Systemic bank runs in a DSGE model

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Very Preliminary. Comments Welcome.
Financial crisis of 2007-2009

- What happened in the financial crisis of 2007-2009 can be interpreted as bank runs on various forms of short-term debt (Gorton and Metrick, 2010).
  - runs on the repurchase agreements (repo) market:
    - Gorton and Metrick (2009), Lucas and Stokey (2011), etc.
  - runs on commercial paper:
    - Arteta, Carey, Correa, and Kotter (2010), Covitz, Liang, and Suarez (2009), Kacperczyk and Schnabl (2010), etc.
  - runs on dealer banks:
    - Duffie (2011).
- Also, it was a “systemic event” in the sense that the financial intermediary sector became insolvent as a whole (Gorton and Metrick, 2009).
- It triggered the “Great Recession.”
Repos and commercial paper

Adrian and Shin (2010)

![Graph showing overnight repos and commercial paper](image_url)

**Figure 11**
Overnight repurchase agreements (repos) and M2. All data have been normalized to equal 1 on July 6, 1994. CP, commercial paper. Data taken from the Federal Reserve, 1994W1–2010W5.
Haircut index

Gorton and Metrick (2010)

Figure 4: The Repo-Haircut Index

Notes: The repo-haircut index is the equally-weighted average haircut for all nine asset classes included in Table I, Panel D.
What we do in this paper

- develop a DSGE model with bank runs.
- focus on
  - "fundamental bank runs" instead of "sunspot bank runs," and
  - "systemic bank runs" rather than "idiosyncratic bank runs."

- analyze how a systemic bank run amplifies a negative productivity shock:
  a sufficiently negative productivity shock
    \[ \implies \] a systemic bank run
    \[ \implies \downarrow \] supply of liquidity
    \[ \implies \downarrow \] working capital
    \[ \implies \uparrow \] labor wedge (gap between MPL and MRS)
    \[ \implies \downarrow \downarrow \] output.
Our model: Banks

Our modeling of banks follows Diamond and Rajan (2001).

- Banks have superior loan collection skills, which are relation specific.
  - Holdup problem: Banks may threaten to withdraw their skills in order to get more rents.
- Demand deposits make banks susceptible to runs, but prevent them from behaving opportunistically.

Liquidity creation by banks:

- Banks obtain funds in the form of demand deposits and make loans to firms; they collect loan payments from firms and provide the economy with liquidity.
Our model: Systemic bank runs

- A systemic bank run is caused by a sufficiently negative productivity shock:
  - A negative technology shock lowers the surplus generated by firms, which lowers the repayments collected by banks.

  
  - If the shock is bad enough, all banks would become insolvent, which leads to a systemic bank run, and damages the economy’s capacity to create liquidity.
Our model: Propagation and amplification

A systemic bank run reduces the supply of liquidity.

\[ \Rightarrow \] lowers the amount of working capital available to firms.

\[ \Rightarrow \] decreases employment and hence output further.

A bank run distorts the economy by enlarging the gap between MPL and MRS (the labor wedge).

\[ \bullet \] consistent with what happened during the Great Recession.
1 Introduction

2 The model economy

3 Numerical results

4 Conclusion
Households

- a continuum of identical households.
- Each household consists of
  - a (standard) infinitely-lived consumer/worker, and
  - overlapping generations of two-period lived firms and banks.
- In every period a firm and a bank are born in each household.
Firms and banks

- A new born firm needs funds to purchase physical capital and to hire labor.
  - It must obtain loans from a bank in a different household.
- A new born bank must raise funds in the form of demand deposits.
  - It should obtain them from other households.
- New born firms and banks in different households are matched randomly in each period.
Match between a firm and a bank

Consider a match formed in period $t$.

The bank:
- $b_t =$ amount of funds that the bank obtains by issuing demand deposits.
- acquires relation-specific loan-collection skills.

The firm:
- in period $t$:
  - borrows $b_t$ from the bank.
  - purchases physical capital $k_t$ in period $t$.
  - deposits the rest, $d^F_t \equiv b_t - k_t$, which becomes the working capital.
- in period $t + 1$:
  - hires labor from other households and produces output:
    \[
    y_{t+1} = A_{t+1} k_t^\alpha l^{1-\alpha}_{t+1}
    \]
  - where $A_{t+1} =$ economy-wide productivity shock.
Repayment of the loan

- Let $\Pi_{t+1}$ = total surplus generated by the firm in period $t + 1$.
- The repayment of the loan is a constant fraction of $\Pi_{t+1}$.
- The bank has superior loan-collection skills.
  - $\Theta\Pi_{t+1}$ = the amount that the bank can take as the repayment of the loan;
  - But if someone else negotiates with the firm on the repayment, it reduces to $\theta\Pi_{t+1}$, where $\theta < \Theta$.
- In particular if a run against the bank occurs,
  - the depositors of the bank become a collective owner of its loans to the firm and they collectively negotiate with the firm on the repayment.
  - As a result, the depositors obtain $\theta\Pi_{t+1}$ from the firm, which is shared equally.
Bank runs

- Let $s_{t+1}$ denote the occurrence of a systemic bank run in period $t + 1$.

$$s_{t+1} = \begin{cases} 
1, & \text{if a systemic bank run occurs in period } t + 1, \\
0, & \text{otherwise}.
\end{cases}$$

- $1 + r_t =$ interest rate on demand deposits between periods $t$ and $t + 1$.
- When a bank run occurs, however, it is reduced to $\xi_{t+1}(1 + r_t)$.
- Define $\widetilde{\xi}_{t+1}$ by

$$\widetilde{\xi}_{t+1} = \begin{cases} 
1, & \text{if } s_{t+1} = 0, \\
\xi_{t+1}, & \text{if } s_{t+1} = 1.
\end{cases}$$
Problem of the firm born in period $t$

- Profit earned by the firm born in period $t$ is

$$\pi_{t+1}^F = (1 - \tilde{\theta}_{t+1})\Pi_{t+1}$$

where

$$\Pi_{t+1} = A_{t+1}k_t^\alpha l_{t+1}^{1-\alpha} - w_{t+1}l_{t+1} + \tilde{\xi}_{t+1}(1 + r_t)d_t^F + (1 - \delta)k_t$$

$$\tilde{\theta}_{t+1} = \begin{cases} \Theta, & \text{if } s_{t+1} = 0, \\ \theta, & \text{if } s_{t+1} = 1. \end{cases}$$

- The firm chooses $\{k_t, d_t^F, l_{t+1}\}$ to maximize:

$$\max_{\{k_t, d_t^F, l_{t+1}\}} E_t \frac{\beta \lambda_{t+1}}{\lambda_t} \pi_{t+1}^F,$$

s.t.

$$k_t + d_t^F \leq b_t,$$

$$w_{t+1}l_{t+1} \leq \tilde{\xi}_{t+1}(1 + r_t)d_t^F,$$

where $\beta^t \lambda_t = \text{stochastic discount factor}$. 
Problem of the bank born in period $t$

- Profit earned by the bank born in period $t$ is

$$\pi^B_{t+1} = \max \left\{ \tilde{\pi}^B_{t+1}, 0 \right\},$$

where

$$\tilde{\pi}^B_{t+1} = \Theta \Pi_{t+1} - (1 + r_t) b_t + T_{t+1},$$

where $T_{t+1}$ denotes the transfer from the government.

- It chooses $b_t$ to solve

$$\max_{b_t} E_t \beta \lambda_{t+1} \frac{\lambda_t}{\lambda_t} \pi^B_{t+1}. $$

Here, The bank takes into account the fact that the paired firm chooses \{ $k_t$, $d_t^F$, $l_{t+1}$ \} as a function of $b_t$ by solving its profit-maximization problem.
Bank run

- If a bank run occurs in period \( t + 1 \), it does so after \( A_{t+1} \) has been realized but before the production process starts.
  - \( \bar{A}_{t+1} = \) threshold level of productivity below which a bank run occurs.
- The firm born chooses \( l_{t+1} \) given \((d^F_t, k_t, A_{t+1}, r_t, w_{t+1}, \tilde{\xi}_{t+1})\) to solve

\[
\max_{l_{t+1}} A_{t+1} k^\alpha_t l^ {1-\alpha}_{t+1} - w_{t+1} l_{t+1}
\]

\[
\text{s.t. } w_{t+1} l_{t+1} \leq \tilde{\xi}_{t+1}(1 + r_t) d^F_t
\]

- Given \((d^F_t, k_t, r_t, w_{t+1})\), consider the hypothetical problem for each \( A' > 0 \):

\[
l^*_{t+1}(A') \equiv \arg \max_{l_{t+1}} A' k^\alpha_t l^ {1-\alpha}_{t+1} - w_{t+1} l_{t+1}
\]

\[
\text{s.t. } w_{t+1} l_{t+1} \leq (1 + r_t) d^F_t
\]
The threshold value $\bar{A}_{t+1}$ is given as the solution to

$$
\Theta\left\{ \bar{A}_{t+1} k_t^\alpha \left[ l^*_t(\bar{A}_{t+1}) \right]^{1-\alpha} - w_{t+1} l^*_t(\bar{A}_{t+1}) + (1 + r_t) d_t^F + (1 - \delta) k_t \right\}
- (1 + r_t) b_t + T_{t+1} = 0.
$$

If $A_{t+1} < \bar{A}_{t+1}$ then $\tilde{\pi}_{t+1}^B < 0$ even without a bank run.

Thus, all banks born in period $t$ become insolvent if $A_{t+1} < \bar{A}_{t+1}$. That is,

$$s_{t+1} = 1 \iff A_{t+1} < \bar{A}_{t+1}$$

When a bank run occurs, the ex-post rate of return on the bank account is determined by

$$\xi_{t+1}(1 + r_t) b_t = \theta \Pi_{t+1} + T_{t+1}.$$
Household’s problem

Preferences:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t). \]

The only financial asset that each household can hold is the bank account in banks owned by other households.

Its flow budget constraint is

\[ c_t + d_t^H = \xi_t (1 + r_{t-1})d_{t-1}^H + w_t l_t + \pi^F_t + \pi^B_t - T_t \]

where \( T_t \) is lump-sum taxes.
Government policy

- Compare two simple policy regimes.
  1. Without policy intervention (laissez-faire):
     \[ T_t = 0, \quad \text{for all } t. \]
  2. With intervention:
     \[
     T_t = \max \left\{ (1 + r_{t-1})b_{t-1} - \Theta [A_t k_{t-1}^{\alpha} (l_t^* (A_t))^{1-\alpha} \right.
     
     - w_t l_t^* (A_t) + (1 + r_{t-1})d_{t-1}^F + (1 - \delta) k_{t-1}], 0 \right\}
     \]

- With the second regime, \( A_t \geq \bar{A}_t \) for all \( t \) so that a systemic bank run never occurs.
Summary: Flow of funds in normal times

- Loans to firms:

\[
\text{depositors} \xrightarrow{d_t^H + d_t^F} \text{banks} \xrightarrow{b_t} \text{firms} \left\{ \begin{array}{l}
k_t \\
d_t^F 
\end{array} \right. 
\]

- Creation of liquidity (without govt subsidies):

\[
\text{firms} \xrightarrow{\Theta \Pi_t} \text{banks} \xrightarrow{(1+r_{t-1})b_{t-1}} \left\{ \begin{array}{l}
\text{households}: (1 + r_{t-1})d_{t-1}^H \\
\text{firms}: (1 + r_{t-1})d_{t-1}^F 
\end{array} \right. 
\]

- The demand for liquidity is predetermined. As long as the supply of liquidity exceeds the demand, a bank run does not occur.

\[
\frac{(1 + r_{t-1})(d_{t-1}^H + d_{t-1}^F)}{\Theta \Pi_t} \leq \frac{\Theta \Pi_t}{\text{demand for liquidity}} \leq \frac{\text{supply of liquidity}}{\text{supply of liquidity}}
\]
Summary: Systemic bank run

- A negative productivity shock reduces $\Pi_t$, and hence the supply of liquidity $\Theta \Pi_t$.

- Under the laissez-faire policy regime, a systemic bank run occurs if the shock is large enough that

$$
\frac{(1 + r_{t-1})(d_{t-1}^H + d_{t-1}^F)}{\Theta \Pi_t} > 1
$$

- As a result, the supply liquidity further reduces to $\theta \Pi_t$:

- Rationing of liquidity: only the fraction $\xi_t$ of the liquidity demand is satisfied.

$$
\xi_t(1 + r_{t-1})(d_{t-1}^H + d_{t-1}^F) = \theta \Pi_t
$$
Summary: Propagation and amplification

- A systemic bank run reduces the amount of working capital available to firms by a factor of $\xi_t$.
  - The working capital is used to pay the wage bill:
    \[
    wtlt \leq \xi_t (1 + rt-1) dt^{F}_{t-1}
    \]
  - Thus the run reduces employment and output further, amplifying the effect of the productivity shock.

- Define the labor wedge as the gap between MPL and MRS:
  \[
  \text{labor wedge} = \frac{\text{MPL}}{\text{MRS}}
  \]
  A systemic bank run distorts the economy by increasing this gap.
Numerical example

- Functional forms:

\[ u(c, l) = \ln(c) + \psi \ln(1 - l). \]

- Parameter values: \( \alpha = 0.4, \beta = 0.98, \delta = 0.1, \psi = 0.75, \Theta = 0.9, \theta = 0.65. \)

- Period 0:
  - at the non-stochastic steady state associated with \( A_t = 1 \) for all \( t \).

- Period 1:
  - there is an unexpected temporary decline in productivity: \( A_1 = 0.95. \)

- Periods \( t \geq 2 \):
  - \( A_t \) returns to the original level: \( A_t = 1 \) for all \( t \geq 2. \)
Numerical results

- **Output**
- **Investment**
- **Consumption**

- **Labor**
- **Labor wedge**
- **Interest rate**
- **Capital**

Lines represent:
- Blue: with intervention
- Red: without intervention

Graphs illustrate the effects of systemic bank runs in a DSGE model.
constructs a DSGE model with systemic bank runs.

- bank run $\Rightarrow \downarrow$ liquidity $\Rightarrow \downarrow$ working capital $\Rightarrow \uparrow$ labor wedge $\Rightarrow \downarrow$ output.

The systemic bank run amplifies the effect of a negative productivity shock in an nonlinear way:

![Diagram](image-url)
Some directions for future research:

- In the current model, the policy intervention to prevent a systemic bank run has no costs at all.
  - Distortionary effects of such policy should be taken into account.
- Consider more rich and realistic ways of policy intervention.
- Consider public liquidity such as money and government bonds.
Related literature

- bank runs:
  - Angeloni and Faia (2010): DSGE model with idiosyncratic bank runs;
  - Uhlig (2010): two-period model of systemic bank runs;

- agency problems between banks and depositors: