On the possibility of deflationary equilibria with monetary expansion: A reconciliation between the fiscal theory of the price level and the quantity theory of money

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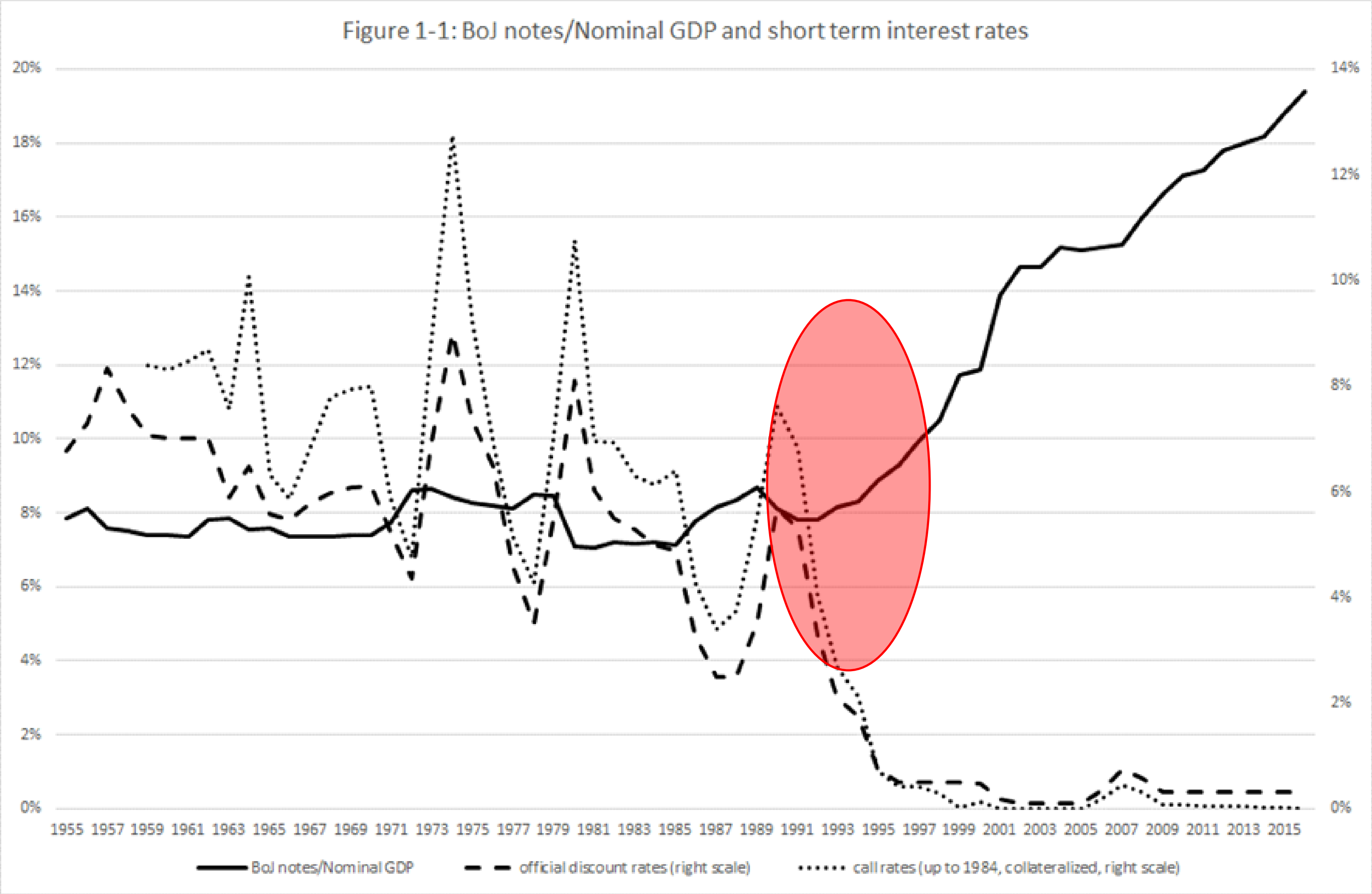
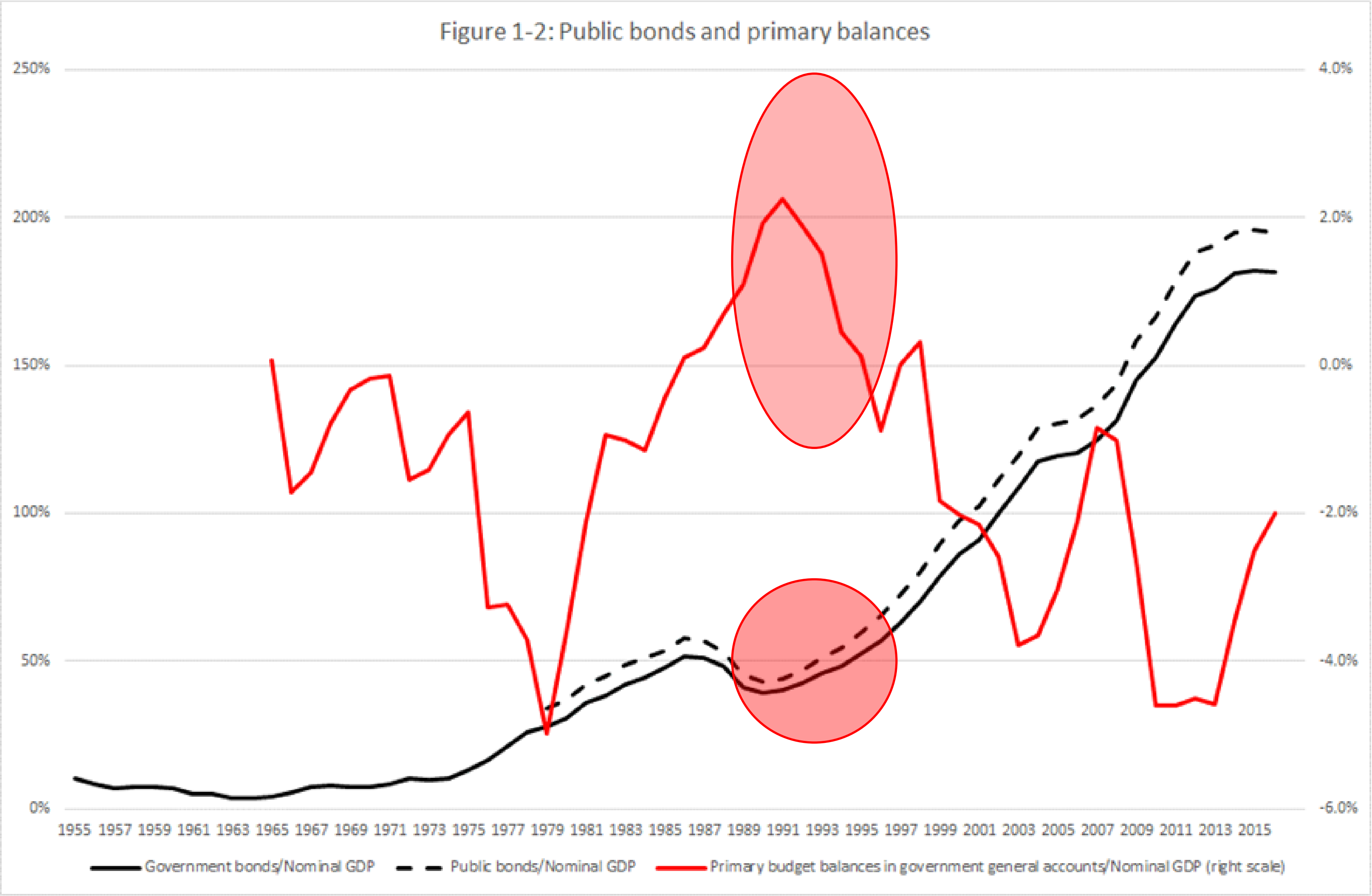
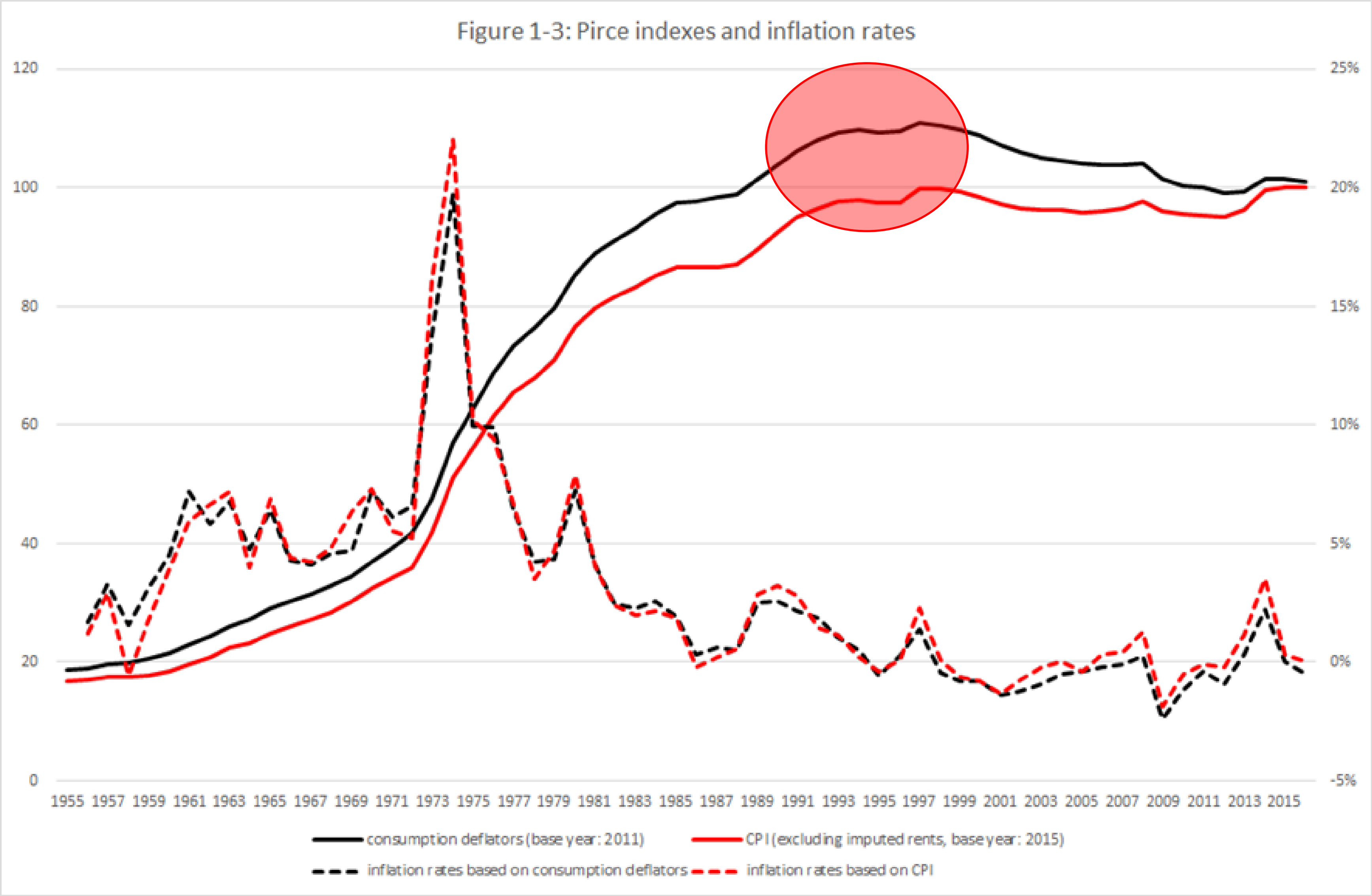
Rapid monetary expansion and declining nominal rates of interest since the early 1990s



Continuing primary deficits and growing public debts since the early 1990s



Deflationary environment since the early 1990s



The Japanese economy as a strong case against standard neo-classical views

- **Mild deflations and near-zero interest rates despite monetary expansion**
  - Inconsistent with zero rates with monetary contraction under the Friedman rule

- **High valuation of the public bonds despite continuing primary fiscal deficits**
  - Far from Ricardian equivalence
  - Inconsistent with the fiscal theory of the price level (FTPL)
    - Weaker fiscal discipline resulted in not inflations, but deflations.

My model building strategy

- The Japanese fiscal policy has been non-Ricardian since the early 1990s.

- But, people are vaguely expecting a drastic fiscal reconstruction to be initiated some day, but they do not know ‘when’ exactly.

- Assuming that the economy will switch from the non-Ricardian regime with the FTPL to the Ricardian regime with the quantity theory of money with a small probability π.
    - At switching, the price level will jump up, and the nominal public bonds will be devalued heavily.
    - People always take into consideration such possible price jump when they form expectations.

- By fixing real sides completely, focusing only on nominal sides.
    - Constant consumption: $c_t = y - g$
    - Constant real interest rates: $r_t = \frac{1}{\beta} - 1$

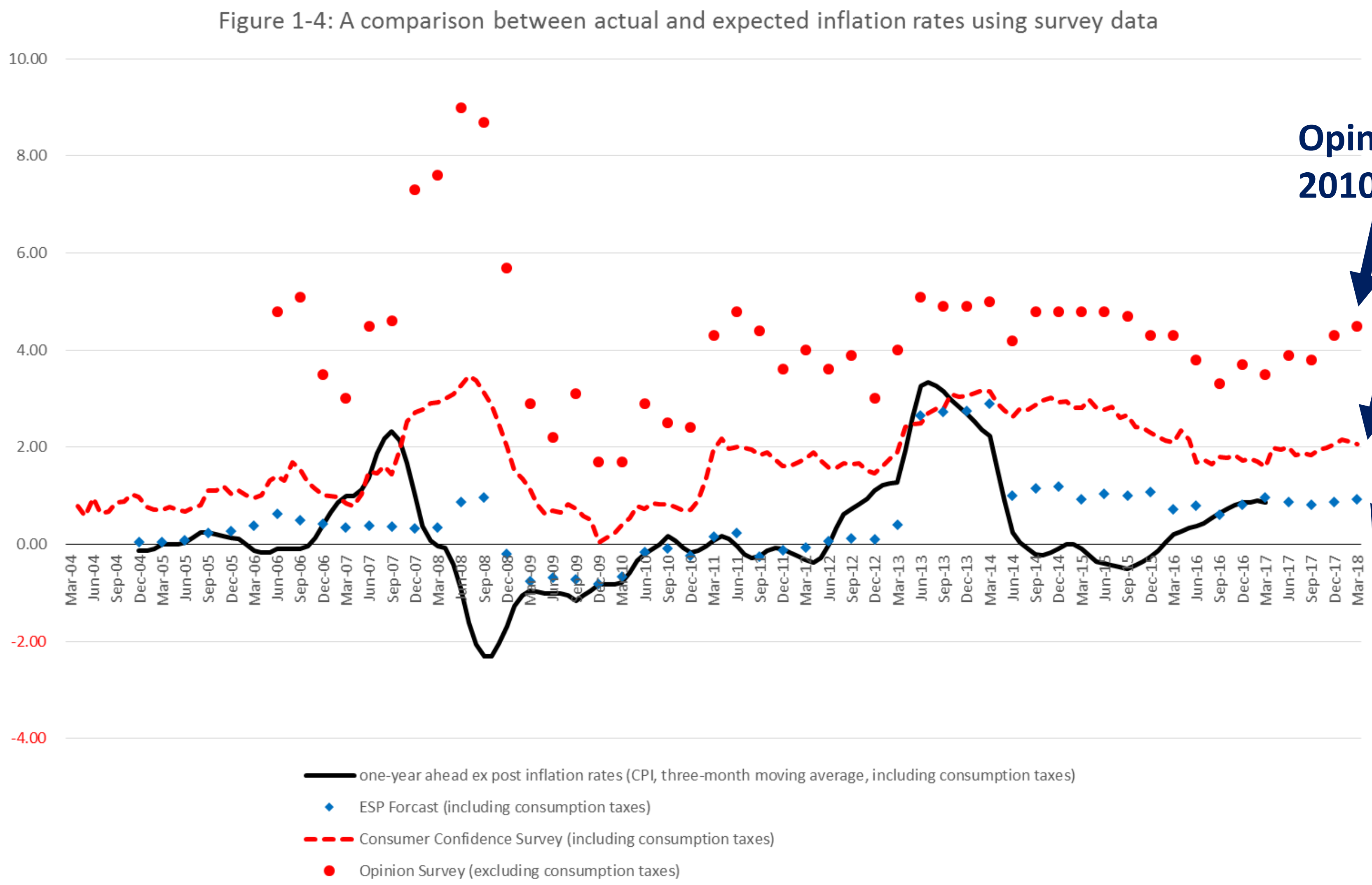
Potentially interesting predictions emerging from the setup with a price jump at switching

- In the non-Ricardian regime, given a small probability of a big price jump at switching,
  - A deflationary pressure is created to balance an one-off big inflations at switching.
  - A gradual appreciation of the public bonds is made to balance their heavy devaluation at switching.

- As long as the non-Ricardian regime continues, the real valuation of the public bonds is backed beyond fiscal surpluses by the stochastic bubbles, which eventually burst at switching, and a government may operate a Ponzi scheme.

- With consideration of unprecedented price jump at switching, the peso problem emerges in the expectation of inflations during the non-Ricardian regime.
  - Actual inflations < Expected inflations
  - Ex post nominal returns < Nominal rates of interest

Circumstantial evidence for the peso problem



Opinion Survey, by 3.5% in 2010-2017

Consumer Confidence Survey, by 1.8% in 2010-2017

ESP Forecast by professional analysts

Existing papers

- FTPL and the bubbles in the government's intertemporal budget constraint (GIBC)
  - LeRoy (2004), Bloise and Regchlin (2008)
- Relaxing the GIBC
  - Sargent and Wallace (1981) on monetary expansion and the price level
- FTPL and lower real returns
  - Bassetto and Cui (2018)
- FTPL and high pricing of public bonds
  - Braun and Nakajima (2012)
  - (though not related to FTPL) Sakuragawa and Sakuragawa (2016), Kobayashi and Ueda (2017)
- Switching among active/passive fiscal/monetary policy rules
  - Davig et al. (2010), Bianchi and Ilut (2017)

Closest

- Davig, Leeper, and Walker, EER, 2011
  - A standard neo-classical case (Active Monetary/Passive Transfers) as an absorbing state
  - Higher expected inflations emerge as the peso problem, reflecting a small probability of sharp inflations in Active Transfers/Passive Monetary

A basic framework (1)

- Life time utility optimization

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v\left(\frac{M_t}{P_t} \right) \right]$$

subject to

$$B_{t+1} = P_{t+1}(y - \tau_{t+1}) - P_{t+1} c_{t+1} - (M_{t+1} - M_t) + R_t B_t$$

A basic framework (2)

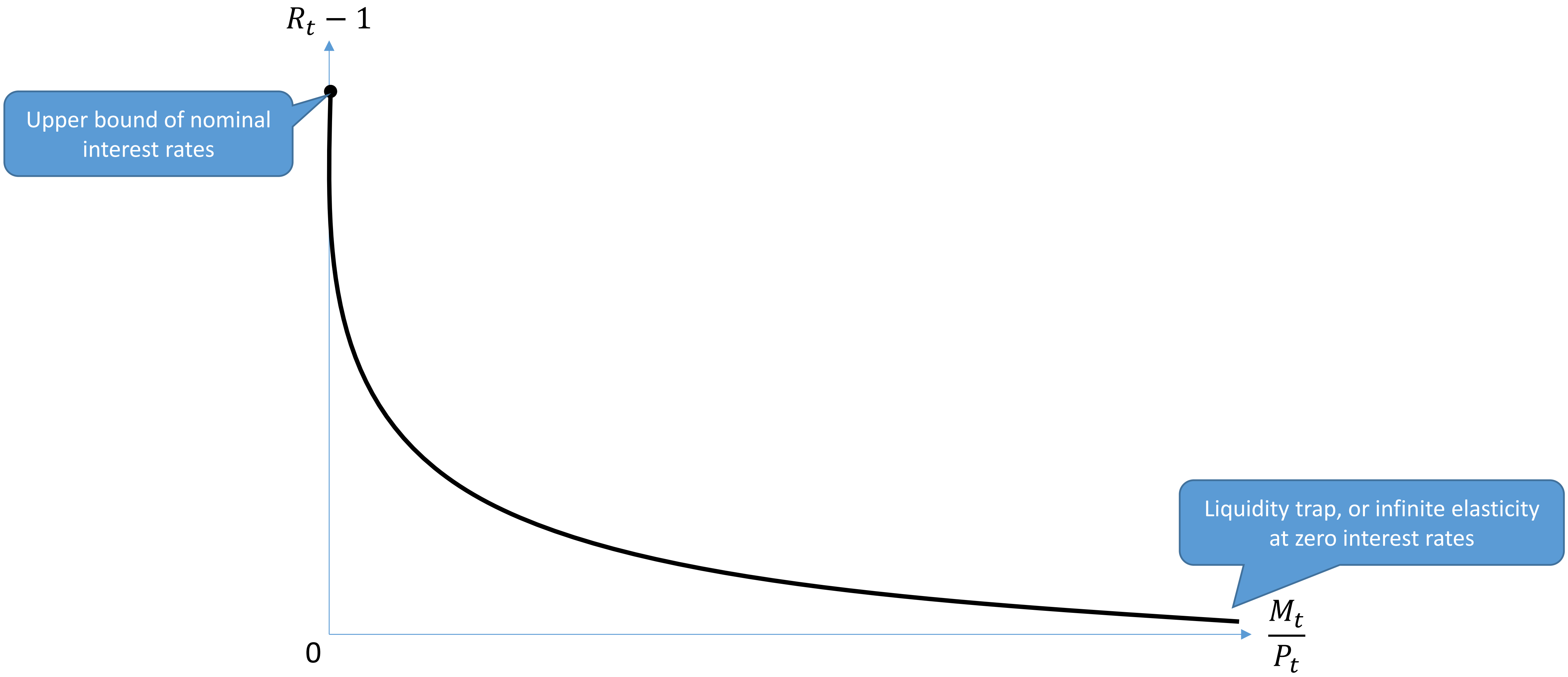
- Functional forms for preferences

$$u(c) = \frac{1}{1-\frac{1}{\sigma}} c^{1-\frac{1}{\sigma}}$$

$$v\left(\frac{M}{P}\right) = \frac{\lambda}{1-\frac{1}{\sigma}} \left(\omega + \frac{M}{P}\right)^{1-\frac{1}{\sigma}}$$

- In this setup with constant consumption ($c_t = y - g$), σ is interpreted as the interest elasticity of money demand.
  - σ may differ between the two specifications.
- Including positive ω in money makes marginal utility finite at $\frac{M}{P} = 0$, and imposing an upper bound of the nominal rate of interest.
  - It is assumed that there is medium of exchange alternative to central bank cash.

Money demand function: An interpretation of σ



Optimality conditions

- FOC's with $c_t = y - g$ (constant consumption).

$$\frac{P_t}{P_{t+1}} = \frac{1}{\beta R_t} = \frac{1}{\beta}\left[1-\lambda\left(1+\frac{M_t}{P_t\left(y-g\right)}\right)^{-\sigma}\right]$$

- Terminal condition

$$\lim_{T\to\infty}\frac{B_T+M_T}{\prod_{s=-1}^{T-1}R_s}+\sum_{\tau=-1}^{\infty}\frac{P_{\tau+1}\left(y-g\right)}{\prod_{s=-1}^{\tau}R_s}=\left(R_{-1}B_{-1}+M_{-1}\right)+\sum_{\tau=-1}^{\infty}\left[\frac{P_{\tau+1}\left(y-\tau_{\tau+1}\right)}{\prod_{s=-1}^{\tau}R_s}-\frac{1}{\prod_{s=-1}^{\tau-1}R_s}\left(1-\frac{1}{R_\tau}\right)M_\tau\right]$$

$$\Rightarrow \lim_{T\to\infty}\frac{B_T+M_T}{\prod_{s=-1}^{T-1}R_s}=0$$

Price dynamics under the QTM

- Prices are determined by monetary policy without any reference to fiscal policy.
  - Prices increase at the rate of monetary growth μ.

$$\frac{P_{t+1}^R}{P_t^R} = 1 + \mu$$

  - Fisher equation holds.

$$R_t = \frac{1+\mu}{\beta}$$

  - The QTM price level (P_t^R) is determined according to the outstanding money stock.

$$P_t^R = \frac{1}{\left[\frac{\lambda(1+\mu)}{1+\mu-\beta}\right]^{\sigma} - 1} \frac{M_t}{y-g}$$

An example of Ricardian fiscal policy with the QTM

- Ricardian fiscal policy ($0 < \gamma < 1$), responsive to the outstanding public bonds.

$$P_t^R(\tau_t - g) = (R_{t-1} - \gamma)B_{t-1} - (M_t - M_{t-1})$$

$$B_t = R_{t-1}B_{t-1} - P_t^R(\tau_t - g) - (M_t - M_{t-1})$$
$$= \gamma B_{t-1} = \gamma^t B_0$$

A continuum of equilibria: Possible hyperinflations

- The initial price: $P_0 > P_0^R$

$$\frac{P_{t+1}}{P_t} \to \frac{\beta}{1-\lambda} > 1+\mu$$

$$\frac{M_t}{P_t} \to 0$$

$$R_t \to \frac{1}{1-\lambda} > \frac{1+\mu}{\beta}$$

- Satisfying terminal condition

$$\lim_{T\to\infty} \frac{\gamma^T B_0 + (1+\mu)^T M_0}{\prod_{s=0}^{T-1} R_s} \le \lim_{T\to\infty} \frac{\gamma^T B_0 + (1+\mu)^T M_0}{\left(\frac{1+\mu}{\beta}\right)^T} = \lim_{T\to\infty} \beta^T \left[\left(\frac{\gamma}{1+\mu}\right)^T B_0 + M_0\right] = 0.$$

A continuum of equilibria: Impossible deflations

- The initial price: $P_0 < P_0^R$

$$\frac{P_{t+1}}{P_t} \to \beta < 1 < 1+\mu$$

$$\frac{M_t}{P_t} \to \infty$$

$$R_t \to 1$$

- Not satisfying terminal condition with monetary expansion ($\mu > 0$)

$$\lim_{T\to\infty} \frac{(1+\mu)^T M_0}{\prod_{s=0}^{T-1} R_s} = \frac{1}{\Lambda(R_0)} \lim_{T\to\infty} (1+\mu)^T M_0 = \infty \quad \text{where} \quad \Lambda(R_0) = \prod_{s=0}^{\infty} R_s > 1$$

Non-Ricardian fiscal policy with the FTPL (1)

- The FTPL works as an instrument to pick up **a particular equilibrium among a continuum of hyperinflationary equilibria** in standard monetary models.

$$P_0^{NR} > P_0^R$$

- Non-Ricardian fiscal policy
  - Fiscal surpluses are irresponsive to the outstanding public debts, and determined in a real term: $\varepsilon > 0$

$$P_t^{NR}\left(\tau_t - g\right) = P_t^{NR}\varepsilon - \left(M_{t+1} - M_t\right)$$

$$B_{t+1} = R_t B_t - P_{t+1}^{NR}\left(\tau_{t+1} - g\right) - \left(M_{t+1} - M_t\right)$$
$$= R_t B_t - P_{t+1}^{NR}\varepsilon.$$

Non-Ricardian policy under the FTPL (2)

- Two interpretations of the government's intertemporal budget constraint.
  - A sort of arbitrage condition for the real valuation of the public bonds.

$$\frac{B_t}{P_t^{NR}} = \beta\left(\varepsilon + \frac{B_{t+1}}{P_{t+1}^{NR}}\right)$$

  - The government's and household's intertemporal budget constraint share the same terminal condition.

$$\frac{B_0}{P_0^{NR}} = \sum_{\tau=0}^{\infty}\beta^{\tau+1}\varepsilon + \lim_{T\to\infty}\beta^T \frac{B_T}{P_T^{NR}} = \frac{\beta\varepsilon}{1-\beta}$$

given

$$\lim_{T\to\infty}\beta^T \frac{B_T}{P_T^{NR}} = \lim_{T\to\infty}\frac{1}{P_0^{NR}}\prod_{s=0}^{T-1}\left(\frac{1}{R_s}\right)B_T = 0$$

Switching from non-Ricardian with the FTPL to Ricardian with the QTM

- With a small annual probability π, the non-Ricardian regime with the FTPL switches back to the Ricardian regime with the QTM.
  - Thus, the economy eventually comes back to the Ricardian economy.

- In the Ricardian regime, the price level is determined according to the money stock.

$$P_t^R = \frac{1}{\left[\frac{\lambda(1+\mu)}{1+\mu-\beta}\right]^{\sigma} - 1} \frac{M_t}{y-g}$$

Equilibrium characterization (1)

- The equilibrium path is deterministic in each regime, but a switching possibility introduces uncertainty into this setup.

- FOC's

$$E_t\left(\frac{P_t^{NR}}{P_{t+1}}\right)=\frac{1}{\beta R_t}=\frac{1}{\beta}\left[1-\lambda\left(1+\frac{M_t}{P_t\left(y-g\right)}\right)^{-\sigma}\right]$$

Equilibrium characterization (2)

- The terminal condition for the household's intertemporal budget constraint (discounting nominal bonds and money by nominal rates)

$$B_0 + M_0 = E_0\left[\sum_{\tau=-1}^{\infty}\frac{P_{\tau+1}\left(\tau_{t+1}-g\right)}{\prod_{s=-1}^{\tau}R_s} + \sum_{\tau=-1}^{\infty}\frac{1}{\prod_{s=-1}^{\tau-1}R_s}\left(1-\frac{1}{R_\tau}\right)M_\tau + \lim_{T\to\infty}\frac{B_T + M_T}{\prod_{s=-1}^{T-1}R_s}\right]$$

$$= \sum_{\tau=0}^{\infty}\frac{\left(1-\pi\right)^{\tau}\pi P_{\tau+1}^{R}\left(\tau_{t+1}-g\right)}{\prod_{s=0}^{\tau}R_s} + \sum_{\tau=0}^{\infty}\frac{\left(1-\pi\right)^{\tau}\pi\left(B_{\tau+1}+M_{\tau+1}\right)}{\prod_{s=0}^{\tau}R_s}$$

$$+\sum_{\tau=-1}^{\infty}\frac{\left(1-\pi\right)^{\tau+1}P_{\tau+1}^{NR}\left(\tau_{t+1}-g\right)}{\prod_{s=-1}^{\tau}R_s} + \sum_{\tau=-1}^{\infty}\frac{\left(1-\pi\right)^{\tau+1}}{\prod_{s=-1}^{\tau-1}R_s}\left(1-\frac{1}{R_\tau}\right)M_\tau + \lim_{T\to\infty}\frac{\left(1-\pi\right)^{T}\left(B_T+M_T\right)}{\prod_{s=-1}^{T-1}R_s}$$

$$\Rightarrow \lim_{T\to\infty}\frac{\left(1-\pi\right)^{T}\left(B_T+M_T\right)}{\prod_{s=-1}^{T-1}R_s} = 0$$

Price dynamics in the non-Ricardian regime

- Depending on the initial price, deflations (inflations) are determined according to

$$E_t\left(\frac{P_t^{NR}}{P_{t+1}}\right)=(1-\pi)\frac{P_t^{NR}}{P_{t+1}^{NR}}+\pi\frac{P_t^{NR}}{P_{t+1}^{R}}$$

$$\frac{P_t^{NR}}{P_{t+1}^{NR}}=\frac{1}{1-\pi}\left[\frac{1}{\beta}\left[1-\lambda\left(1+\frac{M_t}{P_t\left(y-g\right)}\right)^{-\sigma}\right]-\pi\frac{P_t^{NR}}{P_{t+1}^{R}}\right]$$

1. No discontinuity at switching given $P_0^{NR}=P_0^R$. → supported

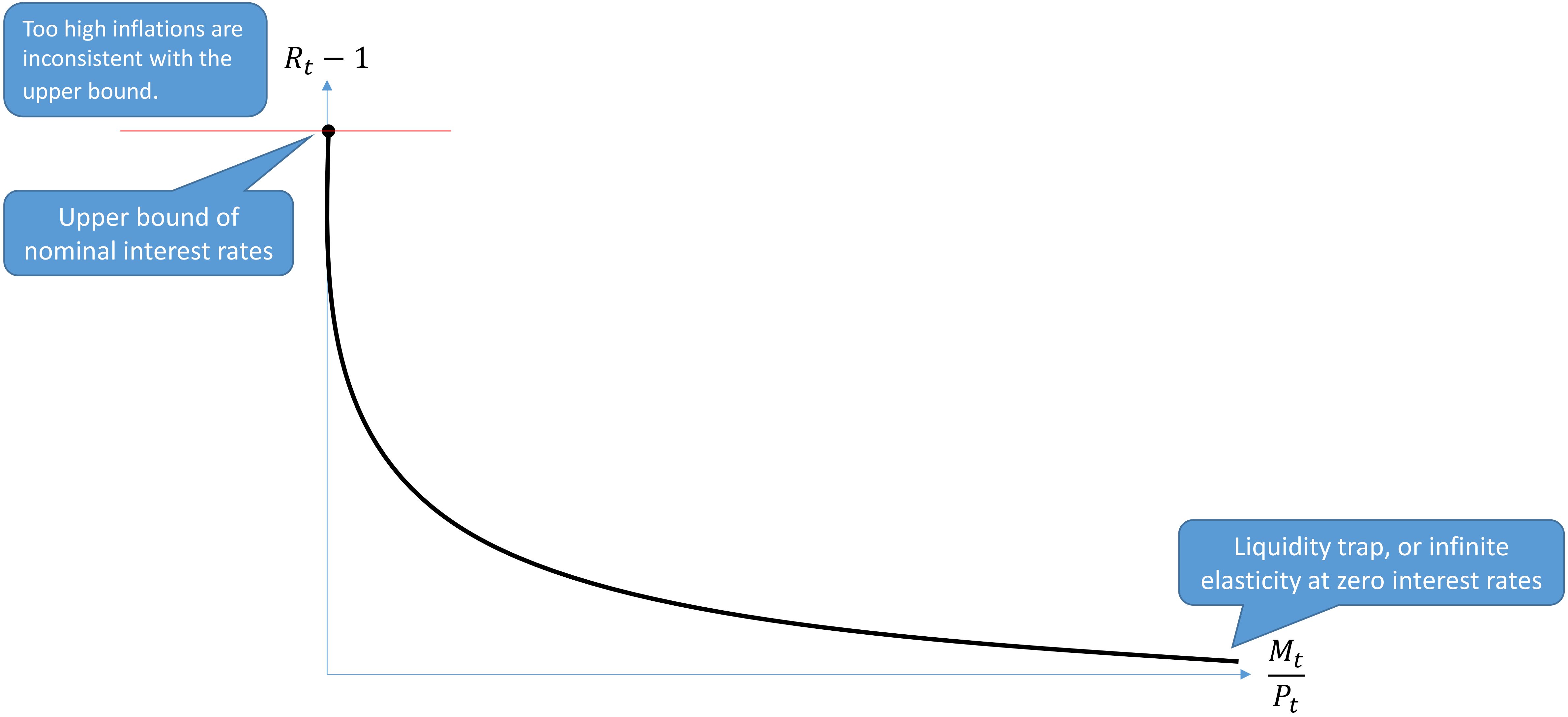
2. At switching, price jumps down given $P_0^{NR}>P_0^R$. → not supported

- In the non-Ricardian regime, too high inflations are inconsistent with the upper bound of the nominal rate of interest.

3. At switching, price jumps up given $P_0^{NR}<P_0^R$. → supported

- never reaching a corner.

Inconsistent with too high inflations in the non-Ricardian regime



Deflationary equilibria with monetary expansion

- The terminal condition associated with the money stock holds in the deflationary environment if $(1+\mu)(1-\pi)<1$, or a switching probability dominates monetary growth. Thus, $\mu>0$ is still possible if $\mu<\frac{\pi}{1-\pi}$.

$$\lim_{T\to\infty}\frac{(1-\pi)^T M_T}{\prod_{s=-1}^{T-1} R_s}\le \lim_{T\to\infty}(1-\pi)^T(1+\mu)^T M_0=0$$

The peso problem: A difference between actual and expected deflations

- A deflationary pressure is created by a rare event of large price jump.

- In the deflationary environment, actual deflations > expected deflations.

$$\frac{P_t^{NR}}{P_{t+1}^{NR}} > E_t\left(\frac{P_t^{NR}}{P_{t+1}}\right)$$

- In the deflationary environment, ex post nominal returns (eventually negative) < nominal interest rates (at least zero).

$$\frac{1}{\beta \frac{P_t^{NR}}{P_{t+1}^{NR}}} < \frac{1}{\beta E_t\left(\frac{P_t^{NR}}{P_{t+1}}\right)} = R_t$$

A sort of arbitrage condition for the real valuation of the public bonds

- The real valuation of the nominal public bonds (discounting real bonds by real rates)

$$\frac{B_t}{P_t^{NR}} = \beta\left[(1-\pi)\frac{B_{t+1}}{P_{t+1}^{NR}} + \pi\frac{B_{t+1}}{P_{t+1}^{R}} + \varepsilon\right]$$

- $\frac{B_t}{P_{t+1}^{NR}}$: gradual appreciation thanks to continuing deflations
- $\frac{B_t}{P_{t+1}^{R}}$: heavy devaluation due to large price jump

- An important note: A non-Ricardian fiscal policy continues one more period after switching.

The government's intertemporal budget constraint in the non-Ricardian regime

- **Black**: backed by non-Ricardian fiscal surpluses
- Blue: backed by a heavy devaluation and Ricardian fiscal surpluses
- Red: backed by stochastic bubbles

$$\frac{B_0}{P_0^{NR}} = \frac{\beta(1-\pi)\varepsilon}{1-\beta(1-\pi)} + \sum_{\tau=0}^{\infty} \beta^{\tau} (1-\pi)^{\tau} \beta\pi \frac{R_\tau B_\tau}{P_{\tau+1}^{R}} + \lim_{T\to\infty} \beta^{T} (1-\pi)^{T} \frac{B_T}{P_T^{NR}}.$$

- Important feature: The initial real valuation is independent of the path of P_t^{NR}.
  - P_t^{R} is determined according to monetary expansion.

Ricardian equivalence in the non-Ricardian regime!

- The initial price P_0^{NR} is independent of non-Ricardian fiscal policy or ε.

$$\frac{B_0}{P_0^{NR}} = \frac{\beta(1-\pi)\varepsilon}{1-\beta(1-\pi)} + \sum_{\tau=0}^{\infty} \beta^{\tau}(1-\pi)^{\tau} \beta\pi \left[\frac{R_{\tau} B_{\tau}}{P_{\tau+1}^{R}} \right] + \lim_{T\to\infty} \beta^{T}(1-\pi)^{T} \frac{B_T}{P_T^{NR}}$$

$$= \sum_{\tau=0}^{\infty} \beta^{\tau}(1-\pi)^{\tau} \beta\pi \left[\frac{R_{\tau} \prod_{s=0}^{\tau-1} R_s B_0}{P_{\tau+1}^{R}} \right] + \lim_{T\to\infty} \beta^{T}(1-\pi)^{T} \frac{\prod_{s=0}^{T-1} R_s B_0}{P_T^{NR}}.$$

  - The initial real value of the public bonds is independent of ε or fiscal surpluses (deficits).
  - With Ricardian equivalence, ε may be negative.
- The initial price is lower to the extent that the stochastic bubbles are larger in the GIBC, but it is not influenced by fiscal policy even in the FTPL environment.

A continuum of deflationary equilibria including pricing by the QTM and possible stochastic bubbles

- With $P_0^{NR} = P_0^{R}$ (the upper bound of the initial price equivalent to the QTM)

$$\lim_{T \to \infty} \beta^T (1-\pi)^T \frac{B_T}{P_T^{NR}} = 0$$

- With $P_0^{NR} < P_0^{R}$ (leading to the deflationary environment)

$$0 < \lim_{T \to \infty} \beta^T (1-\pi)^T \frac{B_T}{P_T^{NR}} < \infty$$

The terminal condition differs between the household's and government's intertemporal budget constraints (IBC)

- Asymmetry between the household's and government's IBC

$$\lim_{T\to\infty}(1-\pi)^T \beta^T \frac{\hat{B}}{P_T^{NR}} > \lim_{T\to\infty}(1-\pi)^T \prod_{s=-1}^{T-1}\left(\frac{1}{R_s}\right)\frac{\tilde{B}_0}{P_0^{NR}} = 0$$

given

$$\lim_{T\to\infty}(1-\pi)^T \prod_{s=0}^{T-1}\left(\beta\frac{P_s^{NR}}{P_{s+1}^{NR}}\right)\frac{B_T}{P_0^{NR}} > \lim_{T\to\infty}(1-\pi)^T \prod_{s=0}^{T-1}\left[\beta E_t\left(\frac{P_s^{NR}}{P_{s+1}^{NR}}\right)\right]\frac{B_T}{P_0^{NR}}$$

- In the deflationary non-Ricardian regime, the government can operate a Ponzi scheme, but the public bonds never serve as net wealth for the household at all.

Tentative summary

- Given a big price jump at switching, deflations and near-zero rates of interest emerge in the non-Ricardian regime.

- In this environment with switching, the FTPL serves as not an equilibrium selection device, but an instrument to create a continuum of deflationary equilibria.

- The stochastic bubbles emerge in the non-Ricardian regime, but they burst at switching.
  - The initial price is lower to the extent that the bubbles are larger in the GIBC, but it is completely independent of non-Ricardian fiscal policy.
  - A government can operate a Ponzi scheme as long as the non-Ricardian regime continues.

Calibration exercises

- It is assumed that in the early 1990s,
  - Fiscal policy switched from Ricardian to non-Ricardian.
  - Fiscal surpluses (ε) were expected to be negative in the non-Ricardian regime.
  - A slightly downward deviation of the 1990 price level from the QTM level by a deflationary shock.
    - $\boldsymbol{P_{1990}^{R} - P_{1990}^{NR} > 0}$ **as only a exogenous disturbance in this model.**
- People believe that the economy will eventually switch back to Ricardian with the QTM.
- The predicted relative positions of money balances and public bonds are matched with the observations.

A set of parameters

- A set of parameters
  - $\beta = 0.99$
  - $\sigma = 0.1, 0.05, \text{and } 0.01$
    - Extremely low interest elasticity is consistent with stable *Marshallian* k under positive interest rates.
  - $\mu = 0.033$ (1980-1989)
  - $\pi = 0.04 \quad (\pi > \mu)$
  - $\varepsilon = -2.9$ (2000-2016) with $y - g = 100$

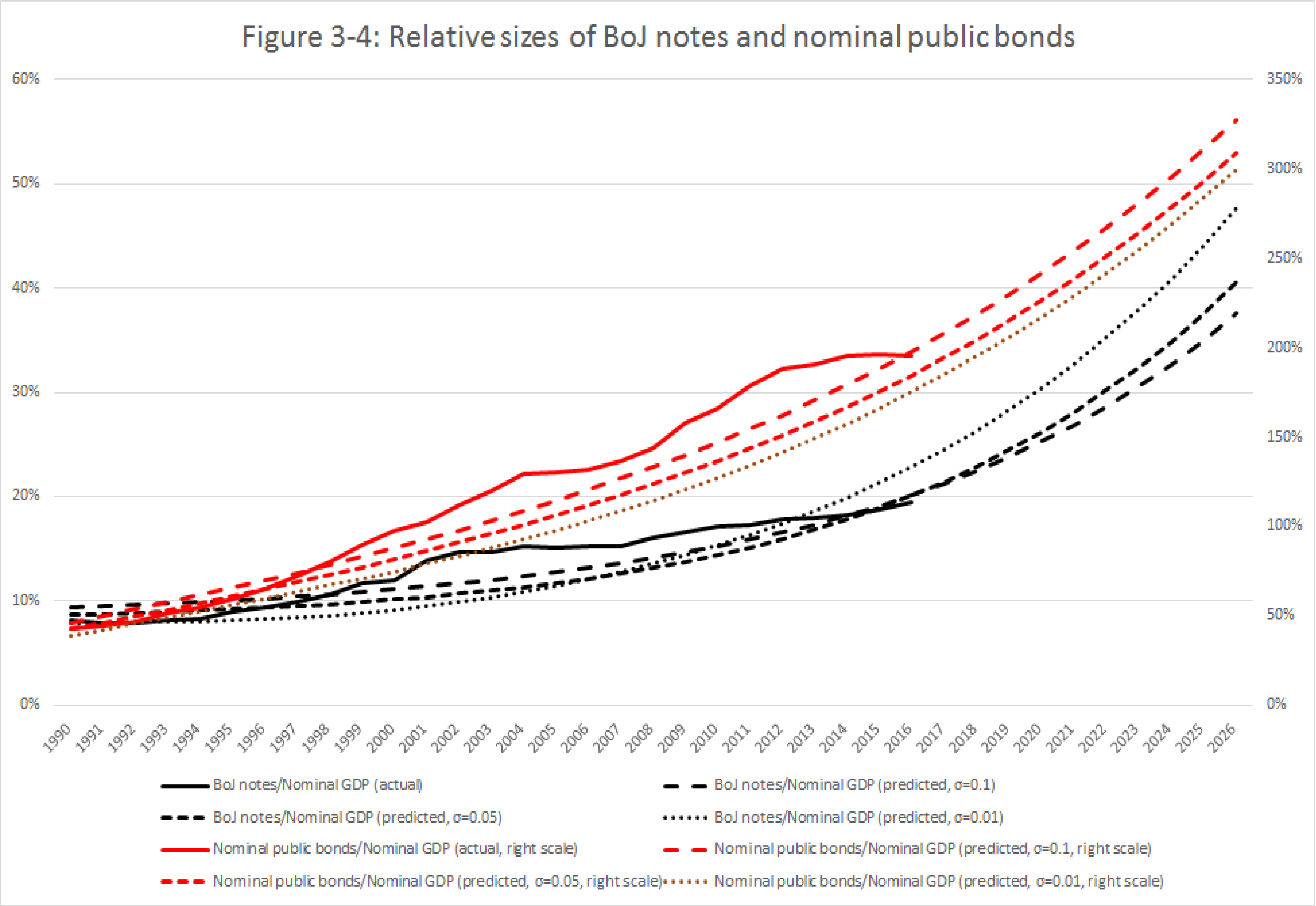
- Given $\kappa = \dfrac{M_t}{P_t^R (y-g)} = 0.078$ (1980-1990),

$$\lambda = (1+\kappa)^{\frac{1}{\sigma}} \frac{1+\mu-\beta}{1+\mu}$$

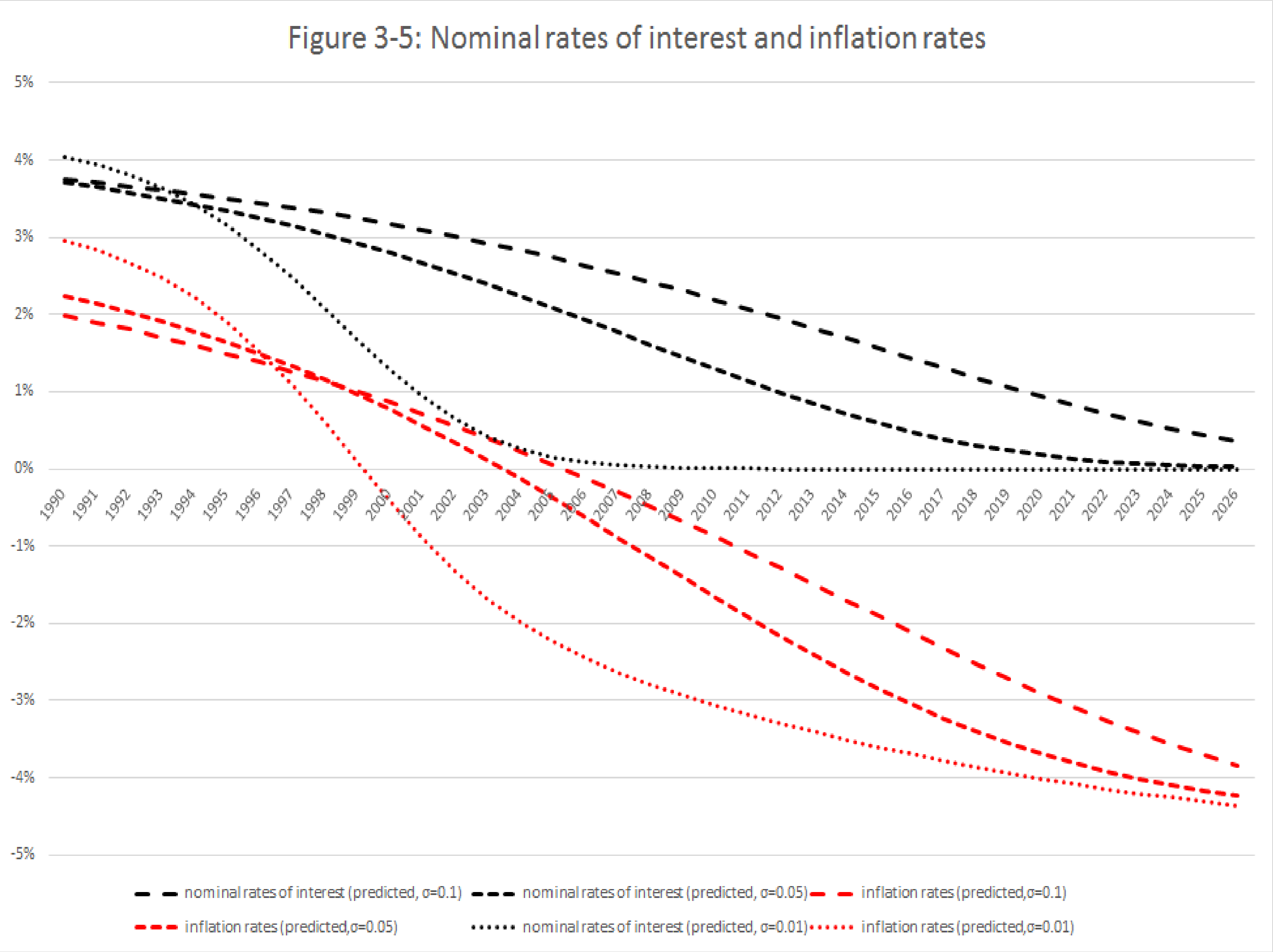
On the initial price of 1990

- To pick up the initial price consistent with the observed relative money stock and public bonds
  - Ricardian
    - $P_{1990}^{R} = 12.8$, given κ = 0.08
  - Non-Ricardian with $P_{1990}^{NR} < P_{1990}^{R}$
    - $P_{1990}^{NR} = 10.7$ for $\sigma = 0.1$
    - $P_{1990}^{NR} = 11.6$ for $\sigma = 0.05$
    - $P_{1990}^{NR} = 12.7$ for $\sigma = 0.01$

Calibration result (1)



Calibration result (2)

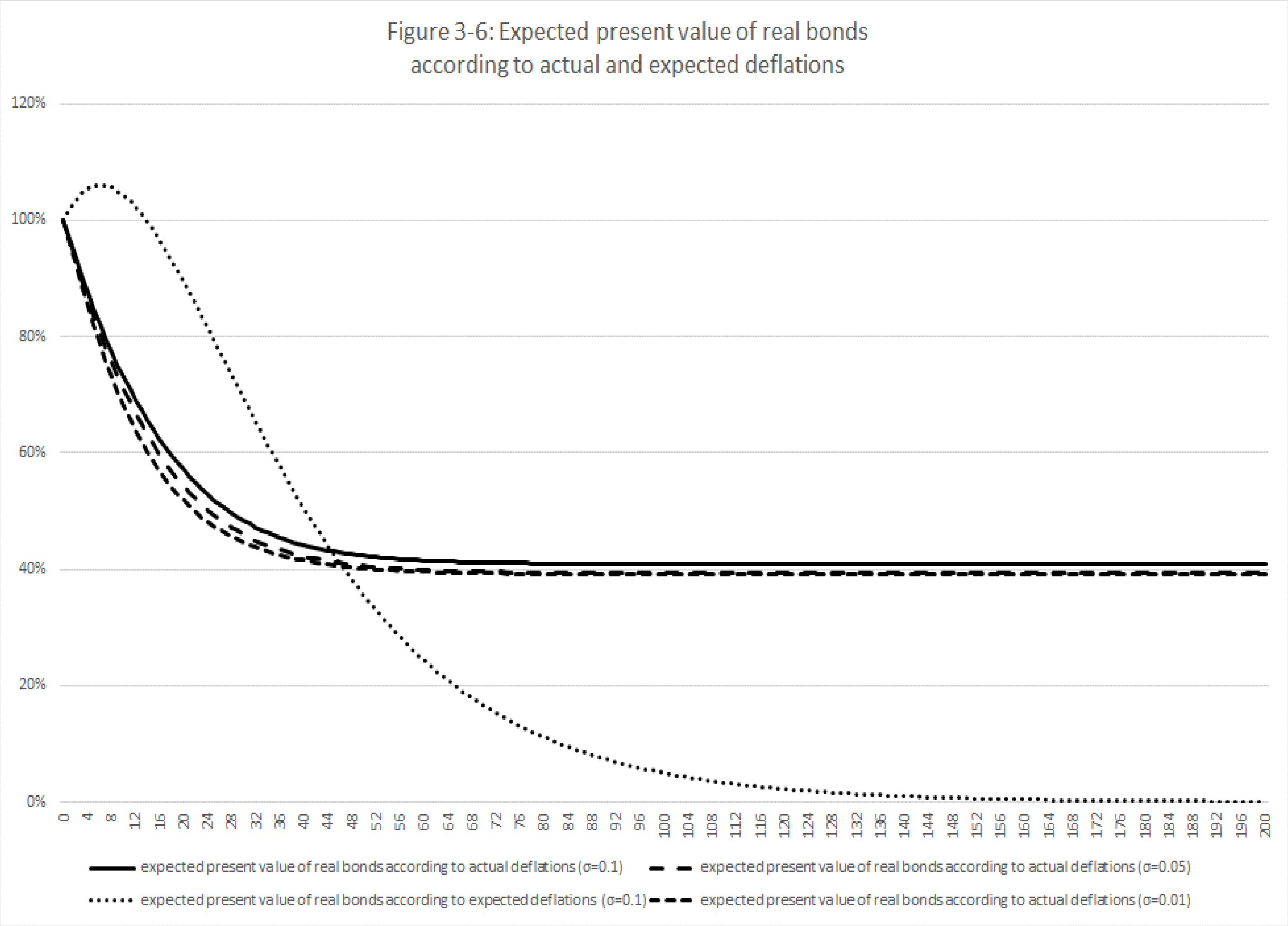


Degree of over-estimation of one-year ahead inflations

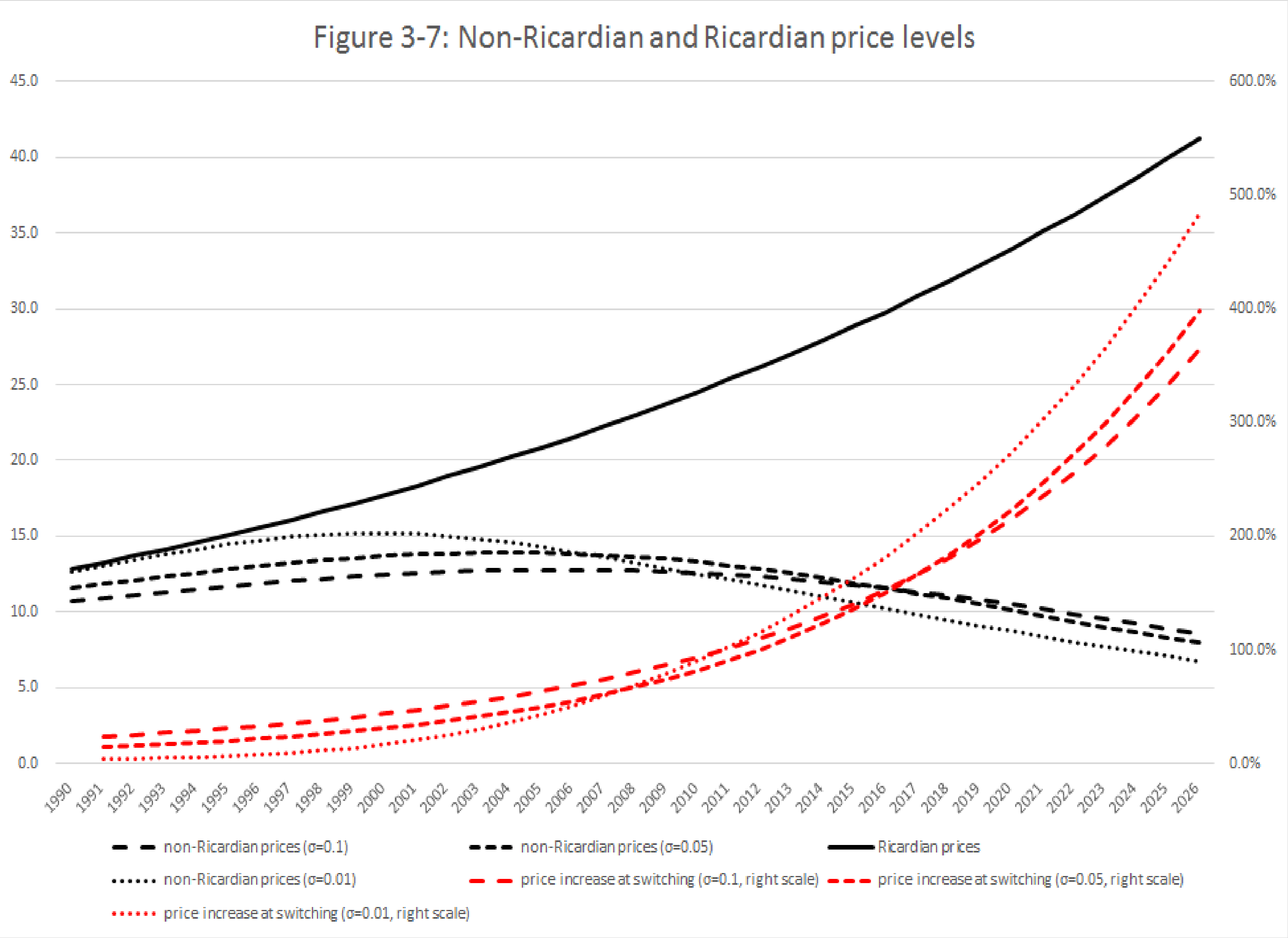
- According to the observation of the years 2010-2017
  - 1.8% in the CCS.
  - 3.5% in the OS.

- According to the calibration,
  - Around 2.4% in either σ.

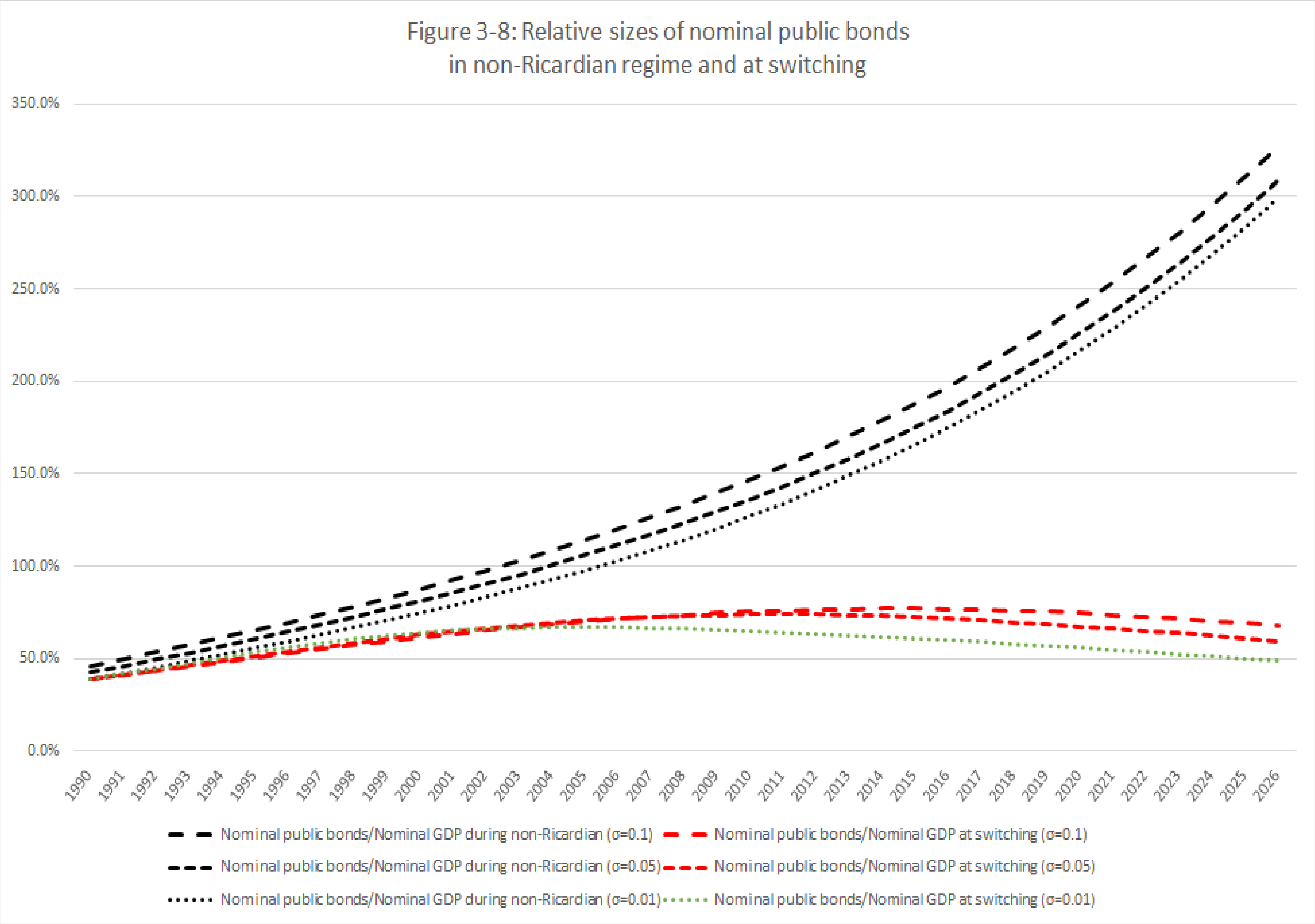
Calibration result (3)



Calibration result (4)



Calibration result (5)



Conclusion

- Deflationary economy despite monetary expansion is possible given a large, one-off increase in the price level at switching in the future.
- High valuation of the public bonds despite continuing primary budget deficits is possible given repayment by a heavy devaluation at switching and by Ricardian fiscal policy after switching.
- Hence, monetary expansion and irresponsible fiscal policy do not help fix a deflationary situation.