Forecasting with DSGE Models: Theory and Practice

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August, 2011

CIGS Conference on Macroeconomic Theory and Policy
Estimated DSGE models are now widely used for
  - empirical research in macroeconomics;
  - quantitative policy analysis and prediction at central banks.

The use of DSGE models at central banks has triggered a strong interest in their forecast performance.

Talk is based on a chapter for *Handbook of Economic Forecasting*

**Goals:**

- Review recent advances in forecasting with DSGE models.
- Present some innovations in regard to the incorporation of external information and the use of DSGE-VARs
Comparison of DSGE forecasts (RMSE, ln det of forecast error covariance, log score) with other forecasts

- AR, VAR, BVAR, DFM, etc.
- Professional forecasts: SPF, Bluechip, Green(Teal)book

Incorporating external information into the DSGE model forecasts

- Projections conditional on alternative instrument-rate paths

Forecasting with hybrid models

- Combination of forecasts from different DSGE models as well as other models
Outline of Presentation

- DSGE model used throughout this talk
- Data for forecast evaluation
- Benchmark Forecasts
- Using External Information
- Conditioning on alternative instrument-rate paths
- Forecasting with DSGE-VARs
DSGE Model

- Small-scale model New Keynesian DSGE model (e.g., Woodford, 2003)
  - Euler equation, NK Phillips curve, monetary policy rule
  - 3 exogenous shocks: technology, government spending, monetary policy

- Keep in mind monetary policy rule:

\[ R_t = R_{*,t}^{1-\rho_R} R_{t-1}^{\rho_R} \exp[\sigma_R \epsilon_R,t], \quad R_{*,t} = (r_\pi^*) \left( \frac{\pi_t}{\pi_*} \right)^{\psi_1} \left( \frac{Y_t}{\gamma_* Y_{t-1}} \right)^{\psi_2}. \]

- Measurement equations:
  \[
  \begin{align*}
  GDP(\text{GR})_t & = 100 \ln \gamma_* + \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \\
  \text{INFLATION}_t & = 100 \ln \pi_* + \hat{\pi}_t \\
  \text{FEDFUNDS}_t & = 100(\ln r_* + \ln \pi_* ) + \hat{R}_t.
  \end{align*}
  \]

- Note: Handbook chapter will also contain results for large-scale model
Generating Forecasts with a DSGE Model

- **DSGE Model = State Space Model**
  - Measurement Eq:
    \[ y_t = \Psi_0(\theta) + \Psi_1(\theta)t + \Psi_2(\theta)s_t \]
  - State Transition Eq:
    \[ s_t = \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t \]
- **Posterior distribution of DSGE model parameters:**
  \[ p(\theta|Y_{1:T}) = \frac{p(Y_{1:T}|\theta)p(\theta)}{p(Y_{1:T})}, \quad p(Y_{1:T}) = \int p(Y_{1:T}|\theta)p(\theta)d\theta. \]
- **Objective of interest is predictive distribution:**
  \[ p(Y_{T+1:T+H}|Y_{1:T}) = \int p(Y_{T+1:T+H}|\theta, Y_{1:T})p(\theta|Y_{1:T})d\theta. \]
- Use numerical methods to generate draws from predictive distribution.
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Data Sets

- Forecast evaluations are pseudo-out-of-sample.

- In the past, forecast evaluations were based on the latest available data vintage at the time the study was conducted.

- More recently: use of real time data sets. Particularly important if DSGE model forecasts are compared to professional forecasts.

Reference: Croushore and Stark (2001)
<table>
<thead>
<tr>
<th>Quarter</th>
<th>Greenbook Date</th>
<th>End of Estimation Sample $T$</th>
<th>Initial Forecast Period $T + 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>Jan 21</td>
<td>2003:Q3 (F)</td>
<td>2003:Q4</td>
</tr>
<tr>
<td></td>
<td>Mar 10</td>
<td>2003:Q4 (P)</td>
<td>2004:Q1</td>
</tr>
<tr>
<td>Q2</td>
<td>Apr 28</td>
<td>2003:Q4 (F)</td>
<td>2004:Q1</td>
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<td></td>
<td>June 23</td>
<td>2004:Q1 (P)</td>
<td>2004:Q2</td>
</tr>
<tr>
<td>Q3</td>
<td>Aug 4</td>
<td>2004:Q2 (A)</td>
<td>2004:Q3</td>
</tr>
<tr>
<td></td>
<td>Sep 15</td>
<td>2004:Q2 (P)</td>
<td>2004:Q3</td>
</tr>
<tr>
<td>Q4</td>
<td>Nov 3</td>
<td>2004:Q3 (A)</td>
<td>2004:Q4</td>
</tr>
<tr>
<td></td>
<td>Dec 8</td>
<td>2004:Q3 (P)</td>
<td>2004:Q4</td>
</tr>
</tbody>
</table>

Notes: (A) denotes *advance* NIPA estimates, (P) refers to *preliminary*, and (F) to *final* NIPA estimates.
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- **Benchmark Forecasts**
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Most common approach: evaluation of univariate point forecasts under a quadratic loss function

Root-Mean-Squared Forecast Errors (RSMEs):

\[
RMSE(h) = \sqrt{\frac{1}{T_{\text{max}} - T_{\text{min}}} \sum_{\tau = T_{\text{min}}}^{T_{\text{max}}} (y_{\tau + h} - \hat{y}_{\tau + h|\tau})^2}.
\]

Begin of estimation sample: 1983:Q1

We use Greenbook dates:

- \( T_{\text{min}} \): January 1992
- \( T_{\text{max}} \): December 2009

References: Edge and Gürkaynak (2010)
RMSEs of Benchmark Forecasts

- RMSEs for QoQ rates in percentages
- Red, Solid = DSGE Model
- Green, Dotted = Blue Chip
Figure depicts RMSE ratios: DSGE (reported in various papers) / AR(2) (authors calculation).
Even though DSGE forecasts are potentially RMSE dominated by other forecasts one can ask: are predictive densities well calibrated?

Roughly: in a sequential forecasting setting events that are predicted to have 20% probability, should roughly occur 20% of the time.

Probability Integral Transforms:
- If $Y$ is cdf $F(y)$, then
  \[ P\{F(Y) \leq z\} = P\{Y \leq F^{-1}(z)\} = F(F^{-1}(z)) = z \]
- PITs
  \[ z_{i,t,h} = \int_{-\infty}^{Y_{i,t+h}} p(\tilde{y}_{i,t+h} | Y_{1:T}) d\tilde{y}_{i,t+h}. \]

September 2008

GDP Growth

INFL Rate

Interest Rate
Contribution of Realized Shocks to Forecast Errors

Del Negro, Schorfheide

Forecasting with DSGE Models: Theory and Practice
• DSGE model RMSEs are decent but not stellar. Clearly dominated over short horizons.

• Probability densities seem to be in line with empirical frequencies in our small model.

• Our DSGE model missed the big drop in output and interest rates in 2008.
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- Forecasting with DSGE-VARs
Professional forecasts beat DSGE forecasts in the short-run with respect to variables for which real time information is available.

How can this information be incorporated?

1. Without modifying the DSGE model:

   \[
   \text{replace } p(Y_{T+1:T+H} | \theta, Y_{1:T}) \text{ by } p(Y_{T+1:T+H} | \theta, Y_{1:T}, Z_T)
   \]

   Interpretation: forecaster (but not the agents!) obtains information about future realization of shocks as well as time $T$ state of economy.

2. By introducing additional shocks – and possibly informational frictions – into the DSGE model.
Utilizing External Information without Model Modification

- **Approach 1**: true $Y_{T+1} = \text{external info } Z_T + \text{noise}$
  Roughly speaking, factorize
  \[
  p(Y_{T+1}, Y_{T+2:T+H}|\theta, Y_{1:T}) = p(Y_{T+1}|\theta, Y_{1:T}) \times p(Y_{T+2:T+H}|\theta, Y_{1:T}, Y_{T+1})
  \]
  replace by $p(Y_{T+1}|\theta, Y_{1:T}, Z_T)$

- **Approach 2**: external info $Z_T = \text{true } Y_{T+1} + \text{noise}$
  For example, let $p(Z_T|Y_{T+1}, Y_{1:T}, Y_{T+2:T+H}) = p(Z_T|Y_{T+1})$ and use Bayes Theorem to determine $p(Y_{T+1:T+H}|\theta, Y_{1:T}, Z_T)$.

- Approaches are the same for hard conditioning: $\text{noise} = 0$.

- Under both approaches the forecaster essentially obtains information about the shocks $\epsilon_{T+1}$ as well as the initial state $s_T$.

- Treatment of parameters $p(\theta|Y_{1:T})$ versus $p(\theta|Y_{1:T}, Z_T)$.

- Hard conditioning on Bluechip nowcasts.
- Use Kalman-filter/smooth to extract information about future shocks provided by external information;
- Generate draws from predictive distribution conditional on the extracted information about future shocks
### Illustration 1 – Greenbook Forecast Dates, Estimation Samples, and External Nowcasts in 2004

<table>
<thead>
<tr>
<th>Greenbook Date</th>
<th>End of Estimation Sample $T$</th>
<th>External Nowcast BlueChip Date</th>
<th>External Nowcast Period $T + 1$</th>
<th>Initial Forecast Period $T + 2$</th>
</tr>
</thead>
</table>
Illustration 1 – RMSE Comparison: Benchmark versus Model with Ext. Nowcasts

- **Red, Solid** = DSGE Model
- **Blue, Dashed** = DSGE Model + Blue Chip Nowcasts
- **Green, Dotted** = Blue Chip
Illustration 2 – Suppose in January 1992 we hard-condition on the belief that interest rates stay constant for 1 year.

<table>
<thead>
<tr>
<th>Period</th>
<th>$\epsilon^g$</th>
<th>$\epsilon^R$</th>
<th>$\epsilon^z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = T + 1$</td>
<td>-0.10</td>
<td>-0.10</td>
<td>-0.07</td>
</tr>
<tr>
<td>$t = T + 2$</td>
<td>-0.24</td>
<td>-0.04</td>
<td>-0.02</td>
</tr>
<tr>
<td>$t = T + 3$</td>
<td>-0.19</td>
<td>-0.04</td>
<td>-0.01</td>
</tr>
<tr>
<td>$t = T + 4$</td>
<td>-0.17</td>
<td>-0.02</td>
<td>-0.01</td>
</tr>
</tbody>
</table>
The likelihood of the structural shocks that are needed to attain the path of observables implied by the external information provides a measure of how plausible this external information is in view of the model. References: Benes, Binning, and Lees (2008)

Rather than using the Kalman filter, one could select a subset of shocks that guarantee that pre-specified future values of observables are attained, e.g., use MP shocks to implement an interest rate path. More on this later... References: Smets and Wouters (2004), Christoffel, Coenen, and Warne (2007)


Incorporating monthly information into the estimation of DSGE models. Giannone, Monti, and Reichlin (2009)
Three approaches:

1. Add measurement equation:
   \[
   \text{external info} = \text{agents’ expectations} + \text{noise}
   \]

2. Introduce additional structural shocks such that agents’ expectations and external information, e.g., survey forecasts, can be equated.

3. External information is interpreted as a signal that agents in the model receive and can use to form expectations.

We shall focus on the second approach:

1. Example 1: incorporate long-run inflation expectations

2. Example 2: incorporate interest-rate expectations

Related literatures: anticipated shocks, DSGE models with informational frictions
Many countries have experienced a decline in the inflation rate.

Inflation forecasts are important for central banks.

In a constant $\pi_*$ model the long-run forecast of inflation is essentially the sample average, which often implies an implausible reversion to a relatively high inflation rate.

Possible solution: incorporate long-run inflation expectations.

References: Aruoba and Schorfheide (2010), Wright (2011)
Example 1: Long-Run Inflation Expectations

- Modify interest-rate feedback rule as follows:

\[ R_t = R_{*,t}^{1-\rho^R} R_{t-1}^{\rho^R} \exp[\sigma^R \epsilon^R, t], \quad R_{*,t} = (r_{\pi*,t}) \left( \frac{\pi_t}{\pi_{\pi*,t}} \right)^{\psi_1} \left( \frac{Y_t}{\gamma Y_{t-1}} \right)^{\psi_2}. \]

- \( \pi_{\pi*,t} \) is the time-varying target inflation rate.

- Assume that agents forecast the target according to

\[ \pi_{\pi*,t} = \pi_{\pi*,t-1} + \sigma_{\pi} \epsilon_{\pi, t}. \]

- Long-run inflation expectations are interpreted as observations on \( \pi_{\pi*,t} \).

- To amplify the effect, we change the beginning of estimation sample from 1983:Q1 to 1965:Q1.
Example 1: RMSE Comparison: Benchmark versus Model with $\pi_{*,t}$

- **Red, Solid** = Benchmark DSGE Model
- **Blue, Dashed** = $\pi_{*,t}$ DSGE Model
- **Green, Dotted** = Blue Chip

Note: gain is a lot smaller if estimation sample starts in 1983:Q1.
Example 2: Using Interest Rate Expectations

- We’ve seen that the interest rate forecasts of the DSGE model are relatively poor.

- Not appealing from a policy maker’s perspective.

- We’ll use interest rate expectations $\mathbb{E}_T[R_{T+1}]$, $\mathbb{E}_T[R_{T+2}]$, ... when generating forecasts.
Example 2: Using Interest Rate Expectations

- To match external interest rate forecast and model-implied expectations, add anticipated monetary policy shocks to the model.

- Simple version of the new policy rule can be expressed as:

\[
\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \psi \hat{\pi}_t + \sigma_R \epsilon_t^R + \sum_{k=1}^{K} \sigma_{R,k} \epsilon_{k,t-k}^R,
\]

where \( \epsilon_t^R \) is the usual unanticipated policy shock, and \( \epsilon_{k,t-k}^R \), \( k = 1, \ldots, K \) is an anticipated policy shock that affects the policy rule \( k \) periods later.

- So far, anticipated shocks are only used during forecasting step. We distributed the estimated variance of the MP shock across the unanticipated and two anticipated MP shocks.
Example 2: How Do Anticipated Policy Shocks Work?

- **Simple Model:**

\[
\begin{align*}
y_t & = \mathbb{E}[y_{t+1}] - (R_t - \mathbb{E}[\pi_{t+1}]) \\
\pi_t & = \beta \mathbb{E}[\pi_{t+1}] + \kappa y_t \\
R_t & = \frac{1}{\beta} \pi_t + \epsilon^R_t + \epsilon^R_{1,t-1}.
\end{align*}
\]

- **Solution:**

\[
\begin{align*}
y_t & = -\psi \left( \epsilon^R_t + \epsilon^R_{1,t-1} + \psi \epsilon^R_{1,t} \right) \\
\pi_t & = -\kappa \psi \left( \epsilon^R_t + \epsilon^R_{1,t-1} + (\psi + \beta) \epsilon^R_{1,t} \right) \\
R_t & = \psi \left( \epsilon^R_t + \epsilon^R_{1,t-1} - \frac{1}{\beta} \kappa (\psi + \beta) \epsilon^R_{1,t} \right)
\end{align*}
\]

where \( \psi = (1 + \kappa / \beta)^{-1} \).
Density Forecasts with and without Interest Rate Expectations – March 2010

Benchmark DSGE Model

Using Nowcasts and Interest Rate Expectations
RMSE Comparison: Benchmark versus Model with Ext. Nowcasts and Interest Rate Expectations

- **Red, Solid** = Benchmark DSGE Model
- **Blue, Dashed** = DSGE Model with Interest Rate Expectations and Blue Chip nowcasts.
- **Green, Dotted** = Blue Chip
- Hard conditioning on nowcasts understates uncertainty in the current quarter.
- One-year ahead forecasts appear well calibrated.
Hard-conditioning on external nowcasts, which improved short-run forecasting performance of DSGE model, but understates uncertainty in predictive densities.

We showed how time-varying inflation targets and anticipated monetary policy shocks can be used to incorporate inflation and interest rate expectations to improve forecasts.
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Policy analysis is one of the potential strength of the DSGE model forecasting approach.

Two approaches:


2. Choose sequence of anticipated policy shocks to attain the desired instrument-rate path. References: Laseen and Svenson (2009).

Only the second approach is consistent with the notion that the central bank credibly announces an interest rate path.
Conditional Projections Using Anticipated Shocks – Constant Interest Rate for 4 Periods

GDP Growth

INFL Rate

Interest Rate
The use of unanticipated MP shocks is not consistent with the notion of a policy that tries to announce interest rate paths.

Anticipated MP shocks appear conceptually attractive but – depending on the properties of the estimated model – might generate fairly implausible predictions for non-policy variables.
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Hybrid Models

- Goal: relax some of the DSGE model restrictions to improve forecasting performance


VAR

\[ y_t = \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + \Phi_c + \Sigma_{tr} \Omega \epsilon_t, \]

with binding functions:

\[ \Phi^*(\theta), \quad \Sigma^*(\theta), \quad \Omega^*(\theta). \]

Allow for deviations from restriction functions...
Hierarchical Hybrid Models

Subspace generated by the DSGE model restrictions

Prior for misspecification parameters \( \Phi^\Delta \): Shape of contours determined by Kullback-Leibler distance.

\( \Phi^*(\theta) \): Cross-equation restriction for given value of \( \theta \)

\( \Phi^*(\theta)+\Phi^\Delta \)

Subspace generated by the DSGE model restrictions
Overall, the setup leads to

\[ p_\lambda(Y, \Phi, \Sigma, \theta) = p(Y|\Phi, \Sigma)p_\lambda(\Phi, \Sigma, \Omega|\theta)p(\theta). \]  \hspace{1cm} (1)

In previous applications \( \Omega|\theta \) is a pointmass centered at \( \Omega^*(\theta) \)

Posterior for \( \lambda \):

\[ p(\lambda|Y) \propto (Y|\lambda) = \int p_\lambda(Y|\theta)p(\theta)d\theta. \]  \hspace{1cm} (2)

Applications: model evaluation, forecasting, policy analysis.

**Prior** can be interpreted as **posterior** calculated from observations generated from DSGE model.

We start out with a Minnesota prior and then “add” artificial observations from the DSGE model.

\[ \lambda = 0 \] means no DSGE model observations to construct the prior, \( \lambda = \infty \) means that we impose DSGE restrictions \( \Phi^*(\theta), \Sigma^*(\theta), \) and \( \Omega^*(\theta) \).

Innovation here: (i) we mix Minnesota and DSGE model prior; (ii) average over hyperparameter \( \lambda \).
Posterior Weights for $\lambda$

- Grid for $\lambda$: 0.1, 0.5, 1, 2, $\infty$
- Red = $P_T\{\lambda \leq 0.1\}$
- Blue = $P_T\{\lambda \leq 0.5\}$
- Green = $P_T\{\lambda \leq 1\}$
RMSE Comparison: Benchmark versus DSGE-VAR

- **Red, Solid** = Benchmark DSGE Model
- **Blue, Dashed** = DSGE-VAR
- **Green, Dotted** = Blue Chip
DSGE model restrictions can be relaxed in many different ways, which opens the door for forecast improvements.

DSGE-VAR approach creates a hybrid model that in many dimensions mimics the underlying DSGE model.

In our illustration there are gains in forecast performance, but they are smaller than those attained by incorporating real time information.
There exists a large literature on forecasting with DSGE models. Most of the forecast evaluation focuses on RMSEs. Some recent work on evaluating density forecasts and predictions comovements, e.g. Herbst and Schorfheide (2010).

Lots to be gained from incorporating real time information. Models with anticipated shocks or informational frictions open interesting avenues to incorporate expectation data into forecasts.

Projections conditional on interest rate paths still problematic.