Debt-Ridden Borrowers and Productivity Slowdown

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Motivation 1: Productivity slowdown after a financial crisis

- Decade after a financial crisis
  - Low growth (Reinhart and Rogoff 2009, Reinhart and Reinhart 2010)
- Why productivity slows down persistently after a financial crisis?
Decade after a crisis: the 1990s Japan

<table>
<thead>
<tr>
<th>Period</th>
<th>HP</th>
<th>KI</th>
<th>JIP2011</th>
</tr>
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<tbody>
<tr>
<td>1971–80</td>
<td>0.83</td>
<td></td>
<td>1.68</td>
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<td>1981–90</td>
<td>1.93</td>
<td>2.06</td>
<td>1.39</td>
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<td>1991–2000</td>
<td>0.36</td>
<td>0.35</td>
<td>0.04</td>
</tr>
<tr>
<td>2001–2007</td>
<td>0.48</td>
<td></td>
<td>1.13</td>
</tr>
</tbody>
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Note: HP, KI, JIP2011 are from updated versions of Hayashi and Prescott(2002), Kobayashi and Inaba (2006), and Fukao and Miyagawa (2008).

Table: TFP growth rate in Japan
Motivation 2: Cause of financial shocks

- Great Recession
  - Financial shocks (Jermann and Quadrini 2012)
  - Risk shock (Christiano, Motto, and Rostagno 2010)
  - Shock to quality of capital (Gertler and Kiyotaki 2009)
  - These shocks tightened the financial constraints after a financial crisis.

- It is assumed that these financial shocks are **exogenous**.

- What causes these shocks?
  (Why are financial constraints tightened?)
**Hypothesis**

1. Exogenous shock (e.g., bubble collapse) makes a large fraction of firms default or debt-ridden.
2. Borrowing constraints for firms with excessive debt (or debt-ridden firms) are endogenously tightened.
   - We can analyze the borrowing constraints for working capital loans of debt-ridden firms by Modification of Jermann-Quadrini’s bargaining
3. Tightened borrowing constraint lowers the aggregate productivity.
   - Tighter constraint on working capital loan lowers observed TFP (Chari, Kehoe, and McGrattan)
**Jermann-Quadrini (JQ)’s bargaining**

- **Borrowing constraint derived from bargaining**
  - Borrowing is bounded by limited enforceability of debt contract.
  - Firm can default on the debt.
  - If firm defaults, the firm and the lender renegotiate on repayment $f$.
    - If renegotiation breaks down the firm is liquidated and the lender confiscates the collateral, $\phi q_t k_t$.
    - If they agree the firm pays $f$ and continues as a normal firm.
  - Nash bargaining

$$\max_f (V - f)^\sigma (f - \phi q_t k_t)^{1-\sigma}$$

- In the limit of $\sigma \to 1$ the bargaining outcome is $f = \phi q_t k_t$
- No-default condition: $m + b \leq f = \phi q_t k_t$. 
What we do: Modification of JQ’s bargaining to derive borrowing constraint of debt-ridden firm

- We define “debt-ridden firm” as an intermediate status between a normal firm and liquidation.
- Debt-ridden firm = defaulted already and not liquidated yet
- Institutional setting motivated by Japan’s 1990s:
  - The status of debt-ridden firm is not derived as outcome of optimal contracting, but is an institutional setting.
  - Lender never releases firm if it defaults on the debt.
  - Defaulter must become either debt-ridden firm or liquidated. It cannot go back to a normal firm unless it repays all the original debt.
What we do: Definition of debt-ridden firm

- Debt-ridden firm
  - Lender can decide whether the firm continue next period or not.
  - Lender allows firm to continue if it promises to pay continuation fee $d_{t+1}$.
  - Continuation fee $d_t$ determined by the bargaining between the debt-ridden firm and the lender.
Summary

Model Setup:

- An exogenous shock makes firms default.
  - Redistribution shock that changes the amount of debt
    \[ b = b' + \Delta. \]
- If firm defaults, it must become either debt-ridden firm or liquidated. It cannot return to normal firm unless repaying all original debt.
Summary

- Result 1: Borrowing constraint for debt-ridden firms
  - The borrowing constraint for working capital loan is tightened. It is tighter for debt-ridden firms than for normal firms.
    - Counterintuitive. Lenders should have more influence on debt-ridden firms than on normal firms.
    - Lenders could obtain more from debt-ridden firms than from normal firms in the bargaining.
    - Then the borrowing constraint for debt-ridden firms could be looser than for normal firms.
    - Our analysis shows the opposite.
Summary

- Result 2: Tight borrowing constraint lowers aggregate productivity
  - Mass default ⇒ Emergence of debt-ridden firms
  - Tighter borrowing constraint on working capital
    - reduces the demand for inputs by debt-ridden firm, leading to inefficient production (Direct effect)
    - raises price of capital $q_t$ because of higher demand for collateralizable asset by debt-ridden firms (Congestion effect)
      ⇒ Higher cost for entry of new firms
      ⇒ No entry of new firms
      ⇒ Aggregate productivity becomes permanently low
Summary

- Result 2’: Endogenous growth version of our model $\Rightarrow$ Zero Growth Path
  - We assume externality that enables endogenous growth.
  - Tight borrowing constraint raises the price of collateralizable capital $q_t$.
  - Higher $q_t$ depresses the new entry of firms
  - No new entry leads to no growth in productivity
    $\Rightarrow$ Zero productivity growth
- Externality induces multiple equilibria
  - Zero Growth Path (ZGP)
  - Balanced Growth Path (BGP)
Literature

- Debt overhang (Myers 1977, Lamont 1995, Philippon 2009)
  Debt holder is different from a lender of new money.
  - Debt holder is the lender of new money in our model.

- Zombie lending (Caballero, Hoshi, Kashyap 2008)
  Zombie firms are intrinsically unproductive. Zombie lending is inefficient subsidy from banks to unproductive firms.
  - Debt-ridden firms are not intrinsically unproductive in our model.

- Bubbles (Hirano and Yanagawa 2010, Aoki and Nikolov 2011)
  Collapse of bubble tightens borrowing constraints.
  - We consider default on debt and bargaining after default.
Model

- Expanding Variety Model (Rivera-Batiz, Romer 1991)
  - Intermediate good $i \in [0, N_t]$: Monopolistic competition.
  - R&D investment $I_t$ expands $N_t$.
  - Capital $K = 1$ and labor $L = 1$ are fixed supply.


- Household:

\[
\max E_0 \left[ \sum_{t=0}^{\infty} \beta^t \ln C_t \right],
\]

subject to

\[
C_t + \frac{b_{t+1}}{1 + r_t} + I_t \leq w_t L + \int_0^{N_t} \pi_{it} \, di + b_t,
\]

\[
N_{t+1} = (1 - \delta)N_t + \chi I_t,
\]

\[
I_t \geq 0.
\]
Model

Household: R&D investment \( I_t = 0 \) if \( \chi V_{nt} < 1 \) and \( I_t > 0 \) if \( \chi V_{nt} = 1 \), where

- \( V_{nt} \) is the value of a new firm,
- \( \chi V_{nt} \) is the value of one unit of R&D investment.

Final good is produced in the competitive market:

\[
\max Y_t - \int_0^{N_t} p_{it} x_{it} di - w_t L_t,
\]

subject to

\[
Y_t = \frac{1}{\eta} \left( \int_0^{N_t} x_{it}^{\eta} di \right) L^{1-\eta}.
\]
Model

- Intermediate good $i$ produced by firm $i$:
  \[
  x_{it} = A_{it} k_{it}^{\alpha} m_{it}^{1-\alpha}.
  \]

- Capital (land) is fixed supply: \( \int_{0}^{N_t} k_{it} \, di = K \). Price is \( q_t \).

- Material input $m_{it}$ is the final good.

Demand  \( p_{it} = p(x_{it}) = L^{1-\eta} x_{it}^{\eta-1} \),

Productivity parameter

\[ A_{it} = A \quad \text{for all } i \text{ and } t. \]
Redistribution shock in this economy

- Firm $i$ borrows inter-temporal debt $\frac{b'_{it+1}}{1+r_t}$ at the end of period $t$
- Redistribution shock changes the amount to be repaid from $b'$ to $b$:

  $$b_{it+1} = b'_{it+1} + \Delta_{it+1}.$$  

- PDF of the redistribution shock $\Delta_{it}$ is known.
Bellman equation for normal firms

- Bellman eq: $k$ and $b'$ chosen at $t$, and $m$ chosen at $t + 1$

$$V_{nt} = \max_{k,b'} \frac{b'}{1 + r} - qk + E \left[ \max_m \frac{\beta \lambda'}{\lambda'} \left\{ p(x)x - m - b + q'k + \tilde{V} \right\} \right],$$

(1)

s. t.  

$$p(x) = L^{1-\eta}x^\eta,$$

(2)

$$x = A_{t+1}k^\alpha m^{1-\alpha},$$

(3)

$$b = b' + \Delta,$$

(4)

$$m \leq \phi q_{t+1}k_{t+1} + V_{nt+1} - V_{zt+1} - b.$$  

(5)
Derivation of Borrowing Constraint

- Borrowing constraint for normal firm:
  \[ m \leq \phi q_{t+1} k_{t+1} + V_{nt+1} - V_{zt+1} - b. \]

- Borrowing is bounded by limited enforceability of debt contract. \( \Rightarrow \) Firm can default on their obligation.

- *Lender cannot forgive debt of defaulter*
  \( \Rightarrow \) If defaults, either firm continues as debt-ridden or it is liquidated, unless it repays all the original debt.
What they do after default?

- Lender and firm have two stage bargaining after default:
  - First Bargaining in the middle of period on repayment of current debt, $f (< m + b)$.
    - If bargaining breaks down $\Rightarrow$ Liquidation. Lender obtains $(\phi + \psi)q_t k_t$
  - Second Bargaining at the end of period on continuation fee, $d$.
    - If bargaining breaks down $\Rightarrow$ Liquidation. Lender obtains $\psi q_t k_t$
  - A part of collateral $\phi q_t k_t$ is diverted by firm before the second stage bargaining.
Structure of Two stage bargaining after default

- First stage in the middle of period on repayment $f$.
  - If they do not agree, firm is liquidated.
    - Lender obtains $(\phi + \psi)q_t k_t$
    - Firm obtains $p(x)x + (1 - \phi - \psi)q_t k_t$
  - If they agree, firm continue as debt-ridden firm.
    - Lender obtains $f + D_t$
    - Firm obtains $p(x)x + q_t k_t - f + V_z$

- Second stage at the end of period on continuation fee $d$
  - If they do not agree, firm is liquidated.
    - Lender obtains $\psi q_t k_t$
    - Firm obtains $(1 - \psi)q_t k_t$
  - If they agree, firm continue as debt-ridden firm.
    - Lender obtains $D_t = \beta E \left[ \frac{\lambda_{t+1}}{\lambda_t} \{d_{t+1} + D_{t+1}\} \right]$
    - Firm obtains $q_t k_t + V_z$
Bargaining outcome

- We assume that firm has all the bargaining power.
  - Lender’s payoff equals the liquidation value at each stage.

First stage: \( f + D_t = (\phi + \psi)q_t k_t \),
Second stage: \( D_t = \psi q_t k_t \)

- First stage bargaining:
  \[ f = \phi q_t k_t. \]

- Second stage bargaining:
  \[ D_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \{d_{t+1} + D_{t+1}\} \right] = \psi q_t k_t. \]
No default condition $\Rightarrow$ Borrowing constraint

- If a normal firm defaults
  - Firm pays $f = \phi q k$.
  - Firm loses $V_{nt+1} - V_{zt+1}$, because it becomes debt-ridden firm inevitably.
- If the normal firm does not default
  - Firm pays $m + b$ and continues operation as a normal firm.
- No default condition for a normal firm:

\[
m + b \leq f + V_{nt+1} - V_{zt+1} = \phi q k + V_{nt+1} - V_{zt+1}.
\]
Appendix: Comparison with the Jermann-Quadrini model

- Jermann-Quadrini: Lender forgive debt if firm defaults.
  - Firm goes back to be a normal firm if it repays \( f \).
- Our model: Lender cannot forgive debt once firm defaults.
  - Firm cannot be released even if it repays \( f \) if \( f < m + b \).
  - Firm can be released only if it repays the original debt \( m + b \).
  - Defaulter must become either debt-ridden or liquidated.

- Borrowing limit in our model:
  - Borrowing limit for normal firms is higher than JQ, because normal firm lose more by defaulting.
  - Borrowing limit is severely low for debt-ridden firm.
Appendix: Derivation of Borrowing Constraint

Assumption 1

- After receiving $p(x_t)x_t$ firm $i$ can default on the debt $m_t + b_t$ and renegotiate on repayment $f_t$. (Bargaining 1 over $f_t$)
  1. Once a firm defaults, it cannot return to a normal firm unless it pays all original debt $m_t + b_t$.
  2. If the lender liquidates the firm she obtains $(\phi + \psi)q_t k_t$.
  3. If agreement is $f_t \geq m_t + b_t$, lender loses liquidation right.
  4. If agreement is $f_t < m_t + b_t$, lender retains liquidation right.

At the end of $t$, they negotiate on $d_{t+1}$. (Bargaining 2 over $d_{t+1}$)

- If they agree on $d_{t+1}$ firm continue in $t + 1$.
- If they do not agree on $d_{t+1}$, firm is liquidated at end of $t$.
  - Lender confiscates only $\psi q_t k_t$.
  - $\phi q_t k_t$ is hidden by the firm.
Appendix: Derivation of Borrowing Constraint

Assumption 1-1 ⇒ Debt forgiveness is infeasible.

Bargaining over $f_t$ after default

$⇒$ Debt-ridden firm or Liquidation?

- If they agree on $f_t$, the firm continues as a debt-ridden firm.
  - Firm obtains \( p(x_t)x_t + q_t k_t - f_t + V_{zt} \).
  - Lender obtains \( f_t + D_t \).
  - \( V_{zt} \): present value of dividend flow of debt-ridden firm
  - \( D_t \): present value of repayment flow to the lender
  - \( V_{zt} \) and \( D_t \) are specified later.

- If they do not agree on $f_t$, the firm is liquidated.
  - Firm obtains \( p(x_t)x_t + (1 - \phi - \psi)q_t k_t \).
  - Lender obtains \( (\phi + \psi)q_t k_t \).
Appendix: Derivation of Borrowing Constraint

▶ If the firm defaults,
  
  ▶ Nash bargaining over $f_t$:

  $$
  \max_{f_t} [(\phi + \psi) q_t k_t + V_{zt} - f_t]^\sigma [f_t + D_t - (\phi + \psi) q_t k_t]^{1-\sigma}.
  $$

  ▶ With $\sigma = 1$, repayment is $f_t = (\phi + \psi) q_t k_t - D_t$
  
  ▶ Firm becomes a debt-ridden firm.

  ▶ We show later that $D_t = \psi q_t k_t$

  $$
  \Rightarrow f_t = \phi q_t k_t.
  $$
Appendix: Derivation of Borrowing Constraint

- No default condition for the firm:
  - If default, firm obtains
    \[ p(x_t)x_t - f_t + q_t k_t + V_{zt} = p(x_t)x_t + (1 - \phi)q_t k_t + V_{zt}. \]
  - If no default, firm obtains
    \[ p(x_t)x_t + q_t k_t - m_t - b_t + V_{nt}. \]
  - No default condition \( \Leftrightarrow \)
    \[ p(x_t)x_t + (1 - \phi)q_t k_t + V_{zt} \leq p(x_t)x_t + q_t k_t - m_t - b_t + V_{nt}. \]

- Borrowing constraint
  \[ m_t + b_t \leq \phi q_t k_t + V_{nt} - V_{zt}. \]
Default and debt-ridden firm

- Suppose that $\Delta_{i t}$ is very large such that

$$\phi q_t k_{i t} + V_{n t} - V_{z t} < b_{i t}.$$ 

- In this case firm $i$ cannot obtain working capital $m_{it}$ and produce nothing.

- Firm $i$ defaults on $b_{i t}$ and becomes a debt-ridden firm.
Bellman equation for debt-ridden firm

Given \( \{d_{t+j}\}_{j=1}^{\infty} \), firm chooses \( k \) in \( t \) and \( m \) in \( t + 1 \).

\[
V_{zt} = \max_k -q_t k + E_t \left[ \max_m \beta \frac{\lambda_{t+1}}{\lambda_t} \left\{ p(x)x - m - d_{t+1} + q_{t+1} k + V_{zt+1} \right\} \right],
\]

(6)

subject to \( x = A_{t+1} k^\alpha m^{1-\alpha} \),

(7)

\( m + d_{t+1} \leq \phi q_{t+1} k_{t+1} \).

(8)
Derivation of Borrowing Constraint

- Debt-ridden firm can default on \( m + d \).
- If debt-ridden firm defaults, the firm and the lender enter the two stage bargaining over repayment \( f_t \) and continuation fee \( d_{t+1} \).
- The bargaining structure is identical to that for normal firms

\[ \Rightarrow \text{Bargaining outcome is} \]

- First stage bargaining:

\[ f = \phi q_t k_t. \]

- Second stage bargaining:

\[ D_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left\{ d_{t+1} + D_{t+1} \right\} \right] = \psi q_t k_t. \]
No default condition $\implies$ Borrowing constraint

- If debt-ridden firm defaults on $m + d$:
  - Firm pays $f = \phi q_k$.
  - Firm continues operation as a debt-ridden firm.
- If firm does not default
  - Firm pays $m_t + d_t$.
  - Firm continues operation as debt-ridden firm.
- No default condition for a debt-ridden firm:

\[ m_t + d_t \leq f = \phi q_k. \]
Appendix: Second stage bargaining over $d_{t+1}$

- Bargaining over $d_{t+1}$ at the end of $t$:
  - If agree, firm obtains $V_{zt}(d_{t+1})$ and lender obtains $D_t \equiv \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \{d_{t+1} + D_{t+1}\} \right]$.
  - If do not agree, firm obtains $(1 - \psi)q_t k_t$ and lender obtains $\psi q_t k_t$ by liquidation.
  - Nash bargaining on $d_{t+1}$ is

$$\max_{d_{t+1}} \{V_{zt}(d) - (1 - \psi)q_t k_t\}^\sigma \left\{ \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \{d + D_{t+1}\} \right] - \psi q_t k_t \right\}^{1-\sigma}$$

- With $\sigma = 1$, the outcome is

$$D_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \{d_{t+1} + D_{t+1}\} \right] = \psi q_t k_t.$$
Appendix: Second stage bargaining over $d_{t+1}$

- The values of $d_t$ and $D_t$ satisfy

$$D_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( d_{t+1} + D_{t+1} \right) \right] = \psi q_t k_t.$$  

- Given $\overline{k}_{t+1}$,

$$d_{t+1} = \frac{\psi q_t k_t}{\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \right]} - \frac{E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \psi q_{t+1} \overline{k}_{t+1} \right]}{E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \right]}.$$
Appendix: First stage bargaining over $m_t$

- After $d_{t+1}$ is agreed at the end of $t$, firm continues in $t+1$.
- Firm borrows $m_{zt+1}$ and produces $x_{t+1} = A_{t+1}k_{t+1}^\alpha m_{zt+1}^{1-\alpha}$.
- After receiving $p(x_{t+1})x_{t+1}$, firm can default on $m_{zt+1} + d_{t+1}$.
- Renegotiation on repayment $f_t$:
  - If agree, firm obtains $p(x_{t+1})x_{t+1} + q_{t+1}k_{t+1} + V_{zt+1} - f$ and lender obtains $f + D_{t+1}$.
  - If do not agree, firm obtains $p(x_{t+1})x_{t+1} + (1 - \phi - \psi)q_{t+1}k_{t+1}$ and lender obtains $(\phi + \psi)q_{t+1}k_{t+1}$.
Appendix: First stage bargaining over $m_t$

- Nash bargaining

$$\max_f \left\{ (\phi + \psi)q_{t+1}k_{t+1} + V_{zt+1} - f \right\}^\sigma \cdot \left\{ f + D_{t+1} - (\phi + \psi)q_{t+1}k_{t+1} \right\}^{1-\sigma}$$

- With $\sigma = 1$, repayment is $f = (\phi + \psi)q_{t+1}k_{t+1} - D_{t+1}$.

- With $D_{t+1} = \psi q_{t+1}k_{t+1}$, this is written as $f = \phi q_{t+1}k_{t+1}$.
Appendix: First stage bargaining over $m_t$

- No renegotiation condition: $m_{zt+1} + d_{t+1} \leq f_t$. 

\[ m_{zt+1} + d_{t+1} \leq \phi q_{t+1} k_{t+1}. \]
Borrowing constraints for normal firm and debt-ridden firm

- Normal firm:

\[ m_{t+1} + b_{t+1} \leq \phi q_{t+1} k_{t+1} + V_{nt+1} - V_{zt+1}. \]

Firm chooses \( b_{t+1} \) to maximize \( V_{nt} \)

- Debt-ridden firm:

\[ m_{t+1} + d_{t+1} \leq \phi q_{t+1} k_{t+1}. \]

\( d_{t+1} \) is determined by the bargaining bw lender and firm.

- In our simulation:
  - Borrowing constraint is non-binding for normal firms.
  - Borrowing constraint is binding for debt-ridden firms.
Appendix: What if debt-ridden firms can make savings?

- Assumption 2: Debt-ridden firms cannot make savings $s_t$.

- Appendix C
  - We assume that firms can accumulate $s_t$.
  - Assumption: Lender can confiscate $s_t$ if she liquidates debt-ridden firm at the end of $t$.
  - Focus on a deterministic equilibrium
  - It is shown that cost and gain of accumulating $s_t$ cancel out with each other.
  - Debt-ridden firm has no incentive to accumulate $s_t$.
  - No savings in the deterministic equilibrium.
    (Corporate savings are neutral in equilibrium.)
Equilibrium with debt-ridden firms

- Initial steady state:
  \[ I_t = 0, \quad N_t = N, \quad k_t = \frac{K}{N} \]. Parameter is \( \chi = \chi_{SS} \).
  All firms are normal and there is no debt-ridden firms.

- Mass default due to one-time redistribution shock at \( t = 0 \):
  At \( t = 0 \), firm \( i \in [0, Z] \) default on the debt, while firm \( i \in [Z, N] \) do not default. \((0 < Z < 1)\)
  - Firm \( i \in [0, Z] \) ⇒ Debt-ridden firms
  - Firm \( i \in [Z, N] \) ⇒ Normal firms

- No more shock ⇒ Steady state equilibrium
Appendix: Equilibrium with debt-ridden firms

Parameters:
\[ \alpha = 0.25, \quad \beta = 0.9, \quad \delta = 0, \quad \phi = 0.15, \quad \eta = 0.7, \quad A = 1.9048, \]
\[ K = 1, \quad L = 1, \quad \psi = 0.4, \quad \chi_{SS}^{-1} = 3.4234, \quad V = \chi_{BGP}^{-1} = 2.9307. \]
Equilibrium with debt-ridden firms

- There is no R&D, i.e., $I_t = 0$, in equilibrium after default. (Verified later.)
  $\Rightarrow$ Equilibrium after default is a steady state equilibrium.
Equilibrium with debt-ridden firms

\begin{align*}
  k_n &= 1 - Z \\
  k_z &= 2 - Z \\
  Y &= 2 - Z \\
  m_n &= 0.7 - 0.35Z \\
  m_z &= 0.5 - 0.45Z \\
  C &= 1.3 - 0.9Z
\end{align*}
Equilibrium with debt-ridden firms

- Observed TFP, \( \tilde{A} \), is decreasing in \( Z \):

\[
\tilde{A} = \frac{C(Z)}{K^\theta L^{1-\theta}},
\]

where \( C(Z) \) is consumption and \( \theta = \frac{\alpha \eta}{1-(1-\alpha)\eta} \).
Equilibrium with debt-ridden firms

\[ d(Z) \]
\[ V_n(Z) \]
\[ V_z(Z) \]
\[ q(Z) \]
\[ \text{RHS–LHS (eq.7)} \]
\[ \text{LHS–RHS (eq.18)} \]
Equilibrium with debt-ridden firms

- $V_{nt}$ decreases as $Z$ increases, and $\chi_{SS}V_{nt} < 1$ for $Z > 0$.
  - Tighter borrowing constraint
    - Debt-ridden firms buy $k$ aggressively.
  - Price $q$ rises. (Congestion effect)
  - Higher cost of $k$ decreases the value of new entry $V_{nt}$.
- $\chi_{SS}V_{nt} < 1$
  - $I_t = 0$ and $N_t$ is constant over time.
    - ($I_t > 0$ if $\chi_{SS}V_{nt} \geq 1$)
  - Steady state equilibrium.
Policy implication

- If all defaulters are liquidated, the economy goes back to efficient steady state by new entry of firms.
- If debt forgiveness is allowed institutionally TFP goes back to normal immediately. (The equilibrium borrowing constraint for normal firms would be tighter. Appendix D.)
Endogenous growth and zero growth path (ZGP)

- Assumption 3
  Externality on productivity from variety: $A_t = \hat{A}N_t^\alpha$.
- We set parameter: $\chi = \chi_{BGP} \ (> \chi_{SS})$.
- Under this assumption, the equilibrium without debt-ridden firms is the balanced growth path (BGP) with $\frac{N_{t+1}}{N_t} = 1 + g$.
- Same parameters as before, except for $\chi$; The value of $\chi$ is chosen such that $g = 0.01$ in the BGP.
Endogenous growth and zero growth path (ZGP)

- Emergence of debt-ridden firm decreases $V_{nt}$.
- Condition for $N_t$ to grow is $\chi V_{nt} = 1$.
- If $Z$ is large than $\underline{Z}$, then the equilibrium is ZGP
  If $Z > \underline{Z}$ then $V_{nt}$ become so small that $\chi_{BGP} V_{nt} < 1$.
  - If $\chi V_{nt} < 1$, no R&D investment takes place.
  - $N_t$ does not grow
  - No productivity growth in equilibrium $\Rightarrow$ ZGP
Endogenous growth and zero growth path (ZGP)

- If \( Z > 0.5439 \), then \( V_{nt} < V \) and equilibrium is ZGP.
- Variables in Figure are those in ZGP.
Endogenous growth and zero growth path (ZGP)

- If $Z > 0.5439$, then $V_{nt} < V$ and equilibrium is ZGP.
- Variables in Figure are those in ZGP.
Conclusion

We consider a model in which

- exogenous shock makes a large proportion of firms default, and
- defaulted firms cannot go back to normal firms unless they repay all original debt. Defaulted firms continue as debt-ridden firms,
  (c.f. defaulted firms can go back to normal after renegotiation in Jermann-Quadrini model.)

In this economy,

- borrowing constraints are tighter for debt-ridden firms,
- TFP decreases as debt-ridden firms increase,
- a sufficient increase of debt-ridden firms lowers TFP growth to zero in the endogenous growth version of our model.
Future research

- Embed this model into a DSGE model with variable capital and labor.
- Money
- Various policy assessments