Bubbles, Banks and Financial Stability*

Kosuke Aoki (University of Tokyo)

Kalin Nikolov (European Central Bank)

May 2012 [First draft: November 2010]

Abstract

We build a model of rational bubbles in a limited commitment economy and show that the impact of the bubble on the real economy crucially depends on who holds the bubble. When banks are the bubble-holders, this amplifies the output boom while the bubble survives but also deepens the recession when the bubble bursts. In contrast, the real impact of bubbles held by ordinary savers is more muted.

1 Introduction

The last 20 years have seen two spectacular episodes of financial instability. First, stock prices experienced a truly unprecedented increase and subsequent collapse around the turn of the millenium. Then came the dramatic rise and fall of world-wide housing prices, culminating in the financial crisis and 'Great

*We are especially grateful to Jaume Ventura whose excellent comments helped us improve the paper a lot. We also thank Henrik Borchgrevink, Wouter den Haan, Piti Disyatat, Martin Eichenbaum, Philipp Hartmann, Jonathan Heathcote, Xavier Freixas, Roman Inderst, Peter Karadi, Nobu Kiyotaki, Alberto Martin, Tomoyuki Nakajima, Helene Rey, Victor Rios-Rull, Immo Schott, Shigenori Shiratsuka, Harald Uhlig, Christopher Waller and Philippe Weil. The views expressed here are those of the authors and should not be interpreted as those of the European Central Bank.

Contraction' of 2008-2009. These movements in financial prices were large enough to reawaken interest in asset price bubbles and the way they affect the real economy. In addition, the near failure of the global banking system during the Lehman Crisis of 2008 has increased awareness of the importance of banks in amplifying macroeconomic fluctuations.

Motivation and economic questions Our paper studies the interaction of asset price bubbles, the banking system and the real economy in a general equilibrium model with credit constraints. The current financial crisis is the most immediate motivation for wanting to study this interaction. Bank losses due to subprime mortgage defaults depleted bank capital positions and led to a severe credit crunch and the deepest recession since the 1930s. This episode is by no means unique in history. In Japan, during the 1980s, real estate prices grew rapidly allowing property developers to borrow heavily from banks using real estate as collateral. When property prices collapsed in the 1990s, Japan went through a protracted banking crisis and a 'lost decade ' of low economic growth. A similar set of events occurred during the housing bust and financial crisis that hit the Scandinavian countries in the early 1990s. In all of these historical episodes, the banking system was exposed to a bursting housing bubble through a higher incidence of household default. As Reinhart and Rogoff (2008) show, using their rich time-series and cross-country data set, the resulting insolvency of the financial system tends to lead to a deep downturn in real activity.

The one bubble episode that stands in clear contrast to all the costly boom-bust cycles described above is the 1998-2000 'technology bubble'. Stock prices rose and then collapsed dramatically without triggering a banking crisis or a deep recession. Why was this bubble episode so different from other (mainly housing) recent bubbles? More generally, what makes some asset price busts more costly than others? These are the main questions we focus on in this research. We build a model which focuses on bank exposures to bubbles as an important reason why some lead to crisis while others do not[1].

Main results Following the insights of the growing literature on rational bubbles with credit constraints (Caballero and Krishnamurthy (2006), Kocherkalota (2009), Martin and Ventura (2011a), Martin and Ventura (2011b), Arce and Lopez-Salido (2008), Farhi and Tirole (2011), Hirano and Yanagawa (2010)) we focus on an environment in which binding borrowing constraints lead to a shortage of means of saving and the use of dynamically inefficient investment technologies in equilibrium. This creates the conditions for bubbles to circulate, helping to improve the supply of liquidity but also exposing the economy to the risks of their bursting. Our innovation relative to the rest of the rational bubbles literature is to model financial intermediation explicitly. In our model credit flows along a chain that starts with a saver, goes via a bank and ends up with a final borrowing firm. This realistic feature of our environment also has the implication, overlooked by the rest of the literature, that asset price bubbles can be held by a variety of agents with very different economic roles in equilibrium. Our paper shows that the identity of the bubble-holder is vital for understanding the effect of bubbles on the real economy. Bubbles held by banks expand output more during the boom phase and then lead to a much more severe contraction in credit and output when they finally burst.

[1] Commercial banks in the US tend not to hold equities and their Commercial and Industrial Loans had fallen to 20% of total loans by the late 1990s. In contrast, almost 60% of their total loans and leases were secured by real estate. This is an important reason why housing busts tend to be more costly for the banking system compared to equity busts.

Martin and Ventura (2011a) argue that a reason why the 'dot com' bust did not lead to a recession is that it was immediately followed by the housing bubble which helped to expand the economy. This argument is plausible and we see it as complementary to our channel which stresses the health of the banking system.

In contrast, bubbles held by ordinary savers have a relatively muted effect on the real economy.

The key to understanding our main result lies in the way the identity of the bubble holder affects the evolution of the wealth distribution in equilibrium. Bubbles in our world are risky assets which deliver a higher return compared to safe assets in order to compensate the holder for the possibility of losing everything in the bust. This implies that the bubble holder experiences strong growth of net worth while the bubble survives and then a sharp fall in net worth when the bubble bursts. These movements in the wealth distribution have large real effects when the affected agents are the credit constrained intermediaries. The gains and losses they make on bubble holdings affect aggregate credit supply and therefore the production scale of the bank's credit constrained borrowers. During the bubble's run up, banks expand their size, increase credit supply and this allows productive firms to increase output. The re-allocation of productive factors towards more efficient firms then boosts aggregate TFP. When the bubble crashes, banks make large losses and inflict a credit crunch on the rest of the economy. In contrast, ordinary savers are credit unconstrained and their net worth does not play the same role in the credit intermediation mechanism. When savers hold the bubble, their wealth also fluctuates but this has a very limited impact on the credit constraints faced by other agents in the economy.

In this paper, banks only undertake bubble exposures when they believe that potential losses will be underwritten by the government (moral hazard). The promise of bailouts gives banks an advantage in holding bubbly assets, pushing the risky exposure into their hands. However, the main result of our paper is general: the identity of the bubble holder is key to understanding the real effects of bubbles during the boom as well as the bust. Bubbles held by banks create a bigger boom and a bigger crash by stimulating a boom-bust cycle in credit supply.

Related Literature Our paper contributes to a large literature that studies the way asset price bubbles affect economic activity. The seminal work of Tirole (1985) showed that, in an environment without credit constraints, bubbles have a contractionary effect on economic activity because they displace inefficient investments in savers' portfolios. This is the famous 'crowding out' effect of bubbles. Subsequent papers (Caballero and Krishnamurthy (2006), Kocherkalota (2009), Martin and Ventura (2011a), Martin and Ventura (2011b), Farhi and Tirole (2011)) have shown that when there are credit market imperfections, bubbles may have an expansionary effect through a variety of mechanisms that help to reduce the severity of credit constraints.

In Martin and Ventura (2011b) the expansionary effect of bubbles arises because bubble creation increases both current and anticipated future profits for productive entrepreneurs who cannot borrow against their tangible assets due to a moral hazard problem. Because the anticipated profits from future bubble sales are collateralisable, this allows the most productive entrepreneurs to increase borrowing and production in the current period, thereby increasing aggregate TFP. Related to the results in our paper, Martin and Ventura (2011b) demonstrate that the effects of bubble creation depend crucially on the identity of the bubble creator. When credit constrained productive firms create bubbles, the bubble is strongly expansionary. When unconstrained savers benefit from the creation, the bubble is contractionary as in Tirole (1985). In our paper, we abstract from bubble creation, instead focusing on the impact of bubble holding and bubble destruction on economic outcomes. We show that this impact is also highly dependent on the identity of the bubble holder.

Similarly to Martin and Ventura (2011b) our paper also relies on a redistribution of productive resources towards more able firms in order to generate an increase in aggregate TFP and output as a result of the bubble. But the way in which resources are channelled into productive hands does not rely on bubble creation. Instead, bubble holdings turn out to be expansionary or contractionary due to the interplay of the 'liquidity effect' and the 'competition effect' identified in Farhi and Tirole (2011) as well as the 'credit supply effect' which is unique to our paper. It is the 'credit supply effect' which really depends on who holds the bubble. It is at its strongest when the bubble is in the banks' hands. There it boosts banks' net worth and expands credit supply. This also exerts a powerful positive externality on the net worth of borrower entrepreneurs, helping to increase their productive activities and the economy's aggregate efficiency. When savers hold the bubble, such credit supply externalities are relatively weak and the bubble is less expansionary.

Our paper's findings are in line with several important stylised facts uncovered by the literature on the empirical regularities around financial crises. Our focus on rational bubbles that can only occur in a low interest rate environment is supported by Schularick and Taylor (2012) who report that growth adjusted real interest rates are very low for several years in the run up to global crises. Schularick and Taylor (2012) present evidence that credit booms are associated with larger cumulative increases and steeper subsequent declines in GDP. Our theoretical results are exactly in line with such empirical regularities.

Plan of the paper Section 2 introduces the economic environment, section 3 describes the equilibrium without government sector and discusses the conditions for the existence of bubbles and bubble ownership in the absence of government bailout policy. Section 4 examines the effect of the financial safety net on the existence and ownership of bubbles, and analyses the implications of bubble ownership on the real effects of bubbly episodes. Finally, section 5 concludes.

2 The Model

The economy is populated with three kinds of agents. There is a continuum of infinitely lived entrepreneurs and a continuum of infinitely lived workers both of measure 1. There is also a continuum of bankers who have finite lives and can die with probability $1-\gamma$ in any period, conditional on being alive in the previous period.

2.1 Entrepreneurs

Each entrepreneur is endowed with a constant returns to scale production function which converts labor h_t into output in the next period y_{t+1}.

$$y_{t+1} = a_t^i h_t, \tag{1}$$

where a_t^i is a productivity parameter which is known at time t.

In each period some firms are productive ($a_t^i = a^H$) and the others are unproductive ($a_t^i = a^L < a^H$). Each entrepreneur shifts stochastically between productive and unproductive states following a Markov process. Specifically, if a productive entrepreneur in this period may become unproductive in the next period with probability δ, and an unproductive entrepreneur in this period may become productive with probability $n\delta$. This probability is independent across entrepreneurs and over time. This Markov process implies that the fraction of productive entrepreneurs is stationary over time and equal to $n/(1+n)$, given that the economy starts with such population distribution. We assume that the probability of the productivity shifts is not too large:

$$\delta + n\delta < 1. \tag{2}$$

This assumption implies that the productivity of each agent is persistent.

Entrepreneurs are ex-ante identical and have log utility over consumption streams

$$U^E = E_t \sum_{t=0}^{\infty} \beta^t \ln c_t \tag{3}$$

Entrepreneurs purchase consumption (c_t), bubbles (m_t^e) at price μ_t and bonds b_t. They also pay wages to the workers they hire $w_t h_t$ in order to receive future revenues $a^i h_t$ which the government taxes at rate τ_t after deducting debt repayments. w_t and h_t denote real wage and labor respectively.

$$c_t + w_t h_t + m_t^e \mu_t - b_t = (1 - \tau_t)\left(a^i h_{t-1} - R_{t-1}^i b_{t-1} + m_{t-1}^e \mu_t\right) \equiv (1 - \tau_t)\, z_t \tag{4}$$

where z_t stands for entrepreneur's net worth. R_t^i is the interest rate which is equal to the loan rate R_t^l when the entrepreneur is a borrower and R_t^d when the household is a saver. The bubble asset in our economy is a durable but intrinsically worthless asset which has no productive or consumption value. In other words, its fundamental value is zero.

Due to limited commitment in the credit market, agents will only honour their promises if it is in their interests to do so. We assume that only a fraction of the value of the firm can be seized by creditors and tax collectors. Furthermore we assume that the tax authorities have a first call on the firm's resources while private creditors are second in line. Hence the collateral constraint is given by:

$$R^l_t b_t + E_t \Upsilon_{t+1} \leqslant \theta E_t y_{t+1}, \quad 0 < \theta < 1 \tag{5}$$

where

$$E_t \Upsilon_{t+1} = E_t \tau_{t+1} \left(a^i h_t - R^i_t b_t + m^e_t \mu_{t+1} \right)$$

stands for expected tax payments in the following period. Entrepreneurs maximize (3) subject to (4) and (5).

2.2 Banks

We assume that only banks can enforce debt repayments in our economy. Consequently, all borrowing and lending will be bank-intermediated. Bankers are risk neutral and live for a stochastic length of time. Once bankers receive an "end of life" shock, they liquidate all their asset holdings and consume their net worth before exiting. This shock hits with probability $1 - \gamma$.

Banks maximize the following objective function:

$$U^B = E_t \sum_{t=0}^{\infty} (\beta\gamma)^t c^B_t \tag{6}$$

subject to the following constraints explained below.

In each period the bank has net worth (n_t). It collects deposits (d_t) from the savers. Then it lends to the borrowers (b_t), purchases bubbles $(\mu_t m^b_t)$, or consumes (c^b_t). We assume that intermediation is entirely costless. The bank's balance sheet constraint is given by

$$c^b_t + b_t + \mu_t m^b_t = n_t + d_t. \tag{7}$$

The evolution of net worth is given by

$$n_{t+1} = R^l_t b_t + \mu_{t+1} m^b_t - R^d_t d_t. \tag{8}$$

when the bubble does not burst and by:

$$n_{t+1} = R^l_t b_t + \rho_{t+1} m^b_t - R^d_t d_t. \tag{9}$$

when it does burst. ρ_{t+1} is the fraction of the banks' bubble investment which is guarranteed by the government. In the event of a bubble collapse, the government transfers these funds to the banks to compensate them for losses made. The parameter ρ_{t+1} is a simple means of capturing the explicit or implicit guarrantees given by the government to the banking system

Following Gertler and Karadi (2009), we model banks subject to limited commitment. More specifically, the banker may divert $1 - \lambda$ fraction of deposits. Once he diverts, he consumes the funds and closes his bank, remaining inactive until his 'death'. The savers can recover the remaining λ fraction of deposits. Since the savers recognize the banker's incentive to divert funds, they will restrict the amount they deposit with the intermediary, according to the following borrowing constraint:

$$(1 - \lambda) d_t \leqslant V(n_t). \tag{10}$$

The left hand side of equation (10) is the value when the banker diverts, while the right hand side is the value when he did not (i.e., the continuation value of the bank). We also assume that the bank cannot short m_t. The bank maximizes (6) subject to (7), (8), (9) and (10).

2.3 Workers

Unlike the entrepreneurs, the workers do not have access to the production technology nor any collateralizable asset in order to borrow. They maximize the following utility

$$U^W = E_t \sum_{t=0}^{\infty} \beta^t \left(c_t^w - \frac{h_t^{1+\eta}}{1+\eta} \right) \tag{11}$$

subject to their flow-of-funds constraint

$$c_t^w + m_t^w \mu_t - b_t^w = w_t h_t + m_{t-1}^w \mu_t - R_{t-1}^d b_{t-1}^w, \tag{12}$$

here superscript 'w' stands for 'workers'.

2.4 The Government

We assume that the only role for the government in this economy is to levy taxes on entrepreneurs and bail out the banking system when it makes losses. We assume that the government follows a balanced budget rule and does not issue government debt. Consequently taxes are only levied whenever bailout spending is necessary. For the rest of the time, taxation is zero.

3 Equilibrium without government

We consider a stochastic bubble that persists with probability π. With probability $1-\pi$ the bubble bursts and its value reverts to zero. We assume this probability is constant over time. Also, we assume that once bubbles burst they never arise again.

In equilibrium, due to the difference in their productivity, productive entrepreneurs borrow from banks and unproductive entrepreneurs make deposits to banks. We focus on equilibria in which the productive entreprenerus borrow up to their borrowing constraint[2]. In this section, we characterise the equilibrium without government. Therefore we set $\tau_{t+1} = 0$ and $\rho_{t+1} = 0$ at all times.

3.1 Entrepreneurs' optimal behavior

The entrepreneurs' problem can be interpreted as a savings problem with uncertain returns. Since utility function is logarithmic and there is no labour income or transfer income, entrepreneurs consume a constant fraction of net worth (z_t).

$$c_t = (1 - \beta)\, z_t. \tag{13}$$

and save the remaining β fraction[3].

The entrepreneur has several possibilities for accumulating net worth. First of all, an entrepreneur with productivity $a_t = a^h, a^L$ can undertake unleveraged investments using his own technology. This yields a return of $\frac{a_t}{w_t}$: the cost of hiring a worker is w_t and the worker produces output a_t in the following period. Secondly, the entrepreneur can deposit in the banking system, earning a return of R_t^d. Thirdly, the entrepreneur can pledge up to θ fraction of future output to banks and borrow at interest rate R_t^l in order to invest in her own production technology. In this case the leveraged rate of return is

$$\frac{a_t(1-\theta)}{w_t - \theta a_t / R_t^l}. \tag{14}$$

By borrowing from banks secured by θ fraction of output, the entrepreneur can finance externally $\theta a_t / R_t^l$ amount (equation (5)). Therefore the denominator

[2] This happens when the borrowing constraints are tight enough. See Aoki et al. (2009).
[3] See, for example, Sargent (1987).

is the required downpayment for the unit labor cost. Finally, the entrepreneur can invest in bubbles. In this case the rate of return is given by μ_{t+1}/μ_t, where μ_{t+1} stands for the market value of the bubble next period. When it bursts, the return is zero.

High productivity entrepreneurs enjoy better returns on production so they are the ones who borrow in equilibrium. When their borrowing constraints bind the rate of return on wealth ($r(a^H)$) is given by:

$$r(a^H) = \frac{a^H(1-\theta)}{w_t - \theta a^H/R_t^l} \geqq R_t^l. \tag{15}$$

From (5) and (4) the investment (employment) of a productive agent is given by

$$h_t = \frac{\beta z_t}{w_t - a^H\theta/R_t^l}. \tag{16}$$

The entrepreneur saves a β fraction of wealth z_t and uses her entire savings as a downpayment for wage payments to the workers she hires.

While the productive entrepreneurs borrow, the unproductive entrepreneurs make deposits. In addition, they have two other means of savings: unleveraged production and investing in bubbles. Notice that both deposits and production are riskless in our environment. When

$$R_t^d > \frac{a^L}{w_t} \tag{17}$$

low productivity agents are inactive in production. However when the credit constraints on banks and borrowing entrepreneurs are tight enough, the productive entrepreneurs cannot absorb all national saving. The supply of deposits is limited and the low productivity technology may be viable in equilibrium.

In such case

$$R_t^d = \frac{a^L}{w_t} \tag{18}$$

and the saver entrepreneurs use both bank deposits and their own production technology to accumulate wealth.

Bubbles are risky. When savers invest in bubbles as well as deposits, the arbitrage condition for bubbles is determined by the savers' state-contingent wealth valuation

$$E_t\left[\frac{1}{c_{t+1}^L}\frac{\mu_{t+1}}{\mu_t}\right] = E_t\left[\frac{1}{c_{t+1}^L}\right] R_t^d, \tag{19}$$

where $1/c_{t+1}^L$ represents the shadow value of wealth at time $t+1$ of the entrepreneur who is unproductive at time t[4], where expectation operator is taken over whether bubble survives or crashes.

$$\mu_{t+1} = \begin{array}{l} \mu_{t+1}^b \text{ with probability } \pi \\ 0 \text{ with probability } 1-\pi \end{array} \tag{20}$$

where μ_{t+1}^b is the market value of the bubble on survival.

3.2 Banks' optimal behaviour

Next, we characterise the optimal behaviour of a representative bank in our economy. The problem of the bank can be represented in recursive form by using the bank's value function representation as follows:

$$V(n_t) = \max_{c_t^b, d_t, b_t, m_t} \left\{c_t^b + \beta E_t\left[\gamma V(n_{t+1}) + (1-\gamma)\, n_{t+1}\right]\right\} \tag{21}$$

[4]Namely, it is given by

$$1/c_{t+1}^L = (1-\beta) Z_{t+1}^L$$

where Z_{t+1}^H is given by equation (34)

$V(n_t)$ is the value of a bank with net worth n_t which chooses current consumption, deposits, bubbles and loans optimally. This value is equal to current consumption and the expected future discounted value of bank net worth $\beta E_t[\gamma V(n_{t+1}) + (1-\gamma)n_{t+1}]$. This value includes the continuation value of being a banker - this happens only if the banker survives with probability γ. With probability $1-\gamma$, the banker receives the death shock and consumes his entire net worth in the following period.

Because of risk neutrality, we can guess that the value of the bank is a linear function of net worth n_t

$$V(n_t) = \phi_t n_t \tag{22}$$

When $R_t^l > R_t^d$, the credit constraint (10) binds and consumption is postponed until death. Then, with equation (10) binding, deposits are given by

$$d_t = \frac{\phi_t}{1-\lambda} n_t. \tag{23}$$

Here $\phi_t/(1-\lambda)$ is the bank's leverage.

Regarding the bank's choice over bubbles and loans,

$$E_t\left[(1-\gamma+\gamma\phi_{t+1})\frac{\mu_{t+1}}{\mu_t}\right] \leq E_t\left[(1-\gamma+\gamma\phi_{t+1})\right]R_t^l, \tag{24}$$

where expectation operator is again taken over the bubble surviving or not. If equation (24) holds with strict inequality, then the bank will not invest in bubbles ($m_t^b = 0$). When the bank invests in bubbles (24) must hold with equality. By substituting (22), (23), and (24) into (21), ϕ_t satisfies

$$\phi_t = \frac{\beta E_t\left[(1-\gamma)+\gamma E_t\phi_{t+1}\right]R_t^l}{1-\beta E_t\left[(1-\gamma)+\gamma E_t\phi_{t+1}\right]\frac{R_t^l - R_t^d}{1-\lambda}}. \tag{25}$$

This expression states that the value of a unit of net worth for a banker is equal to the value of the returns on its loan book (the numerator), suitably boosted by leverage (the denominator). The banker issues one unit of loans but the downpayment he has to make is only given by the denominator of (25) because he can pledge some of the future expected excess returns from intermediation ($\beta E_t \left[(1-\gamma) + \gamma E_t \phi_{t+1} \right] \frac{R_t^l - R_t^d}{1-\lambda}$) to depositors who finance a large part of the loan outlay. Note that the above formulas show that ϕ_t increases with ϕ_{t+1}. This implies that the current leverage depends on the future franchise value of the bank which is reflected by the leverage next period.[5] It also shows that ϕ_t is an increasing function of the spread $R_t^l - R_t^d$.

3.3 Workers' optimal behaviour

Workers are risk-neutral and consequently their consumption-savings behaviour is a knife-edge one. When the loan interest rate is lower than the rate of time preference, workers want to borrow unlimited quantities. Because the workers cannot operate the production technology, they cannot pledge collateral to lenders. Hence workers cannot borrow.

Of course workers could save at the deposit rate but they only want to do this when

$$R_t^d \geqslant \beta^{-1}. \tag{26}$$

If this condition is not satisfied, workes will consume their entire net worth (financial wealth and labour income) and save nothing. Their labour supply h_t^s is given by

$$h_t^s = w_t^\eta. \tag{27}$$

Because ours is a limited commitment economy, we guess and verify that $R_t^d <$

[5] See Nikolov (2010), who considers a similar problem for firms.

β^{-1} at all times along the equilibrium paths we consider. Hence our workers are hand-to-mouth consumers at all times.

3.4 Aggregation and market clearing

Let the total supply of the bubble asset be normalized to 1. In other words,

$$m_t^e + m_t^b = 1, \tag{28}$$

where m_t^e and m_t^b, respectively, denote the shares of the bubble held by unproductive entrepreneurs and banks.

Let Z_t^H and Z_t^L, respectively, denote aggregate wealth of the productive and unproductive entrepreneurs. Then we can characterise the aggregate equilibrium as follows. From (16) the aggregate employment of the productive entrepreneurs is given by

$$H_t^H = \frac{\beta Z_t^H}{w_t - \theta a^H / R_t^l}. \tag{29}$$

When (17) holds, the unproductive entrepreneurs are indifferent between making deposits and producing, thus their aggregate saving is split as follows

$$H_t^L = \beta Z_t^L - D_t - m_t^e \mu_t \tag{30}$$

where D_t denotes aggregate deposit.

Let us turn to banks. Under the banks binding borrowing constraint, the aggregate deposit is given by

$$D_t = \frac{\phi_t}{(1-\lambda)} \gamma N_t. \tag{31}$$

Notice that $1-\gamma$ fraction of banks exits in each period by liquidating all their net worth. Therefore the aggregate net worth of the operating banks is given by γN_t. The aggregate balance sheet of the operating banks is given by

$$D_t + \gamma N_t = B_t + m_t^b \mu_t. \tag{32}$$

Let us turn to the transition of state variables. Note that the unproductive entrepreneurs become productive in the next period with probability $n\delta$ and the productive entrepreneurs continues to be productive with probability $1-\delta$. Their rates of return are given by (15) and (17). Therefore the net worth of the productive entrepreneurs evolves from (15) and (13) as

$$Z_{t+1}^H = (1-\delta)\frac{a^H(1-\theta)}{w_t - \theta a^H/R_t^l}\beta Z_t^H + n\delta\left[R_t^d\left(\beta Z_t^L - m_t^e\mu_t\right) + m_t^e\mu_{t+1}\right] \tag{33}$$

Similarly, the aggregate net worth of the unproductive entrepreneurs evolves as

$$Z_{t+1}^L = \delta\frac{a^H(1-\theta)}{w_t - \theta a^H/R_t^l}\beta Z_t^H + (1-n\delta)\left[R_t^d\left(\beta Z_t^L - m_t^e\mu_t\right) + m_t^e\mu_{t+1}\right] \tag{34}$$

From aggregating production function, aggregate output is given by

$$Y_t = a^H H_{t-1}^H + a^L H_{t-1}^L. \tag{35}$$

Finally, aggregate bank net worth is given by

$$N_{t+1} = \gamma\left[R_t^l B_t + m_t^b\mu_{t+1} - R_t^d D_t\right] \tag{36}$$

The markets for goods, labour, capital, loan and deposit must clear. Goods market clearing implies that aggregate saving must equal to aggregate investment.

$$\beta(Z_t^H + Z_t^L) + \gamma N_t = w(H_t^H + H_t^L) + \mu_t. \tag{37}$$

From (27), labour market clearing implies

$$w_t^\eta = H_t^H + H_t^L. \tag{38}$$

Now equations (18), (19), (24), (25), (28)-(38) jointly determine 15 variables R_t^d, R_t^l, w_t, H_t^H, H_t^L, Y_t, ϕ_t, D_t, B_t, Z_{t+1}^H, Z_{t+1}^L, N_{t+1}, μ_t, m_t^e, m_t^b with three states Z_t^H, Z_t^L, N_t[6]. At $t = 0$, Z_0^H is given by (33).

Definition 1 *Competitive bubbly equilibrium without government is a sequence of decision rules* $\left\{H_t^H, H_t^L, Y_t, D_t, B_t, m_t^e, m_t^b\right\}_{t=0}^\infty$*, aggregate state variables* $\left\{Z_{t+1}^H, Z_{t+1}^L, N_{t+1}\right\}_{t=0}^\infty$ *and a price sequence* $\left\{R_t^d, R_t^l, w_t, \phi_t, \mu_t\right\}_{t=0}^\infty$ *such that: (i) entrepreneurs, banks and workers optimally choose decision rules* $\left\{H_t^H, H_t^L, Y_t, D_t, B_t, m_t^e, m_t^b\right\}_{t=0}^\infty$ *taking the evolution of aggregate states, prices and idiosyncratic productivity opportunities as given; (ii) the price sequence* $\left\{R_t^d, R_t^l, w_t, \phi_t, \mu_t\right\}_{t=0}^\infty$ *clears the goods, labor, capital, loan, bubble and deposit markets and (iii) the equilibrium evolution of state variables* $\left\{Z_{t+1}^H, Z_{t+1}^L, N_{t+1}\right\}_{t=0}^\infty$ *is consistent with the individual choices of entrepreneurs, banks and workers and with the exogenous evolution of productive opportunities at the individual firm level.*

3.5 Calibration

We have 8 parameters $\left\{\eta, a^H/a^L, \delta, n, \theta, \gamma, \beta, \lambda\right\}$ we need to calibrate before we proceed to examine the quantitative predictions of our model economy. There is little consensus regarding η, the Frisch elasticity of labour supply. Micro-data evidence suggests a value close to zero based on the labour supply

[6] By Warlas law one of these equations is redundant.

behavior of primary earners. The real business cycles literature usually sets a much higher value in the region of 3 or even higher. The differences is justified by the presence of labour market frictions that ensure that aggregate labour is highly elastic even though individuals are relatively unwilling to vary their market hours over time. Gertler and Kiyotaki (2010) make this argument and set the Frisch elasticity to 10 in their model. We pick a value of $\eta = 5$, which is within the range set in calibrating macro models.

a^H/a^L is an important parameter, whose value is also highly uncertain. As studies such as Bernard et al. (2003) and Syverson (2004) have documented, the dispersion of plant level productivity in US manufacturing is enormous, with the most productive plants having more than 4 times more productive compared to the least productive. But as Aoki et al. (2009) argue, it is hard to believe that such a huge dispersion of productivity levels is entirely due to the presence of credit constraints. More likely, inputs could be mismeasured in a number of ways. For example, intangible assets such as managerial quality could be an important missing input which could explain some of the huge differences in measured plant level TFP. Following Aoki et al. (2009) we set a value for $a^H/a^L = 1.1$ implying a substantial cross-sectional dispersion in plant level TFP in the model.

We calibrate the remaining 6 parameters in order to match the steady state predictions of the model in the absence of bubbles to 7 moments in the US data. These are (1) the real loan rate minus the growth rate of real GDP and minus intermediation costs; (2) the real deposit rate minus the growth rate of real GDP; (3) commercial bank leverage; (4) average corporate leverage; (5) average leverage for highly leveraged corporates; (6) the rate of return on bank equity and (7) the ratio of M2 to GDP.

Calibration targets (1) and (2) deserve further discussion. For simplicity, in our model we assume there is no growth and no intermediation costs. In reality these two assumptions of course do not hold. Growth in the US economy has averaged close to 3% per annum since the second world war. Since we are interested in the dynamic efficiency of the investments of US savers and banks, we want to know whether the real return on these investments exceeds the economy's growth rate. This is why we subtract the real growth rate from the real return on deposits and loans.

In addition, when it comes to evaluating the dynamic efficiency of banks' loan investments, we need to take intermediation costs into account. FDIC data on US commercial banks' cash flow sources reveals that there are substantial intermediation costs (80% of those are labour costs). Here we assume that all of these costs arise due to loan issuance rather than deposit taking. This assumption is not entirely unreasonable given the labour intensive nature of arranging loans, monitoring them and then recovering them if they become non-performing. It does, however, err on the side of assuming that banks' real loan returns are more dynamically inefficient. We subtract these loan costs from banks' real loan returns to get the final numbers shown in Table 1 below. Full details of data sources and construction are available in Appendix A.

Table 1: Model and data moments

Moment (Model concept)	Data	Model
Real deposit rate - real GDP growth (R^d)	0.950	0.971
Real loan rate - real GDP growth - costs/Assets (R^l)	0.982	0.982
Ratio of M2 to GDP (D/Y)	0.500	0.464
Bank leverage (D/N)	10.00	10.00
Average corporate leverage (L/Z)	0.500	0.530
Leverage of indebted corporates ($L/(sZ)$)	2.000	2.000
Bank rate of return on equity ($R_t^l + \frac{\phi_t\left(R_t^l - R_t^d\right)}{(1-\lambda)}$)	1.100	1.103

Table 2 below presents the values of the parameters chosen to match the moments.

Table 2: Baseline calibration

Parameter	Value
δ	0.167
n	0.011
a^H/a^L	1.100
η	5.000
θ	0.622
λ	0.788
γ	0.907
β	0.958

3.6 Bubbly equilibria without government

3.6.1 Credit frictions and existence of bubbly equilibria

Now we characterise the deposit rate R_t^d and loan rate R_t^l in the steady state without bubbles and discuss when bubbles can circulate. For bubbles to circulate, two conditions are needed. Firstly, bubbles should be attractive. For savers, the opportunity cost of investing in bubbles is the deposite rate, and for banks it is the lending rate. Secondly, bubbles should be affordable. This implies that the rate of return of bubbles conditional on survival is no larger than the rate of economic growth. In our economy the gross growth rate of the economy is unity because there is no technological growth.

As a benchmark case here we show the condition for the existence of bubbly equilibria when $\pi = 1$. Credit frictions suppress the interest rates and those rates are lower than β^{-1} when the credit constraints bind.[7] Similarly to Farhi and Tirole (2011), when $\pi = 1$ whether a bubbly steady state exists and who owns bubbles depend on whether the two interest rates are lower than the growth rate in the 'no bubbles' steady state[8].

In our economy, the severity of credit frictions is represented by two parameters, λ and θ. Figure 1a shows the region of λ and θ in which the deposit rate is less than one and low productivity agents produce in equilibrium (the red area). In this case, the savers (unproductive entrepreneurs) have incentive to buy bubbles in order to boost the rate of return they receive on their savings. The blue parts of the graph show parts of the parameter space where the economy is very credit constrained. At such low values of λ and θ low productivity entrepreneurs are active but wages are so low that even such inefficient projects deliver a rate of return greater than unity. As a result, savers have no incentive to hold bubbles in such economies. The white parts of the graph

[7] See Aoki et al. (2009)) for the general discussion of the relationship between the interest rate and credit frictions.

[8] In Martin and Ventura (2011a) emergence of bubbles itself can creates a "pocket of dynamic inefficiency" so that bubbly equilibria may exist even when the interest rate is greater than the growth rate in no-bubble equilibrium. This result depends on their model setting that credit-constrained agents can create bubbles in subsequent periods. We abstracts from new bubble creation. Because of this assumption, the condition for existence of bubbly equilibria reduces to whether the interest rates in no-bubble equilibrium are smaller than the growth rate.

(very high values of λ and θ) shows parts of the parameter space where low productivity entrepreneurs do not produce because the financial system is well developed. Here again, the rate of return on deposits is greater than unity and savers have no incentive to hold bubbles. So it should be clear from Figure 1b that the conditions for the existence of bubbles is satisfied at intermediate levels of financial development.

[Figure 1a here]

The red area of Figure 1b shows the region in which the loan rate is less than one. Then the banks have an incentive to buy bubbles. It is natural that the part of the parameter space where banks bubbles can exist is more limited compared to the parts of the parameter space where saver bubbles exist. Because banks' borrowing constraints bind, this introduces a positive spread between lending and deposit rates. Hence the parameter space where bank bubbles exist is a subset of the space where savers have an incentive to invest in bubbly assets. Since the deposit rate is always lower than the loan rate, the savers also have incentive to hold bubbles at these parameter values. In equilibrium, it turns out that only the savers have bubbles because their arbitrage implies that the rate of return on bubbles must be equal to the deposit rate, which is lower than the loan rate. Therefore the banks are crowded out from the market for the bubble.

[Figure 1b here]

3.6.2 Who holds risky bubbles: stochastic steady state without government policy

In the previous section we show that in the deterministic steady state, bank deposits and the bubble become perfect substitute and only the savers hold bubbles. Now we allow for risky bubbles and consider an environment in which bubbles only survive with a certain probability ($\pi < 1$.) We focus on a stochastic steady state in which bubbles are traded at a positive value and all the endogenous variables including bubbles are constant. In such a steady state, agents take the probability of bubble bursting into account when they make their investment decision. In that case, bubbles, loans and deposits are no longer perfect substitutes. Also, note that the savers and banks have different preferences for risk. Then it is of interest to analyse who holds risky bubbles.

Table 3 below shows the share of bubbles held by banks. The table looks at banks' participation under different probabilities of the bubble's survival (π). The dashes in the table show economies in which the bubble is too risky for a stochastic steady state to exist in which such a bubble trades at a positive price. This occurs when the survival probability is 0.965. Such a risky bubble would need to command a very high return conditional on survival in order to compensate its holders for the losses when it eventually collapses. But when this 'risk premium' is too high, the bubble follows an explosive path and violates feasibility constraints in finite time. Consequently such a bubble is never valued in equilibrium.

It turns out that banks mostly stay out of the bubble market. Their holdings are zero for very high or very low values of π and barely reach 1.6% of the total stock of bubbly assets when $\pi = 0.985$.

Table 3: Bank bubble holdings in the stochastic steady state

	$\pi = 0.965$	$\pi = 0.975$	$\pi = 0.985$	$\pi = 0.995$
Fraction of bubbles held by banks	-	0.000	0.016	0.000
Ratio of bubbles to total wealth	-	0.118	0.317	0.403
Expected saver loss (fraction of wealth)	-	0.004	0.006	0.003
Expected bank loss (fraction of wealth)	-	0.000	0.002	0.000
$\phi^b_{t+1}/\phi^s_{t+1}$	-	0.984	1.060	0.987

The table also shows the size of bubbles as percentage of total wealth. It shows that bubbles are not large. As a result, the expected losses when bubbles burst are also small for both savers and banks.

The reason why banks hold very few bubbles is straightforward. When banks' borrowing constraints bind, $R^l > R^d$ and banks' investment opportunities (R^l) are superior to savers' (R^d). It is natural therefore that savers should be more willing to take on the risk of saving via the bubble rather than via deposits or own production.

Here, it is worth noting that banks' linear utility does not necessarily make them the natural risk-bearers in equilibrium. Due to binding borrowing constraints and time-varying loan spreads, banks behave as if they are risk-averse. Similar to Gertler and Karadi (2009), periods when bank capital is low are periods when lending spreads are high. Therefore banks' marginal value of an extra unit of net worth (ϕ_t) is high when they choose to hold bubbles and bubbles burst. Indeed, Table 3 shows that, when the banks choose to hold bubbles ($\pi = 0.9875$), the marginal value when bubbles burst at time $t+1$ (ϕ^b_{t+1}), is larger than the marginal value when bubbles survive (ϕ^s_{t+1}). Consequently they have a strong profit motive to ensure some stability in their net worth. Holding a very large quantity of the bubbly asset is not optimal since the excess return (which is pinned down by savers' first order condition too) is not big enough to justify the risks.

4 Equilibrium with government bailouts

4.1 Modification of optimal behavior

Now we characterize the economy with an active government. Introducting government bailout policy changes agents' behaviour in an important way. Firstly, consider productive entrepreneurs. We continue to focus our analysis on the case in which they borrow up to their borrowing constraint. However, their rate of return of production with maximum borrowing (after taxation) is now given by

$$(1-\tau_{t+1})\,\frac{a^H(1-\widetilde{\theta}_t)}{w_t-\widetilde{\theta}_t a^H/R^l_t},\tag{39}$$

where $\widetilde{\theta}_t$ is the maximum amount that can be pledged to private creditors after taking into account expected tax payments. By borrowing from banks secured by

$$\widetilde{\theta}_t=\frac{\theta-E_t\tau_{t+1}}{1-E_t\tau_{t+1}}\leqslant\theta$$

fraction of output, the entrepreneur can finance externally $\widetilde{\theta}_t a_t/R^l_t$ amount (equation (5)). The fraction of output that can be pledged to outside private creditors ($\widetilde{\theta}_t$) is less than the total amount of pledgable returns θ. This is because the government is expected to call on the resources of the firm in the form of taxation $E_t\tau_{t+1}$. Because the government holds the senior claim to the firm's pledgable returns, private creditors can only rely on the residual fraction $\theta-E_t\tau_{t+1}$ which is boosted to some extent by the fact that debt repayments are shielded from taxation (hence the division by $1-E_t\tau_{t+1}$). Also, note that the rate of return on wealth is potentially subject to taxation which is why (39) contains a $1-\tau_{t+1}$ term.

In order for the productive entrepreneurs to borrow up to limit, the following inequalities must hold:

$$\frac{a^H(1-\widetilde{\theta})}{w_t - \widetilde{\theta}a^H/R_t^l} E_t\left[\{(1-\tau_{t+1})\}\frac{1}{c_{t+1}^H}\right] > R_t^d E_t\left[(1-\tau_{t+1})\frac{1}{c_{t+1}^H}\right]$$

and

$$\frac{a^H(1-\widetilde{\theta})}{w_t - \widetilde{\theta}a^H/R_t^l} E_t\left[\{(1-\tau_{t+1})\}\frac{1}{c_{t+1}^H}\right] > E_t\left[(1-\tau_{t+1})\frac{\mu_{t+1}}{\mu_t}\frac{1}{c_{t+1}^H}\right],$$

where $1/c_{t+1}^H$ is the shadow value of wealth at time $t+1$ of the entrepreneur who is productive at time t[9]. The expectation operator is taken over whether bubbles survive or not (and therefore taxes are levied or not).

The equilibrium portfolio choice of bubble owners also changes in an important way. Since they are subject to taxation, the arbitrage condition for the savers is now given by

$$E_t\left[\frac{1}{c_{t+1}^L}(1-\tau_{t+1})\frac{\mu_{t+1}}{\mu_t}\right] = E_t\left[\frac{1}{c_{t+1}^L}(1-\tau_{t+1})\right]R_t^d \tag{40}$$

The arbitrage condition for the bank is also modified:

$$E_t\left[(1-\gamma+\gamma\phi_{t+1})\frac{\tilde{\mu}_{t+1}}{\mu_t}\right] = E_t\left[(1-\gamma+\gamma\phi_{t+1})\right]R_t^l, \tag{41}$$

where

$$\tilde{\mu}_{t+1} = \begin{matrix} \mu_{t+1}^b \text{ with probability } \pi \\ \rho_{t+1}\mu_t \text{ with probability } 1-\pi \end{matrix}. \tag{42}$$

Notice that when the bubble bursts, banks receive a fraction ρ_{t+1} of their origi-

[9] Namely, it is given by

$$1/c_{t+1}^H = (1-\beta)Z_{t+1}^H$$

where Z_{t+1}^H is given by equation (33) nal bubble investment. This is due to a bailout payment from the government.

The portfolio balance condition for entrepreneurs differs in two crucial aspects from (41) above. First of all, the state income valuations differ due to the different preferences of bankers (assumed to be linear in consumption) and entrepreneurs (assumed to be logarithmic). But secondly, the risks faced by the groups of potential bubble investors are very different because of their different access to government bailouts. Here we assume that banks may get at least partially bailed out in the event of losses on direct bubble holdings ($\rho_{t+1} \geqslant 0$) whereas ordinary savers will not.

4.2 Aggregation and market clearing

The employment of the productive entrepreneurs Z_t^H is given by (29) but θ is replaced by $\tilde{\theta}$. Net worth of the productive entrepreneurs is modified as

$$Z_{t+1}^H = (1-\tau_{t+1})\left[(1-\delta)\frac{a^H(1-\widetilde{\theta})}{w_t - \theta a^H/R_t^l}\beta Z_t^H + n\delta\left[R_t^d\left(\beta Z_t^L - m_t^e \mu_t\right) + m_t^e \mu_{t+1}\right]\right]. \tag{43}$$

Notice that μ_{t+1} is equal to $\mu_{t+1}^b > 0$ with probability π. In this case $\tau_{t+1} = 0$. With probability $1-\pi$, $\mu_{t+1} = 0$ and $\tau_{t+1} > 0$. Similarly, the aggregate net worth of the unproductive entrepreneurs evolves as

$$Z_{t+1}^L = (1-\tau_{t+1})\left[\delta\frac{a^H(1-\widetilde{\theta})}{w_t - \theta a^H/R_t^l}\beta Z_t^H + (1-n\delta)\left[R_t^d\left(\beta Z_t^L - m_t^e \mu_t\right) + m_t^e \mu_{t+1}\right]\right]. \tag{44}$$

Aggregate bank net worth is given by

$$N_{t+1} = \gamma\left[R_t^l B_t + m_t^b \tilde{\mu}_{t+1} - R_t^d D_t\right] \tag{45}$$

Finally the government's budget constraint implies that taxes will be levied on entrepreneurs in order to bail out the banks's bubble holdings in the event of a bust. This implies that

$$\tau^c_{t+1} = \rho_{t+1} m^b_t \mu_t \tag{46}$$

where $m^b_t \mu_t$ is the value of the bank's bubble purchase in period t. The tax rate is zero whenever the bubble survives and no bailout is needed. The other equilibrium conditions remain the same as Section 3.

Now equations (18), (25), (28)-(32), (35), (37), (38), (40)-(46) jointly determine 16 variables R^d_t, R^l_t, w_t, H^H_t, H^L_t, Y_t, ϕ_t, D_t, B_t, Z^H_{t+1}, Z^L_{t+1}, N_{t+1}, μ_t, m^e_t, m^b_t $\tau^c_t$with three states Z^H_t, Z^L_t, N_t[10]. At $t = 0$, Z^H_0 is given by (33).

Definition 2 *Competitive bubbly equilibrium with government is a sequence of decision rules* $\left\{H^H_t, H^L_t, Y_t, D_t, B_t, m^e_t, m^b_t\right\}^{\infty}_{t=0}$*, aggregate state variables* $\left\{Z^H_{t+1}, Z^L_{t+1}, N_{t+1}\right\}^{\infty}_{t=0}$ *and a price sequence* $\left\{R^d_t, R^l_t, w_t, \phi_t, \mu_t\right\}^{\infty}_{t=0}$ *and government policy* τ^c_{t+1} *such that: (i) entrepreneurs, banks and workers optimally choose decision rules* $\left\{H^H_t, H^L_t, Y_t, D_t, B_t, m^e_t, m^b_t\right\}^{\infty}_{t=0}$ *taking the evolution of aggregate states, prices, government policy and idiosyncratic productivity opportunities as given; (ii) the price sequence* $\left\{R^d_t, R^l_t, w_t, \phi_t, \mu_t\right\}^{\infty}_{t=0}$ *clears the goods, labor, capital, loan, bubble and deposit markets and (iii) governmenta taxes* τ^c_t *satisfies the government budget constraint given exogenous* ρ_t*; (iv) the equilibrium evolution of state variables* $\left\{Z^H_{t+1}, Z^L_{t+1}, N_{t+1}\right\}^{\infty}_{t=0}$ *is consistent with the individual choices of entrepreneurs, banks and workers and with the exogenous evolution of productive opportunities at the individual firm level.*

[10] By Warlas law one of these equations is redundant.

4.3 Moral Hazard, Bubble Ownership and Amplification

Having described the way anticipation of bailout policy modifies the structure of our model economy, we proceed to examine its effect on the economic allocation by using numerical simulations with our calibrated model. Our focus is on two main questions. What determines banks' ownership of bubbly assets in equilibrium? How does bubble ownership affect the size of the bubble-driven boom-bust cycle?

4.3.1 Moral Hazard Comparative Statics

We start by considering the effect of the financial safety net on the bubbly equilibrium in which bubbles are only expected to burst with probability 0.5% per annum (i.e. bursts happen once every 200 years). The results are presented in Table 4 below.

Table 4: Bailouts and bank risk with low probability of bursting ($\pi = 0.995$)

	$\rho = 0.00$	$\rho = 1/3$	$\rho = 2/3$
Fraction of bubbles held by banks	0.000	0.000	0.000
Bubble to GDP ratio	0.674	0.674	0.674
E(Bubble Return|bank) - R^{l}[11]	-0.004	-0.002	-0.001
E(Bubble Return|saver) - R^d	0.006	0.006	0.006
Bank NW/GDP (pre-crash)	0.057	0.057	0.057
Bank Loss/GDP	0.000	0.000	0.000
% fall in bank NW[12]	0.000	0.000	0.000

[11] This is the expected return to a bank of holding the bubble. It may differ from the expected return for the saver because of bailouts.

[12] The percentage fall in bank net worth is computed after the receipt of government assistance. In other words, in many of the more extreme scenarios considered, the banking system would have negative net worth without a government bailout.

The first row of the table shows that the percentage of bubbles held by banks remains close to zero even for relatively generous financial safety nets (high values of ρ). There, the presence of financing constraints ensures that the bank enjoys a spread of lending over deposit rates, giving rise to a 'franchise value' of the bank. When ordinary savers hold bubbles which are not expected to burst (or are expected to burst with a very low probability), the return on bubbles and deposits must be approximately equal in equilibrium. For banks, in contrast, the indifference condition implies an equality between the loan rate and the return on bubbles. When the spread $R^l - R^d$, is large enough, banks do not find it profitable to hold bubbles under this value of π.

We also compute the effects of the bubble's collapse on banks' capital positions. The 6th and 7th rows of Table 4 shows banks' loss of net worth as percentage of GDP and the percentage fall in bank net worth when the bubbles burst. Since banks are not exposed to bubbly assets, bank losses are zero. Despite the large bubble size (around 67.5% of GDP), the bursting of the bubble does not cause a systemic banking crisis.

Table 5: Bailouts and bank risk with medium probability of bursting

($\pi = 0.98$)

	$\rho = 0.00$	$\rho = 1/3$	$\rho = 2/3$
Fraction of bubbles held by banks	0.000	0.123	0.515
Bubble to GDP ratio	0.296	0.385	0.415
E(Bubble Return|bank) - R^l[13]	-0.002	0.007	0.010
E(Bubble Return|saver) - R^d	0.010	0.011	0.005
Bank NW/GDP (pre-crash)	0.055	0.067	0.092
Bank Loss/GDP	0.000	0.032	0.071
% fall in bank NW[14]	0.000	0.469	0.779

In Table 5 above we consider the effect of the financial safety net when the probability of the bubble bursting is equal to 2% (i.e. the bubble's expected life span is 50 years). Here again banks do not hold bubbles when $\rho = 0$. Expanding the financial safety net (increasing ρ towards unity) increases the incentive for banks to hold bubbles because it shields them from an increasing fraction of the potential losses. The share of bubbles held by banks increases to 51.5% as ρ rises to 2/3 and the size of the bubble grows from 30% to 41.5% of GDP. As the banks' bubble holdings grow, the banking sector expands to absorb these and take advantage of the government guarantee on its risky bubble holdings. The net worth of the banking system relative to GDP increases from 5.5% to 9.2% as ρ increases from 0 to 2/3.

As banks' bubble holdings expand, bank risk grows substantially. The last two rows of Table 5 presents some statistics measuring the impact on bank balance sheets when the bubble bursts. We see that as the financial safety net

[13] This is the expected return to a bank of holding the bubble. It may differ from the expected return for the saver because of bailouts.

[14] The percentage fall in bank net worth is computed after the receipt of government assistance. In other words, in many of the more extreme scenarios considered, the banking system would have negative net worth without a government bailout.

expands, bank losses relative to GDP increase from zero ($\rho = 0$) to 7.1% of GDP ($\rho = 2/3$). Bank capital also experiences much larger falls during the crisis when ρ is high. For example, when $\rho = 2/3$, bank capital falls by 78%. In the event that the expected government assistance does not materialise, the banking system would be deeply insolvent.

Table 6: Bailouts and bank risk with high probability of bursting ($\pi = 0.965$)

	$\rho = 0.00$	$\rho = 1/3$	$\rho = 2/3$
Fraction of bubbles held by banks	-	-	1.000
Bubble to GDP ratio	-	-	0.254
E(Bubble Return|bank) - R^l[15]	-	-	-0.001
E(Bubble Return|saver) - R^d	-	-	0.015
Bank NW/GDP (pre-crash)	-	-	0.057
Bank Loss/GDP	-	-	0.109
% fall in bank NW[16]	-	-	0.774

Finally, Table 6 presents the case of a bubble with a 3.5% probability of bursting (expected life span of about 30 years). As is explained in Table 3, such a bubble would not be valued in the absence of a financial safety net. The dashes in the table show economies in which the bubble is too risky for a stochastic steady state to exist in which such a bubble trades at a positive price.

However, once ρ rises to 2/3, the equilibrium changes. The insurance available to banks means that these bubbles are now sufficiently safe for them and they no longer require such a high risk premium in order to hold them. This ensures that for high values of ρ a stochastic steady state with bubbles exists

[15] This is the expected return to a bank of holding the bubble. It may differ from the expected return for the saver because of bailouts.

[16] The percentage fall in bank net worth is computed after the receipt of government assistance. In other words, in many of the more extreme scenarios considered, the banking system would have negative net worth without a government bailout.

even at high values of the probability of bursting (π). The value of the bubble (entirely held by banks) rises sharply to 25% of GDP and so does bank risk. Potential losses reach 10% of GDP and bank capital falls by more than three quarters in the event of a crisis.

Table 6 reveals an interesting and important result. The introduction of a financial safety net widens the range of bubbles that exist in equilibrium. Fragile bubbles now exist and trade at potentially large valuations amongst banks who hold them due to the explicit or implicit guarantees given to them by governments and central banks. The result that bailout anticipation increases the valuation of risky assets is of course well-known in the literature (see for example Kareken and Wallace (1978)). The novel contribution of our paper is to show how risk-shifting can relax the conditions for the existence of stochastic rational bubbles.

4.3.2 Bubble ownership and amplification

In the previous subsection we showed how the financial safety net affected bank bubble ownership and bank losses in the event of the bubble collapsing. We now examine what effect moral hazard and bubble ownership have on the real effect of the bubble.

Figure 3 below compares the dynamics of the economy starting at the stochastic steady state and tracking the economy's evolution following the bubble's collapse. We compare two scenarios. In one, a generous financial safety net ($\rho = 2/3$) ensures that the bubble asset is held only by banks. In the other, there is no financial safety net ($\rho = 0$) and the bubble is optimally held by savers while banks stay out. In each case, we set the probability that the bubble survives into the following period in order to generate a bubble which is exactly 20% of GDP. Therefore the difference between the two scenarios shown in Figure 3 is due to different bubble ownership rather than different bubble size. The vertical axis of the figure shows the percentage deviation of each variable from its no-bubble steady state value.

[Figure 3 here]

The main feature of the simulations shown in the figure is that the economy with more bank guarantees experiences a more volatile path for output and net worth during the bubbly episode. While the collapse of the saver-held bubble actually generates an expansion in output, output falls under the bank bubble. This difference comes from the degree of banks' exposure to bubbly assets. When bubbles collapse, banks lose a large portion of net worth, causing a sharp lending contraction and an increase in the lending-deposit rate spread. This results in a credit crunch and pushes down the investment of the productive entrepreneurs[17].

Table 7 below tries to go deeper into the underlying mechanism which generates the large positive real effects of bank bubbles during their survival. The table decomposes the increase in output relative to the bubbleless steady state according to the following identity

$$Y = a^L H_t^L + a^H H_t^H \equiv \frac{1}{w_t} \left[a^L w_t \left(H_t^L + H_t^H \right) + \left(a^H - a^L \right) w_t H_t^H \right] \tag{47}$$

which shows that output is determined by total investment $w_t \left(H_t^L + H_t^H \right)$ and the investment of productive agents $w_t H_t^H$ as well as the impact of the cost of employment $\frac{1}{w_t}$ holding investment fixed[18]. We can use the aggregate resouce constraint (37) to substitute total investment $w_t \left(H_t^L + H_t^H \right)$ out from (47)

[17] Output expands after bubbles collapse in the case of saver-held bubble because the savers increase their own inefficient production. Bursting of bubbles causes shortage of means of savings. Then the savers increase their production because this represents another means of savings.

[18] Other things equal, higher wages reduce output and employment because entrepreneurs can only afford to operate on a smaller scale.

and the balance sheet identity of the productive agents in order to substitute $w_t H_t^H$ out from (47). The resulting expression is given by (48) below. This is what we base our output decomposition on.

$$Y = \frac{1}{w_t}\left[a^L\left(\beta Z_t + \gamma N_t\right) - a^L \mu_t + \left(a^H - a^L\right)\left(\beta s_t Z_t + L_t\right)\right] \tag{48}$$

Here $s_t \equiv Z_t^H / Z_t$ represents the share of wealth in the hands of productive agents. The $a^L\left(\beta Z_t + \gamma N_t\right)$ is the 'liquidity effect ' stressed by Farhi and Tirole (2011). $\beta Z_t + \gamma N_t$ is the economy's aggregate saving (comprising of total entrepreneurial saving and bank saving). Bubbles help to increase wealth and (other things equal) this tends to boost investment. The negative $a^L \mu_t$ term is the traditional 'bubble crowding out' effect: when the economy (both savers and bankers) hold bubbles, they hold fewer real productive assets and this (other things equal) tends to reduce output.

The $\left(a^H - a^L\right)\left(\beta Z_t^H + L_t\right)$ term is the investment composition effect discussed by Martin and Ventura (2011a). In a Kiyotaki-Moore environment like ours, a greater share of wealth in the hands of productive agents (s_t) and improved access to bank credit (L_t) lead to a re-allocation of resources from unproductive to productive agents. Even holding total investment constant, this would tend to increase output because aggregate TFP rises due to the fact that factors of production are used by more productive firms. Finally, higher labour costs reduce output for given investment: this is captured by the $\frac{1}{w_t}$ term in (48).

The different columns of the table correspond to different values of ρ but in each case we vary the probability of bubble survival (π) in order to ensure that the bubble is always equal to 20% of GDP.[19] This allows us to focus on

[19] When $\rho = 0$, $\pi = 0.978$; when $\rho = 1/3$, $\pi = 0.976$; when $\rho = 2/3$, $\pi = 0.963$.

the real effect of bubble ownership holding bubble size fixed.

Table 7: Decomposing the real effects of bubbles

	$\rho = 0$	$\rho = 1/3$	$\rho = 2/3$
Bank bubble holdings (fraction of tot. bubble value)	0.00	0.11	1.00
% increase in output relative to 'no bubble' SS	1.03	1.32	3.11
(1) Liquidity effect, %	19.64	20.01	21.78
(2) Bubble 'crowding out' effect, %	- 19.54	- 19.67	- 19.99
(3) Investment composition effect, %	0.95	1.04	1.65
...of which			
(3.1.) Productive net worth, %	0.31	0.34	0.54
(3.2.) Bank lending, %	0.64	0.70	1.11
(4) Labour costs, %	- 0.02	- 0.06	- 0.32

Note: Rows (1)-(4) show the percentage point contributions of various channels to the total increase in output relative to the bubbleless steady state.

The table shows that the larger real effect of bank-held bubbles is largely due to the fact that it generates a bigger liquidity (wealth) effect and due to the fact that it stimulates bank lending to a greater extent.

Table 8: Cost of funds (percentage point deviation from bubbleless steady state)

	$\rho = 0$	$\rho = 1/3$	$\rho = 2/3$
Bank net worth (% increase)	14.43	25.03	113.7
Bank lending to entrepreneurs (% increase)	13.03	14.35	22.69
Lending-Deposit spread (percentage points)	0.02	- 0.01	- 0.22

Table 8 above provides some further intuition on the reasons behind the expansion of credit in the bank bubble case. The first row of the table clearly shows that the increase in bank capital relative to the bubbleless steady state is much larger in the case when banks hold the bubble. This is because banks' ownership of the risky bubble asset boosts the rate of return on wealth while the bubble survives. Bubbles carry a risk premium to compensate the holder for the risk of bursting. Conditional upon no crisis occuring, this risk premium allows the bubble holder to expand its balance sheet substantially. When the bubble holder is a bank, the expansion of its balance sheet also leads to greater credit supply. This is shown in the second row of Table 8 - lending to corporates increases by more when banks are the bubble holders (the $\rho = 2/3$ column) compared to the case when savers are the holders (the first and second columns).

This credit expansion lowers lending-deposit spreads (the third row of the table) helping to boost the leverage of productive entrepreneurs and allowing them to expand the scale of their operations. Note again, that this effect is only present when banks hold the bubble (the third column of the table). When savers are the main (or only) bubble holders, the expansion in bank lending is more limited and lending spreads do not change much.

5 Conclusions

In this paper we build a model with explicit financial intermediation in which rational asset price bubbles arise due to credit frictions. Our aim is to model the interaction between banks' net worth and the value of asset price bubbles which has been a key feature in many historical episodes of boom and bust. This interaction turns out to be very important in understanding the real impact of asset price bubbles on the wider economy.

We show that in the baseline version of our economy, banks have better investment opportunity than savers. As a result, savers have a stronger incentive to 'search for yield' by holding bubbles. Nevertheless, we invoke the possibility that in some cases, banks may be the only bubble holders due to technological or informational constraints on savers' direct participation. We then proceed to compare the way the bubble's impact on the real economy depends on who the main holder is. It turns out that the identity of the holder is very important. When banks are the bubble-holders, this amplifies both the boom while the bubble survives and the bust when it finally bursts. This is because the bubble delivers a high return conditional on not bursting in order to compensate for the tail risk of total loss. During the boom phase, this high return leads to very high profits for the bubble holder. When the banks are the recipients of these profits, this helps to expand credit and boost the net worth of other credit constrained agents. When the savers are the recipients of these profits, no such relaxation of borrowing constraints occurs and the impact of the bubble is relatively limited.

In the second part of the paper, we explore the possibility that government guarantees a part of banks' bubble investments against loss. This can make banks the natural bubble holders even in an environment of full participation in the bubble market by all agents. When bailouts are expected, the banking system expands rapidly to exploit its advantage in holding the bubble, bidding up its price very substantially. This subsidy to bank risk-taking leads to a dramatic increase in the riskiness of bank balance sheets: bank capital falls very sharply when the bubble eventually bursts. Finally, banks' diminished sensitivity to risk increases the range of parameter values for which bubbly equilibria exist. In particular, bank-held bubbles with a very high probability of bursting exist only when the financial safety net is sufficiently generous.

6 Appendix A: Data Sources

In this section we provide details of the sources of the data used for calibrating the model. This is given in Table A1 below:

Table A1:

Theor. concept	Data concept	Source
Real bank loan rate	Real prime loan rate-GDP growth-costs	FRB, Table H.15, FDIC, BEA
Real deposit rate	Real M2 own rate - GDP growth	FRED
Expected inflation	Average CPI inflation (All Urban Consumers)	FRED
Expected real GDP growth	Average real GDP growth (chained measure)	FRED
Deposit stock	M2	FRED
Nominal GDP	Nominal GDP	FRED
Bank leverage	Bank Debt Liabilities/Bank Net Worth	FRB, Table H.8
Average corporate leverage	Corporate Debt/Corporate Net Worth	Welch (2004)
Leverage of indebted corporates	Debt/Corp Net Worth for high leverage corporates	Welch (2004)
Bank rate of return on equity	Bank rate of return on equity	FDIC

7 Appendix B: Computational procedure

In this Appendix, we outline the computational procedure we use to solve for the stochastic steady state in the general case of full participation in the bubble market for all agents.

(1) Start by solving the deterministic steady state of the model in the absence of bubbles. This is where the economy will converge to after the bubble bursts because we do not consider any fundamental shocks in our simulations.

(2) We have one significant forward looking variable in the model ϕ_t and we need to characterise the way it evolves after the bubble bursts. This will give us an expected value of ϕ_t conditional on bursting. We start by guessing such a path.

(3) Solve the system of equations that defines the stochastic steady state, taking the future assumed path of ϕ_t as given. We use Matlab's own non-linear equation solver fsolve.m in order to do this.

Within step (3) we need to determine which regime our economy operates in. In particular we check whether (a) only savers hold bubbles, (b) only banks hold bubbles or (c) both hold bubbles. In addition we need to check whether unproductive agents are active producers in equilibrium[20]. We do this by trial and error:

(3.1) First we check who holds the bubble

(a) First we try to solve the model under the assumption that only savers hold bubbles. This means that savers's first order condition determines the equilibrium growth rate of bubbles conditional upon survival. Once we solve the model under this assumption we check whether banks would like to invest in bubbly assets on the margin by examining the difference in the expected

[20] There are a multitude of other regimes the economy can operate in, depending on whether the banks' and the entrepreneurs' borrowing constraints bind or not. We only consider parameter values for which both of these constraints bind.

utility for bankers from investing in one unit of bubbles and loans. If we find that banks stictly prefer to invest in loans on the margin, this confirms our assumption that only savers buy bubbles and we move on to step (4). If we find that banks strictly prefer to invest in bubbles, then not only savers hold bubbles in equilibrium and we need to try a different assumption.

(b) Only banks hold bubbles. This means that banks's first order condition determines the equilibrium growth rate of bubbles conditional upon survival. Once we solve the model under this assumption we check whether savers would like to invest in bubbly assets on the margin by examining the difference in the expected utility for savers from investing in one unit of bubbles and deposits. If we find that savers stictly prefer to invest in deposits on the margin, this confirms our assumption that only banks buy bubbles and we move on to step (4). If we find that savers strictly prefer to invest in bubbles, then not only banks hold bubbles in equilibrium and we need to try a different assumption.

(c) Both hold bubbles. This means that both banks and savers hold bubbles and each holds a strictly positive fraction of the total bubble stock.

(3.2) Check whether the low productivity agents produce in equilibrium

(a) We solve the model, assuming that the deposit rate is equal to the unleveraged rate of return on the low productivity technology. We then check if the investment of the low productivity firms is positive. If not we reject this equilibrium and assume that low productivity agents are inactive as producers.

(b) We solve the model, assuming that the employment (investment) of low productivity entrepreneurs is zero. We check the resulting equilibrium to ensure that the deposit rate is higher than the unleveraged rate of return on the low productivity technology. If this condition holds, this confirms our initial assumption that saver entrepreneurs prefer not to produce.

(4) Use the output in (3) to compute the initial conditions of the econ-

omy when the bubble bursts. We then solve for a perfect foresight path that converges to the no-bubble deterministic steady state, following the bubble's collapse. We do this by iterating on the ϕ_t path after the bubble's collapse until it converges

(5) Return to step (1) and keep iterating until the entire path of ϕ_t (including its stochastic steady state value) has converged to within error tolerance.

References

Aoki, K., Benigno, G., Kiyotaki, N., 2009. Adjusting to capital liberalization. Working Paper, LSE.

Arce, O., Lopez-Salido, D., 2008. Housing bubbles. Banco de Espania Working Paper No. 0815.

Bernard, A., Eaton, J., Jensen, B., Kortum, S., 2003. Plants and productivity in international trade. American Economic Review 93(4), 1268–1290.

Caballero, R., Krishnamurthy, A., 2006. Bubbles and capital flow volatility: Causes and risk management. Journal of Monetary Economics 53, 35–53.

Farhi, E., Tirole, J., 2011. Bubbly liquidity. Review of Economic Studies forthcoming.

Gertler, M., Karadi, P., 2009. A model of unconventional monetary policy. Working Paper.

Gertler, M., Kiyotaki, N., 2010. Financial intermediation and credit policy in business cycle analysis. In: Woodford, M. (Ed.), Handbook of Monetary Economics. Elsevier.

Hirano, T., Yanagawa, N., 2010. Asset bubbles, endogenous growth, and financial crisis. Working Paper, University of Tokyo.

Kareken, J., Wallace, N., 1978. Deposit insurance and bank regulation: A partial equilibrium exposition. Journal of Business 51, 412–438.

Kocherkalota, N., 2009. Bursting bubbles: Consequences and cures. Working Paper, Federal Reserve Bank of Minneapolis.

Martin, A., Ventura, J., 2011a. Economic growth with bubbles. American Economic Review forthcoming.

Martin, A., Ventura, J., 2011b. Theoretical notes on bubbles and the current crisis. IMF Economic Review 59 (1), 6–40.

Nikolov, K., 2010. A model of borrower reputations as intangible collateral. Working paper, LSE.

Reinhart, C., Rogoff, K., 2008. This time is different: A panoramic view of eight centuries of financial crisis. Working paper, University of Maryland.

Sargent, T., 1987. Dynamic Macroeconomic Theory. Harvard University Press, Cambridge, USA.

Schularick, M., Taylor, A., 2012. Credit booms gone bust: Monetary policy, leverage cycles and financial crises, 1870-2008. American Economic Review forthcoming.

Syverson, C., 2004. Market structure and productivity: A concrete example. Journal of Political Economy 112 (6), 1181–1222.

Tirole, J., 1985. Asset bubbles and overlapping generations. Econometrica 53 (6), 1499–1528.

Welch, I., 2004. Capital structure and stock returns. Journal of Political Economy 112 (1), 106–131.

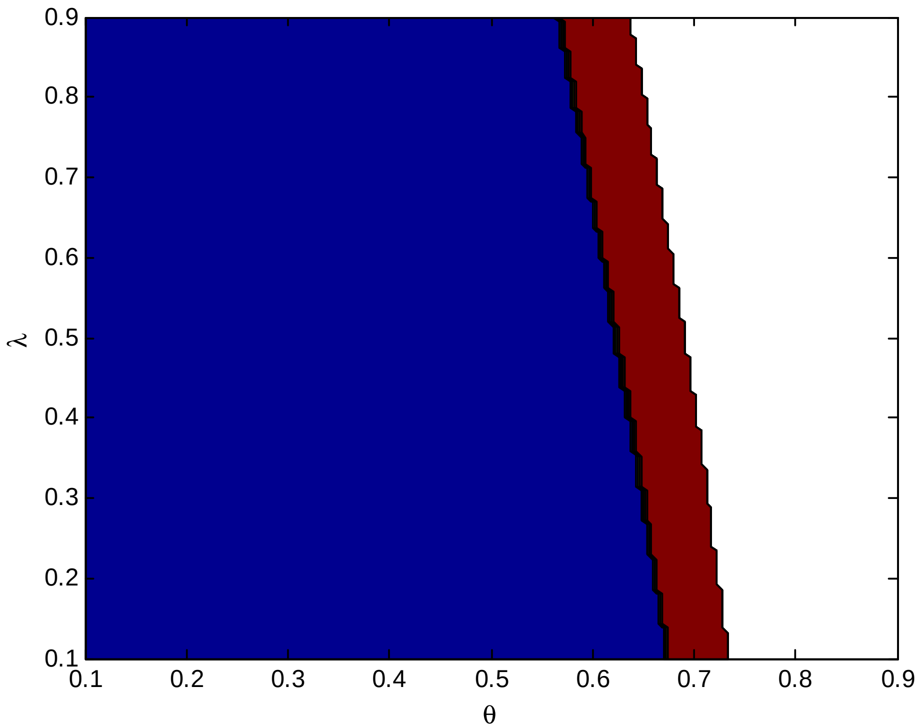

Figure 1a. Deposit rate less than one (red area)

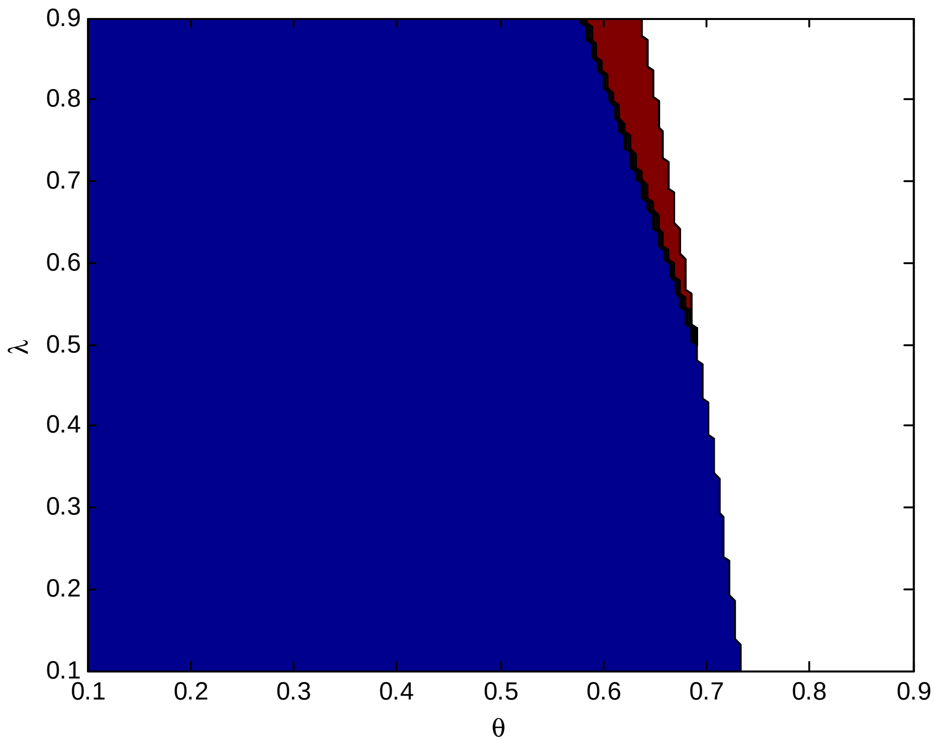

Figure 1b. Lending rate less than one (red area)

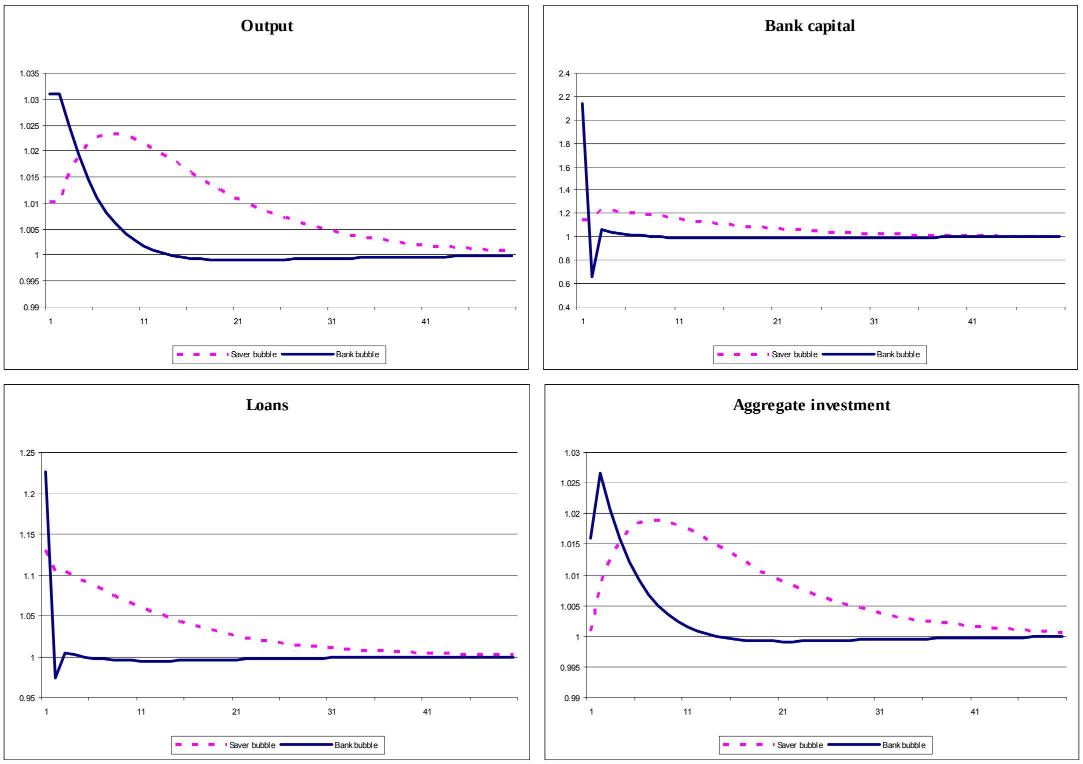

Figure 3: Comparing a bank-held (solid line) and a saver-held (dashed line) bubble