Paralyzed by Fear: Rigid and Discrete Pricing Under Demand Uncertainty

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Motivation

- Why do we care?
  - Price rigidity: crucial to understanding propagation mechanism of monetary policy and business cycle fluctuations

- How to model?
  - Taylor, Calvo, menu costs, sticky information, rational inattention, etc.

- How to choose between models?
  - to guide us, large empirical literature on documenting price stickiness
  - rich set of 'overidentifying' restrictions on the theory

- This paper: a new model of rigid prices
  - intuitive and parsimonious
Key Mechanism: uncertainty about competition

1. Uncertainty about demand function
   - Not confident about potentially complex shape of demand curve
   - Learn through noisy demand signals at posted price
   - Reduction in uncertainty: stronger locally, not confident to extrapolate
   - Uncertainty aversion $\rightarrow$ kinks in as if expected demand at past prices
     - If increase price $\Rightarrow$ worry demand is very elastic
     - If decrease price $\Rightarrow$ worry demand is very inelastic
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     - If decrease price \(\Rightarrow\) worry demand is very inelastic

2. Uncertainty about relevant relative price (the argument of demand)
   - Relevant price index of competition is unknown; review it infrequently
   - Short run: unknown relation b/w price index and observed aggr. price
   - Firm takes action robust to worst-case demand schedule
     - action: relative price against last observed competition price index
     - worst case: agg prices are uninformative about competition price index
Key Implications

- Kinks from lower uncertainty at previously posted prices $\Rightarrow$ prices that are endogenously:
  1. sticky: do not want to move and face higher uncertainty
  2. discrete: conditional on price change, move to 'safer' prices
  3. increasingly attractive: larger kinks if posted more often
  4. both flexible and sticky: endogenous cost of adjustment

- Novel empirical implications: prices with unusually high demand realizations are stickier
Literature

1. Sticky prices
   1. Empirical
   2. Theory: pricing rigidities
      - Real: Ball & Romer (1990), Kimball (1995), kinked demand curves (Stigler 1947, Stiglitz 1979)

2. Pricing under demand uncertainty

3. Knightian uncertainty
Outline

1. **Analytical Model**
   - Learning under ambiguity
   - Optimal pricing
     - static and dynamic tradeoffs
     - policy functions

2. **Quantitative Model**
   - Nominal Rigidity
   - Quantitative Results
   - Novel Empirical Implications
   - Monetary Policy
Information structure

- The firm faces log marginal cost $c_t$, sells single good for price $p_t$
- Time $t$ profit:
  \[ v(p_t, q_t, c_t) = (e^{p_t} - e^{c_t})e^{q(p_t)} \]
  - demand:
    \[ q_t = x(p_t) + z_t \]
- Information:
  - not observe $x(p_t)$ and $z_t$ separately
  - $z_t$ is purely risky - i.e. know that
    \[ z_t \sim iidN(0, \sigma_z^2) \]
  - $x(.)$ is ambiguous – not know its probability distribution
  - the firm learns about $x(p_t)$ through past sales data $\{q^{t-1}, p^{t-1}\}$
Learning Framework

- Prior is a Gaussian Process distr: for any price vector \( \mathbf{p} = [p_1, \ldots, p_N]' \)

\[
\mathbf{x}(\mathbf{p}) \sim N \left( \begin{bmatrix} m(p_1) \\ \vdots \\ m(p_N) \end{bmatrix}, \begin{bmatrix} K(p_1, p_1) & \cdots & K(p_1, p_N) \\ \vdots & \ddots & \vdots \\ K(p_N, p_1) & \cdots & K(p_N, p_N) \end{bmatrix} \right)
\]

1 Ambiguity – the firm entertains a set of priors \( \Gamma \)
   - Priors have different mean function \( m(p) \)
   - Same covariance function (infinitely differentiable):
     \[
     K(p, p') = \sigma^2 x \exp(-\psi(p - p')^2)
     \]

2 Non-parametric – not restricted to a parametric family, just:
   - Lay inside some bounds
     \[
     m(p) \in [\gamma_L - bp, \gamma_H - bp]
     \]
   - Non-increasing, i.e. is a demand curve
     \[
     m(p') \leq m(p), \text{ for } \forall p' > p
     \]
   - Maximum derivative (ensures continuity): \( |m'(p)| \leq b_{\text{max}} \)
Admissible Prior Mean Functions
Learning: Prior-by-prior Bayesian updating

- The firm uses data $\varepsilon^{t-1} = (p^{t-1}, q^{t-1})$ to update each prior.
- Recursive multiple priors utility (Epstein-Schneider (2007))

$$V(\varepsilon^{t-1}, c_t) = \max_{p_t} \min_{m(p)} E^{\hat{x}_{t-1}(p_t;m(p))} [v(\varepsilon_t, c_t) + \beta V(\varepsilon^{t-1}, \varepsilon_t, c_{t+1})]$$

- Min operator is conditional on price choice.
  - The firm looks for the $p_t$ most robust to the set of possibilities it faces.
- Price choice – affects profits today and information set tomorrow.
Learning: Prior-by-prior Bayesian updating

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- Min operator is conditional on price choice
  - The firm looks for the $p_t$ most robust to the set of possibilities it faces
- Price choice – affects profits today and information set tomorrow

- Worst-case $m(p)$ – lowest expected demand $\hat{x}_{t-1}(p_t; m(p))$:

$$m^*(p; p_t) = \arg\min_{m(p) \in \gamma} \hat{x}_{t-1}(p_t; m(p))$$
Imagine firm has observed $p_0$ for $N_0$ times, with avg demand

$$q_0 = x(p_0) + \frac{1}{N_0} \sum_i z_i$$

Then signal-to-noise ratio for a given $p'$ is

$$\alpha(p', p_0) = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_z^2/N_0} \exp(-\psi(p' - p_0)^2)$$
Kinks in expected demand

- **Set** of conditional expectations, indexed by priors

\[ \hat{x}_0(p'; m(p)) = (1 - \alpha(p', p_0)) m(p') + \alpha(p', p_0) [q_0 + m(p') - m(p_0)] \]

- Prior demand at \( p' \)
- Signal + \( \Delta \) in Demand between \( p' \) and \( p_0 \)
Kinks in expected demand

- **Set** of conditional expectations, indexed by priors

\[
\hat{x}_0(p'; m(p)) = (1 - \alpha(p', p_0)) m(p') + \alpha(p', p_0) [q_0 + m(p') - m(p_0)]
\]

- Prior demand at \( p' \)
- Signal + \( \Delta \) in Demand between \( p' \) and \( p_0 \)

- Worst-case priors: minimize

1. Prior demand at \( p' \):
   \[
m^*(p') = \gamma_L - bp'
\]

2. Change in demand from \( p' \) to \( p_0 \): worst-case is conditional on price \( p' \)

   - For \( p' > p_0 \): worry demand is elastic between \( p' \) and \( p_0 \)
     \[
m^*(p') - m^*(p_0) = -b_{\text{max}}(p' - p_0)
\]

   - For \( p' < p_0 \): worry demand is inelastic between \( p' \) and \( p_0 \)
     \[
m^*(p') - m^*(p_0) = 0
\]
Worst-case is conditional on price
Worst-case expected demand

- Kink in worst-case expected demand at $p_0$: from endogenous switch in worst-case prior

  - Demand elasticity to the left ($p' \rightarrow p^-_0$):
    
    $$-(1 - \frac{\sigma^2_x}{\sigma^2_x + \sigma^2_z/N_0})b$$

  - Demand elasticity to the right ($p' \rightarrow p^+_0$)
    
    $$-(1 - \frac{\sigma^2_x}{\sigma^2_x + \sigma^2_z/N_0})b - \frac{\sigma^2_x}{\sigma^2_x + \sigma^2_z/N_0}b_{\max}$$
As if kinked expected demand
As if kinked expected demand: $b_{max} \to \infty$
As if kinked expected demand: more exercises
As if kinked expected demand: 2 past prices
Optimal pricing: Myopic (static) maximization

- Perceived kinks lead to price stickiness
  - Intuition: higher uncertainty at new prices ⇒ kink at $p_0$ ⇒ stickiness

- Inaction regions (stickiness) are price and history specific
  - Increase with information precision ($N$) and level of past demand ($q$)

- Past price not only ‘sticky’ but also attractive – i.e. ‘reference’ prices
  1. Memory / discreteness (positive probability of revisiting past prices)
  2. Declining hazard – prob. of revisit increases with $N$
  3. Flexibility and stickiness – small price changes could be optimal

- Theory of endogenous, time-varying cost of price change
Myopic Optimal Price: kinked expected demand

Optimal Price, Static Problem

- Blue: Ambiguity - $N_0 = 1$
- Black: Ambiguity - $N_0 = 2$
- Orange: Rational Expectations

Log Marginal Cost

Log Price

$p_0$

$c$

Log Marginal Cost

Log Price
Myopic Optimal Price: kinked expected demand

Optimal Price, Static Problem

- Ambiguity Aversion
- Rational Expectations

$p_0$

$\bar{c}$
Dynamics: Experimentation Motive

- Full model infeasible: infinite state space
  - Whole history of prices and demand observations
- Consider instead
  - Firm understands how action at \( t = 1 \) affects information set at \( t \geq 2 \)
    \[
    \max_{p_1} E(\pi(p_1, c_1) + \beta V(c_2, \mathcal{I}_1)|\mathcal{I}_0)
    \]
    s.t.
    \[
    \mathcal{I}_1 = \mathcal{I}_0 \cup \{p_1, q_1\}
    \]
  - But thinks there are no updates to information for \( t \geq 2 \), so
    \[
    V(c_t, \mathcal{I}_1) = \max_{p_t} E_t(\pi(p_t, c_t)) + \beta E_t(V(c_{t+1}, \mathcal{I}_1),
    \]
- Puts an upper bound on experimentation motive
  - Today is last period in which you can acquire new information
Forward looking policy

\[ N_0 = 1 \]

- Log Marginal Cost
  - -0.5
  - -0.4
  - -0.3
  - -0.2
  - -0.1
  - 0
  - 0.1
  - 0.2
  - 0.3
  - 0.4
  - 0.5

- Log Price

Graph showing the relationship between Log Marginal Cost and Log Price, with dynamic and myopic policies represented by blue and red lines respectively.

\[ p_0 \]
Experimentation Motive: existing information matters

$N_0 = 5$

![Graph showing the relationship between log price and log marginal cost for dynamic and myopic cases.](image-url)
Reduced benefits of experimentation

1. Better information about $x(p_t) \Rightarrow$ get closer to true optimal price
   - More useful if you set price further away from $p_0$ (influential point)
   - Here: information is local, reducing effect of influential points

2. Option value of new information: if bad signal, go back to “safe” $p_0$
   - Higher value if close to $p_0$ (marginal cost is persistent)
   - Here (unlike independent arm bandit models): $x(p)$ and $x(p')$ are correlated $\Rightarrow p_1 \approx p_0$ carries little new information
   - Likely to set $p_0$ again (sticky price) $\Rightarrow$ best to draw new signal there

Why does higher $N_0$ reduce experimentation motive?

1. Cost of forgone profit of large experimentation is large
2. New signal at $p_1 \approx p_0$ will have little effect on beliefs
Move a little when $\text{corr}(x(p), x(p'))$ is low.
Low experimentation motive with low signal-to-noise ratio

\[ \text{Log Price} \]

\[ \text{Log Marginal Cost} \]

\[ p_0 \]

\[ \text{Low } \sigma_x^2 \]

- Blue: Dynamic
- Orange: Myopic
Outline

1. Analytical Model

2. Quantitative Model
   - Nominal Rigidity
   - Calibration and Quantitative Results
   - Novel Empirical Implications
   - Monetary Policy Effects
A monopolistically competitive model with nominal prices

- Household: CES aggregator over goods produced by industries $j$
  \[ P_t = \left( \int P_{j,t}^{1-b} \, dj \right)^{\frac{1}{1-b}} \]

- Industry $j$: aggregates over interm. goods $\Rightarrow$ demand for good $i$
  \[ q_{i,j,t} = h(p_{i,t} - p_{j,t}) - b(p_{j,t} - p_t) + c_t + z_{i,t} \]
  \[ = \text{demand for industry } j \]

1. Firm $i$ observes aggregate and own realizations: \{\(p_t, c_t, p_{i,t}, q_{i,j,t}\}\)
2. Firm $i$ observes relevant prices $p_{j,t}$ infrequently, with prob. $\lambda_T$
3. Firms exit with exogenous probability $\lambda_\phi$

- Ambiguity about competition: two layers
  - **demand function**: functional form of industry demand $h(.)$
  - **argument of demand function**: ambiguity about $p_{j,t}$
Ambiguous demand \[ y_{i,j,t} = h(p_{i,t} - p_{j,t}) - bp_{j,t} + bp_t + c_t + z_{i,t} \]

- Relation between \( p_{j,t} \) and \( p_t \). If \( p_{j,s} \) last observed ind price

\[ p_{jt} - p_{js} = \phi(p_t - p_{js}) + \nu_{jt} \]

- Long-run cointegrated but in short-run ambiguous relationship:

\[ \phi(p_t - p_{js}) \in [-\gamma_p, \gamma_p], \text{ for } |p_t - p_{js}| \leq K. \]

- We empirically document imprecise industry - aggregate inflation link

Inflation Evidence
Ambiguous demand \( y_{i,j,t} = h(p_{i,t} - p_{j,t}) - bp_{j,t} + bp_{t} + c_{t} + z_{i,t} \)

- Relation between \( p_{j,t} \) and \( p_{t} \). If \( p_{j,s} \) last observed ind price

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p_{j,t} - p_{j,s} = \phi(p_{t} - p_{js}) + \nu_{jt}
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\]

- We empirically document imprecise industry - aggregate inflation link

- Identification problem: nature draws jointly \( h(.) \) and \( \phi(.) \)

\[
h(p_{it} - p_{jt}) = h(p_{it} - p_{js} - \phi(p_{t} - p_{js}) + \nu_{jt}) = \hat{r}_{it}
\]
Ambiguous demand $y_{i,j,t} = h(p_{i,t} - p_{j,t}) - bp_{j,t} + bp_{t} + c_{t} + z_{i,t}$

- Relation between $p_{j,t}$ and $p_{t}$. If $p_{j,s}$ last observed ind price

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- Identification problem: nature draws jointly $h(.)$ and $\phi(.)$

$$h(p_{it} - p_{jt}) = h(p_{it} - p_{js} - \phi(p_{t} - p_{js}) + \nu_{jt})$$

$$= \hat{r}_{it}$$

- Firm’s action is robust against worst-case demand schedule:

$$h^*(\hat{r}_{it}, \nu_{jt}) \approx x(\hat{r}_{it}) + \varepsilon_{it}; \varepsilon_{it} \sim N(0, \sigma^2)$$

- no $p_{t}$ because nature chooses some (unidentifiable) $\phi^*(p_{t} - p_{js}) = \bar{\phi}$
Nominal rigidity from learning the worst-case demand

- Demand signals

\[ y_{i,j,t} = x(\hat{r}_{it}) + c_t + b(p_t - p_{js}) + \varepsilon_{it} + z_{it} \]

- Ambiguity about competition

1. **demand function**: kinks formed in relative prices \( \hat{r}_{it} = p_{it} - p_{js} \)

2. **argument of demand function**: \( p_{jt} \) beliefs constant in the short-run \( \Rightarrow \) nominal rigidity

- Potential for 'pricing regimes': sticky nominal prices with memory
Quantitative model

- GE model with measure zero of ambiguity-averse firms
  - Aggregate shocks: money supply and TFP
  - Endogenous aggregates evolve as with flex prices

- Ergodic distribution: beliefs of firms converge to a stable distribution
  - Learning friction still present at aggregate & individual firm level
    - Endogenous reference prices means firms select from coarse set
    - Never learns demand at all possible prices, friction remains in long-term

- Parameters:
  - macro: calibrate to standard moments on inflation and aggregate TFP
  - micro: use micro-data pricing and quantity moments (IRI dataset)
  - take out sales (V-shape filter)
  - some direct evidence:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\phi}$</td>
<td>0.0075</td>
<td>mean lifespan of a product 2.5 yrs (Argente-Yeh 2017)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.61</td>
<td>median demand forecast error</td>
</tr>
</tbody>
</table>
Calibration micro parameters: SMM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_w$</td>
<td>0.784</td>
<td>Persistence of idiosyncratic productivity</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>0.047</td>
<td>St. dev. of idiosyncratic productivity shock</td>
</tr>
<tr>
<td>$\nu$</td>
<td>1.15</td>
<td>Ambiguity parameter</td>
</tr>
<tr>
<td>$\psi$</td>
<td>4</td>
<td>Prior covariance function smoothing parameter</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>0.51</td>
<td>Prior variance of $x(.)$</td>
</tr>
<tr>
<td>$b_{max}$</td>
<td>3.4*b</td>
<td>Maximum derivative</td>
</tr>
<tr>
<td>$\lambda_T$</td>
<td>0.015</td>
<td>Frequency of price reviews</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Target Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of price changes</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>Fraction of price increases</td>
<td>0.54</td>
<td>0.55</td>
</tr>
<tr>
<td>Mean size of abs price changes</td>
<td>0.19</td>
<td>0.20</td>
</tr>
<tr>
<td>Lower quartile of abs price change distribution</td>
<td>0.069</td>
<td>0.069</td>
</tr>
<tr>
<td>Upper quartile of abs price change distribution</td>
<td>0.27</td>
<td>0.28</td>
</tr>
<tr>
<td>Frequency of modal price change (13 week window)</td>
<td>0.027</td>
<td>0.029</td>
</tr>
<tr>
<td>Mean duration of pricing regimes (weeks - Stevens, 2017)</td>
<td>29.1</td>
<td>32.1</td>
</tr>
</tbody>
</table>
### Additional Implications: discrete prices with memory

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of revisiting a price (last 26 weeks)</td>
<td>0.62</td>
<td>0.68</td>
</tr>
<tr>
<td>Avg # uniq. prices (26 weeks) / (# price changes + 1)</td>
<td>0.77</td>
<td>0.73</td>
</tr>
<tr>
<td>Fraction of time at modal price</td>
<td>0.83</td>
<td>0.85</td>
</tr>
<tr>
<td>Prob. price change goes to modal price</td>
<td>0.43</td>
<td>0.51</td>
</tr>
</tbody>
</table>
Price change hazard

Hazard Functions

Model
Data

Price Spell Length

Prob. Price Change

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Additional Implications: demand signals matter for pricing

- Model predicts that stickiness is stronger for a price:

1. posted more often (’high $N$’)
   - already some evidence to this: e.g. declining hazard

2. with unusually positive demand realizations (’high $\bar{z}$’)
   - intuitive: more likely to remain at prices that appear ’profitable’
   - stronger effect at young prices: kink mostly driven by $\bar{z}$
   - at older prices: $\bar{z}$ changes little the large kink that comes from $N$
   - novel empirical implications: link quantity data to stickiness
Demand signals matter for pricing: data and model

- Regression that tests those predictions

\[ I(p_{i,t} \neq p_{i,t-1}) = \alpha_i + \xi_t + \beta_Z z_{i,t-1} + \beta_N N_{i,t-1} + \epsilon_t \]

- \( \beta_Z < 0 \) (\( \beta_N < 0 \)): less likely to change a price \( p_{i,t-1} \) with high \( z \) (\( N \))
- subsample with young prices: effects stronger for \( Z \)

<table>
<thead>
<tr>
<th></th>
<th>Young ((N_{i,t-1} \leq 8))</th>
<th>All ((N_{i,t-1} \leq 26))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Pr(\Delta &gt; 0) )</td>
<td>( Z ) effect</td>
</tr>
<tr>
<td>Data</td>
<td>0.14</td>
<td>-7.9%</td>
</tr>
<tr>
<td>Model</td>
<td>0.15</td>
<td>-6.1%</td>
</tr>
</tbody>
</table>

Regression
Monetary Policy IRF

- Output Response (% of MP shock)

Weeks

% of MP shock

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Conclusions

- Firm exploits demand curve under ambiguity
  - learning about non-parametric demand
  - firm acts as if kinked expected demand at previously observed prices
  - generates 'price memory’ and makes them endogenously:
    sticky, discrete, increasingly attractive

- With imperfect info on competitors’ prices: nominal rigidity

- Endogenous cost of price change: rigidity is history and state dependent
  - implications for policy
Evidence on weak aggregate - industry prices link

**Figure:** 3-year rolling regressions of 3-month industry inflation on 3-month aggregate inflation.
Demand regression

Regression to recover $z_{ijt} > 0$ realizations

$$q_{ijt} = \beta_0 + \beta_1 q_{i,j,t-1} + \beta_2 p_{ijt} + \beta_3 p_{ijt}^2 + \beta_4 cpi_t +$$
$$+ week_t' \theta_1 + store_j' \theta_2 + item_i' \theta_3 + z_{ijt}$$

where

- $q_{ijt}, p_{ijt}$ are quantities and prices in logs
- $cpi_t$ is the consumer price index for food and beverages
- $week_t$ is a vector of week dummies
- $store_j$ is a vector of store dummies
- $item_i$ is a vector of item dummies