Banking, Liquidity and Bank Runs in an Infinite Horizon Economy

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Recent financial crisis started in summer 2007

Despite of many attempts, DSGE models with financial friction forecasted no deep recession until fall 2008

After 2008Q4, models with financial friction predict deeper recession than the models without

To explain financial crisis, we need "bank run" or "sudden stop"

Liquidity mismatch opens up the possibility of run →

Inefficient liquidation of assets, loss of intermediation, and deep recession
We develop a simple macro model of banking crisis

Financial accelerator / Credit cycles

Banks runs

Macroeconomic conditions affect whether runs are feasible

Bank leverage ratio

Liquidation prices

An increase in the likelihood of run contracts the economy severely
Basic Model

Capital is either intermediated by banks or directly held by households

\[ K_t^b + K_t^h = \bar{K} \]

\[ \begin{align*} 
  \text{date } t & \quad \text{date } t+1 \\
  K_t^b \text{ capital} & \rightarrow \left\{ 
  K_t^b \text{ capital} \\
  Z_{t+1}K_t^b \text{ output} 
  \right\} \\
  f(K_t^h) \text{ goods} & \rightarrow \left\{ 
  K_t^b \text{ capital} \\
  Z_{t+1}K_t^b \text{ output} 
  \right\}
\]
Households

Banks

Intermediated Finance

\[ Q_t K^b_t = N_t + D_t \]

Direct Finance

\[ Q_t K^h_t \]

Deposit

Business: \( Q_t \bar{K} \)

Households
Deposit contract

Short term

Promised rate of return $\overline{R}_{t+1}$ is non-contingent

With run, the returns is the minimum of $\overline{R}_{t+1}$ and total realized bank assets per deposit

In Basic Model, bank run is unanticipated $\rightarrow$

Realized return: $R_{t+1} = \overline{R}_{t+1}$: Promised return
Households maximize

\[ U_t = E_t \left( \sum_{i=0}^{\infty} \beta^i \ln C_{t+i}^h \right) \]

subject to:

\[ C_t^h + D_t + Q_t K_t^h + f(K_t^h) = Z_t W^h + R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h \]

\[ 1 = E_t (\Lambda_{t,t+1}) R_{t+1} \]

\[ 1 = E_t \left( \Lambda_{t,t+1} \frac{Z_{t+1} + Q_{t+1}}{Q_t + f'(K_t^h)} \right) \]

\[ \Lambda_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \]
Many bankers

Each has an i.i.d. survival probability of $\sigma$

Banker consumes wealth upon exit: $c^b_t = n_t$

Preferences are linear in "terminal" consumption

$$V_t = E_t \left[ \sum_{i=1}^{\infty} \beta^i \sigma^{i-1} (1 - \sigma) c^b_{t+i} \right]$$

Each exiting banker replaced by a new banker with an endowment $w^b = n_t$

Bank balance sheet

$$Q_t k^b_t = d_t + n_t$$

Net worth $n_t$ of surviving bankers

$$n_t = (Z_t + Q_t) k^b_{t-1} - R_t d_{t-1}$$
Agency Problem:

After the banker raises funds, it may divert a fraction of $\theta$ of loans at the end of period $t$.

If the banker does not repay its debt in period $t + 1$, the creditors shut the bank down.

Incentive constraint

$$\theta Q_t k_t^b \leq V_t$$
Bank chooses \( k_t^b \) and \( d_t \) to maximize

\[
V_t = \beta E_t [(1 - \sigma) n_{t+1} + \sigma V_{t+1}]
\]

subject to \( \theta Q_t k_t^b \leq V_t \iff \)

Bank chooses "leverage multiplier" \( \phi_t = \frac{Q_t k_t^b}{n_t} \) to maximize

\[
\frac{V_t}{n_t} = \psi_t = \beta E_t \left\{ (1 - \sigma + \sigma \psi_{t+1}) \frac{n_{t+1}}{n_t} \right\}
\]

\[
= \beta E_t \left\{ (1 - \sigma + \sigma \psi_{t+1}) \left[ \phi_t \left( \frac{Q_{t+1} + Z_{t+1}}{Q_t} - R_{t+1} \right) + R_{t+1} \right] \right\}
\]

subject to \( \theta \phi_t \leq \psi_t \)
Aggregate leverage constraint

\[ Q_t K_t^b = \phi_t N_t \]

Aggregate net worth

\[ N_t = \sigma \left[ (Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right] + (1 - \sigma) w^b \]

Goods market

\[ C_t^h + (1 - \sigma) \left[ (Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right] + f(K_t^h) \]
\[ = Z_t \overline{K} + Z_t W_t^h + (1 - \sigma) w^b \]
Bank Runs

Ex ante, zero probability of a run

If depositors do not roll over the deposits ("run"), the bank sells its capital to households who are less efficient in managing capital

In addition to an equilibrium without run, bank run equilibrium exists if:

\[(Z_t + Q_t^*) K_{t-1}^b < R_tD_{t-1}\]

\[Q_t^* \equiv \text{the liquidation price of the bank's assets}\]
After a bank run at $t$:

$$K^h_t = \bar{K},$$

$$N_{t+1} = (1 - \sigma)w^b$$

$$N_s = \sigma \left[ (Z_s + Q_s)K^b_{s-1} - R_sD_{s-1} \right] + (1 - \sigma)w^b, \; \forall \; s \geq t+2$$

Household condition for direct capital holding $\rightarrow$

$$Q^*_t = E_t \left\{ \sum_{i=1}^{\infty} \Lambda_{t,t+i} [Z_{t+i} - f'(K^h_{t+i})] \right\} - f'(\bar{K})$$
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount rate</td>
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<tr>
<td>$\sigma$</td>
<td>0.95</td>
<td>Bankers survival probability</td>
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<tr>
<td>$\theta$</td>
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<td>Seizure rate</td>
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<tr>
<td>$\alpha$</td>
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<td>$\rho$</td>
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<td>Serial correlation of productivity shock</td>
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<td>$Z$</td>
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<td>Steady state productivity</td>
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<tr>
<td>$W_b$</td>
<td>0.0007</td>
<td>Bankers endowment</td>
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<tr>
<td>$W^h$</td>
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<td>Household endowment</td>
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<tr>
<td>$\bar{\rho}$</td>
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<td>Endogenous Run Probability Parameter</td>
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<tr>
<td>$\delta$</td>
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<td>Endogenous Run Probability Parameter</td>
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### Table 2: Steady State Values

<table>
<thead>
<tr>
<th>Steady State Values</th>
<th>Baseline</th>
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<tr>
<td>K</td>
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<tr>
<td>Q</td>
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<td>$R^h$</td>
<td>1.0404</td>
</tr>
<tr>
<td>$R$</td>
<td>1.0404</td>
</tr>
</tbody>
</table>
Extension: Anticipated Bank Runs

Deposit returns $R_{t+1} = \begin{cases} 
\bar{R}_{t+1} & \text{if no bank run} \\
x_{t+1}\bar{R}_{t+1} & \text{if bank run} 
\end{cases}$

$$x_{t+1} = \frac{(Q_{t+1}^* + Z_{t+1}) K^b_t}{\bar{R}_{t+1} D_t}$$

Household attaches the probability of bank run as

$$p_t = 1 - E_t \left[ \min(1, x_{t+1}) \right]$$

FONC for deposits is

$$1 = \bar{R}_{t+1}[(1 - p_t)E_t (\Lambda_{t,t+1}) + p_tE_t(\Lambda_{t,t+1}^* x_{t+1})]$$
Bank’s leverage \( \phi_t = \frac{Q_t k_t^b}{n_t} \) maximizes

\[
\frac{V_t}{n_t} = \psi_t = \beta(1-p_t)E_t \left\{ (1-\sigma+\sigma\psi_{t+1}) \left[ \phi_t \left( \frac{Q_{t+1} + Z_{t+1}}{Q_t} - \overline{R}_{t+1} \right) + \overline{R}_{t+1} \right] \right\}
\]

subject to \( \theta \phi_t \leq \psi_t \).

An increase in likelihood of run is contractionary in two ways

leverage \( \phi_t \) declines when the franchise value falls

\( N_{t+1} \) decreases even without run since \( \overline{R}_{t+1} \) increases
Some Remarks About Policy

Deposit insurance can eliminate bank run equilibrium

But may have moral hazard of risk-taking, particularly by investment banks

Capital requirement reduces bank risk-taking and likelihood of bank run

Can increase intermediation cost if capital is costly to raise

Lender-of-last resort stabilizes liquidation price and reduces likelihood of bank run

Can purchase or lend against a good quality securities e.g. AMBS