Monetary Independence and Rollover Crises

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Concerns about the risk of a rollover crisis and “bad equilibrium”:

- Investors refuse to rollover $\Rightarrow$ liquidity problems for govt....
- Liq. problems $\Rightarrow$ govt. default $\Rightarrow$ investors don’t rollover...

$\Rightarrow$ self-fulfilling rollover crisis
Eurozone Crisis

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- Missing: aggregate demand channel and monetary policy
Central to Eurozone crisis: countries lack of monetary autonomy

- Spain and Portugal close to default despite *low debt-GDP* ratios compared to economies *not* in a currency union
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Can monetary autonomy help to deal with rollover crisis?
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  - This channel hinges on debt being in domestic currency
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**This paper:** Theory linking monetary autonomy and rollover crisis based on aggregate demand channel
What we do

Canonical sovereign default model with foreign currency debt featuring rollover crisis & downward wage rigidity
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Show how rollover risk depend on monetary policy regime
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**Key result:** Lack of monetary autonomy make an economy more vulnerable to rollover crises
What we do

Canonical sovereign default model with foreign currency debt featuring rollover crisis & downward wage rigidity

Show how rollover risk depend on monetary policy regime

**Key result:** Lack of monetary autonomy make an economy more vulnerable to rollover crises

Quantitatively (preliminary):

- With flexible exchange rate, economy remains relatively immune to rollover crisis.
- With fixed exchange rate, much higher exposure
When investors “run” on govt. bonds, repayment requires large reduction in aggregate demand...⇒ unemployment
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• Ability to depreciate enables govt. to break self-fulfilling loop
Rollover Crises and Downward Wage Rigidity

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⇒ Additional cost from losing monetary independence
**Related Literature**

**Classic papers on rollover crises:** Alesina, Tabellini and Pratti; Giavazzi and Pagano; Cole and Kehoe

**Recent quantitative models on rollover crises:** Chatterjee and Eygunoor; Bocola and Dovis; Aguiar, Chatterjee, Cole and Stangebye; Roch and Uhlig; Conesa and Kehoe

**Other types of multiplicity in sovereign debt:** Calvo, Lorenzoni and Werning, Ayres, Navarro, Nicolini and Teles, Aguiar and Amador

**Monetary models with multiple equilibria in sovereign debt:** Da Rocha, Gimenez and Lores; Araujo, Leun and Santos; Aguiar, Amador, Farhi and Gopinath, Corsetti; Camous and Cooper; Bacchetta, Perazzi and van Wincoop;

**Sovereign default model with nominal rigidities:** Na, Schmitt-Grohe, Uribe and Yue, Bianchi, Ottonello and Presno
Main elements of the model

- Small open economy with tradable and non-tradable goods
  - Stochastic endowment of tradable goods $y^T$
  - Non-tradable goods produced with labor $y^N = F(h)$
- Law of one price for tradable goods $P^T_t = P^*_t e_t$.
  - Assume $P^*_t = 1 \Rightarrow P^T_t = e_t$
- Wages are downward rigid in domestic currency $W_t \geq \bar{W}$
  - With fixed exchange rate regime $\Rightarrow$ real wage rigidity
- Government issues defaultable long-term debt, $b$, in foreign currency
Households

- Preferences
  \[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t) \right] \]
  \[ c = [\omega(c^T)^{-\mu} + (1 - \omega)(c^N)^{-\mu}]^{-1/\mu} \]

- \( c^T, c^N \): consumption of tradables and non-tradables

- Budget constraint (in domestic currency)
  \[ e_t c^T_t + P_t^N c^N_t = e_t y^T_t + \phi^N_t + W_t h^s_t - T_t \]

- \( \phi^N \) firms’ profits, \( T_t \) lump sum taxes

- Total endowment of hours \( \bar{h} \)
Households

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Households (ctd)

Optimality

\[
\frac{P_t^N}{e_t} = \frac{1 - \omega}{\omega} \left( \frac{c_t^T}{c_t^N} \right)^{1+\mu}
\]
Firms

- Produce using labor: $y^N = F(h)$
- Profit maximization
  \[
  \phi^N_t = \operatorname{Max}_{h_t} \left\{ P^N_t F(h_t) - W_t h_t \right\}
  \]
- Optimality
  \[
  W_t = P^N_t F'(h_t)
  \]
Downward wage rigidity

Wages in domestic currency cannot fall below $\bar{W}$

$$W_t \geq \bar{W}$$
Downward wage rigidity

Wages in domestic currency cannot fall below \( \bar{W} \)

\[
W_t \geq \bar{W}
\]

If \( \bar{W} \) is higher than market clearing wage \( \Rightarrow \) unemployment

If \( \bar{W} \) is lower than market clearing wage \( \Rightarrow h = \bar{h} \)

\[
(W_t - \bar{W})(\bar{h} - h_t) = 0
\]
• Government issues long-term bonds at price $q_t$

• Bond payoff structure: $\delta \left[1, (1 - \delta), (1 - \delta)^2, ..., (1 - \delta)^t\right]$

• Law of motion for bonds $b_{t+1} = b_t(1 - \delta) + i_t$
Government

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$$\delta e_t b_t (1 - d_t) = e_t q_t i_t + T_t$$

$d_t = 0(1)$ if government repays (defaults)
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- If default, utility loss and exclusion with stochastic reentry
- Focus on fixed exchange rate regime $e_t = e \ \forall t$
International Investors

- International investors are risk-neutral and competitive.
- Besides the defaultable bonds, they can invest in real risk-free security at rate $r$.
- Bond prices satisfy no-arbitrage condition

$$q_t(1 + r) = \mathbb{E}_t[(1 - d_{t+1})(\delta + (1 - \delta)q_{t+1})]$$

when government repays
Equilibrium conditions

• Recall households’ and firm optimality

\[
\frac{P_t^N}{e_t} = \frac{1 - \omega}{\omega} \left( \frac{c_t^T}{c_t^N} \right)^{1+\mu}
\]

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W_t = P_t^N F'(h_t)
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- Real equilibrium wage given by

\[
\mathcal{W}_t(c_T^t, h) = \frac{1 - \omega}{\omega} \left( \frac{c_T^t}{F(h_t)} \right)^{1+\mu} F'(h_t)
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W_t \left( c_t^T, h \right) = \frac{1 - \omega}{\omega} \left( \frac{c_t^T}{F(h_t)} \right)^{1+\mu} F'(h_t) \geq \frac{W_t}{e_t}
\]

where \( \frac{\partial W}{\partial c_t} > 0 \), and \( \frac{\partial W}{\partial h} < 0 \)

• If wage rigidity binds: \( \downarrow c^T \Rightarrow \downarrow h \)
Definition: Competitive eq. given govt. policies

Given $b_0$, and govt. policy $\{e_t, b_{t+1}, d_t\}_{t=0}^{\infty}$, a *competitive equilibrium* is given by households and firms’ allocations $\{c_t^T, c_t^N, h_t\}_{t=0}^{\infty}$, and prices $\{P_t^N, W_t, q_t\}_{t=0}^{\infty}$, such that

i. Households and firms solve their optimization problems

ii. Government budget constraint holds

iii. Bond pricing schedule satisfies investors’ optimality

iv. NT market clears $c_t^N = y_t^N$ and resource constraint for $T$

$$c_t^T - q_t (b_{t+1} - (1 - \delta)b_t) = y_t^T - \delta(1 - d_t)b_t$$

v. Labor market equilibrium conditions hold
Where are going?

We defined equilibrium for given government policies

Next, we will study markov equilibria: government chooses repayment and borrowing without commitment
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Three distinct “zones” (Cole-Kehoe)

• Safe zone: government always repays
• Default zone: government always defaults
• Crisis zone: government repayment depends on investors’ expectations
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Goal: study how $\overline{W}$ and monetary policy affect zones
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**Goal:** study how $\overline{W}$ and monetary policy affect zones
• States: \((b, s) = (y^T, \zeta)\) where \(\zeta\) is a sunspot

• Government problem in good credit standing

\[
V(b, s) = \text{Max} \left\{ V_D(y^T), V_R(b, s) \right\}
\]
Values of repayment and default

\[ V_R (b, s) = \max_{c^T h, b'} \left\{ u(c^T, F(h)) + \beta \mathbb{E} \left[ V(b', s') \right] \right\} \]

s.t. \[ c^T = y^T - \delta b + q(b', b, s) \left[ b' - (1 - \delta) b \right] \]
\[ \mathbb{W}(c^T, h) \bar{e} \geq \overline{W} , \]
\[ h \leq \bar{h} \]
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\[ V_D(y^T) = u(y^T, F(h)) - \kappa(y^T) + \beta \mathbb{E}[\psi V(0, s') + (1 - \psi) V_D(y^{T'})] \]

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\[ s.t. \quad \mathcal{W}(y^T, h) \bar{e} \geq \bar{W}, \]

\[ h \leq \bar{h} \]

Optimal exchange rate eliminates wage rigidity
Values of repayment and default **good sunspot**

\[ V_R^+(b, y^T) = \max_{c^T h, b'} \left\{ \left. u(c^T, F(h)) + \beta \mathbb{E} \left[ V(b', s') \right] \right\} \right. \]

s.t. \[ c^T = y^T - \delta b + q(b', b, s) \left[ b' - (1 - \delta) b \right] \]

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s.t. \[ \mathcal{W}(y^T, h) \bar{e} \geq \bar{W}, \]

\[ h \leq \bar{h} \]

If government is not issuing debt \( \hat{b}_R^+ < (1 - \delta) b \) \( \Rightarrow V_R^+ = V_R^- \)
Values of repayment and default bad sunspot

\[ V_R^-(b, y^T) = \max_{c^T, h, b'} \left\{ u(c^T, F(h)) + \beta \mathbb{E} \left[ V(b', s') \right] \right\} \]

s.t. \[ c^T = y^T - \delta b \]
\[ \mathcal{W}(c^T, h) \bar{e} \geq \bar{W}, \]
\[ h \leq \bar{h} \]
\[ b' = b(1 - \delta) \]

\[ V_D(y^T) = u(y^T, F(h)) - \kappa(y^T) + \beta \mathbb{E} \left[ \psi V(0, s') + (1 - \psi) V_D(y^T') \right] \]

s.t. \[ \mathcal{W}(y^T, h) \bar{e} \geq \bar{W}, \]
\[ h \leq \bar{h} \]

If government is not issuing debt \( \hat{b}_R^+ < (1 - \delta)b \) \( \Rightarrow V_R^+ = V_R^- \)
A Markov perfect equilibrium is defined by value functions \( \{ V(b, s), V_R(b, s), V_D(y^T) \} \), policy functions \( \{ d(b, s), c^T(b, s), b'(b, s), h(b, s) \} \), and a bond price schedule \( q(b', b, s) \) such that

i. Given the bond price schedule, the policy functions solve the government problem

ii. The bond price schedule satisfies no arbitrage given future government policies
Consider a state \((b, y^T)\) in which government wants to issue debt:

1. If each lender expects other lenders' to extend credit, government can rollover debt and obtains value \(V_R\). If \(V_R + \) is greater than \(V_D\), government repays.

2. If each lender expects other lenders to refuse to extend credit, government cannot rollover debt and obtains value \(V_R\) and defaults if \(V_R - \) is less than \(V_D\).

In the second case, default is entirely due to self-fulfilling beliefs: if lenders refuse to lend, government is unwilling/unable to cut down consumption and defaults.
Consider a state \((b, y^T)\) in which government wants to issue debt:

1. If each lender expects other lenders’ to **extend** credit
   - Government can rollover debt and obtains value \(V_R\)
   - If \(V_R^+ > V_D\), government **repays**
Multiplicity of Equilibria as in Cole-Kehoe

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2. If each lender expects other lenders to refuse to extend credit
   - Government cannot rollover debt and obtains value \(V_R\)
   - If \(V_R^- < V_D\), government defaults
Multiplicity of Equilibria as in Cole-Kehoe

Consider a state \((b, y^T)\) in which government wants to issue debt:

1. If each lender expects other lenders’ to **extend** credit
   - Government can rollover debt and obtains value \(V_R\)
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2. If each lender expects other lenders to **refuse** to extend credit
   - Government cannot rollover debt and obtains value \(V_R\)
   - If \(V_R^- < V_D\), government **defaults**

In second case, default is entirely due to self-fulfilling beliefs: if lenders refuse to lend, government is unwilling/unable to cut down consumption and defaults
Three Zones

• Safe zone (govt. always repays)

\[ S \equiv \left\{ (b, y^T) : V_D(y^T) \leq V_R^-(b, y^T) \right\} \]

• Default zone (govt. always defaults)

\[ D \equiv \left\{ (b, y^T) : V_D(y^T) > V_R^+(b, y^T) \right\} \]

• Crisis zone (govt. repayment depends on beliefs)

\[ C \equiv \left\{ (b, y^T) : V_D(y^T) > V_R^-(b, y^T) \right. \]

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Three Zones

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  \[ \land \ V_D(y^T) \leq V_R^+(b, y^T) \]

Will show how wage rigidity enlarges “crisis zone”
Policy functions and value functions & zones with flexible wages
Policy for Borrowing: good and bad sunspot

\[ \hat{b}^+ \]

\[ \hat{b}^- = (1 - \delta)b \]
Value Functions

$V_D$
Value Functions

$V^+_R$

$V_D$
Value Functions

\[ V_R^+ \quad V_R^- \quad V_D \]

Debt
Value Functions and Zones
Zones: Flexible Wages

The diagram illustrates the relationship between tradable endowment and debt, with three zones:

- **Safe Zone**: Where the tradable endowment is sufficient to cover debt without default.
- **Crisis Zone**: Where debt is high and tradable endowment is low, indicating potential default.
- **Default Zone**: Where debt is unmanageable and tradable endowment is insufficient, leading to default.
Comparison: flexible vs. sticky wages

- Assume wage rigidity is introduced for *only one period*
  - Same continuation values and bond price schedule
- How do three zones change?
  - High and low wage rigidities, $\bar{w}_{\text{high}} > \bar{w}_{\text{low}}$
- Later, will study permanent changes in wage rigidity
Recall crisis region with flexible wages

Note that default region does not change in this example.
$V_D$ is unaffected with $\bar{w}_{low}$
$V^+$ is reduced with $\bar{W}_{low}$.
\( V^- \) is reduced by more than \( V^+ \)
Wage rigidity leaves zones unaffected
Recall flexible wage
Higher wage rigidity affects crisis and default regions

\[ V_R^+ \]

\[ V_R^- \]

\[ h = \bar{h} \quad h < \bar{h} \]

\[ V_D \]
Higher wage rigidity affects crisis and default regions.
Higher wage rigidity affects crisis and default regions.

Graph showing the relationship between debt and the zones of crisis, default, and safety.
Explaining the increase in crisis region: the role of unemployment
Unemployment
Zones: Flexible Wages

![Graph showing zones: Safe Zone, Crisis Zone, Default Zone](image)
Zones: Low Wage Rigidity

- Safe Zone
- Crisis Zone
- Default Zone

Tradable Endowment vs Debt graph.
Zones: High Wage Rigidity

- Safe Zone
- Crisis Zone
- Default Zone

Tradable Endowment vs. Debt
Theoretical Characterization

We show in the paper

- Safe region shrinks with wage rigidity
- Default region expands with wage rigidity
- For given level of $y^T$, higher wage rigidity implies that:
  - Economy enters in crisis zone with lower debt
  - There exists $\hat{w}$ such that length of crisis zone is increasing in $\bar{w} \land \bar{w} < \hat{w}$
Quantitative analysis
Calibration Strategy

• Spain 1996-2015 as a case of study
• A period is a year.
• Calibrate directly:
  • Preference elasticities (intra- and inter-temporal) and discount factor
  • Production parameters and process for $y^T$
  • Maturity
  • For now, sunspot process is iid with probability $\pi = 0.03$
• Calibrate by simulation two cost of default parameters to match average spread and average debt
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• A period is a year.
• Calibrate directly:
  • Preference elasticities (intra- and inter-temporal) and discount factor
  • Production parameters and process for $y^T$
  • Maturity
  • For now, sunspot process is iid with probability $\pi = 0.03$
• Calibrate by simulation two cost of default parameters to match average spread and average debt
• First, we look at flex economy. Calibration of fixed wage economy in progress
## Benchmark Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.750</td>
<td>Labor share in nontradable production</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.905</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.176</td>
<td>Maturity of debt</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.240</td>
<td>Probability of Re-entry</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.205</td>
<td>Elasticity of substitution</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.300</td>
<td>Share of tradables</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.000</td>
<td>Risk aversion</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.100</td>
<td>Sunspot probability</td>
</tr>
<tr>
<td>$r$</td>
<td>0.020</td>
<td>Risk-free rate</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>1.000</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.777</td>
<td>Persistency of shock</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.029</td>
<td>Standard deviation of shock</td>
</tr>
<tr>
<td>$\kappa_0$</td>
<td>0.375</td>
<td>Mean spread 1.05%</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>1.825</td>
<td>Debt-GDP 22%</td>
</tr>
</tbody>
</table>
Statistics

Rigidity $\bar{w}$ is set 10% above the lowest wage in flex economy

- No unemployment along equilibrium path

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Flexible wage</th>
<th>$\bar{w} = 1.50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu(r^* - r)$</td>
<td>1.03</td>
<td>1.78</td>
</tr>
<tr>
<td>$\mu(\bar{b}/y)$</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>$\mu(\bar{h} - h)$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\rho(y, c)$</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho(y, r^* - r)$</td>
<td>-0.81</td>
<td>-0.76</td>
</tr>
<tr>
<td>$\rho(y, TB)$</td>
<td>-0.54</td>
<td>-0.60</td>
</tr>
<tr>
<td>$\sigma(\hat{c})/\sigma(\hat{y})$</td>
<td>1.3</td>
<td>1.4</td>
</tr>
<tr>
<td>$\sigma(r^* - r)$</td>
<td>0.2</td>
<td>0.73</td>
</tr>
<tr>
<td>$\sigma(\bar{h} - h)$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Defaults due to rollover crisis</strong></td>
<td>0.02</td>
<td>0.16</td>
</tr>
</tbody>
</table>
Fundamental and Non-Fundamental Defaults

![Graph showing the relationship between share and wage rigidity for Overall Defaults and Non-Fundamental Defaults. The graph illustrates how the share increases with increasing wage rigidity, with two distinct curves: one for Overall Defaults and another for Non-Fundamental Defaults.](image-url)
Conclusion

Uncover new cost from currency unions:

• Lack of monetary independence makes an economy more prone to rollover crisis

Lender of last resort is more important than we thought

Avenues ahead:

• Applications to ZLB, managed exchange rates
• Interactions with fiscal policies
Mario Draghi: “The assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a “bad equilibrium”, namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios. So, there is a case for intervening, in a sense, to “break” these expectations”
Proposition. *(Safe zone shrinks with \( \bar{w} \))

There exist a \( \bar{w}^* \) such that for every \( \bar{w}_2, \bar{w}_1 \in [0, \bar{w}^*] \), if \( \bar{w}_2 > \bar{w}_1 \), the safe zone compresses \( S(\bar{w}_2) \subset S(\bar{w}_1) \).
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Proposition.  (*Default zone expands with \( \bar{w} \))
There exist a \( \bar{w}^* \) such that for every \( \bar{w}_2, \bar{w}_1 \in [0, \bar{w}^*] \), if \( \bar{w}_2 > \bar{w}_1 \), the default zone expands \( D(\bar{w}_1) \subset D(\bar{w}_2) \).
Safe and Default Zones and $\tilde{w}$

**Proposition.** (*Safe zone shrinks with $\tilde{w}$*)
Theorem: There exist a $\tilde{w}^*$ such that for every $\tilde{w}_2, \tilde{w}_1 \in [0, \tilde{w}^*]$, if $\tilde{w}_2 > \tilde{w}_1$, the safe zone compresses $S(\tilde{w}_2) \subset S(\tilde{w}_1)$.

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Next, results on crisis zone
• For every $y^T$, there is an interval of debt in crisis region

$$C_{yT} \equiv \left[ \bar{B}^S_{yT}, \bar{B}^D_{yT} \right] \quad \& \quad \Delta C_{yT} \equiv \bar{B}^D_{yT} - \bar{B}^S_{yT}$$

$\bar{B}^S_{yT}, \bar{B}^D_{yT}$ are the thresholds for the default and safe zones

Assumption. Autarchy after default, i.i.d. shock for $y^T$, and one-period wage rigidity shock $\bar{w} > 0$

Proposition. There exists a $\bar{w}^*$ such that for every $\bar{w}_2, \bar{w}_1 \in [0, \bar{w}^*]$, if $\bar{w}_2 > \bar{w}_1$, then, for all $y_T$, $\Delta C_{yT}$ increases and $\frac{d\bar{B}^S_{yT}}{d\bar{w}} \leq 0$
Crisis zone expands with \( \bar{w} \)

- For every \( y^T \), there is an interval of debt in crisis region

\[
C_{yT} \equiv \left( \bar{B}^S_{yT}, \bar{B}^D_{yT} \right) \quad \& \quad \Delta C_{yT} \equiv \bar{B}^D_{yT} - \bar{B}^S_{yT}
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**Proposition.** There exists a \( \bar{w}^* \) such that for every \( \bar{w}_2, \bar{w}_1 \in [0, \bar{w}^*] \), if \( \bar{w}_2 > \bar{w}_1 \), then, for all \( y_T \), \( \Delta C_{yT} \) increases and \( \frac{d\bar{B}^S_{yT}}{d\bar{w}} \leq 0 \)

Starting from \( w^{FLEX} \), crisis region expands with higher \( \bar{w} \)
Why crisis region expands with $\bar{\nu}$?

\[
V^R(S) = \max_{c^T h, b'} \{ u(c) + \beta \mathbb{E} [V(b', s')] \}
\]

subject to

\[
c = \left( \omega \left( c^T \right)^{-\mu} + (1 - \omega) (F(h))^{-\mu} \right)^{-\frac{1}{\mu}}
\]

\[
c^T = y^T - \delta b + q(b', S) \left[ b' - (1 - \delta) b \right]
\]

\[
\bar{\nu} \leq \mathcal{W}_t \left( c^T, F(h), h \right)
\]

\[
h \leq \bar{h}
\]
Why crisis region expands with $\bar{w}$?

Value of repayment during rollover crisis, $V^C$, is reduced considerably more than $V^F$ and $V^D$

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$$c^T = y^T - \delta b$$

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$$h \leq \bar{h}$$

Even if unemployment not “observed”, rigidity can trigger crisis