Deflation and debt:
A neoclassical framework for monetary policy analysis

Keiichiro Kobayashi
Keio University/CIGS

June 25, 2013 at CIGS
Very Preliminary. Comments Welcome.
Can New Keynesian (NK) model account for Japan’s Deflation?

- Deflation continued for more than a decade in Japan from 1998–present.
- New Keynesian models are not fully satisfactory in analyzing decade-long deflation.
  - Was the price stickiness a problem in Japan’s 1990s and in the global crises?
What is necessary for a framework of monetary policy analysis

- Can we consider an alternative framework without price stickiness?
- Three features in the New Keynesian model
  1. Suboptimality of the Friedman rule:
     zero or moderate inflation maximizes social welfare.
  2. Phillips curve:
     a positive correlation between inflation and output.
  3. Liquidity effects:
     a negative correlation between nominal interest rate and output (or
     money supply).
What we do in this paper

- Construct a neoclassical model with flexible prices, which has the following features:
  1. Suboptimality of the Friedman rule: **input-smoothing effect**
     - Suppose that firms with loose constraints and tight constraints coexist. Distortionary tax on loosely-constrained firms can be welfare enhancing (input-smoothing effect).
     - Inflation works as a device for input-smoothing effect.
  2. Phillips curve
  3. Liquidity effect

- These features are generated from heterogeneous financial constraints.

- Our model may provide a new account for a decade-long deflation in Japan.
Overview of the model

- Closed economy, representative consumer, heterogeneous firms (firms 1, firms 2), central bank (CB).

- Heterogeneity in financial constraints
  - Consumer can transfer cash to firm 1 as internal funds.
    - Consumer is owner-manager of firm 1.
  - Consumer cannot transfer cash to firm 2.
    - Consumer is owner but not manager of firm 2. The manager of firm 2 can divert cash for private purposes without getting penalty.

- CB can choose intra-period interest rate (nominal rate), $j$, and the amount of intra-period loan, $\Delta$.

- The policy $(j, \Delta)$ decides the inflation rate $\pi$ as an equilibrium outcome.
1 Introduction

2 Model Economy

- Setup
- Steady-state equilibrium
- Suboptimality of the Friedman rule
- Phillips curve and the liquidity effect

3 The Fisherian Deflation

4 Conclusion
Setup

- Time is discrete: \( t = 0, 1, 2, \cdots \).
- Closed economy with representative consumer who owns two types of firms: firm 1 and firm 2.
- Firms can produce \( y_t \) from labor \( L_t \):
  \[
  y_t = A_t L_t^\alpha,
  \]
  where the wage \( w_t L_t \) must be financed by cash and/or credit subject to a liquidity constraint, described later.
- The measure of consumers is normalized to one.
- There are continuum of firms 1 with measure \( \phi \) and continuum of firms 2 with measure \( 1 - \phi \).
A consumer can invest his cash in firm 1 as internal funds, while he cannot invest cash in firm 2.

The manager of firm 2 can divert cash for private purposes without getting penalty if the consumer invests cash in firm 2.

Optimization of a consumer is

\[
\max \sum \beta^t [\ln c_t + \gamma \ln(1 - l_t)],
\]

s.t. \( c_t + \phi m_t + b_t \leq w_t l_t + \phi \Pi \left( \frac{m_{t-1}}{\pi_t} + \Delta_t \right) + (1 - \phi) \Pi(0) - (1 + j_t) \phi \Delta_t + (1 + r_t) b_{t-1}, \)

where \( \Pi(m) \) is the profit from his own firm with internal funds \( m \).

Consumer can invest cash, \( \phi \left\{ \frac{m_{t-1}}{\pi_t} + \Delta_t \right\} \), in his own firm 1.

- \( m_{t-1} \) is the real money carried over from \( t - 1 \),
- \( \pi_t = P_t/P_{t-1} \) is the inflation from \( t - 1 \) to \( t \),
- \( \Delta_t \) is intra-period loan from Central Bank, for which \( j_t \) is interest rate.
Cash $\frac{m_{t-1}}{\pi_t} + \Delta_t$ is transferred from its owner (the consumer).

Given $\frac{m_{t-1}}{\pi_t} + \Delta_t$, the firm solves

$$V^1_{t-1} = \beta E_{t-1} \left[ \frac{\lambda_t}{\lambda_{t-1}} \left\{ \max\left[AL^\alpha_{1t} - w_tL_{1t}\right] + \frac{m_{t-1}}{\pi_t} + \Delta_t + V^1_t \right\} \right],$$

s.t. $w_tL_{1t} \leq \frac{m_{t-1}}{\pi_t} + \Delta_t + \theta V^1_t \quad (\mu_1)$

where $\Pi\left(\frac{m_{t-1}}{\pi_t} + \Delta_t\right) = AL^\alpha_{1t} - w_tL_{1t} + \frac{m_{t-1}}{\pi_t} + \Delta_t$. 
Liquidity constraint

\[ wL_{1t} \leq \frac{m_{t-1}}{\pi_t} + \Delta_t + \theta V_{1t} \]  \hfill (\mu_1)

is derived from the commitment problem (Kiyotaki and Moore 1997, Jermann and Quadrini 2012):

- Before production, firm 1 pays cash to the worker and promises to pay the remaining wage after production.
- After production, if the firm breaks the promise, the worker destroys the firm with probability \( \theta \), in which case the firm cannot operate from \( t + 1 \) on.
- As the firm loses the expected value \( \theta V_{1t} \) by breaking the promise, it can credibly pay the remaining wage as long as it is less than \( \theta V_{1t} \).
The firm solves

\[ V_{t-1}^2 = \beta E_{t-1} \left[ \frac{\lambda_t}{\lambda_{t-1}} \{ \max [AL_{2t}^\alpha - w_t L_{2t}] + V_t^2 \} \right], \]

\[ \text{s.t. } w_t L_{2t} \leq \theta V_t^2 \quad (\mu_2) \]

where \( \Pi(0) = AL_{2t}^\alpha - w_t L_{2t} \).
This time, we do not specify the objective of the government. We assume the following assumption.

- Government follows the exogenous policy rule:

\[ j_t = J(A_t, \theta_t; I_t), \]
\[ \Delta_t = D(A_t, \theta_t; I_t), \]

where \( I_t \) is all information available at time \( t \).

- The government is subject to the budget constraint:

\[ (1 + r_t)B_{t-1} + \frac{M_{t-1}}{\pi_t} = B_t + M_t + j_t \phi \Delta_t, \]

where \( B_t \) and \( M_t \) are supplies of bonds and cash. The upper case variables do not represent nominal variables, but they are real variables.
Setup – Summary

Consumer:  \[ \max \sum \beta^t [\ln c_t + \gamma \ln(1-l_t)], \]

s.t.  \[ c_t + \phi m_t + b_t \leq w_t l_t + \phi \Pi \left( \frac{m_{t-1}}{\pi_t} + \Delta_t \right) + (1 - \phi) \Pi(0) - (1 + j_t) \phi \Delta_t + (1 + r_t) b_{1t-1}, \quad (\lambda_{1t}) \]

Firm 1:  \[ V_{t-1}^1 = \beta E_{t-1} \left[ \frac{\lambda_t}{\lambda_{t-1}} \left\{ \max \left[ A_L^{\alpha} L_{1t} - w_t L_{1t} \right] + \frac{m_{t-1}}{\pi_t} + \Delta_t + V_t^1 \right\} \right], \]

s.t.  \[ w_t L_{1t} \leq \frac{m_{t-1}}{\pi_t} + \Delta_t + \theta V_t^1 \quad (\mu_1) \]

Firm 2:  \[ V_{t-1}^2 = \beta E_{t-1} \left[ \frac{\lambda_t}{\lambda_{t-1}} \left\{ \max \left[ A_L^{\alpha} L_{2t} - w_t L_{2t} \right] + V_t^2 \right\} \right], \]

s.t.  \[ w_t L_{2t} \leq \theta V_t^2 \quad (\mu_2) \]

Government:  \[ (1 + r_t) B_{t-1} + \frac{M_{t-1}}{\pi_t} = B_t + M_t + j_t \phi \Delta_t, \]
Equilibrium conditions

Equilibrium conditions

\[ c = Y = \phi AL_1^\alpha + (1 - \phi) AL_2^\alpha, \]
\[ l = \phi L_1 + (1 - \phi) L_2, \]
\[ \phi m = M, \]
\[ b = B. \]

Nominal interest rate: The consumer’s FOCs wrt \( \Delta \) and \( m_t \), and the envelope condition for \( \Pi(m) \) imply that

\[ j_t = \mu_{1t}. \]

The short-term nominal interest rate \( j_t \) equals the tightness of liquidity constraint for firm 1.
Introduction

Model Economy

- Setup
- Steady-state equilibrium
- Suboptimality of the Friedman rule
- Phillips curve and the liquidity effect

The Fisherian Deflation

Conclusion
Steady-state equilibrium

Given policy parameters \((j, \Delta)\), the steady-state equilibrium is given as a solution to the following 14 equations for 14 unknowns \((c, w, l, L_1, L_2, \pi, \mu_2, m, V^1, V^2, r, b, M, B)\).

\[
\begin{align*}
w &= \frac{\gamma c}{1 - l}, \\
1 + j &= 1 + \mu_1 = \frac{\pi}{\beta}, \\
wL_1 &= \frac{\alpha AL_1^\alpha}{1 + \mu_1}, \\
wL_1 &= \frac{m}{\pi} + \Delta + \theta V^1, \\
V^1 &= \frac{\beta AL_1^\alpha}{1 - (1 - \theta)\beta}, \\
wL_2 &= \theta V^2,
\end{align*}
\]
Steady-state equilibrium

\[ V^2 = \frac{\beta A L_2^\alpha}{1 - (1 - \theta)\beta}, \]  
\[ wL_2 = \frac{\alpha A L_2^\alpha}{1 + \mu_2}, \]  
\[ Y = c = \phi A L_1^\alpha + (1 - \phi) A L_2^\alpha, \]  
\[ l = \phi L_1 + (1 - \phi) L_2, \]  
\[ 1 + r = \beta^{-1}, \]  
\[ rB = \left(1 - \frac{1}{\pi}\right) M + \phi j \Delta, \]  
\[ b = B, \]  
\[ \phi m = M. \]
The degree of inefficiency is pinned down by $\pi$. The other policy variable ($\Delta$) determines the value of $m$.

- Variables $(w, L_1, L_2, Y, V^1, V^2)$ depend only on $\pi$, and independent of $\Delta$.
- The value of $w$ is given by

$$w^{\frac{1}{1-\alpha}} = \gamma \phi A^{\frac{1}{1-\alpha}} \left\{ \left( \frac{\alpha \beta}{\pi} \right)^{\frac{1}{1-\alpha}} + \left( \frac{\alpha \beta}{\pi} \right)^{\frac{\alpha}{1-\alpha}} \right\} + (1 - \phi) \gamma A^{\frac{1}{1-\alpha}} \left\{ \left( \frac{\theta \beta}{1 - (1 - \theta) \beta} \right)^{\frac{1}{1-\alpha}} + \left( \frac{\theta \beta}{1 - (1 - \theta) \beta} \right)^{\frac{\alpha}{1-\alpha}} \right\}.$$ (17)

- Redundancy of $\Delta$:
  - $\Delta$ does not affect the welfare.
  - There are continuously infinite combinations of $(m, \Delta)$ for given $\pi$. 

Welfare analysis on the steady state

- We focus on the deterministic steady-state equilibrium in which $j$ and $\Delta$ are constant.

- Inflation rate $\pi$ is pinned down by

$$1 + j = 1 + \mu_1 = \frac{\pi}{\beta}.$$ 

- Tightness of constraint for firm 2 does not depend on monetary policy:

$$1 + \mu_2 = \frac{\{1 - (1 - \theta)\beta\} \alpha}{\theta \beta}.$$ 

- We define the social welfare $W$ by

$$W = \frac{1}{1 - \beta} U(c, l) = \frac{1}{1 - \beta} \{\ln c + \gamma \ln(1 - l)\}.$$
Parameter values are

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\phi$</th>
<th>$\gamma$</th>
<th>$A$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.89</td>
<td>0.95</td>
<td>0.25</td>
<td>1.8</td>
<td>1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

As shown in the next slide, $W$ is maximized by the policy: $j = 0.11$, which implies $\pi = 1.0552$.

$\Rightarrow$ A moderate inflation is optimal!
1 Introduction

2 Model Economy
   • Setup
   • Steady-state equilibrium
   • Suboptimality of the Friedman rule
   • Phillips curve and the liquidity effect

3 The Fisherian Deflation

4 Conclusion
The welfare level in the steady state is maximized at
\[ \Delta = 0, \quad \pi^* = 1.0552, \quad Y^* = 0.349: \]
Suboptimality of the Friedman rule

- Output is maximized at the Friedman rule.
Why is the Friedman rule suboptimal?

- In the equilibrium,
  - all monetary distortion, which is represented by $\mu_1$, can be completely eliminated by setting $j = 0$ (or $\pi = \beta$) because $j = \mu_1$ in equilibrium.
  - The Friedman rule seems to be optimal...

- ... But it turns out that the Friedman rule is suboptimal in our model.
Rigorous derivation of suboptimality of the Friedman rule

- Social welfare is \( W = U/(1 - \beta) \), where

\[
U = \ln Y + \gamma \ln(1 - L),
\]

\[
Y = \phi AL_1^\alpha + (1 - \phi)AL_2^\alpha,
\]

\[
L_1 = \left( \frac{\alpha \beta A}{w(\pi)\pi} \right)^{\frac{1}{1-\alpha}},
\]

\[
L_2 = \left( \frac{\theta \beta A}{\{1 - (1 - \theta)\beta\}w(\pi)} \right)^{\frac{1}{1-\alpha}}.
\]

- We can show for the elasticity of wage rate wrt inflation \( \varepsilon(\pi) \),

\[
0 < \varepsilon(\pi) < 1,
\]

that

\[
d\frac{U}{d\pi} = [(1 - \phi)\varepsilon(\pi)\mu_2L_2 - (1 - \varepsilon(\pi))\phi\mu_1L_1] \frac{w}{(1 - \alpha)\pi Y}.
\]

- Suboptimality of the Friedman rule: \( \frac{dU}{d\pi} > 0 \) at \( \mu_1 = 0 \).
Economic intuition: input-smoothing effect

- Inflation tax on firm 1 is welfare enhancing: Input-smoothing effect
  - Inflation tax is imposed selectively on firm 1, not on firm 2.
    - Firm 1 can use cash to relax the liquidity constraint.
    - Firm 2 cannot use cash to relax the liquidity constraint.
  - Input-smoothing effect:
    - Inflation tax on firm 1 reduces firm 1’s demand for labor and decreases the wage rate $w$, which in turn increases firm 2’s demand for labor.
    - $\text{MPL of firm 2} > \text{MPL of firm 1}$. 
    - Decrease in total output is moderate compared to the decrease in labor.
    - Thus the inflation tax reduces disutility from labor more than utility from consumption of the goods.
  - Overall effect is welfare enhancing.
## Economic intuition: input-smoothing effect (cont’d)

- **Inflation:** second-best policy to reallocate labor from firm 1 to firm 2.
  - If there exists a policy that can reallocate labor from firm 1 to firm 2 without reducing total labor, it should be better than inflation.
    - Inflation reduces total labor.
    - Inflation reallocate the labor from firm 1 to firm 2.
    - The second effect dominates the first and improve total welfare.

- **Firm 1** represents old and traditional sectors, and **firm 2** young and emerging industries.
  - Old firms have easy access to funds, while young firms do not.
  - Inflation reallocates resources from old firms to young firms.
1 Introduction

2 Model Economy
   • Setup
   • Steady-state equilibrium
   • Suboptimality of the Friedman rule
   • Phillips curve and the liquidity effect

3 The Fisherian Deflation

4 Conclusion
Stochastic shocks

We consider two stochastic shocks in the economy.

- $A_t$: the productivity shock.
- $\theta_t$: the financial shock (Jermann and Quadrini 2012).

Given the policy rules:

$$j_t = \mu_{1t} = J(A_t, \theta_t; I_t),$$

$$\Delta_t = D(A_t, \theta_t; I_t),$$

we can calculate the dynamic response of the economy to these shocks $(A_t, \theta_t)$. 
2 Model Economy

2.4 Phillips curve and the liquidity effect

Dynamics

\[ w_t = \frac{\gamma c_t}{1 - l_t}, \quad (18) \]

\[ w_t L_{1t} = \frac{\alpha A_t L_1^{\alpha}}{1 + j_t}, \quad (19) \]

\[ w_t L_{2t} = \frac{\alpha A_t L_2^{\alpha}}{1 + \mu_{2t}}, \quad (20) \]

\[ w_t L_{1t} - \frac{m_{t-1}}{\pi_{t-1}} - \Delta_t = \theta_t V_t^1, \quad (21) \]

\[ w_t L_{2t} = \theta_t V_t^2, \quad (22) \]

\[ V_t^1 = \beta E_t \left[ \frac{c_t}{\tilde{c}_{t+1}} \left\{ \tilde{A}_{t+1} \tilde{L}_{1t+1}^{\alpha} - \tilde{w}_{t+1} \tilde{L}_{1t+1} + \frac{m_t}{\tilde{\pi}_t} - \tilde{j}_{t+1} \tilde{\Delta}_{t+1} + \tilde{V}_{t+1}^1 \right\} \right], \quad (23) \]

\[ V_t^2 = \beta E_t \left[ \frac{c_t}{\tilde{c}_{t+1}} \left\{ \tilde{A}_{t+1} \tilde{L}_{2t+1}^{\alpha} - \tilde{w}_{t+1} \tilde{L}_{2t+1} + \tilde{V}_{t+1}^2 \right\} \right], \quad (24) \]
\[ Y_t = c_t = \phi A_t L_{1t}^\alpha + (1 - \phi) A_t L_{2t}^\alpha, \quad (25) \]
\[ l_t = \phi L_{1t} + (1 - \phi) L_{2t}, \quad (26) \]
\[ (1 + r_t)B_{t-1} + \frac{M_{t-1}}{\pi_{t-1}} = B_t + M_t + j_t \phi \Delta_t, \quad (27) \]
\[ 1 = \beta E_t \left[ \frac{c_t}{\tilde{c}_{t+1}} \frac{(1 + \tilde{j}_{t+1})}{\tilde{\pi}_t} \right], \quad (28) \]
\[ 1 = \beta E_t \left[ \frac{c_t}{\tilde{c}_{t+1}} (1 + \tilde{r}_{t+1}) \right], \quad (29) \]
\[ M_t = \phi m_t, \quad (30) \]
\[ B_t = b_t. \quad (31) \]
Consider the dynamic response of the model in which

- $\theta_t$ changes exogenously,
- $(j_t, \Delta_t)$ are kept at steady-state values ($j_t = \mu_{1t} = \bar{\mu}_1$, $\Delta = 0$).
Phillips curve
Phillips curve
Intuition for the Phillips curve

Phillips curve relationship is an artifact generated by the responses of output and inflation to the financial shocks.

\[ \theta \uparrow \implies Y \uparrow \quad \text{and} \quad \theta \uparrow \implies \pi \uparrow \]

\[ \theta \downarrow \implies Y \downarrow \quad \text{and} \quad \theta \downarrow \implies \pi \downarrow \]
Liquidity effect

Consider the dynamic response of the model in which

- there is no exogenous shock: $\theta_t$ and $A_t$ are constant,
- $j_t = \mu_{1t}$ is changed by monetary policy,
- $\Delta_t$ is kept at the steady-state value ($\Delta_t = 0$).
Liquidity effect

\[ \mu_1, \mu_2, M, Y, \pi, w \]
Liquidity effect
Intuition for the liquidity effect

- As $j_t = \mu_{1t}$ increases, liquidity constraints for firms 1 become tighter.
- Output decreases as liquidity constraints become tighter.
- Money demand decreases as the nominal short-term rate $j_t$ increases.

\[
\begin{align*}
  j_t \uparrow &\implies Y \downarrow \quad \text{and} \quad j_t \uparrow &\implies M \downarrow \\
  j_t \downarrow &\implies Y \uparrow \quad \text{and} \quad j_t \downarrow &\implies M \uparrow
\end{align*}
\]
1 Introduction

2 Model Economy
   • Setup
   • Steady-state equilibrium
   • Suboptimality of the Friedman rule
   • Phillips curve and the liquidity effect

3 The Fisherian Deflation

4 Conclusion
Accounting for decade-long deflation

- A simple explanation: The Fisher relation in the steady state

\[ 1 + j = \pi(1 + r) = \frac{\pi}{\beta}. \]

- If \( j \) is fixed at zero, then the inflation rate should be negative (\( \pi < 1 \)), as the real rate of interest \( r = \beta^{-1} - 1 \) is positive.
- If the following Ricardian expectations on fiscal policy prevail, then permanent deflation is compatible with increasing money supply.

- Ricardian expectations:
  In the far future, tax will be increased so that the government budget constraint is satisfied under permanent ZIRP.
Why deflation is associated with low output?

- In response to a negative financial shock the zero nominal interest rate policy (ZIRP, i.e., the Friedman rule) can maximize the output by relaxing the liquidity constraint.

- If the negative financial shock is permanent,
  - ZIRP can maximize the output of firm 1, while the level of total output is permanently lower than in the initial steady state,
  - ZIRP creates the equilibrium deflation by the Fisher relation,
  - ZIRP is not the long-run “optimal” policy. (ZIRP maximizes output, while it does not maximize welfare.)

- Permanent financial shock may represent structural changes in financial sector or in financial regulations.
1 Introduction

2 Model Economy
   - Setup
   - Steady-state equilibrium
   - Suboptimality of the Friedman rule
   - Phillips curve and the liquidity effect

3 The Fisherian Deflation

4 Conclusion
We construct a flexible-price model for monetary policy analysis. The model features:

1. suboptimality of the Friedman rule due to the input-smoothing effect,
2. the Phillips curve created by equilibrium response to financial shocks,
3. the liquidity effect.

ZIRP (i.e., the Friedman rule) enhances efficiency by relaxing the liquidity constraint, whereas it generates the equilibrium deflation in the long run by the Fisher relation.