \kappa x + \beta \pi^e} -bx^2 - \beta (\pi^e)^2.
\end{align*}
\]

- \(\pi_0^e(\theta_0)\) increasing in \(\theta_0 \Rightarrow \pi_1(\theta_0, \theta_1)\) increasing in \(\theta_0\).
Full-information solution

- Properties that hold in private info. cases:
  - Solution is history-dependent.
  - State variable = previous period’s inflation promise.
  - \((x_0, \pi_0^e, \pi_1)\) is “efficient” given \(\pi_0\).

- Properties that do not hold in private info. cases:
  - Period \(t\) inflation is strictly increasing in \((\pi_{t-1}^e, \theta_t)\).
  - \(\pi_1 \neq \theta_1\) except when \(\pi_0^e = 0\).
When $\theta_1$ is private information: Period 1 problem

$$W_0(\pi_0^e) = \max_{\pi} -E\left[(\pi(\theta_1) - \theta_1)^2\right].$$

subject to

"Promise-keeping" \quad $\pi_0^e = E[\pi(\theta_1)],$

Incentive-compatibility \quad $-(\pi(\theta_1) - \theta_1)^2 \geq -(\pi(\theta') - \theta_1)^2,$ \quad $\forall \theta_1, \theta'.$

- If the solution is differentiable at $\theta_1$, then IC implies

$$\left(\pi_1(\theta_1, \pi_0^e) - \theta_1\right) \frac{\partial \pi_1(\theta_1, \pi_0^e)}{\partial \theta_1} = 0.$$

- Either $\pi_1(., \pi_0^e)$ is flat at $\theta_1$ or equal to "discretionary best response" $\pi_{DBR}(\theta_1) = \theta_1$.

- $\pi_1(\theta_1) = \theta_1$ for all $\theta_1$ is not feasible unless $\pi_0^e = E[\theta] = 0$.

- Tighter first constraint $\Rightarrow$ bigger deviation of $\pi_1(\theta_1, \pi_0^e)$ from $\theta_1$. 
\( \pi_1 \) is continuous in \( \theta_1 \)

Relaxed problem:

\[
\max_{\pi, \delta} E \left[ -(\pi(\theta_1) - \theta_1)^2 + \delta(\theta_1) \right].
\]

subject to

\[
\pi^e_0 = E[\pi(\theta_1)],
\]

\[
-(\pi(\theta_1) - \theta_1)^2 + \delta(\theta_1) \geq -(\pi(\theta') - \theta_1)^2 + \delta(\theta'), \quad \forall \theta_1, \theta',
\]

\[
\delta(\theta) \leq 0, \quad \forall \theta.
\]

- Under the single-crossing condition and the monotone hazard condition, theorems in Athey, Atkeson, and Kehoe (2005) imply:
  - the solution is continuous and satisfies \( \delta(\theta) = 0 \) for all \( \theta \), and
  - the solution has a “cut-off” property.
Cut-off property: Policy function as a function of type

Dependence on $\theta$ limited.

$\pi_{DBR}(\theta) = \theta$ and $\pi^e_* := E[\pi_{DBR}] = 0$. 

\[ \pi_{DBR}(\theta) = \theta \text{ and } \pi^e_* := E[\pi_{DBR}] = 0. \]

(1) $\pi^e_- < \pi_{DBR}(\theta)$

(2) $\pi^e_- > \pi_{DBR}(\bar{\theta})$
History-dependence limited. 
\( \pi_{DBR}(\theta) = \theta \) and \( \pi^{e*} := E[\pi_{DBR}] = 0 \).
When $\theta_1$ is private information

- Value $W_0(\pi_0^e)$ is strictly concave and peaked at $\pi_0^e = E[\pi_{DBR}]$.
- $(x_0, \pi_0^e, \pi_1)$ is “efficient” given $\pi_0$ in that

  Welfare from $(x_0, \pi_1)$ for report $\theta_0 = \max_{(x, \pi^e) : \pi_0(\theta_0) = \kappa x + \beta \pi^e} -bx^2 + \beta W(\pi^e)$. 
Relation to Time-inconsistency

Definition

Optimal degree of discretion at $\pi^e_0 = \text{prob. of } \pi_1(\theta_1; \pi^e_0) = \pi_{DBR}(\theta_1)$.

- This is one when $\pi^e_0 = 0$ and decreases toward zero as $|\pi^e_0 - \pi^e_0^*| \uparrow$.

Definition

Severity of time-inconsistency at $\pi^e_0 = \overline{W}(\pi^e_0^*) - \overline{W}(\pi^e_0)$.

- I.e. gains from reneging the inflation promise.
- This is strictly convex & bottomed at $\pi^e_0^*$.

$\Rightarrow$ They are negatively linked.
Gov’t doesn’t need a direct mechanism: a history-dependent inflation targeting can achieve the second-best.

In period 1,

1. Gov’t sets a range of permissible inflation rates conditional on $\pi^e_0$.
2. Central bank freely chooses inflation from this range.

$\Gamma_1(\pi^e_0) = [\min_\theta \pi_1(\theta; \pi^e_0), \max_\theta \pi_1(\theta; \pi^e_0)]$ is one example.

1. CB’s best choice = optimal mechanism’s prescription.
2. Inflation promise $\pi^e_0$ is delivered.

History-dependence through $\pi^e_0$ is crucial.
When $\theta_0$ is also private information

$(x_0, \pi^e_0, \pi_1)$ may be inefficient given $\pi$, i.e. for some $\theta_0$,

\[
\text{Welfare from } (x_0, \pi_1) \text{ for report } \theta_0 < \max_{(x, \pi^e) : \pi_0(\theta_0) = \kappa x + \beta \pi^e} -bx^2 + \beta \overline{W}(\pi^e).
\]

Why? This potentially helps the planner incentivize the CB in $t = 0$.

However this doesn’t happen under the optimal mechanism.

Idea:

- When inefficient for some $\theta_0$, it is essentially a penalty for reporting $\theta_0$.
- Consider a relaxed problem in which the planner can arbitrary penalize any report $\theta_0$ and apply AAK’s theorems, then it is shown optimal to have zero penalty for all $\theta_0$. 

Policy function properties still hold true

- Cut-off property holds in period 0, with “discretionary best response” appropriately defined.
- Amnesia property holds in period 0.
- When CB chooses \((\pi_0, x_0, \pi^e_0, \pi_1)\) subject to
  1. inflation targeting rule \(\Gamma_t(\pi^e_{t-1}) = [\min_\theta \pi_t(\theta; \pi^e_{t-1}), \max_\theta \pi_t(\theta; \pi^e_{t-1})]\) for all \(t\), and
  2. New Keynesian Phillips curve: \(\pi_0(\theta_0) = \kappa x_t(\theta_0) + \beta \pi^e_0(\theta_0)\),
- CB’s optimal choice coincides with the optimal mechanism, and
- \(\pi^e_{t-1} = E[\pi_t]\) is satisfied.
- \(\rightarrow\) No rule needed for CB’s choice of \(x_0\) and \(\pi^e_0\).
Recursive formulation

The following recursive formulation is justified:

\[
\overline{W}_{-1}(\pi_{-1}^e) = \max_{\pi_0, x_0, \pi_0^e, w_0} E \left[ -(\pi(\theta_0) - \theta_0)^2 - bx_0(\theta_0)^2 + \beta w_0(\theta_0) \right].
\]

subject to

\[
\begin{align*}
\pi_{-1}^e &= E[\pi(\theta_0)], \\
\pi_0(\theta_0) &= \kappa x_0(\theta_0) + \beta \pi_0^e(\theta_0), \\
-(\pi(\theta_0) - \theta_0)^2 - bx_0(\theta_0)^2 + \beta w_0(\theta_0) &\geq -(\pi(\theta') - \theta_0)^2 - bx_0(\theta')^2 + \beta w_0(\theta'), \quad \forall \theta_0, \theta', \\
w_0(\theta) &\leq \overline{W}_0(\pi_0^e(\theta_0)), \quad \forall \theta.
\end{align*}
\]

- Solution satisfies the last inequality with equality.
- Generalizes to the infinite horizon case.
- Can solve by VFI.
Numerical Experiments
Motivation

- Examine the long-run behavior of optimal degree of discretion.
- Our numerical example suggests no “no-discretion” in the long-run.
Parameter Values

Standard parameterization of a NK model with Calvo price setting.

- Calvo parameter $\gamma = 0.75$.
- Risk-aversion $\sigma = 1$; Labor elasticity parameter $\eta = 1$:
  $$u(c, h) = \log c - \frac{1}{2} h^2.$$
- $\beta = 0.99$.
- $\kappa = (1 - \gamma)(1 - \beta \gamma)/\gamma \times (\sigma + \eta) \approx 0.17$ (Calvo model with a single labor market),
- CES elasticity $\epsilon = 5$. (25% of markup)
- $\Theta$ is approximated as 31 equally-spaced grid points between -0.5% and 0.5%.
- $R(\pi, x, \theta) = -\frac{1}{2} \left[ \frac{\kappa}{\epsilon} x^2 + (\pi - \theta)^2 \right]$. 
Transition Dynamics

Dynamics of the degree of discretion as the state variable changes over time: Some discretion in the ergodic set.

\[ \theta = -0.5, -0.26667, 0, 0.26667, 0.5 \]
Conclusion
Summary

- Optimal degree of discretion when CB has private information and is unable to commit.
- Optimal mechanism is dynamic and utilizes private information, but its dependence on private information is limited (cut-off) and history-dependence is limited (amnesia).
- History-dependent inflation targeting is desirable. → Permissible range widens as the time-inconsistency becomes less severe.
- In the long-run some discretion is granted.
Future Work

Some other useful analysis:

- Impulse response analysis to quantify the history-dependence of optimal mechanism.

A little bit more ambitious things:

- Analyze general dynamic delegation problems.
- Lack of commitment on the side of the planner.
- “Biased” agent (non-benevolent central bank).