Liquidity, Assets and Business Cycles

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1. What Do I Try to Do?

- Formulate a hypothesis on the role of asset market liquidity in the business cycle

- Calibrate the model to evaluate the hypothesis
Figure 1.1. Deviation of stock price and investment from trend (%)
Figure 1.2. Deviation of stock price and GDP from trend (%)

- stockdev
- gdpdev x10
Figure 1.3. Deviation of stock price and bond price from trend (%)
An intuitive explanation/hypothesis:

Liquidity shocks in asset market are an independent cause of the business cycle.

- sudden drop in liquidity depresses equity price

- tightens financing constraints on investment

- investment and output fall

- demand for liquid assets rises; bond price increases
Policy implication of this hypothesis:

Central banks should and can supply liquidity to the asset market to reduce or eliminate recessions.

Examples:
- bailouts,
- QE1,
- QE2, ....
Hypothesis formulated by N. Kiyotaki and J. Moore (08):

- two frictions in the equity market:
  - difficulty in issuing new equity
  - difficulty in re-selling equity
- liquidity shocks occur in the resale market for equity

Calibrated versions:
Ajello (10): liquidity shocks are important for business cycles
Del Negro et al. (10): Fed policy prevented a greater recession
The tasks:

• simplify the model to capture Kiyotaki-Moore hypothesis:
  – to facilitate aggregation
  – to construct a recursive competitive equilibrium

• calibrate the model to evaluate the hypothesis

What do I find?

• shocks to equity market liquidity can generate large fluctuations in investment, output and employment

• but not all the effects are what one may expect
2. The Model

2.1. The model environment

A large representative household:

- many members share assets at the beginning of a period
- in the period, members are separated from each other, and realize the role as entrepreneurs or workers
- carry out household’s instructions that maximize:

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \pi u(c_t^e) + (1 - \pi) [U(c_t^w) - h(l_t)] \} 
\]

entrepreneur’s worker’s u
A worker has:

one unit of labor; no investment project

An entrepreneur has:

• no labor endowment

• an investment project:
  one unit of good as input $\implies$ one unit of capital

• financing/liquidity constraints (specified later)
Snapshots at different points of time in a period:

- **Beginning of the period:**
  - aggregate state of the economy is realized

  - a household has:
    - physical capital: $k_t$; equity claims: $s_t$; liquid assets: $b_t$

  - a household:
    - divides assets among the members; gives instructions

  - then members are separated until beginning of next period
● Investment/production stage:
  – each member realizes whether he is an entrepreneur (prob $\pi$) or a worker (prob $1 - \pi$)
  – a worker supplies labor $\ell_t$ to produce goods:
    \[ y_t = A_t \ F(k^d_t, \ell^d_t) \]
  – an entrepreneur raises funds for investment $i_t$

● Consumption stage:
  – worker: consumes $c^w_t$ and holds portfolio $(s^w_{t+1}, b^w_{t+1})$
  – entrepreneur: consumes $c^e_t$ and holds portfolio $(s^e_{t+1}, b^e_{t+1})$
Equity market frictions (Kiyotaki-Moore, 08):

- only $\theta \in (0, 1)$ of investment can be financed by new equity
- only a fraction $\phi_t \in (0, 1)$ of existing equity can be re-sold

Equity liquidity constraint:

$$s_{t+1}^e \geq (1 - \theta) i_t + (1 - \phi_t) \sigma s_t$$

- equity at the end
- unsold new equity
- unsold old equity
2.2. A household’s dynamic programing problem

- **Combined liquidity constraint** (shadow price $\lambda^e$):

\[
(r + \phi \sigma q) s + \left( b - p_b b^e_{+1} \right) - \tau \geq (1 - \theta q) i + c^e
\]

rental and adjust downpayment
resale liquid assets on investment

Optimal investment:

\[
q - 1 = (1 - \theta q) \lambda^e
\]

benefit of cost of
ew equity downpayment
2.3. Recursive competitive equilibrium

• components:
  – asset price functions: \((q, p_b)(K, Z)\)
  – factor price functions: \((r, w)(K, Z)\)
  – policy functions: \(x(s, b; K, Z), x \in (i, c^e, s^e_{+1}, b^e_{+1}, \ell, c, s_{+1}, b_{+1})\)
  – value function: \(v(s, b; K, Z)\)

• requirements:
  – optimization by individual households and firms
  – clearing of markets for goods, labor, capital, and assets
  – dynamics of aggregate capital: \(K_{+1} = \sigma K + \pi i(K, B; K, Z)\)
3. Equilibrium responses to shocks

3.1. Calibration

\[ U(c^w) = \frac{(c^w)^{1-\rho} - 1}{1 - \rho}, \quad u(c^e) = u_0 U(c^e) \]

\[ h(\ell) = h_0 \ell^\eta, \quad F(K, (1 - \pi)\ell) = K^\alpha [(1 - \pi)\ell]^{1-\alpha} \]

\[ \log A_{t+1} = (1 - \delta_A) \log A^* + \delta_A \log A_t + \varepsilon_{A,t+1} \]

\[ -\log\left(\frac{1}{\phi_{t+1}} - 1\right) = -(1 - \delta_\phi) \log \left(\frac{1}{\phi^*} - 1\right) \]

\[ -\delta_\phi \log \left(\frac{1}{\phi_t} - 1\right) + \varepsilon_{\phi,t+1} \]
<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>calibration target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$: prob of investment</td>
<td>0.06</td>
<td>annual fraction of investing firms $= 0.24$</td>
</tr>
<tr>
<td>$B$: stock of liquid assets</td>
<td>2.020</td>
<td>fraction of liquid assets in portfolio $= 0.12$</td>
</tr>
<tr>
<td>$\phi^*$: steady st. resaleability</td>
<td>0.276</td>
<td>annual return to liquid assets $= 0.02$</td>
</tr>
<tr>
<td>$\theta$: finance by new equity</td>
<td>0.276</td>
<td>set to equal to $\phi^*$</td>
</tr>
<tr>
<td>$\delta \phi$: $\phi$ persistence</td>
<td>0.9</td>
<td>exogenously chosen</td>
</tr>
<tr>
<td>other</td>
<td></td>
<td>standard targets</td>
</tr>
</tbody>
</table>
3.2. Response to a negative liquidity shock

Experiment:

• at $t = 0$: economy is in non-stochastic steady state

• at the beginning of $t = 1$:
  $\phi$ falls from $\phi^* = 0.276$ to $\phi_1 = 0.05$

• for all $t \geq 2$: $\phi_t$ follows the process with $\varepsilon_{\phi,t} = 0$

• $A$ is fixed at $A^*$, and $\theta$ is fixed, throughout
Figure 2.1. Equity resaleability and investment
Figure 2.2. Employment and output
Figure 2.3. Equity price and bond price
A large and persistent negative shock to equity liquidity generates:

- large and persistent reductions in investment
- large and persistent reductions in output and employment
- problem: large and persistent equity price BOOM
3.3. What is the source of this problem?

Some suspects:

- glitch in Matlab programs
- shock is too large: non-linearity messed up things
- $\theta$ (friction in new equity) is fixed: $\theta$ should fall
- model is unrealistic because it omits:
  - wage/price rigidity; adjustment costs; habit persistence
The simple reason:

• Optimal investment requires:

\[ q - 1 = (1 - \theta q) \lambda^e \]

benefit of equity \quad cost of downpayment

• negative liquidity shock tightens the liquidity constraint, and increases the shadow price of the constraint, \( \lambda^e \)

• equity price \( q \) must rise to restore the balance
\[ q - 1 = (1 - \theta q) \lambda^e \]

The equity price boom is even \textbf{LARGER} if

- \( \theta \) falls: difficulty in issuing new equity increases

- wages are sticky:
  rental income falls, tightening liquidity constraint further

- consumption has habit persistence:
  an entrepreneur also needs to maintain high consumption
Adjustment cost in investment won’t help much either:

- adjustment in investing $i$: $i^*\Psi(i/i^*)$
- optimal investment:
  \[
  q - (1 + \Psi') = (1 + \Psi' - \theta q)\lambda^e
  \]
- $\Psi'$ needs to be large to make a difference, but then
  - investment does not fall by much
  - liquidity constraint is tighter,
  - $\lambda^e$ increases by a lot, and so $q$ increases
Assumptions that reduced the equity price boom:

• structure of large households:
  – pooling assets at the beginning of a period eliminates persistence in heterogeneity in asset holdings
  – this should reduce tightness of liquidity constraint

• rental income is immediately available to entrepreneurs:
  – this relaxed the liquidity constraint
4. Some Solutions to the Problem

For equity price to fall after a negative liquidity shock, the equity liquidity constraint must become \textbf{LESS} tight.

- Need other shocks to sufficiently reduce the need for investment

- Some candidates:
  - negative shock to productivity $A$
  - negative shock to quality of capital
  - negative shock to investment opportunities: a fall in $\pi$
Figure 3.1. Negative shocks to $\phi$ and $A$
Figure 3.2. Investment, output and consumption
Figure 3.3. Equity price and bond price
5. Conclusion

- Liquidity shocks to the asset market
  - can amplify and propagate business cycles: they generate large and persistent changes in macro variables
  - cannot be the primary driving force of business cycles: negative liquidity shocks generate equity price boom!

- Other shocks are needed to reduce equity price in recessions

- Problem exists in ALL models where equity financing is important
• Did the Fed policy help?
  – It might have;

  – but it may not be the cure

• Important to model why asset market liquidity fluctuates
2.2. A household’s maximization problem

- aggregate state \((K, Z)\), \(Z = (A, \phi)\)
  - \(A\): total factor productivity; \(\phi\): equity resaleability

- household’s value function: \(v(s, b; K, Z)\)

- household’s choices of:
  - an entrepreneur’s
    - investment \(i\), consumption \(c^e\), portfolio: \((s^e_{+1}, b^e_{+1})\)
  - quantities per member: \(c, s_{+1}, b_{+1}\)
  - a worker’s labor supply: \(\ell\)
A household’s maximization problem (cont’d):

\[ v(s, b; K, Z) = \max \begin{cases} \pi u(c^e) + (1 - \pi) [U(c^w) - h(\ell)] \\ + \beta \mathbb{E}v(s_{+1}, b_{+1}; K_{+1}, Z_{+1}) \end{cases} \]

(i) household’s resource constraint:

\[ \begin{bmatrix} (q - 1)\pi i + rs + (1 - \pi)w\ell \\ +q(\sigma s - s_{+1}) + (b - p_{b}b_{+1}) - \tau \end{bmatrix} \geq c \]
(ii) equity liquidity constraint: \[ s_{e+1}^e \geq (1 - \theta)i + (1 - \phi)s \]

(iii) an entrepreneur’s resource constraint:

\[ rs + q(i + \sigma s - s_{+1}^e) + (b - p_b b_{+1}^e) - \tau \geq i + c^e \]

**New liquidity constraint** (eliminate \( s_{+1}^e \) from above):

\[
\underbrace{(r + \phi \sigma q)}_{\text{rental and}} \underbrace{s}_{\text{resale}} + \underbrace{(b - p_b b_{+1}^e)}_{\text{adjust liquid assets}} - \tau \geq \underbrace{(1 - \theta q)}_{\text{downpayment on investment}} \underbrace{i}_{\text{on investment}} + \underbrace{c^e}_{\text{on investment}}
\]
Price of liquid assets:

\[ p_b = \beta \mathbb{E} \left[ \frac{U'(c_{+1}^w)}{U'(c_w)} (1 + \pi \lambda_{+1}^e) \right] \]

Equity price:

\[ q = \beta \mathbb{E} \left\{ \frac{U'(c_{+1}^w)}{U'(c_w)} \left[ r_{+1} + \sigma q_{+1} + \pi \lambda_{+1}^e (r_{+1} + \phi_{+1} \sigma q_{+1}) \right] \right\} \]

Equity premium:

\[ \frac{r_{+1} + \sigma q_{+1}}{q} - \frac{1}{p_b} \]
Compute a recursive equilibrium:

- Step 1: given asset price functions \((q, p_b)(K, Z)\),
  firm’s optimal conditions \(\Rightarrow\) factor prices;
  household’s optimization \(\Rightarrow\) policy functions

- Step 2:
  asset pricing equations \(\Rightarrow\) new functions \(T(q, p_b)(K, Z)\)

- Iterate to find a fixed point of mapping \(T\)
<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>calibration target</th>
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<tbody>
<tr>
<td>$\beta$: discount factor</td>
<td>0.992</td>
<td>exogenously chosen</td>
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<tr>
<td>$\rho$: risk aversion</td>
<td>2</td>
<td>exogenously chosen</td>
</tr>
<tr>
<td>$u_0$: utility parameter</td>
<td>44.801</td>
<td>capital stock/annual output $= 3.32$</td>
</tr>
<tr>
<td>$h_0$: scale in disutility</td>
<td>17.005</td>
<td>hours of work $= 0.25$</td>
</tr>
<tr>
<td>$\eta$: curvature of disutility</td>
<td>1.5</td>
<td>labor supply elasticity $1/(\eta - 1) = 2$</td>
</tr>
<tr>
<td>parameter</td>
<td>value</td>
<td>calibration target</td>
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<tr>
<td>$\alpha$: capital share</td>
<td>0.36</td>
<td>labor income share $(1 - \alpha) = 0.64$</td>
</tr>
<tr>
<td>$\sigma$: capital survival</td>
<td>0.981</td>
<td>annual investment/capital = 0.076</td>
</tr>
<tr>
<td>$A^*$: steady state TFP</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td>$\delta_A$: TFP persistence</td>
<td>0.95</td>
<td>persistence in TFP = 0.95</td>
</tr>
<tr>
<td>$g$: gov’t spending</td>
<td>0.193</td>
<td>government spending/GDP = 0.18</td>
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