

A Theory of Public Debt Overhang

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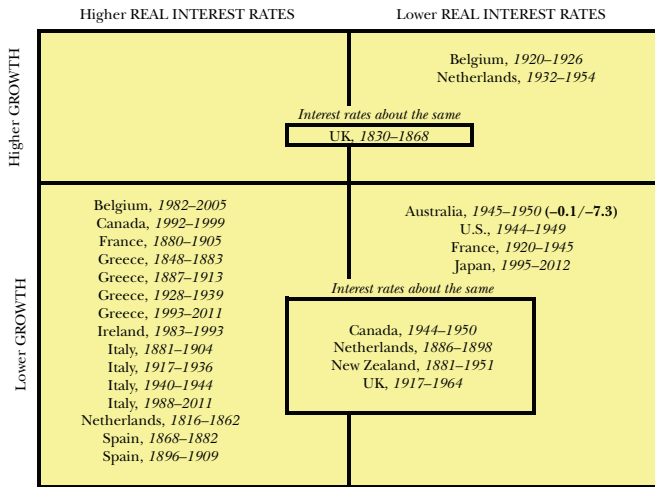
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Public debt overhang – Evidence

- Reinhart, Reinhart, and Rogoff (2012)
 - Decade-long stagnation in debt-ridden economy.
 - Growth rate declines by 1 % as public debt exceeds 90 % of GDP.
 - In some cases, low growth is associated with low interest rates (eg., Japan).



Source: Reinhart, Reinhart, Rogoff(2012)

What we do

- we propose a simple model in which
 - unsustainable debt undermines credibility of government's commitment,
 - commitment problem discourages adoption of new technology, leading to low growth and a low interest rate.
 - fiscal consolidation restores sustainability of public debt,
 - and leads to high growth and a high interest rate by restoring credibility of government commitments (basic model).
 - but cannot restore credibility of government commitments if it comes too late (second model).

Related literature

- Empirics

- Reinhart, Reinhart and Rogoff (2012)
26 episodes of advanced economies
- Checherita-Westphal and Rother (2012)
12 euro countries
- Baum, Checherita-Westphal and Rother (2013)

- Crowding out

- expansionary fiscal policy leads to low investment associated with high interest rates. Galí, López-Salido and Vallés (2007), Romer (2011).

Related literature

- Non-Keynesian effect (Giavazzi and Pagano 1990, Bertola and Drazen 1993, Perotti 1999)
 - expansionary fiscal policy leads to low consumption.
 - inefficiency is caused by expectations of one-time tax distortion in the future.
 - it is not consistent with a decade of low growth.

Related literature

- Growth theory
 - Diamond (1960): public debt enhances growth in a neoclassical model.
 - Saint-Paul (1992): public debt lowers growth in an endogenous growth model.
 - Brauning (2005): multiple equilibria.
 - Arai, Kunieda, Nishida (2012); Futagami, Hori, and Ohdoi (2010): crowding out and higher interest rate when growth slows down.
- Political economy (Acemoglu and Robinson 2005, Acemoglu, 2009)
 - commitment problem due to political conflict lowers investment.

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Benchmark model without public debt overhang

- Continuous time AK model. Capital does not depreciate.
- a consumer, a government, and N firms. ($N \gg 1$)
- Firms are owned by the consumer.
- the consumer's utility:

$$\int_0^{\infty} e^{-\rho t} \ln c_t, \quad \text{where } 0 < \rho.$$

- the incumbent government wants to maximize the length of its tenure T . If $T = \infty$ the government maximizes welfare of consumers.
- A firm borrows $\frac{k_t}{N}$ from the consumer and produce the consumption good in period t .

Benchmark model without public debt overhang

- Two technologies, A and B

$$0 < B < A.$$

- Technology A: a firm with k_t can produce Ak_t in period t . The government can impose the output tax $\tau_{kt}Ak_t$, where $0 \leq \tau_{kt} \leq 1$.
- Technology B: a firm with k_t can produce Bk_t in period t . The government cannot impose the output tax on Bk_t .
- Without public debt overhang, the government has no incentive to impose output tax.
- Thus firms choose technology A.

Benchmark model without public debt overhang

- consumer's problem

$$\begin{aligned} \max_{c_t, k_t} \int_0^{\infty} e^{-\rho t} \ln c_t, \\ \text{s.t. } c_t + \dot{k}_t = R_t k_t + X_t. \end{aligned}$$

- a firm's problem

$$\begin{aligned} \max_y y - R_t \frac{k_t}{N}, \\ \text{s.t. } y \in \left\{ A \frac{k_t}{N}, B \frac{k_t}{N} \right\} \end{aligned}$$

Benchmark model without public debt overhang

- government's problem
 - tenure of incumbent government: $T = \infty$.
 - tax policy does not change the length of tenure.
 - government has no incentive to set $\tau_{kt} \neq 0$.
 - It is credible that $\tau_{kt} = 0$.
⇒ Firms choose technology A.

Benchmark model without public debt overhang

- Equilibrium
 - Firms choose technology A .
 - Government chooses $\tau_{kt} = 0$
 - standard AK outcome:

$$R_t = A,$$

$$X_t = 0,$$

$$k_t = k_0 e^{(A-\rho)t},$$

$$c_t = \rho k_t,$$

$$\zeta_t = \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = A - \rho.$$

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Public debt overhang

- Outstanding government debt b_0 .
- No tax is imposed in $t = 0$.
- For $0 \leq t (< T)$, consumer's problem is

$$\begin{aligned} \max_{c_t, \dot{k}_t, \dot{b}_t} \int_0^{\infty} e^{-\rho t} \ln c_t, \\ \text{s.t. } c_t + \dot{k}_t + \dot{b}_t = R_t k_t + r_t b_t + X_t. \end{aligned}$$

- Debt evolves by

$$\dot{b}_t = r_t b_t.$$

Two taxes

- Lump-sum tax and output tax
 - Lump-sum tax, τ_{t+1} , on the consumer.
 $\tau_t \in [0, \tau]$.
 - The government loses power if the lump-sum tax is imposed.
 - If the lump-sum tax is imposed at T , the tenure of incumbent government is terminated at T .
 - Output tax, $\tau_{kt}Ak_t$, on the firms.
 $\tau_{kt} \in [0, 1]$.
 - Government can impose output tax without any cost.
 - Output by technology A is taxable, while output by technology B is **not** taxable.
 - Output tax can be imposed after observing production.

Transversality condition

- Government can continue until T , where

$$T = \arg \max_T b_T,$$

$$\text{s.t. } \dot{b}_t = r_L b_t,$$

$$r_H b_T \leq \tau,$$

where $r_H = A$ and $r_L = B$.

Transversality condition

- Lemma. Given that the lump-sum tax $\tau_t = \tau$ is imposed in period T , the TVC from T on is satisfied iff

$$r_H b_T \leq \tau.$$

- Debt evolves by $\dot{b}_t = r b_t - \tau_t$ for $t \geq T$, where $\tau_t = \tau$ if $b_t > 0$ and $\tau_t = 0$ if $b_t = 0$.
- Thus $b_{t+T} = \max\left\{0, \frac{\tau}{r} + \left(b_T - \frac{\tau}{r}\right) e^{rt}\right\}$ if $b_T \leq \frac{\tau}{r}$;
and $b_{t+T} = \frac{\tau}{r} + \left(b_T - \frac{\tau}{r}\right) e^{rt}$ if $b_T > \frac{\tau}{r}$.
- TVC is $\lim_{t \rightarrow \infty} e^{-rt} b_t = 0$, which is equivalent to $b_T \leq \frac{\tau}{r}$.
- can show $r \neq r_L$ by contradiction: if $r = r_L$ and TVC is satisfied, the government has no incentive to impose output tax; then firms choose technology A, leading the interest rate to $r = r_H$.
- The equilibrium interest rate is $r_t = r_H$ for $t \geq T$.

Transversality condition

- Define $r_H b_T = \tau$. Incumbent government cannot stay for $t > T$.
 - Consumer does not buy b_t unless τ is introduced.
 - If τ is introduced at T , incumbent loses power.
 - If τ is not introduced at T , incumbent loses power because default on government bond occurs at T .
- Consumers accept to buy b_t for $t > T$ iff τ is introduced and TVC is satisfied.

Lack of Commitment

- Firms choose technology B due to political economy distortion (Acemoglu 2009):
 - At every period t , government decides whether to introduce output tax after observing firms' choice of technology (A or B).
 - Government cannot precommit **not to introduce output tax**.
 - Equilibrium is Markov Perfect Equilibrium. Punishment strategy is excluded.

Lack of Commitment

- If firms choose technology A, government imposes output tax and sets $\tau_{kt} = 1$. (government takes all output.)
 - Government can extend the tenure T by imposing output tax if technology A is adopted.
 - In continuous time model, tax revenue at t is infinitesimally small and does not affect the length of tenure T ?
We can justify the above statement by the following argument:
 - We assume that if firms choose technology A at t they cannot change technology until $t + \Delta$, where Δ ($\ll 1$) is a very short time interval.
 - We assume that capital stock allocated to one firm cannot be reallocated to other firms; in other words, capital stock of each firm k_t must satisfy $k_s \geq k_t$ for all $s \geq t$.
 - Suppose that a firm that owns $\frac{k_t}{N}$ chooses technology A at t .
 - At $t' = t + \Delta$, the government debt $b_{t'}$ becomes

$$b_{t'} \approx b_t + \left(r_t b_t - A \frac{k_t}{N} \right) \Delta < b_t + r_t b_t \Delta.$$

- Since the amount of debt is lower by approximately $A \frac{k_t}{N} \Delta$ with output tax than without it, the tenure can be extended by $O(\Delta)$ by imposing output tax if firms adopt technology A.
- Anticipating this, all firms choose technology B for $0 \leq t < T$.

Equilibrium with Public Debt Overhang

- $T = \arg \max_t b_t$ subject to $\dot{b}_t = r_L b_t$ and $r_H b_t \leq \tau$.
- Firms choose technology B for $0 \leq t < T$.

$$r_t = \begin{cases} r_L = B, & \text{for } 0 \leq t < T, \\ r_H = A, & \text{for } t \geq T, \end{cases}$$

$$b_t = \begin{cases} b_0 e^{r_L t}, & \text{for } 0 \leq t < T, \\ b_T, & \text{for } t \geq T, \end{cases}$$

$$k_t = \begin{cases} k_0 e^{(r_L - \rho)t}, & \text{for } 0 \leq t < T, \\ k_T e^{(r_H - \rho)(t - T)}, & \text{for } t \geq T, \end{cases}$$

$$c_t = \rho k_t.$$

Equilibrium with Public Debt Overhang

- Growth rate is low before fiscal consolidation, while it is high after fiscal consolidation.

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \zeta_t = r_L - \rho, \quad \text{for } 0 \leq t < T,$$
$$\frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \zeta_t = r_H - \rho, \quad \text{for } t \geq T,$$

- Interest rate is low before fiscal consolidation, while it is high after fiscal consolidation.
- Output tax is not imposed in equilibrium.

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Modified model – No credible fiscal policy

- AK model
- Output tax τ_k is already imposed.
- Two technology. A and B

$$A - \gamma < B < A$$

- Technology A: Firm produces Ak_t . Firm needs to pay output tax $\tau_k Ak_t$ and political rent γk_t to consumer.
 - γk_t represents rent associated with education of workers and/or investment in infrastructure for new technology A.
- Technology B: Firm produces Bk_t . Firm needs to pay output tax $\tau_k Bk_t$. Firm with technology B does not have to pay political rent.

Modified model – Description of fiscal policy

- Fiscal policy g_t : government pays $g_t k_t$ to firms that use technology A.
 - $g_t k_t$ represents redistribution associated with education and/or public investment in infrastructure for new technology.
 - Profit of firm that uses technology A

$$[(1 - \tau_k)A - \gamma + g_t]k_t$$

- Profit of firm that uses technology B

$$(1 - \tau_k)Bk_t$$

- Firm chooses A if

$$g_t \geq \gamma - (1 - \tau_k)(A - B).$$

- Tax revenue decrease if government pays g_t :

$$\tau_k A k_t - g_t \leq \tau_k B.$$

Modified model – Government debt

- Large initial value $d_0 > \frac{\tau_k B k_0}{\rho} > 0$.
- Debt evolves by $\dot{b}_t = r_t b_t - \tau_k B k_t$. Therefore,

$$b_t = \frac{\tau_k B k_0}{\rho} e^{(r_L - \rho)t} + \left(b_0 - \frac{\tau_k B k_0}{\rho} \right) e^{r_L t}.$$

Debt diverges at the rate r_L .

- Lump-sum tax, τ
 - Government can impose lump-sum tax τ on consumer to restore TVC.
 - If lump-sum tax is introduced at T , the incumbent's tenure is terminated at T .

Modified model – Lack of commitment

- Government decides whether to pay $g_t k_t$ after observing choice of technology A or B.
- If firms choose technology A,
 - government revenue increases: $\tau_k A k_t$ ($> \tau_k B k_t$) if the government does not pay $g_t k_t$.
 - government revenue decreases: $(\tau_k A - g_t) k_t$ ($< \tau_k B k_t$) if it pays $g_t k_t$, where $g_t > \gamma - (1 - \tau_k)(A - B)$.

Modified model – Lack of commitment

- government wants to extend its tenure, T , by increasing revenue.
- government does not pay g_t if firms choose A.
 - In continuous time model, tax revenue at t is infinitesimally small and does not affect the length of tenure T ? We can justify the above statement by the following argument:
 - We assume that if firms choose technology A at t they cannot change technology until $t + \Delta$, where Δ ($\ll 1$) is a very short time interval.
 - Observing firms' choice of technology at t , government decides whether to pay gk_s for $s \geq t$.
 - The amount of debt is lower by approximately $gk_t\Delta$ when government pays subsidy than when it does not.
 - The tenure is strictly longer by $O(\Delta)$ when government does not pay gk_t than when it does, whatever technology firms choose.
 - Government does not pay gk_t .
- anticipating this, all firms choose B.

Modified model – Optimization problems

- Consumer's problem

$$\max \int_0^{\infty} e^{-\rho t} \ln c_t,$$

$$\text{s.t. } c_t + \dot{k}_t + \dot{b}_t = R_t k_t + r_t b_t + X_t.$$

- Firm's problem: $\max [\max \{(1 - \tau_k)A - \gamma + g_t, (1 - \tau_k)B\} - R_t, 0]$.

Firms choose A iff $g_t \geq \gamma - (1 - \tau_k)(A - B)$.

- Government's problem

$$\max_{g_t} T,$$

$$\text{s.t. } \begin{cases} \dot{b}_t = r_t b_t - (\tau_k A - g_t) k_t \mathbf{1}(g_t) - \tau_k B k_t (1 - \mathbf{1}(g_t)), \\ g_t \geq \gamma - (A - B), \\ \text{and TVC.} \end{cases}$$

Modified model – Transversality condition

- TVC is either

$$b_T \leq \frac{\tau}{r_H} + \frac{(\tau_k A - g)k_T}{\rho} \equiv B_E(T), \quad (1)$$

or

$$b_T \leq \frac{\tau}{r_L} + \frac{\tau_k B k_T}{\rho} \equiv B_L(T). \quad (2)$$

- $B_E(T) < B_L(T)$ for all T .
- TVC is determined by the expectations on the path after introduction of lump-sum tax.

Modified model – Transversality condition

- After imposition of lump-sum tax, debt evolves by

$$\dot{b}_t = rb_t - \tau - \Gamma k_t, \quad \text{for } t \geq T,$$

where $(r, \Gamma) = (r_H, \tau_k A - g)$ or $(r_L, \tau_k B)$, and $k_t = k_T e^{(r-\rho)(t-T)}$.

- $(r, \Gamma) = (r_H, \tau_k A - g)$
if expectation is that technology A is dominant for $t \geq T$.
- $(r, \Gamma) = (r_L, \tau_k B)$
if expectation is that technology B is dominant for $t \geq T$.
- Solution is

$$b_{t+T} = x + ye^{(r-\rho)t} + (b_T - x - y)e^{rt},$$

where $x = \frac{\tau}{r}$,

$$y = \frac{\Gamma k_T}{\rho}.$$

Modified model – Transversality condition

- Consumer's Transversality condition is

$$\lim_{t \rightarrow \infty} b_{t+T} e^{-rt} = 0.$$

- TVC is equivalent to

$$b_T \leq x + y,$$

where $x + y$ is either $B_E(T)$ or $B_L(T)$.

Modified model – Good Equilibrium

- Lump-sum tax is imposed at T_E , which is defined by $b_T = B_E(T)$.
- Tenure of new government is ∞ . Credibility of fiscal policy is restored.
- Firms choose technology A and new government pays gk_t to firms for $t \geq T_E$, where $g = \gamma - (1 - \tau_k)(A - B)$.

$$r_t = \begin{cases} r_L = B, & \text{for } 0 \leq t < T_E, \\ r_H = A, & \text{for } t \geq T_E, \end{cases}$$

$$b_t = \begin{cases} \tau_k B k_0 \rho^{-1} e^{(r_L - \rho)t} + (b_0 - \tau_k B k_0 \rho^{-1}) e^{r_L t}, & \text{for } 0 \leq t < T_E, \\ \tau r_H^{-1} + (\tau_k A - g) k_T \rho^{-1} e^{(r_H - \rho)t}, & \text{for } t \geq T_E, \end{cases}$$

$$k_t = \begin{cases} k_0 e^{(r_L - \rho)t}, & \text{for } 0 \leq t < T_E, \\ k_T e^{(r_H - \rho)(t - T)}, & \text{for } t \geq T_E, \end{cases}$$

$$c_t = \rho k_t.$$

Modified model – Bad Equilibrium

- Lump-sum tax is imposed at T_L , which is defined by $b_T = B_L(T)$.
- Tenure of new government becomes ∞ .
- Too late to restore credibility of fiscal policy.
 - Proof by contradiction: Suppose fiscal policy is credible for $t \geq T_L$; firms would choose A and interest rate be $r_t = r_H$; because of high rate r_H , debt would increase exponentially; the equilibrium path would be unsustainable for $t \geq T_L$.
- Firms choose technology B and new government does not pay gk_t to firms for $t \geq T_L$.

Modified model – Bad Equilibrium

- Bad equilibrium:

$$r_t = r_L = B, \quad \text{for } t \geq 0,$$

$$b_t = \begin{cases} \tau_k B k_0 \rho^{-1} e^{(r_L - \rho)t} + (b_0 - \tau_k B k_0 \rho^{-1}) e^{r_L t}, & \text{for } 0 \leq t < T_L, \\ \tau r_L^{-1} + \tau_k B k_T \rho^{-1} e^{(r_L - \rho)t}, & \text{for } t \geq T_L, \end{cases}$$

$$k_t = k_0 e^{(r_L - \rho)t}, \quad \text{for } t \geq 0,$$

$$c_t = \rho k_t.$$

Modified model – Comparison

- Good Equilibrium

- Growth rate

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \zeta_t = \begin{cases} r_L - \rho, & \text{for } 0 \leq t < T_E, \\ r_H - \rho, & \text{for } t \geq T_E. \end{cases}$$

- Interest rate

$$r_t = \begin{cases} r_L, & \text{for } 0 \leq t < T_E, \\ r_H, & \text{for } t \geq T_E. \end{cases}$$

- Bad Equilibrium

- Growth rate

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \zeta_t = r_L - \rho, \quad \text{for } t \geq 0.$$

- Interest rate

$$r_t = r_L, \quad \text{for } t \geq 0.$$

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Conclusion

- We analyzed
 - effect of unsustainable public debt on technology choice and economic growth.
- We have shown
 - unsustainable debt undermines credibility of government policy because government will do whatever possible to postpone fiscal consolidation.
 - lack of commitment makes choice of technology inefficient.
- Fiscal consolidation can restore credibility and high growth (basic model).
- Fiscal consolidation may not be able to restore credibility and growth if it comes too late (second model).