Systemic bank runs in a DSGE model

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Abstract

We consider systemic bank runs in the DSGE framework. The demand deposit contract is used as a commitment device for banks to create liquidity, but at the same time it makes banks susceptible to runs. In our model bank runs amplify the effects of a bad shock to the economy as follows. A sufficiently negative productivity shock makes the banking sector insolvent as a whole, leading to a systemic bank run. It creates aggregate liquidity shortages, and reduces the amount of working capital available to firms. This worsens the labor wedge, leading to a further decline in employment and output.

Keywords: Financial crisis, bank run, working capital, labor wedge, amplification.

JEL Classification numbers: E32, E60, G01, G21.

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1 Introduction

What happened in the financial crisis of 2007-2009 can be interpreted as bank runs on various forms of short-term debt, as emphasized, for instance, by Gorton and Metrick (2010). For example, there were runs on the repurchase agreements (repo) market as documented by Gorton and Metrick (2009) and Lucas and Stokey (2011). There were also runs on other short-term debt such as commercial paper and money markets funds (see, for instance, Arteta, Carey, Correa, and Kotter 2010, Covitz, Liang, and Suarez 2009, Kacperczyk and Schnabl 2010, Investment Company Institute 2009). Also notable is that the crisis was a “systemic event” in the sense that the financial intermediary sector became insolvent as a whole (Gorton and Metrick, 2009). In response to the crisis, the Fed has taken various measures to provide liquidity and act as a lender of last resort. Nevertheless, it triggered the largest economic downturn since World War II.

There is large literature on bank runs, such as Bryand (1980), Diamond and Dybvig (1983), Allen and Gale (1998), and Diamond and Rajan (2001), among others. But the previous research on bank runs is mostly based on two- or three-period models. Here we develop a dynamic stochastic general equilibrium (DSGE) model with bank runs, as a step toward understanding the macroeconomic effects of the financial crisis and assessing the policy responses to it. As in Allen and Gale (1998), we restrict attention to bank runs caused by shocks to fundamentals (“fundamental bank runs”) rather than those due to sunspot shocks (“sunspot bank runs”). Also we focus on “systemic bank runs,” in which the banking sector as a whole is subject to runs, as opposed to “idiosyncratic bank runs,” in which runs occur against an individual bank (or a fraction of banks).

Our model has two important features. The first is concerned with how a systemic bank run occurs, and the second is regarding how a banking crisis exacerbates a recession. In terms of the mechanism that generates a bank run, our model is based in particular on Diamond and Rajan (2001). Banks play two roles. First, they obtain funds in the form of demand deposits and make loans to firms. Second, they collect loan payments from firms and provide the economy with liquidity. As in Diamond and Rajan (2001), we assume that banks have superior loan-collection skills compared to other agents. Suppose that a negative productivity shock hits the economy. It lowers the surplus generated by firms, which, in turn, reduces the revenue of banks. If the shock is large enough, the banking sector as a whole

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1Overviews of the crisis are given by Adrian and Shin (2010), Brunnermeier (2009), and Gorton (2010), among many others.

2As emphasized by Diamond and Rajan (2001), the demand deposit contract can be interpreted as a commitment device for banks to create liquidity.
becomes insolvent, triggering a systemic bank run.

In terms of the mechanism that propagates a banking crisis to a deep recession, we consider a form of working capital constraint, as in Jermann and Quadrini (2006), Kobayashi, Nakajima and Inaba (2007), and Mendoza (2010). Specifically, we assume that the amount of liquidity created in each period limits the size of working capital available for firms in that period. Since banks have superior skills in transforming firms’ surplus into liquidity, a systemic bank run significantly damages the economy’s ability to create liquidity, and hence to provide working capital. The reduction in working capital forces firms to reduce employment. In this way, a banking crisis amplifies the effect of a productivity shock in our model. Also, we would like to note that tightening the working capital constraint is translated into a worsening of the labor wedge in our model. The prediction of our model is consistent with the sharp deterioration in the labor wedge observed in the US economy after the Lehman Collapse in September 2008.

Regarding related literature, there are several recent papers which consider bank runs in the macroeconomic context. For instance, Angeloni and Faia (2010) study bank runs in the DSGE framework as in this paper, but they focus on idiosyncratic bank runs, rather than systemic ones. Systemic bank runs are studied by Uhlig (2010) in a two-period model, and by Kato and Tsuruga (2011) in an overlapping-generations model with two-period lived individuals. To our knowledge, this paper is the first attempt to consider systemic bank runs in the standard DSGE framework with infinitely lived individuals.

We should also emphasize that the bank run is only one aspect of the recent financial crisis. There are of course other important features, and thus different approaches to analyze the crisis are possible. One example of such approaches is to focus on financial frictions due to agency problems between banks and depositors, such as Gertler and Karadi (2011), Gertler and Kiyotaki (2011), and Gertler, Kiyotaki, and Queralto (2010). We view those alternative approaches which shed light on different aspects of the financial crisis as complements rather than substitutes.

The rest of the paper is organized as follows. The model is described in the next section. Numerical simulations are shown in Section 3. We discuss the relevance of the model in details in Section 4. Section 5 concludes.

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3The labor wedge is defined as the gap between the marginal rate of substitution between consumption and leisure and the marginal product of labor. Its importance of accounting for business cycles has been emphasized, for instance, by Shimer (2010).
2 The model economy

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. In each period a single commodity is produced, which can be used for consumption and investment. There exists a continuum of identical households. Each household consists of a worker/consumer, a firm, and a bank. The worker/consumer is infinitely lived, consumes the consumption good and supplies labor in each period. The household’s objective is to maximize the utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

where $E_0$ is the expectation operator conditional on the period-0 information, $c_t$ is the amount of consumption in period $t$, $l_t$ is the amount of labor supply in period $t$, $\beta \in (0, 1)$ is the discount factor, and $u(c, l)$ is the period utility function satisfying the standard properties.

In every period, a firm and a bank are born in each household. They live for two periods. To make the model work, we make the following assumptions.\(^4\) First, a new born firm needs funds to purchase physical capital, and to hire labor, but its own household does not provide them. Instead, it should obtain loans from a bank in a different household. Second, to make loans, a new born bank needs to obtain funds in the form of demand deposits.\(^5\) Its own household does not provide funds for the bank. It should obtain them from elsewhere.

New born firms and banks in different households are matched randomly in each period. Consider such a match formed in period $t$. The bank becomes a relationship lender as in Diamond and Rajan (2001). Let $b_t$ be the amount of loanable funds which the bank raises by issuing demand deposits. In its first period of life, the firm purchases physical capital $k_t$ using the loan from the bank $b_t$. The remaining, $d_t^F \equiv b_t - k_t$, is deposited in a bank, which is assumed to be different from the bank it is paired. In its second period of life, the firm produces the good using the capital obtained in the previous period, and labor supplied by workers in other households according to the production technology:

$$y_{t+1} = A_{t+1}k_t^{\alpha}l_{t+1}^{1-\alpha},$$

where $l_{t+1}$ is the labor input, and $A_{t+1}$ is the economy-wide productivity shock.

\(^4\)These assumptions are commonly made in the literature on financial frictions (e.g., Gertler and Kiyotaki 2011) in order to preserve the representative-agent framework.

\(^5\)The demand deposit contract gives a depositor the unilateral right to withdraw her deposit, the amount of which is prefixed, unconditionally at anytime. Our reasoning why demand deposits are used by banks follows Diamond and Rajan (2001). They show that the demand deposit contract is used as a commitment device for banks to use their loan collection skills on behalf of their depositors, and thereby to create liquidity.
The bank has relation-specific skills to collect loans. Let \( \Theta \) denote the fraction of the surplus generated by the firm which the bank can take as a repayment of the loan. If the depositors of the bank directly negotiate with the firm on the repayment of the loan, they can take only a smaller fraction of the firm’s surplus. We let \( \theta < \Theta \) be the fraction that they can take.

Because banks issue demand deposits, bank runs might occur in this economy. Here we restrict attention to bank runs caused by shocks to fundamentals, rather than those due to sunspot shocks. We also focus on “systemic bank runs,” in which all banks are subject to a run simultaneously. When a systemic bank run occurs in period \( t \), it does so after the aggregate productivity, \( A_t \), has been realized but before the production of the consumption good begins. In such an event, the depositors of each bank become a collective owner of its loans to a firm and they collectively negotiate with the firm on the repayment. As a result, the depositors obtain the fraction \( \theta \) of the firm’s surplus, which will be shared equally among them.

The interest rate on demand deposits between periods \( t-1 \) and \( t \) is \( 1+r_{t-1} \) for each bank, as long as it is solvent. When it is subject to a run, however, the rate of return on demand deposits reduces to \( \xi_t(1+r_{t-1}) \). How \( \xi_t < 1 \) is determined will be discussed later. In each period a systemic bank run might occur with some (very low) probability. We let \( s_t \) denote its occurrence in period \( t \): \( s_t = 1 \) when the systemic bank run occurs, and \( s_t = 0 \) otherwise. Let us define the random variable \( \tilde{\xi}_t \) by

\[
\tilde{\xi}_t = \begin{cases} 
1, & \text{if } s_t = 0, \\
\xi_t, & \text{if } s_t = 1.
\end{cases}
\]

Consider a representative firm born in period \( t \). Its profit in period \( t+1 \) is given by

\[
\pi_{t+1}^F = (1 - \tilde{\theta}_{t+1}) \left\{ A_{t+1}k_{t+1}^{\alpha}l_{t+1}^{1-\alpha} - w_{t+1}l_{t+1} + \tilde{\xi}_{t+1}(1+r_t)d_t^F + (1-\delta)k_t \right\},
\]

where

\[
\tilde{\theta}_t = \begin{cases} 
\Theta, & \text{if } s_t = 0, \\
\theta, & \text{if } s_t = 1.
\end{cases}
\]

The firm chooses \( \{k_t, d_t^F, l_{t+1}\} \) to maximize its profit \( \pi_{t+1}^F \) in (4):

\[
\max_{\{k_t,d_t^F,l_{t+1}\}} \mathbb{E}_t \frac{\beta \lambda_{t+1}}{\lambda_t} \pi_{t+1}^F,
\]

s.t. \( k_t + d_t^F \leq b_t \),

\[
w_{t+1}l_{t+1} \leq \tilde{\xi}_{t+1}(1+r_t)d_t^F,
\]
where $\lambda_t$ is the marginal utility of wealth of the representative household, which is used as the stochastic discount factor. Note that the allocation of the loan $b_t$ into $k_t$ and $d^F_t$ is decided in period $t$, and the labor input $l_{t+1}$ is in period $t+1$. Constraint (8) states that wages in period $t+1$ must be paid using the liquid asset the firm obtains in period $t$.

Now consider a representative bank born in period $t$. It collects demand deposits of amount $b_t$ and lends them to the paired firm. The bank’s profit in period $t+1$ is then given by

$$\pi_B^{t+1} = \max \left\{ \tilde{\pi}_B^{t+1}, 0 \right\},$$

where

$$\tilde{\pi}_B^{t+1} = \Theta \left[ A_{t+1} k_t^{\alpha} l_{t+1}^{1-\alpha} - w_{t+1} l_{t+1} + \xi_{t+1} (1 + r_t) d^F_t + (1 - \delta) k_t \right] - (1 + r_t) b_t + T_{t+1},$$

where $T_{t+1}$ denotes the transfer from the government, which we shall discuss later. Note that the bank’s profit is nonnegative because it can walk away if $\tilde{\pi}_B^{t+1} < 0$. When $\tilde{\pi}_B^{t+1} < 0$, the bank run occurs. We assume that there are startup costs for banks. Let $\gamma(b_t)$ be the amount of initial investment that is required to start a bank which makes a loan $b_t$ in period $t$. Then the problem of the bank is to choose $b_t$ to solve

$$\max_{b_t} E_t \beta^{\lambda_{t+1}} \pi_B^{t+1} - \gamma(b_t),$$

where the bank takes into account the fact that the paired firm chooses $\{k_t, d^F_t, l_{t+1}\}$ as a function of $b_t$ by solving problem (6).

In our model a fundamental bank run occurs when there is a bad technology shock $A_{t+1}$, making the firm’s profit lower than the certain threshold level. Consider a firm born in period $t$. Given the pair $(d^F_t, k_t)$ chosen in period $t$, let us consider the hypothetical problem in which the firm chooses labor input $l_{t+1}$ in period $t+1$ for each realization of $A_{t+1}$ under the hypothesis that there is no bank run:

$$l^*_{t+1}(A_{t+1}) \equiv \arg \max_{l_{t+1}} A_{t+1} k_t^{\alpha} l_{t+1}^{1-\alpha} - w_{t+1} l_{t+1}$$

s.t. $w_{t+1} l_{t+1} \leq (1 + r_t) d^F_t$

The threshold value $\tilde{A}_{t+1}$ is then defined as the solution to

$$\Theta \left[ \tilde{A}_{t+1} k_t^{\alpha} (l^*_{t+1}(\tilde{A}_{t+1}))^{1-\alpha} - w_{t+1} l^*_{t+1}(\tilde{A}_{t+1}) + (1 + r_t) d^F_t + (1 - \delta) k_t \right]$$

$$- (1 + r_t) b_t + T_{t+1} = 0.$$  

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6We implicitly assume that a firm cannot promise a worker to pay the wage after it produces the consumption good. The wage payment must be done before the production begins and therefore it must be in the form of the credible claims, i.e., the bank deposit or the loan to the firm, the value of which is discounted by $\xi_t$. 

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We set $\bar{A}_{t+1} = 0$ if this equation has no solution. We see that if $A_{t+1} < \bar{A}_{t+1}$, the bank would necessarily be insolvent. Since all banks born in the same period are identical, it causes a systemic bank run. That is, $s_{t+1} = 1$ if and only if $A_{t+1} < \bar{A}_{t+1}$.

When there is a bank run, the depositors are left with the bank’s loan asset, while the bank itself walks away. Thus after a bank run, the holders of the account in each bank would have to negotiate with the firm by themselves on the repayment of the loan. The ex-post rate of return on the bank account in this case, $\xi_{t+1}(1 + r_t)$, is determined as

$$\xi_{t+1}(1 + r_t) = \theta \left\{ A_{t+1} k_{t+1}^{\alpha} l_{t+1}^{1-\alpha} - w_{t+1} l_{t+1} + \xi_{t+1}(1 + r_t) d_{t+1}^F + (1 - \delta) k_t \right\} + T_{t+1}, \quad (14)$$

where $l_{t+1}$ is the solution to (6)–(8), given $s_{t+1} = 1$.

The only financial asset that the representative household can hold is the bank account (in those banks owned by other households). Thus the flow budget constraint of the representative household is

$$c_t + d_t^H = \tilde{\xi}_t (1 + r_{t-1}) d_{t-1}^H + w_t l_t + \pi_t^F + \pi_t^B - \gamma(b_t) - T_t, \quad (15)$$

where $T_t$ is lump-sum taxes used to subsidize banks, and $d_t^H$ is the amount of the bank account held by the household between periods $t$ and $t+1$. The representative household maximizes the lifetime utility (1) subject to the flow budget constraint (15) and a debt constraint $d_t^H \geq 0$.

In equilibrium, we have

$$d_t^H + d_t^F = b_t = k_t + d_t^F$$

so that $d_t^H = k_t$. The economy-wide resource constraint is

$$y_t = c_t + k_t - (1 - \delta) k_{t-1} + \gamma(b_t) \quad (16)$$

For the government policy, we consider the following two regimes. The first one is the regime without policy intervention, in which

$$T_t = 0, \quad \text{for all } t \text{ and under all contingencies.} \quad (17)$$

The second one is the regime with policy intervention, in which

$$T_t = \max \left\{ (1 + r_{t-1}) b_{t-1} - \Theta \left[ A_t k_{t-1}^{\alpha} (l_{t-1}^*)^{1-\alpha} - w_t l_t^* (A_t) + (1 + r_{t-1}) d_{t-1}^F + (1 - \delta) k_{t-1} \right], 0 \right\} \quad (18)$$

With policy intervention, the threshold value defined in (13) satisfies $A_t \geq \bar{A}_t$ for all $t$. That is, a systemic bank run never occurs.
3 Numerical results

In this section we consider numerical simulations to illustrate how a systemic bank run amplifies the effects of an exogenous shock to the economy. For this we assume:

\[ u(c, l) = \ln(c) + \psi \ln(1 - l), \quad \text{and} \quad \gamma(b) = \gamma b. \]

We suppose that one period in our model corresponds to a year, and choose the parameter values as \( \alpha = 0.4, \beta = 0.98, \delta = 0.1, \gamma_b = 0.01, \psi = 0.75, \Theta = 0.9, \) and \( \theta = 0.65. \)

Suppose that the economy is at the non-stochastic steady state in period 0 with \( A_0 = 1. \) In particular, all agents believe that \( A_t = 1 \) for all \( t \geq 0 \) in period 0. In period 1, however, there is an unexpected decline in the level of productivity so that \( A_1 = 0.95. \) This is a large enough decline in the level of productivity so that without policy intervention a systemic bank run occurs. We show two cases where the shock is permanent and where it is temporary. When the shock is permanent, \( A_t = 0.95 \) for all \( t \geq 1. \) When it is temporary, \( A_1 = 0.95 \) and \( A_t = 1 \) for all \( t \geq 2. \) In each case, all agents realize in period 1 how \( A_t \) evolves for \( t \geq 1. \)

Figure 1 plots the response of the economy to the permanent decline in productivity under the two policy regimes. Figure 2 plots the case of the temporary shock. These examples show large amplifications of the shocks without policy intervention. In both cases, the labor wedge, defined as the marginal product of labor divided by the wage rate, significantly deteriorates due to the systemic bank runs. It is consistent with the sharp deterioration of the labor wedge observed in the US economy after the Lehman Collapse in September 2008. These figures show that the policy intervention is effective enough to nullify the nonlinear amplifications due to the systemic bank run.

4 Discussion

5 Conclusion

In this paper we develop a DSGE model with bank runs. The demand deposit contract is used as a commitment device for banks to create liquidity, but at the same time it makes banks susceptible to runs. We restrict attention to bank runs caused by shocks to fundamentals, rather than sunspot shocks. Also we focus on systemic bank runs, where the banking sector becomes insolvent as a whole. In our model a systemic bank run amplifies the effects of a bad shock to the economy as follows. When a negative productivity shock lowers the revenue of the banking sector, a systemic bank run occurs. It damages the economy’s capacity to create liquidity, which, in turn, reduces the amount of working capital available to firms. It then
requires firms to cut down employment, aggravating the recession originally caused by the productivity shock. We also note that a systemic bank run in our model worsens the labor wedge, which is consistent with what happens in the U.S. economy after the Lehman Collapse in September 2008.

Needless to say, our model is highly stylized and should be extended in many directions, in particular, to examine how policy should respond to a financial crisis like the one in 2007-2009. Here we discuss a few examples of such extensions. First, in our model bailing out banks in distress is the only way to provide liquidity to prevent a systemic bank run. The model should be enriched so that there are alternative policy instruments to provide liquidity in the market. Second, in the current model the policy intervention to prevent a systemic bank run has no costs whatsoever. This may not be realistic. Such a policy may distort the behavior of banks. Third, our model abstracts from public liquidity such as money and government bonds. Introducing it would allow us to conduct more realistic policy analysis. Those extensions are left for future research.

References


Figure 1: Effects of a permanent decline in the level of productivity with and without policy intervention.
Figure 2: Effects of a temporary decline in the level of productivity with and without policy intervention.