Cross-Sectional and Aggregate Labor Supply

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Modern Business Cycle Analysis

- Early representative agent models
  - Kydland-Prescott, ...
  - Largely ignore cross-sectional moments

- Recent advances in hetero agent models
  - Aiyagari (1994), ...
  - Heterogeneity: idiosyncratic productivity shocks
  - Testable **cross-sectional** implications
Labor Supply in Macro Models

- **Representative agent models**
  - Offsetting income and substitution effects
  - So-called “Balanced Growth Path” preferences

- **Aiyagari-type Hetero agent models**
  - Idiosyncratic productivity shocks
  - Inherits “Balanced Growth Path” preferences
  - Cross-sectional $\text{cor}(w, h) \approx 0.7 \leftrightarrow 0$ in data
Standard heterogeneous agent macro models that highlight idiosyncratic productivity shocks do not generate the near \textit{zero cross-sectional correlation between hours and wages}.

Ask whether matching this moment matters for business cycle properties of these models.
Two Extensions

We consider two extensions from standard model

- Departure from balanced growth path preferences.
- Introduction of idiosyncratic shocks to the opportunity cost of working.
Both extensions can match the empirical correlation.

Large and opposing effects on the cyclical volatility of the labor market.

Cross-sectional moments are important for business cycle analysis.
Illustrative Example

Consider a household with utility:

\[
\frac{c^{1-\sigma}}{1 - \sigma} - \frac{h^{1+1/\gamma}}{1 + 1/\gamma}
\]

F.O.C. for labor supply:

\[
h = \left\{ \frac{w}{c^{\sigma}} \right\}^\gamma
\]
Examine this issue using Chang-Kim-Kwon-Rogerson (2019)

A heterogeneous agent model that features both intensive and extensive margins of labor.

Benchmark: “One-Shock” Model

\[
\max_{\{c_t, h_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - B \frac{h_t^{1+1/\gamma}}{1 + 1/\gamma} \right\} \right]
\]

\[
c_t + a_{t+1} = (1 + r_t) a_t + w_t z_t g(h_t)
\]

\[
g(h_t) = \max\{0, h_t - \hat{h}\}, \quad h_t \in [0, 1]
\]

\[
a_{t+1} \geq \bar{a}
\]

\[
\ln z_{t+1} = \rho_z \ln z_t + \epsilon_{zt}, \quad \epsilon_{zt} \sim \mathcal{N}(0, \sigma_z^2)
\]

- \(z\): idiosyncratic productivity
- minimum hours \(\hat{h}\) reflects setup costs, commuting, etc.
- Both margins are chosen optimally.
Technology: Representative Firm

\[
\max_{L_t, K_t} \ Y_t = Z_t L_t^\alpha K_t^{1-\alpha}.
\]

\[
\ln Z_{t+1} = \rho_Z \ln Z_t + \varepsilon_Z t, \quad \varepsilon_Z t \sim N(0, \sigma_Z^2).
\]

\[
K_t = \int a_t \ d\mu : \text{Aggregate Capital}
\]

\[
L_t = \int h_t z_t \ d\mu: \text{Aggregate Efficiency unit of labor}
\]

\[
\mu(a, z) \text{ Cross-sectional distribution of workers}
\]
Calibration: One-Shock Model

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$B$</th>
<th>$\rho_z$</th>
<th>$\sigma_z$</th>
<th>$\hat{h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36</td>
<td>0.976</td>
<td>1.00</td>
<td>0.025</td>
<td>18.9</td>
<td>0.975</td>
<td>0.165</td>
<td>0.151</td>
</tr>
</tbody>
</table>

- $\rho_z$, $\sigma_z$: from panel data on wages (e.g., Floden-Linde, 2001)
- $\gamma$: Frisch Elast of labor supply
- $B$: chosen to match employment rate (70%)
- $\hat{h}$: to match the average hours (0.33)
- $\beta$: to match 4% annual rate of return
## Cross-Sectional Dispersion

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.D. of Annual Hours</td>
<td>0.32</td>
<td>0.45 (CPS)</td>
</tr>
<tr>
<td>Earnings Gini</td>
<td>0.59</td>
<td>0.63 (SCF)</td>
</tr>
<tr>
<td>Wealth Gini</td>
<td>0.71</td>
<td>0.78 (SCF)</td>
</tr>
<tr>
<td>$Corr(w, h)$</td>
<td>0.78</td>
<td>0 (PSID, SIPP)</td>
</tr>
</tbody>
</table>

- Does well on earnings and wealth dist.
- Not enough dispersion in hours
- Too high correlation b/w wages and hours
The only change:

\[ \ln c - B x \frac{h^{1+1/\gamma}}{1 + 1/\gamma} \]

\[ \log x_{it+1} = \rho_x \log x_{it} + \varepsilon_{xit+1}, \quad \varepsilon_{xit} \sim N(0, \sigma_x^2). \]

- \( x \): opportunity cost of working  
  (preference for leisure, home productivity)
- \( \text{cor}(\varepsilon_z, \varepsilon_x) = \rho_{zx} \)
Intensive Margin:

\[ h = \left\{ B \frac{wz}{cx} \right\}^{\gamma} \]

Extensive Margin (Work if):

\[ w(h - \hat{h}) \frac{z}{x} \geq B h^{\frac{1}{\gamma}} c \]
What matters is comparative advantage \((z/x)\).

Shape of cross-sectional distribution of \((z,x)\) crucial for aggregate labor supply.
Calibration: Two-Shock Model

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$B$</th>
<th>$\rho_z$</th>
<th>$\sigma_z$</th>
<th>$\rho_x$</th>
<th>$\sigma_x$</th>
<th>$\hat{h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.977</td>
<td>20.6</td>
<td>0.975</td>
<td>0.165</td>
<td>0.975</td>
<td>0.103</td>
<td>0.133</td>
</tr>
</tbody>
</table>

- $\rho_x = \rho_z = 0.975$
- $\rho_{zx} = corr(x, z) = 0$
- $\sigma_x$ to match the disperson of hours
- $B$: chosen to match employment rate (70%)
- $\hat{h}$: to match the average hours (0.33)
- $\beta$: to match 4% annual rate of return
## Annual Hours Transition: PSID

<table>
<thead>
<tr>
<th></th>
<th>Not Work</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Work</td>
<td>83.57</td>
<td>12.25</td>
<td>1.69</td>
<td>0.91</td>
<td>0.99</td>
<td>0.60</td>
</tr>
<tr>
<td>1st</td>
<td>21.08</td>
<td>49.45</td>
<td>14.91</td>
<td>6.15</td>
<td>5.29</td>
<td>3.12</td>
</tr>
<tr>
<td>2nd</td>
<td>4.77</td>
<td>15.40</td>
<td>45.77</td>
<td>18.27</td>
<td>11.15</td>
<td>4.63</td>
</tr>
<tr>
<td>3rd</td>
<td>2.81</td>
<td>6.75</td>
<td>19.77</td>
<td>46.24</td>
<td>17.88</td>
<td>6.54</td>
</tr>
<tr>
<td>4th</td>
<td>2.26</td>
<td>5.14</td>
<td>10.63</td>
<td>19.91</td>
<td>42.42</td>
<td>19.64</td>
</tr>
<tr>
<td>5th</td>
<td>1.81</td>
<td>3.41</td>
<td>4.69</td>
<td>6.80</td>
<td>19.77</td>
<td>63.52</td>
</tr>
</tbody>
</table>
## Annual Hours Transition: Model

<table>
<thead>
<tr>
<th></th>
<th>Not Work</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Work</td>
<td>72.49</td>
<td>17.64</td>
<td>6.53</td>
<td>2.39</td>
<td>0.81</td>
<td>0.14</td>
</tr>
<tr>
<td>1st</td>
<td>21.12</td>
<td>34.43</td>
<td>24.53</td>
<td>12.60</td>
<td>5.61</td>
<td>1.72</td>
</tr>
<tr>
<td>2nd</td>
<td>8.46</td>
<td>20.54</td>
<td>37.54</td>
<td>23.19</td>
<td>7.88</td>
<td>2.39</td>
</tr>
<tr>
<td>3rd</td>
<td>3.24</td>
<td>11.03</td>
<td>17.67</td>
<td>37.28</td>
<td>24.68</td>
<td>6.09</td>
</tr>
<tr>
<td>4th</td>
<td>1.69</td>
<td>6.99</td>
<td>7.57</td>
<td>19.31</td>
<td>39.95</td>
<td>24.48</td>
</tr>
<tr>
<td>5th</td>
<td>0.49</td>
<td>3.57</td>
<td>3.83</td>
<td>5.37</td>
<td>20.49</td>
<td>66.25</td>
</tr>
</tbody>
</table>
Two-Shock Model

- Does well on transition of hours
- More importantly, $\text{corr}(w, h) = 0.53$. 
To achieve $corr(w, h) \approx 0$, consider two extensions:

- Departure from balanced growth path preferences: $\sigma \neq 1$.
- Departure from $\rho_{zx} = 0$
Extension I: $\sigma \neq 1$

\[ h = \left\{ \frac{w z}{c^\sigma x} \right\}^\gamma \]

- $\sigma \uparrow$ making the wealth effect in labor supply stronger $\text{corr}(w, h) \downarrow$.
- With $\sigma = 2$, we achieve $\text{corr}(w, h) \approx 0$
- Pijoan-Mas (2006), Heathcote et al. (2016), ...
Calibration of Extension I Model

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$B$</th>
<th>$\rho_z$</th>
<th>$\sigma_z$</th>
<th>$\rho_x$</th>
<th>$\sigma_x$</th>
<th>$\hat{h}$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.96946</td>
<td>77.0</td>
<td>0.975</td>
<td>0.165</td>
<td>0.975</td>
<td>0.144</td>
<td>0.144</td>
<td>2</td>
</tr>
</tbody>
</table>
Extension II: $\rho_{zx} \neq 0$

$$h = \left\{ \frac{w z}{c x} \right\}^\gamma$$

- $corr(z, x) = \rho_{zx} \uparrow \rightarrow corr(w, h) \downarrow$.  
- With $\rho_{zx} = 0.9$, we achieve $corr(w, h) \approx 0$.
- Weak cross-sectional comparative advantage

<table>
<thead>
<tr>
<th>$\rho_{zx}$</th>
<th>$-0.9$</th>
<th>$-0.5$</th>
<th>$0$</th>
<th>$0.5$</th>
<th>$0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$corr(w, h)$</td>
<td>$0.78$</td>
<td>$0.66$</td>
<td>$0.53$</td>
<td>$0.34$</td>
<td>$-0.004$</td>
</tr>
</tbody>
</table>
Calibration of Extension II Model

<table>
<thead>
<tr>
<th>$\rho_{zx}$</th>
<th>$\beta$</th>
<th>$B$</th>
<th>$\hat{h}$</th>
<th>$\sigma_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.9</td>
<td>0.97537</td>
<td>19.0</td>
<td>0.128</td>
<td>0.0825</td>
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<tr>
<td>-0.5</td>
<td>0.976</td>
<td>19.2</td>
<td>0.130</td>
<td>0.09</td>
</tr>
<tr>
<td>0.5</td>
<td>0.97818</td>
<td>22.6</td>
<td>0.135</td>
<td>0.124</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9814</td>
<td>26.2</td>
<td>0.139</td>
<td>0.147</td>
</tr>
</tbody>
</table>
# Wealth and Earnings

## Gini Coefficient: Wealth

<table>
<thead>
<tr>
<th>PSID</th>
<th>SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.76</td>
<td>0.78</td>
</tr>
</tbody>
</table>

\[ \sigma = 2, \ \rho_{zx} = 0 \]

<table>
<thead>
<tr>
<th>( \rho_{zx} )</th>
<th>-0.9</th>
<th>-0.5</th>
<th>0.0</th>
<th>0.5</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.63</td>
<td>0.64</td>
<td>0.65</td>
<td>0.67</td>
<td>0.68</td>
</tr>
</tbody>
</table>

## Gini Coefficient: Earnings

<table>
<thead>
<tr>
<th>PSID</th>
<th>SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.53</td>
<td>0.63</td>
</tr>
</tbody>
</table>

\[ \sigma = 2, \ \rho_{zx} = 0 \]

<table>
<thead>
<tr>
<th>( \rho_{zx} )</th>
<th>-0.9</th>
<th>-0.5</th>
<th>0.0</th>
<th>0.5</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.60</td>
<td>0.59</td>
<td>0.59</td>
<td>0.57</td>
<td>0.53</td>
</tr>
</tbody>
</table>
# Wealth Share by Quintile

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSID</td>
<td>-0.52</td>
<td>0.50</td>
<td>5.06</td>
<td>18.74</td>
<td>76.22</td>
</tr>
<tr>
<td>SCF</td>
<td>-0.39</td>
<td>1.74</td>
<td>5.72</td>
<td>13.43</td>
<td>79.49</td>
</tr>
<tr>
<td>$\sigma = 2$, $\rho_{zx} = 0$</td>
<td>0.07</td>
<td>2.19</td>
<td>9.80</td>
<td>24.77</td>
<td>63.17</td>
</tr>
<tr>
<td>$\rho_{zx} = -0.9$</td>
<td>0.16</td>
<td>2.66</td>
<td>9.51</td>
<td>23.80</td>
<td>63.88</td>
</tr>
<tr>
<td>$\rho_{zx} = 0.0$</td>
<td>0.08</td>
<td>2.10</td>
<td>8.75</td>
<td>23.20</td>
<td>65.87</td>
</tr>
<tr>
<td>$\rho_{zx} = 0.9$</td>
<td>0.04</td>
<td>1.16</td>
<td>7.54</td>
<td>22.51</td>
<td>68.76</td>
</tr>
</tbody>
</table>
## Earnings Share by Wealth Quintile

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSID</td>
<td>7.51</td>
<td>11.31</td>
<td>18.72</td>
<td>24.21</td>
<td>38.23</td>
</tr>
<tr>
<td>SCF</td>
<td>7.05</td>
<td>14.50</td>
<td>16.48</td>
<td>20.76</td>
<td>41.21</td>
</tr>
<tr>
<td>$\sigma = 2, \rho_{zx} = 0$</td>
<td>10.05</td>
<td>14.97</td>
<td>18.79</td>
<td>23.38</td>
<td>32.81</td>
</tr>
<tr>
<td>$\rho_{zx} = -0.9$</td>
<td>5.56</td>
<td>11.14</td>
<td>16.91</td>
<td>24.87</td>
<td>41.52</td>
</tr>
<tr>
<td>$\rho_{zx} = 0.0$</td>
<td>6.26</td>
<td>11.95</td>
<td>17.06</td>
<td>24.43</td>
<td>40.31</td>
</tr>
<tr>
<td>$\rho_{zx} = 0.9$</td>
<td>10.27</td>
<td>13.09</td>
<td>18.65</td>
<td>23.21</td>
<td>34.78</td>
</tr>
</tbody>
</table>
Both extensions can match the empirical correlation ($\text{corr}(w, h) \approx 0$).

Large and opposing effects on the cyclical volatility of the labor market.
**Cross-sectional moments are important for business cycle analysis.**

- \( H = E \times h \), \( E \): Employment, \( h \): Hours per worker
- \( L \): Efficiency units
<table>
<thead>
<tr>
<th>$\rho_{zx}$</th>
<th>$\sigma_Y$</th>
<th>$\sigma_H$</th>
<th>$\frac{\sigma_H}{\sigma_Y}$</th>
<th>$\sigma_E$</th>
<th>$\sigma_h$</th>
<th>$\sigma_L$</th>
<th>$\sigma_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.9$</td>
<td>1.59</td>
<td>0.66</td>
<td>0.41</td>
<td>0.47</td>
<td>0.22</td>
<td>0.83</td>
<td>0.87</td>
</tr>
<tr>
<td>$-0.5$</td>
<td>1.62</td>
<td>0.68</td>
<td>0.42</td>
<td>0.51</td>
<td>0.19</td>
<td>0.87</td>
<td>0.86</td>
</tr>
<tr>
<td>$0$</td>
<td>1.65</td>
<td>0.72</td>
<td>0.44</td>
<td>0.58</td>
<td>0.17</td>
<td>0.93</td>
<td>0.85</td>
</tr>
<tr>
<td>$0.5$</td>
<td>1.72</td>
<td>0.77</td>
<td>0.45</td>
<td>0.68</td>
<td>0.14</td>
<td>1.04</td>
<td>0.83</td>
</tr>
<tr>
<td>$0.9$</td>
<td>1.92</td>
<td>0.93</td>
<td>0.48</td>
<td>0.95</td>
<td>0.11</td>
<td>1.47</td>
<td>0.84</td>
</tr>
</tbody>
</table>
What matters is **comparative advantage** \((z/x)\).

Shape of cross-sectional distribution of \((z,x)\) crucial for aggregate labor supply.
Cross-Sectional Comparative Advantage and Aggregate Employment Response

\[ \log(z) \]

\[ \log(x) \]
Special Case: No Heterogeneity

No heterogeneity in \( z \) and \( x \).

- Hansen-Rogerson Lottery Economy
- Infinitely elastic aggregate labor supply
Special Case: $x = \psi \ast z$

$z$ and $x$ are perfectly correlated.

- No Comparative Advantage ($z/x$ is a constant)
- Reservation wage distribution is degenerate
- Infinitely elastic aggregate labor supply
Heterogeneity in $z$ only: e.g., Aiyagari (1994).

- Reservation wage depends on market productivity and wealth.
Cross-Sectional Comparative Advantage

z and x are positively correlated: \( \text{cor}(z, x) > 0 \).

- “Weak” Comparative Advantage
- Workers with high productivity in the market are also good at home production
- Sensitive to the change in relative return
- Elastic aggregate labor supply
\( \text{cor}(z, x) > 0 \)
Celebrating 30 Years at the University of Rochester
and a Happy 60th Birthday for Mark Bils
Cross-Sectional Comparative Advantage

z and x are **negatively** correlated: $\text{cor}(z, x) < 0$.

- **“Strong” Comparative Advantage**
- Workers with high productivity in the market are bad at home production
- Not willing to move between activities
- $z/x$ distribution is dispersed.
- Inelastic aggregate labor supply
$\text{cor}(z, x) < 0$
Conclusion

- Standard heterogeneous agent model fail to match the near zero cross-sectional corr\((w,h)\).

- Economy with weak comparative advantage.
  - Match cross-sectional corr\((w,h)\).
  - Exhibits an elastic aggregate labor supply.

- Cross-sectional moments are important for business cycle analysis.