The Intertemporal Keynesian Cross

Adrien Auclert   Matthew Rognlie   Ludwig Straub

Stanford   Northwestern   MIT

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Motivation: the government spending multiplier

- How large is the government spending multiplier?
  - Crucial macro question, vast theoretical and empirical literatures
  - Important dialogue: theory $\rightarrow$ testable predictions $\rightarrow$ theory
- Main theoretical answers:
  - Representative agent (RA) models
    - Stress the **response of monetary policy**
    - Large at the zero lower bound
    - [Eggertsson 2004; Christiano, Eichenbaum, Rebelo 2011]
  - Two agent (TA) models
    - Stress the **aggregate MPC** (as proxied by % of constrained agents)
    - Large when MPC is high and spending is deficit-financed
    - [Galí, López-Salido, Vallés 2007; Coenen et al 2012]

What we do

- Implications of heterogeneous agent (HA) models for fiscal policy:

1. Theoretical characterization of impulse response in special case
    - No capital + 'neutral' monetary policy: constant real rate
    - Main results:
        1. Balanced budget multiplier is 1 [Haavelmo 1945, Woodford 2011]
        2. Intertemporal MPCs (*IMPCs*) characterize impulse in other cases
    - Logic: intertemporal Keynesian cross

2. Quantitative investigation away from special case
    - General monetary and fiscal policy rules
        - (eventually: capital + two assets + sticky prices and wages)
    - Can match data IMPCs, contrary to HA and TA models
    - Robust result: deficit-financed government spending has large and persistent effects, irrespective of monetary policy

Related literature

- **Theory**
  - IS-LM: Gelting 1941, Haavelmo 1945, Blinder-Solow 1973, ...
  - Rep agent (RA): Baxter-King 1993, Aiyagari-Christiano-Eichenbaum 1992, Christiano-Eichenbaum-Rebelo 2011, ...
  - Two agents (TA): Galí, López-Salido, Vallés 2007, Coenen et al. 2012, Farhi-Werning 2012, Drautzburg-Uhlig 2015, ...
  - Heterogeneous agent (HA): Oh-Reis 2010, McKay-Reis 2016, Ferrière-Navarro 2017, Hagedorn-Manovski-Mitman 2017, ...
- **Empirics**
  - Aggregate evidence: Ramey-Shapiro 1998, Blanchard-Perotti 2002, Mountford-Uhlig 2009, Ramey 2011, Barro-Redlick 2011, ...
  - State dependence: Auerbach-Gorodnichenko 2012, Ramey-Zubairy 2018, ...
  - Cross-sectional multipliers: Shoag 2010, Chodorow-Reich et al. 2012, Nakamura-Steinsson 2014, ...

Outline

1. Baseline model
2. Benchmark fiscal policy results
3. IMPCs in model vs. data
4. Quantitative model
5. Conclusion

Households

- GE economy in discrete time $t = 0 \ldots \infty$

- Heterogeneous agents in incomplete markets
  - Face idiosyncratic risk to skills e_{it} (no aggregate risk)
  - Maximize $\mathbb{E}\left[\sum \beta^t \left\{u\left(c_{it}\right) - v\left(n_{it}\right)\right\}\right]$ s.t. trade in one-period real a_{it},

$$c_{it} + a_{it} = \left(1 + r_t\right) a_{it-1} + \tau_t \left(\frac{W_t}{P_t} e_{it} n_{it}\right)^{1-\lambda}$$

$$a_{it} \geq \underline{a}$$

  - r_t is real rate, P_t aggregate price level, W_t nominal wage, n_{it} labor hours, τ_t and $1 - \lambda$ scale and elasticity of after-tax retention function, taken as given
  - Equivalently, take net income z_{it} as given, where

$$z_{it} \equiv \tau_t \left(\frac{W_t}{P_t} e_{it} n_{it}\right)^{1-\lambda}$$

Employment, firms and labor market

- Sticky nominal wage W_t
  - Employment n_{it} of each agent determined by aggregate labor demand
  - Assume proportionality:

$$n_{it} = L_t$$

- Perfectly competitive final goods firm, constant productivity

$$Y_t = L_t$$

- Perfectly flexible prices. Profit maximization implies

$$P_t = W_t$$

  and zero profits
- Unions set W_t to max average of h.h. utility s.t. Rotemberg costs
  - Implies local Phillips curve for price inflation $\pi_t = \log\left(\frac{P_{t+1}}{P_t}\right)$ Details

$$\pi_t = \kappa \int \left(\omega_{it} \frac{v'(n_{it})}{u'(c_{it})} - 1 \right) di + \beta \pi_{t+1}$$

Government

- To partial out monetary policy, assume a constant-r rule

$$r_t = r$$

  (Neutral Taylor rule: coefficient of 1 on expected inflation)

- Government follows a fiscal policy rule:
  - sets exogenous paths for spending G_t and tax revenue T_t obeying intertemporal budget constraint

$$(1+r)B_{-1} + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t G_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t T_t$$

  - adjusts slope τ_t of retention function to satisfy

$$T_t \equiv \mathbb{E}_I \left[\frac{W_t}{P_t} e_{it} n_{it} - \tau_t \left(\frac{W_t}{P_t} e_{it} n_{it} \right)^{1-\lambda} \right]$$

General equilibrium and the consumption function

Definition

Given $\{G_t, T_t\}$, a **general equilibrium** is a set of prices, hh decision rules and quantities s.t. at all t: firms optimize, households optimize, fiscal and monetary policy rules are satisfied, and the goods market clears.

- To characterize eqbm, define the **aggregate consumption function**

$$C_s = \mathbb{E}_I\left[c_{is}\right] = C_s(\{Z_t\}, \{r\})$$

where Z_t is aggregate after-tax labor income

$$Z_t = \tau_t L_t^{1-\lambda} \mathbb{E}_I[e_{it}^{1-\lambda}] = Y_t - T_t$$

- Note *individual* after-tax income z_{it} is $z_{it} = \frac{e_{it}^{1-\lambda}}{\mathbb{E}_I[e_{it}^{1-\lambda}]} Z_t$

Characterizing equilibrium output

Lemma

General equilibrium output $\{Y_t\}$ *is a* **fixed point** *of the equation*

$$Y_s = C_s(\{Y_t - T_t\}, \{r\}) + G_s \quad \forall s$$

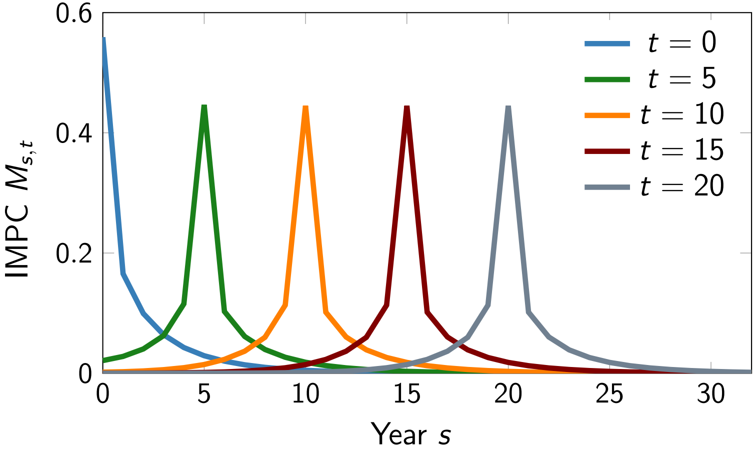
Corollary

Impulse responses from steady state solve the linear fixed point equation

$$dY_s = \sum_{t=0}^{\infty} \frac{\partial C_s}{\partial Z_t} \cdot (dY_t - dT_t) + dG_s \quad \forall s$$

- Path $\{dY_t\}$ entirely characterized by the set of $M_{s,t} \equiv \frac{\partial C_s}{\partial Z_t}$
  - *Partial equilibrium* derivatives—intertemporal MPCs, or **IMPCs**
- Logic: intertemporal Keynesian cross

Shape of IMPCs in baseline model



- Let $Q_t \equiv \left(\frac{1}{1+r}\right)^t$. Budget constraints imply $\sum_{s=0}^{\infty} Q_s M_{s,t} = Q_t$.
- Tent shape typical of models with incomplete markets

The intertemporal Keynesian cross

Proposition

There exists a matrix $\mathbf{M}$, *satisfying* $\mathbf{Q}'\mathbf{M} = \mathbf{Q}'$, *such that the output impulse response from steady state* $d\mathbf{Y}$ *to any fiscal shock* $(d\mathbf{G}, d\mathbf{T})$ *satisfying the GBC* $\mathbf{Q}'d\mathbf{G} = \mathbf{Q}'d\mathbf{T}$ *solves the fixed point equation*

$$d\mathbf{Y} = \mathbf{M}d\mathbf{Y} - \mathbf{M}d\mathbf{T} + d\mathbf{G} \tag{IKC}$$

- All the complexity of GE is in aggregate IMPC matrix $\mathbf{M}$
  - Model 'signature' that can be mapped to data
- When unique, the solution to (IKC) is

$$d\mathbf{Y} = \mathcal{G} \cdot (-\mathbf{M}d\mathbf{T} + d\mathbf{G})$$

  where $\mathcal{G}$ a linear map that depends only on $\mathbf{M}$
  - see Auclert-Rognlie-Straub "Determinacy with Incomplete Markets"

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Benchmark for balanced-budget spending multiplier

Proposition (Haavelmo, 1945)

Assume a unique eqbm. The constant-r balanced-budget multiplier is 1:

$$d\mathbf{Y} = d\mathbf{G} = d\mathbf{T} \tag{1}$$

- Generalizes Woodford's rep agent result to heterogeneous agents
  - Heterogeneity is irrelevant for the effects of fiscal policy !
  - Similar to Werning (2015)'s result for monetary policy
- Proof: (1) is unique solution to

$$d\mathbf{Y} = \mathbf{M} \cdot d\mathbf{Y} + (I - \mathbf{M}) \cdot d\mathbf{G}$$

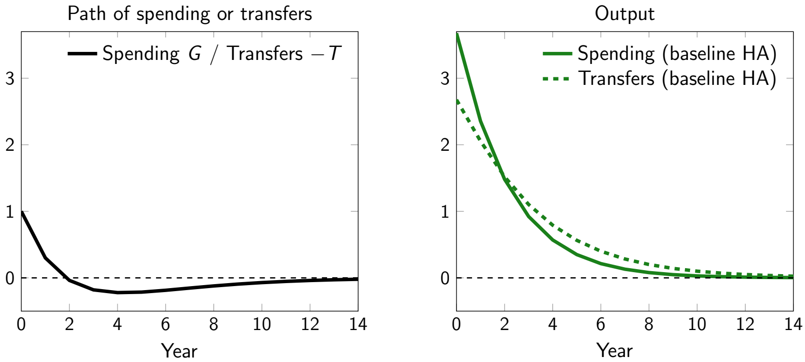
- When are IMPCs relevant?
  1. Delayed taxation, with burden of taxation falling on later taxpayers
  2. Non-proportional incidence, with burden falling on low MPC agents
  3. Monetary policy adjustment
- Consider 1 now, 1—3 in quantitative model

Effects of deficits and transfer multiplier

Proposition

The output effect is the sum of the spending and consumption response. The latter only depends on the path of primary deficits $d\mathbf{G}-d\mathbf{T}$*:*

$$d\mathbf{Y} = d\mathbf{G} + \mathcal{G}\cdot\mathbf{M}\cdot(d\mathbf{G}-d\mathbf{T})$$

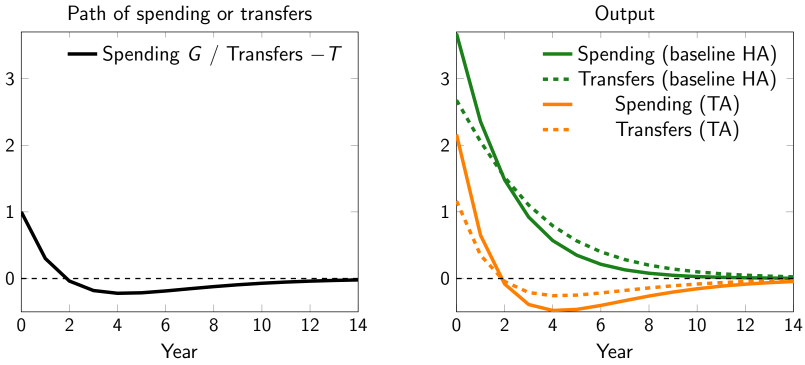


Comparison to TANK model

Proposition

Consider a TANK model with a share μ of constrained consumers. Details
In the equilibrium with $\lim_t d\mathbf{Y}_t = 0$, *output is given by the static Keynesian cross in each period:*

$$d\mathbf{Y} = d\mathbf{G} + \frac{\mu}{1-\mu}\left(d\mathbf{G} - d\mathbf{T}\right)$$



Conclusion: importance of IMPCs

- Under constant real rate, impulse response of consumption to fiscal policy depends *only* on interaction between **path of primary deficits** and **IMPC matrix**
- HA and TA models can have very different amplification and persistence properties
- How do we choose? Compare model and data IMPCs

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Relating aggregate and individual IMPCs

- Object of interest: aggregate IMPC

$$M_{s,t} = \frac{\partial C_s}{\partial Z_t} (\mathbf{Y} - \mathbf{T}, \mathbf{r})$$

where $C_s = \mathbb{E}_I [c_{is}]$

- Since individual post-tax income is $z_{it} = \frac{e_{it}^{1-\lambda}}{\mathbb{E}_I [e_{it}^{1-\lambda}]} Z_t$, at date 0

$$M_{s,0} = \mathbb{E}_I \left[\frac{\partial c_{is}}{\partial z_{i0}} \frac{z_{i0}}{\mathbb{E}_I [z_{i0}]} \right]$$

average of MPCs weighted by date-0 post-tax income

- More general insight: need to weigh individual IMPCs by incidence of aggregate income shocks

Mapping to data

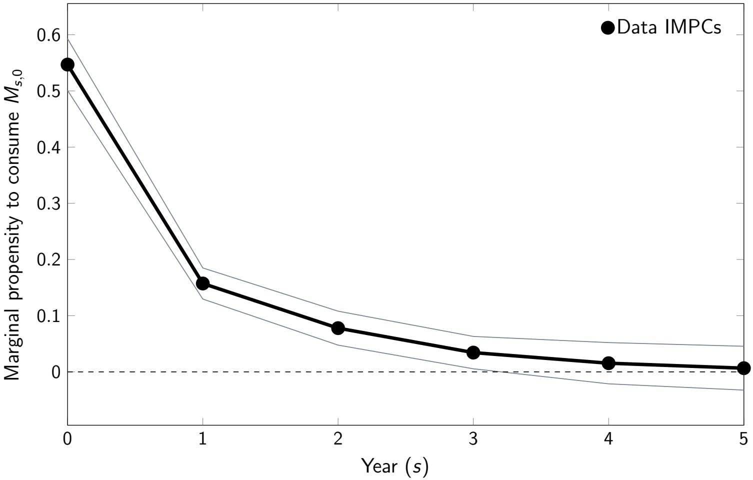
- Obtain date-0 IMPCs from Fagereng-Holm-Natvik (2018)

$$c_{is} = \alpha_i + \tau_s + \sum_{k=0}^{5} \gamma_k \text{lottery}_{i,s-k} + \theta x_{is} + \epsilon_{is}$$

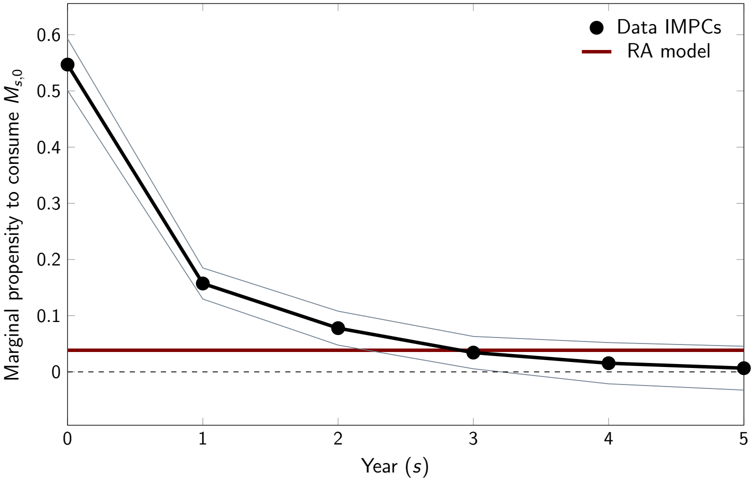
  using WLS with weights = income in year of lottery receipt

- Consider battery of models, including
  - Standard HA calibration (liquidity $B/Y = 140\%$)
  - Baseline HA matching IMPC impulse (with $B/Y = 11\%$) Details
  - In progress: β-heterogeneity and two-asset model

IMPCs in the data



Using IMPCs for model discrimination



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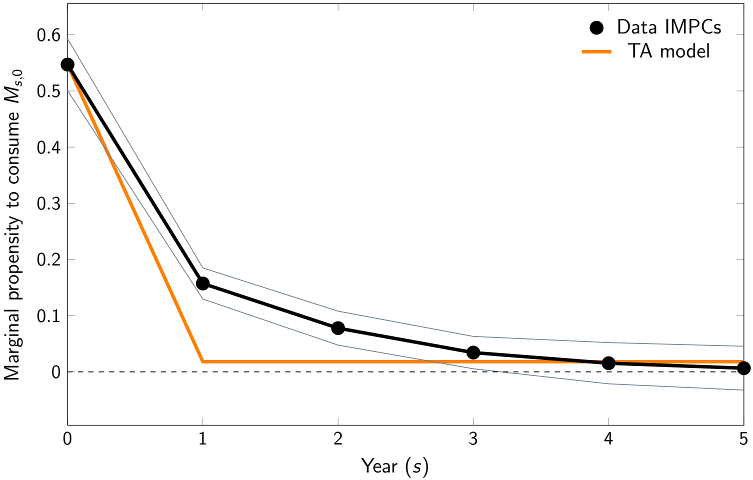

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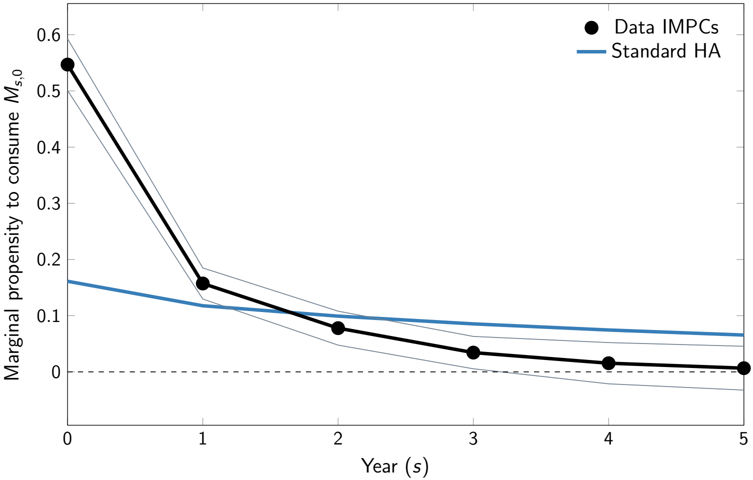

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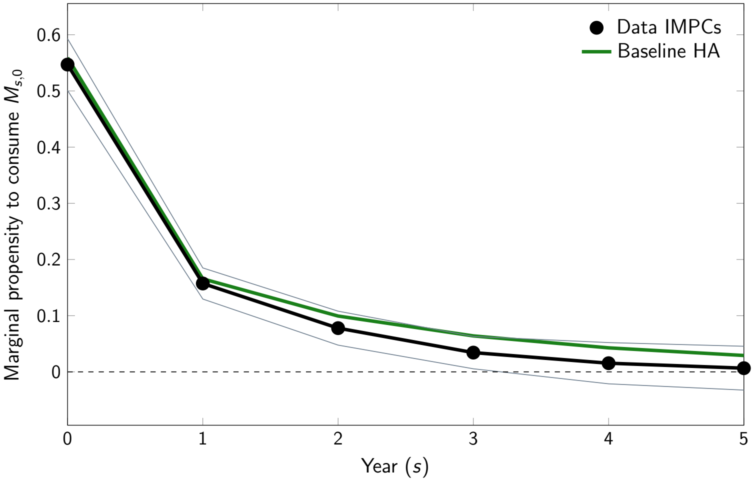
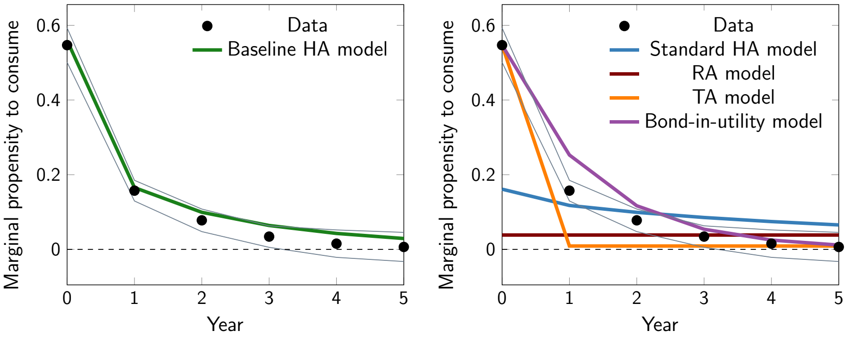


Using IMPCs for model discrimination

- Date-0 IMPC data favors baseline HA against battery of alternatives



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Quantitative model

- **For today**: benchmark model as above, except:
  - Monetary rule: constant-r replaced by Taylor rule

$$i_t = r_{ss} + \phi \pi_t$$

  - Fiscal rule: AR(1) process for $\{G_t\}$

$$\frac{dG_t}{Y_{ss}} = \rho \frac{dG_{t-1}}{Y_{ss}} + \epsilon_t$$

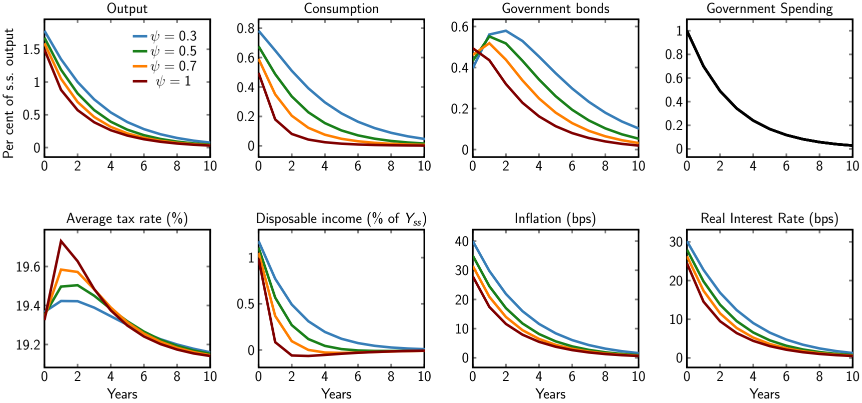
    with three fiscal rules for taxes:

    1. Balanced-budget: $dT_t = dG_t$, levied by changing τ_t
    2. Balanced-budget, but dT_t levied lump-sum
    3. Automatic stabilizer: $d\tau_t = -\psi \frac{dB_{t-1}}{Y_{ss}}$

- **In progress**: sticky prices + capital + two assets

Impulse response: role of deficit financing (ψ)

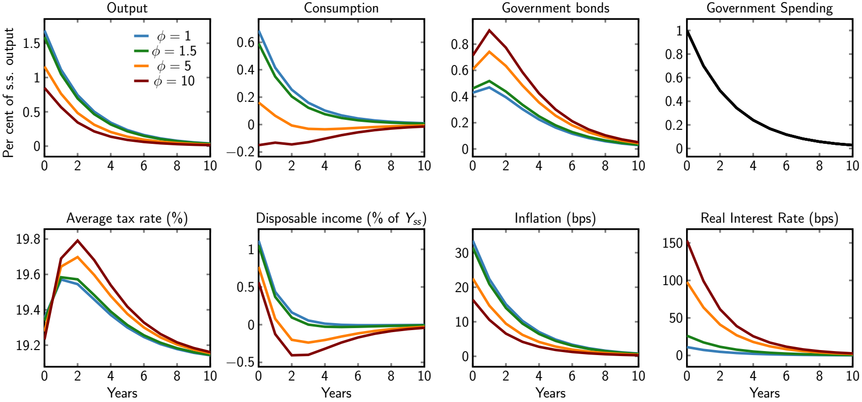
- Deficit financing generates large positive consumption multipliers



Calibration: $\rho = 0.7$, $\kappa = 0.1$, $\phi = 1.5$, and vary ψ in rule $d\tau_t = -\psi \frac{dB_{t-1}}{Y_{ss}}$. BB Capital

Impulse response: role of monetary policy (ϕ)

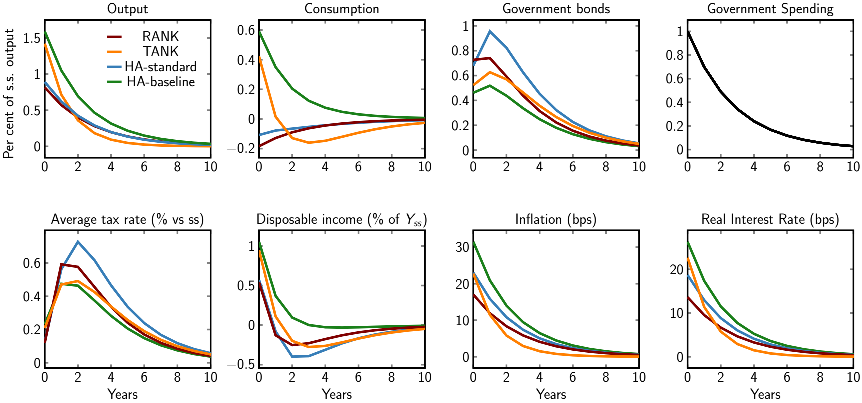
- True irrespective of monetary policy, unless response is very large



Calibration: $\rho = 0.7$, $\kappa = 0.1$, $\psi = 0.7$ and vary ϕ in Taylor rule.

Impulse responses: alternative models

- Amplification and persistence specific to IMPC-based calibration



Calibration: $\rho = 0.7$, $\kappa = 0.1$, $\phi = 1.5$, $\psi = 0.7$.

Amplification: impact output multipliers

Monetary rule	Fiscal rule	RA	TA	HA-standard	HA-IMPC
	BB	**1**	**1**	**1**	**1**
Constant-r	BB + lump-sum	1	1	0.74	0.21
	Auto. stab.	1	1.56	1.08	1.76
	BB	**0.81**	**0.81**	**0.79**	**0.83**
Taylor, $\phi = 1.5$	BB + lump-sum	0.81	0.81	0.56	0.16
	Auto. stab.	0.81	1.44	0.89	1.59

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Taylor, $\phi = 1.5$	BB + lump-sum	0.81	0.81	0.56	0.16
	Auto. stab.	0.81	**1.44**	**0.89**	**1.59**

Persistence: output multipliers at year 2

Monetary rule	Fiscal rule	RA	TA	HA-standard	HA-IMPC
	BB	0.49	0.49	0.49	0.49
Constant-r	BB + lump-sum	0.49	0.49	0.34	0.10
	Auto. stab.	0.49	**0.42**	0.53	**0.78**
	BB	0.4	0.4	0.38	0.40
Taylor, $\phi = 1.5$	BB + lump-sum	0.4	0.4	0.24	0.07
	Auto. stab.	0.40	**0.27**	0.42	**0.69**

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Other applications of the IKC

- IMPC matrix $\mathbf{M}$ relevant in other contexts

1. Amplification and propagation of other shocks: Go

$$d\mathbf{Y} = \mathbf{M}d\mathbf{Y} + \partial\mathbf{Y}$$

   - where $\partial\mathbf{Y}$ is the 'partial equilibrium' effect of shock to preferences, borrowing constraints, inequality, monetary policy...

2. Determinacy:
   - Depends on behavior of $\mathbf{M}_{s,t}$ for large t
     - Taylor principle may involve $\phi \lessgtr 1$
     - See "Determinacy with Incomplete Markets"

Conclusion

- How large is the government spending multiplier?
  - HA models stress the **incidence** and the **timing of taxation**
- Theory: relevance of IMPCs
  - Sufficient statistics in special case
  - Always matter for amplification and persistence
  - Empirical agenda: IMPC evidence to discipline quantitative models
- Quantitative evaluation: large fiscal multipliers despite active m.p.
  - Delayed taxation + proportional incidence is enough
  - Empirical agenda: confront these predictions to data

Thank you!

Phillips curve

- Continuum of unions that each employ every individual, $n_i \equiv \mathbb{E}_J [n_{ij}]$
- Each union $j \in J$
  - produces task $l_j = \mathbb{E}_I [e_i n_{ij}]$ from member hours
  - pays common wage w_j per efficient unit of work e
  - requires that individuals with skills e_i work $n_{ij} = l_j$
- Final good firms aggregate $L \equiv \left(\int_0^1 l_j^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$
- Union j sets w_{jt} each period to maximize

$$\max_{w_{jt}} \sum_{\tau \geq 0} \beta^\tau \left\{ \int \{ u(c_{it+\tau}) - v(n_{it+\tau}) \} \, di - \frac{\psi}{2} \left(\frac{w_{jt+\tau}}{w_{jt+\tau-1}} \right)^2 \right\}$$

- Yields wage and price Phillips curves (where $T'(y_{it}) \equiv$ MTR of i)

$$\pi_t^w = \kappa \int \left(\frac{\epsilon}{\epsilon - 1} \frac{1}{T'(y_{it})} \frac{v'(n_{it})}{u'(c_{it})} - \frac{W_t}{P_t} \right) di + \beta \pi_{t+1}^w$$

$$\pi_t = \kappa \int \left(\omega_{it} \frac{v'(n_{it})}{u'(c_{it})} - 1 \right) di + \beta \pi_{t+1}$$

Back

Two agent New Keynesian (TANK) model

- Two types of agents $i \in \{c, u\}$
  - Fraction μ of permanently constrained agents c
  - Fraction $1-\mu$ of infinitely-lived unconstrained agents u
- Both agents maximize $\mathbb{E}\left[\sum \beta^t \{u(c_{it}) - v(n_{it})\}\right]$ s.t. budget:

$$c_{ct} = \frac{W_t}{P_t} n_{ct} - t_{ct}$$

$$c_{ut} + a_{ut} = \frac{W_t}{P_t} n_{ut} - t_{ut} + (1 + r_t) a_{ut-1}$$

- Assume proportional incidence:

$$n_{ct} = n_{ut} = L_t \qquad t_{ct} = t_{ut} = T_t$$

- Market clearing:

$$Y_t = L_t = \mu c_{ct} + (1-\mu) c_{ut} + G_t$$

- Consider equilibrium with $\beta(1+r) = 1$ and $\lim dY_t = 0$
  - Must have $dc_{ut} = 0$ and $dc_{ct} = dY_t - dT_t$
  - Hence $dY_t = \mu(dY_t - dT_t) + dG_t$: static keynesian cross

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Calibration

- CES utility $u(c) = \frac{c^{1-\nu^{-1}}}{1-\nu^{-1}}$
- Gross income process AR(1) with $\rho = 0.91$ as in Floden-Linde
- Baseline calibration: find model discount factor β that solves

$$\min_{\beta} \sum_{k=0}^{5} \left(M_{k,0}(\beta) - \widehat{\gamma_k}\right)^2$$

Parameters	Description	Baseline	Standard
ν	Elasticity of intertemporal substitution	0.5	
β	Discount factor	0.75	0.89
r	Real interest rate	2%	
B/Y	Government debt to GDP	11%	140%
$\underline{a}/Y$	Borrowing constraint to GDP	0%	
G/Y	Government spending to GDP	18.9%	
λ	Retention function curvature	0.181	

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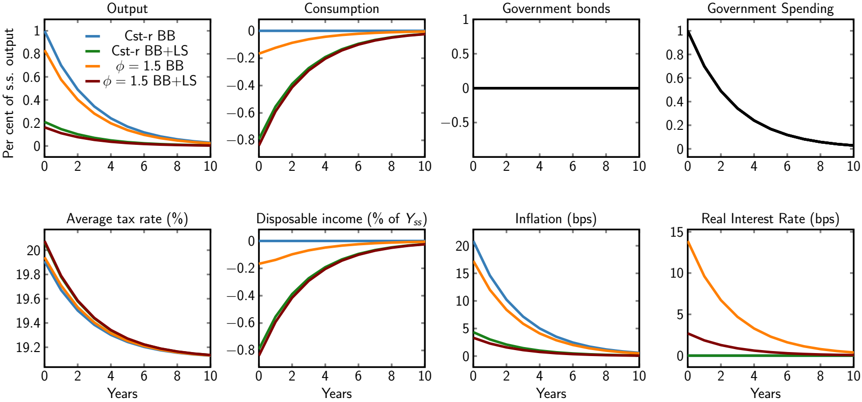
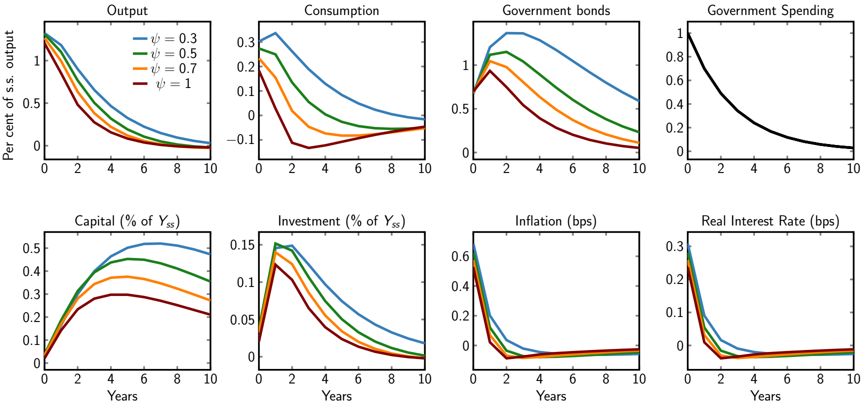
Impulse responses under balanced-budget rule



$\rho = 0.7$, $\kappa = 0.1$, vary mp rule and financing of bb rule Back

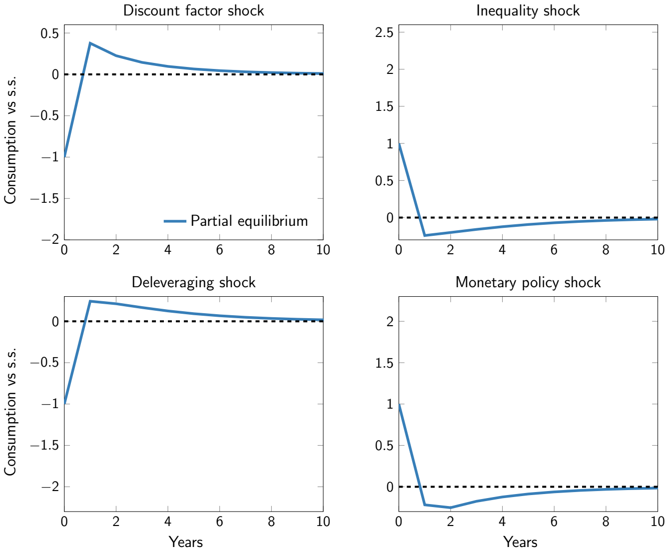
Impulse responses with capital and sticky prices



$\rho = 0.7$, $\kappa = 0.1$ for prices and wages, $\phi = 1.5$, and vary ψ in rule $d\tau_t = -\psi \frac{dB_{t-1}}{Y_{ss}}$ Back

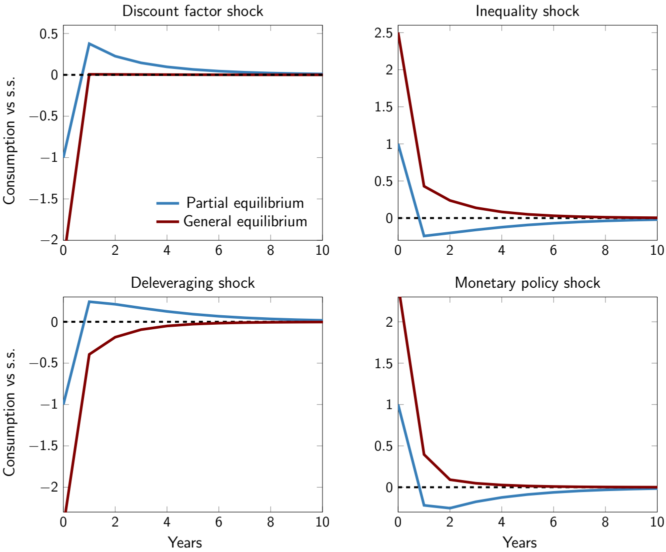
Other shocks

- Different PE effects $\partial \mathbf{Y}$...



Other shocks

- Different PE effects $\partial \mathbf{Y}$, *same* amplification $d\mathbf{Y} = \mathbf{M}d\mathbf{Y} + \partial \mathbf{Y}$



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