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Asset-Price Collapse and Macroeconomic Debt Overhang*

(Incomplete and preliminary)

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Abstract

We provide a tractable two-period model of financial crises that replicates empirical regularities that credit-fueled asset-price booms are often followed by the busts and deep and persistent recessions associated with productivity declines. We argue that the risk-shifting booms of asset prices tend to collapse, and resultant debt overhang lowers productivity and output by discouraging borrowers from expending efforts. This inefficiency is amplified by externality of a decrease in aggregate demand. Larger asset-price booms lead to deeper recessions. Ex-post government intervention to facilitate debt restructuring can be welfare improving, because it mitigates the demand externality and the associated time inconsistency may not have dominant effects.

Key words: Risk shifting, aggregate demand externality, financial crisis.

JEL Classification:

1 Introduction

Recent empirical studies show the following empirical regularities about booms and collapses of asset prices: when the asset-price boom is associated with credit boom or is fueled with an increase of credit supply, the asset boom tends to end up with bust, followed by a deep and persistent recession with lower observed total factor productivity (TFP). See, for example, Jordà, Schularick and Taylor (2015). One purpose of this study is to provide a parsimonious and tractable model to replicate these empirical regularities. In particular, a unique feature of our model is to relate the productivity declines after the asset-price burst to debt overhang. *Debt overhang* in this study indicates the situation that the contractual amount of debt is larger than the repayable amount and the repayment is yet to

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be settled. On the other hand, *default* indicates the situation that the debt repayment has been formally settled and the lender has written off the unpaid amount. In our model, the inefficiency of debt overhang naturally arises after asset-price collapse, because the asset works as an input for production.

Another motivation of this paper is to raise a demand-side perspective on financial crises. Our study is different from the existing literature in three respects. The first is the source of inefficiencies. We focus on debt overhang that discourages borrowers from undertaking new projects and reduces the demand for new credit, while most of the existing studies focus on pecuniary externality due to borrowing constraints and coordination failures like bank runs. The second point is the amplification mechanism of inefficiency. In our model, debt overhang discourages production activities and resultant demand shortage amplifies this inefficiency further. In contrast, the inefficiencies of financial crises in the existing literature are amplified primarily by an increase in the cost of credit or shortage of supply, not demand, of liquidity due to financial frictions (i.e., credit crunch). The third point is the policy interventions. Ex-post policy such as government subsidy to banks to facilitate debt restructuring is not emphasized in the recent literature, which rather pays marked attention to time inconsistency that arises from the ex-post bailouts. However, the ex-post debt restructuring policy was obviously of crucial importance in resolving financial crises such as Global Financial Crisis (GFC) in 2008 or the 1990s in Japan. In this study we make the point that ex-post debt restructuring is welfare improving as it mitigates the aggregate demand externality and that time inconsistency may not have dominant effects under some circumstances.

What we do in this paper are the following. We construct a simple two-period model, in which we unify the model of risk-shifting booms of asset prices (Allen and Gale 2000; Allen, Barlevy and Gale 2022) and the model of macroeconomic debt overhang due to spillover effect through aggregate demand (Lamont 1995), that can explain the productivity declines subsequent to the asset-price collapse. The key ingredient that enables unification of the two theories is our assumption that the risky asset, the price of which can be driven up by risk-shifting, is also used as an input for production by borrowers who potentially suffer from debt overhang. We show that when A_H, the parameter representing the degree of optimistic expectations, is small, the price of the asset is low and there are no debt overhang and no recession in equilibrium. We call this situation the Normal Equilibrium (NE). When A_H is large, there emerges the Debt Overhang Equilibrium (DOE) where the asset price is initially higher, and then it collapses if the productivity of the asset turns out to be low. The asset-price collapse is followed by recession due to debt overhang.

In the DOE, the asset price is driven up by investors who buy the asset by borrowed money. The borrowers bid up the asset price because they can push the cost on the lenders by defaulting on the debt, when the productivity of the asset turns out to be not high enough. This is the risk-shifting boom of asset prices (Allen and Gale 2000; Allen, Barlevy and Gale 2022). Since the asset price is driven up excessively by the borrowing investors, it is quite likely to collapse. The asset-price collapse generates debt overhang and TFP declines disproportionately because debt overhang discourages borrowing investors from expending efforts. They are discouraged because the lenders cannot commit to reward their effort as the lenders have the legitimate right to take all as long as debt is larger than the borrowers income (Kobayashi, Nakajima and Takahashi 2022). In addition to the debt overhang due to the lack of lenders' commitment, there exist a spillover effect through shrinkage of aggregate demand in our economy of the monopolistic competition, which we call the *aggregate demand externality.* We define the aggregate demand externality as the effect of an exit (or entry) of one firm that decreases (or increases) the other firms' revenues by reducing (or increasing) the aggregate demand. In our model, this aggregate demand externality is generated from the "love-for-variety" nature in the Dixit-Stiglitz market of the monopolistic competition (see Section 2.2 for the details). This externality discourages a firm from continuing production when some other firms exit due to debt overhang. This aggregate demand externality causes declines of macroeconomic productivity.[1] It is also shown that a larger asset-price boom may lead to a deeper recession: When the asset-price boom is larger in the first period, the resulting debt overhang due to the asset-price collapse becomes larger, leading to a larger number of exiting firms, which implies a lower aggregate productivity, as the TFP in the monopolistic competition is decreasing in the number of exiting firms.

This study provides a simple policy implication: A policy intervention to facilitate restructuring of debt overhang may increase the recovery of debt for lenders and also improve productivity and social welfare. The result that the lenders are better off by reducing the face value of debt is the same as the classical argument of debt overhang or the debt Laffer curve (Sachs 1988; Krugman 1988), which is about the sovereign debt, while our focus in this paper is on private debt. As argued by Sachs (1988) and Krugman (1988), lenders may know that restructuring of debt overhang increases their payoff, and reduce debt on their own. However, because there exists the aggregate demand externality in our setup, the amount of debt reduction is smaller than the socially optimal level in our model. Therefore, a policy intervention to facilitate debt reduction is welfare improving. To facilitate debt restructuring, the government can subsidize the lenders to partially compensate the loss of debt write-off so that the optimal amount of debt reduction is

[1] Lamont (1995) argue that the investment is reduced by the macroeconomic debt overhang due to the spillover effect, that is similar to the aggregate demand externality in our model. The difference is the following: in Lamont's model, the spillover effect discourages the investment and does not change productivity because there is no exit of firms in his model, while the demand externality causes endogenous productivity declines in our model because firms can exit the market.

realized. Our result that debt forgiveness improves productivity of the borrowers can be seen as complementary to that of Caballero, Hoshi and Kashyap (2008). They stress that zombie firms with debt overhang should be intrinsically inefficient and should be liquidated. Our result points to the possibility that there may exist zombie firms that can become productive if their debts are forgiven. We also show that ex-post policy to facilitate debt restructuring does not necessarily distort ex-ante incentives, that is, the time-inconsistency problem may not arise when the ex-post policy is subsidy to the lenders while the ex-ante allocation is decided by the borrowers.

The subsidy for banks of debt restructuring may be financed by the government debt or tax. The debt restructuring with subsidy to the lenders can be interpreted as bank recapitalization usually observed as a policy response to a financial crisis. The bank subsidy to facilitate debt restructuring is a kind of fiscal policy that is consistent with the recommendations of active fiscal policy in the low interest rate environment after the Great Recession (see Blanchard 2019).

1.1 Literature

Empirical regularities: There are a large empirical literature that report empirical regularities concerning asset-price and credit booms and their effects on the subsequent economic growth. Most noteworthy is Jordà, Schularick, and Taylor (2015), who analyze data of 17 countries for the past 140 years and show that the asset-price booms fueled by credit booms tend to end up with financial crisis, followed by deep and persistent recession. Greenwood, Hanson, Shleifer and Sørensen (2021) also report that a rapid growth in private credit and asset prices predicts a financial crisis.

There is a literature that credit booms alone can be problematic for economic performance. Schularick and Taylor (2012) analyze data on 14 countries for 140 years and report that credit booms tend to lead to financial crises. Giroud and Mueller (2021) also report that a buildup in firm leverage is associated with a boom-bust in employment. It is also shown that credit deepening in the long-run and credit booms in the short-run have opposite effects on economic growth: Credit deepening leads to higher long-term economic growth (King and Levine 1993), while Verner (2019) reports based on the data of 143 countries for 60 years that credit booms in the short-run are usually driven by credit-supply expansion and lead to financial crises.[2] Justiniano, Primiceri and Tambalotti (2019) also argue that the empirical facts about the housing boom preceding the Great Recession are consistent with the explanation that the boom was caused by an increase in credit supply, not in credit demand. Adverse effect of credit supply shock is also reported by Mian, Sufi

[2]Easlerly, Islam and Stiglitz (2000) report a nonlinearity in the relationship between credit and economic performance. They show that the GDP volatility decreases and then increases with an increase in the private credit.

and Verner (2017). They show that a credit supply shock induces a decrease in the interest rate and an increase in household debt with consumption boom, followed by persistently lower GDP growth. There are studies that point to distinction between good credit booms with high economic growth and bad credit booms with low growth (Gorton and Ordoñez 2020). Müller and Verner (2023) report, based on the data of 116 countries for 80 years, that bad credit booms are mostly debt booms in non-tradable sector.

It is also well known that financial crises tend to be followed by persistent productivity slowdown. Duval, Hong and Timmer (2020) argue that financial frictions might have caused the great productivity slowdown during the Great Recession. Adler et al. (2017) report that productivity growth fell sharply after the GFC.[3] Related literature is on the great depressions, a decade-long deep recessions observed in the 20th century. It is said that deep and persistent productivity declines are the major cause of the great depressions (Hayashi and Prescott 2002, Kehoe and Prescott 2002). Caballero, Hoshi and Kashyap (2008) argue that zombie lending causes the lower productivity because intrinsically inefficient firms survive thanks to the zombie lending. Nakamura and Fukuda (2013) report that significant portion of zombie firms in non-tradable sector that had difficulties in repaying debt in the 1990s have recovered and become productive in the 2000s, implying that debt-ridden zombie firms may not have been intrinsically unproductive. This implication is consistent with our theory that the productivity of a firm can increase as debt overhang is forgiven.

Risk-shifting effect on asset prices: This study is related to the literature on risk-shifting booms of asset prices, which are theoretically analyzed by Allen and Gale (2000) and Allen, Barlevy and Gale (2022). They demonstrate that asset-price booms can be driven by risk shifting by investors who buy the asset with borrowed money. In their models, the cost of default is exogenous and no policy response is possible ex-post, whereas in our model the ex-post debt reduction can reduce the inefficiency. Our theory is also related to Biswas, Hanson and Phan (2020), in which a collapse of the asset-price bubble brings a persistent recession, which is aggravated by the nominal wage rigidity, but there is no role for ex-post policy intervention in their model either.

Debt overhang: In our theory, the asset, i.e., capital stock, is used as an input for production, and the production becomes inefficient due to debt overhang, which is caused by the asset-price collapses. Thus, our study is related to the broad literature of debt overhang. As Kobayashi, Nakajima and Takahashi (2023) argue, debt overhang can be categorized into two types. The first type of debt overhang is due to the lack of borrowers'

[3]There is an opposite view that labor productivity increased in GFC. See Lazear, Shaw and Stanton (2013) who argue that people tend to work harder during recessions.

commitment, and the second type is due to the lack of lenders' commitment. The debt overhang in this paper is the second type. The first type of debt overhang is analyzed by, e.g., Albuquerque and Hopenhayn (2004), Kovrijnykh and Szentes (2007), and Aguiar, Amador, and Gopinath (2009). In these models, the inefficiency is generated from the lenders' offer of back-loading payoff schedule to the borrowers in order to prevent the borrowers' default at the early stage. The second type of debt overhang is argued by Sachs (1988), Krugman (1988), and Kobayashi, Nakajima, and Takahashi (2023). In the second type, the inefficiency arises because borrowers choose not to expend effort because the lenders cannot commit to reward their effort. The lack of lenders' commitment is caused by the fact that the lenders have legitimate right to take all when the amount of debt is larger than the borrowers' revenues. In this case, the lenders cannot credibly commit to give positive amounts to the borrowers to reward their efforts.

Aggregate demand externality: Korinek and Simsek (2016) and Farhi and Werning (2016) posit theories of financial crises followed by recessions due to the "aggregate demand externality," which is different from our definition. In their models, financial crises give rise to inefficient redistribution under the zero lower bound and nominal rigidities that transfer wealth from borrowers to savers who have less marginal propensity to consume (MPC). The aggregate demand externality in their models are the externality that the combination of the zero lower bound and nominal rigidity reduces the aggregate demand through transfer to less MPC agents. The aggregate demand externality is due to nominal rigidities in their models, whereas it is due to the monopolistic competition and debt overhang in our model. The similar externality as our aggregate demand externality is argued in Lamont (1995), though there is no exit of firms in Lamont's model, whereas exits of firms endogenously lower the productivity in our model. See also Blanchard and Kiyotaki (1987) for similar spillover effect as our aggregate demand externality.

Theoretical studies on financial crises and policy responses: This paper is related to the vast literature on financial crises and corresponding policy interventions. We can clarify the difference of our model from the existing studies in three aspects: The source of inefficiencies, the nature of inefficiencies, and the relationship between ex-ante and ex-post policy interventions. First, concerning the source of inefficiency, the literature primarily focus on pecuniary externality due to borrowing constraints (Aguiar and Amador 2011; Benigno et al. 2023; Bianchi 2011, 2016; Bianchi and Mendoza 2010; Farhi, Golosov, and Tsyvinski 2009; Gertler, Kiyotaki, and Queralto 2012; Lorenzoni 2008; Lorenzoni and Werning 2019) or coordination failure such as bank runs (Diamond and Dybvig 1983; Gertler and Kiyotaki 2015; Keister 2016; Keister and Narasiman 2016). On the other hand, the source of inefficiency in our model is debt overhang, which can emerge from various reasons such as news shocks, asset bubbles and overconfidence, even if pecuniary externality or coordination failure are nonexistent. Second, concerning the nature of inefficiencies, many existing models feature allocative inefficiencies in consumption allocations (Bianchi 2011; Chari and Kehoe 2016; Farhi, Golosov, and Tsyvinski 2009; Jeanne and Korinek 2020; Keister 2016) or inefficient production due to increases in the cost of credit, that is, the credit crunch (Bianchi 2016; Bianchi and Mendoza 2010; Gertler, Kiyotaki, and Queralto 2012; Lorenzoni 2008). In contrast to them, our model features inefficient production due to shortage of the aggregate demand and the demand for credit. Third, concerning the policy interventions, the existing literature primarily focus on the trade-off that the bailout policy induces between ex-ante incentive and ex-post efficiency, that is, the time inconsistency (Bianchi 2016; Chari and Kehoe 2016; Green 2010; Keister 2016: Keister and Narasiman 2016). Chari and Kehoe (2016) argues that bailouts can be welfare reducing because of the time inconsistency, while Bianchi (2016), Green (2010), Keister (2016), and Keister and Narasiman (2016) make the case that welfare improving effects of bailout policies overwhelm the adverse effects of time inconsistency. It is shown in our model that the time inconsistency of ex-post policy disappears and only welfare-improving effects survive under some circumstances where ex-post policy is subsidy to lenders and ex-ante allocation is decided by borrowers.

2 Model

The model is a two-period closed economy, where households and firms are inhabited. In period 1, firms buy capital from households on credit, that is, they promise to pay consumer goods to households in period 2 in exchange for receiving capital in period 1. Firms install capital for specialization though its productivity, which is an aggregate shock, has not been revealed yet. In period 2, the productivity of capital is revealed. After the productivity is revealed, the lending households have a chance to reduce the borrowing firms' debt, given that the firms can choose to exit the market and default on the restructured debt. The production and consumption take place only in period 2.

2.1 Setup

There are two periods, period 1 and period 2, in the economy. There inhabits a unit mass of identical households and each household owns a firm. Thus, the measure of the firms is also unity. The firms can produce consumer goods from capital only in period 2, and the households can consume the goods only in period 2. Each household is endowed with K units of capital at the beginning of period 1. The total amount of capital in the economy is thus K. Firms can produce consumer goods from capital, while households cannot produce anything. In period 1, firms choose the amount of capital, k, where $k \leq K$, to use for production in S-sector which is explained below shortly. The amount k is endogenously determined in equilibrium. Each firm has to buy k from (another) household and install k in period 1 to prepare for specialized production in period 2. As firms have nothing to pay for k in period 1, they issue debt D to buy k. That is, a firm purchases k units of capital from a household in exchange for a promise to pay $D \equiv Qk$ units of period-2 consumer goods to the household, where Q is the price of capital in terms of period-2 consumer good. We simply posit that debt contract is the optimal contract in this economy, in which it is implicitly assumed that there exist asymmetric information and agency problems a lá Townsend (1979) or Gale and Helwig (1985).

Production technologies: Initially in period 1, all firms are in S-sector, which stands for "Specialized production." They install capital in period 1 for production in period 2. In period 2, lending households can reduce debt D to $\hat{D}$ ($\leq D$) using a costly financial technology (see the paragraph titled "financial technology"). Then, the firms can choose whether to produce output in S-sector or to exit S-sector. The exited firms move to C-sector, which stands for "Common production." After producing output in S- or C-sector, the firms repay $\hat{D}$ if revenues are larger than $\hat{D}$. If revenues are smaller than $\hat{D}$, they repay all revenues to the lenders and default on the remaining debt.

- **S-sector:** In S-sector, each firm produces specialized intermediate goods in the monopolistically competitive market. Productivity parameter in S-sector, A_s, is common for all firms. A_s is stochastic and revealed at the beginning of period 2. There are two states $s \in \{M, H\}$ in period 2. The state s becomes $s = H$, where $A_s = A_H$, with probability p_H, and becomes $s = M$, where $A_s = A_M$, with probability $p_M = 1 - p_H$. We consider the case where $A_M \ll A_H$ and $p_H \ll 1$. The state M is the medium or "normal" state, whereas H is the high or "good" state. Given the realization of A_s in period 2, firm i, where $i \in [0, 1]$, produces the intermediate goods

$$y_i = A_s k_i,$$

where k_i is the amount of capital that firm i installed in period 1. To use k_i for production in S-sector, firm i must install k_i in period 1, and no more capital can be added in period 2. The consumption goods Y_S is produced from the intermediate goods y_i by the Dixit-Stiglitz aggregator:

$$Y_S = \left(\int_0^n y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

where $n \in [0, 1]$ is the number of remaining firms in S-sector, which is endogenously decided as a result of firms' choice of exit at the beginning of period 2. The firms who exit S-sector goes to C-sector.

- **C-sector:** In C-sector, a firm can produce A_L units of consumption goods from one unit of capital. The firms need not install capital in period 1 for production in C-sector. Households can sell capital in period 2 for the use of C-sector, or firms in S-sector can move to C-sector and can use their capital k for production in C-sector in period 2, although they were installed in period 1 for production in S-sector. Productivity parameter in C-sector, A_L, is deterministic and satisfies

  $$0 < A_L < A_M \ll A_H.$$

  Firm i who enters C-sector at the beginning of period 2 can produce $A_L k_i$ units of the consumption goods. C-sector is a perfectly competitive market and firms do not have monopoly power there. In the symmetric equilibrium where $k_i = k$ for all i, the total output in C-sector, Y_C, is given by

  $$Y_C = A_L(K - nk),$$

  where n is the number of S-sector firms, k is the amount of capital per one S-sector firm, and thus nk is the total amount of capital used in S-sector.

- **Utility cost:** Concerning specialized production in S-sector and simple production in C-sector, we assume the following assumption:

  Assumption 1. The firm needs to expend an infinitesimally small utility cost in period 2 when it produces output in S-sector, while there is no utility cost in C-sector. The consumption equivalence of the infinitesimal utility cost of S-sector is ε, where

  $$0 < \varepsilon \ll A_L K.$$

Total consumption in the economy, Y, is given by

$$Y = Y_S + Y_C.$$

Financial technology to restructure debt: In period 2, after the state s and the aggregate productivity in S-sector A_s are revealed and before production takes place, the lending households are given a chance to reduce debt. When a lender i ($\in [0, 1]$) reduces the debt from D to $\hat{D}$ ($\leq D$), where they are measured in terms of period-2 consumer goods, she has to pay the dead-weight cost:

$$z_i(D - \hat{D})^\phi,$$

in terms of the period-2 consumer goods, where $\phi \geq 1$ and the cost parameter z_i distributes over $[0, z_{\max}]$ with the cumulative distribution function $F(z)$ and the density function $f(z) = F'(z)$. For simplicity of the analysis, we assume that z_i is revealed in period 2, and all lenders have the identical expectations $\Pr(z_i \leq z) = F(z)$ in period 1 about their own z_i.

We solve the model backward.

2.2 Decision making in period 2

In the previous period (period 1), capital stock of each firm k and the debt for each firm $D = Qk$ were already determined. In period 2, the debt D is restructured to $\hat{D}_i$ by lender $i \in [0,1]$ and the borrowing firm i decide whether to exit. We use the same subscript for a lender and her borrower. What is to be determined in period 2 is the amount of restructured debt $\hat{D}_i$ for $0 \leq i \leq 1$ and the number of continuing firms in S-sector, n.

We assume a standard Dixit-Stiglitz monopolistic competition as the market structure for S-sector. The demand function for firm i's good is given as the solution to $\max_{y_i} Y_S - \int_0^n p_i y_i di$, where $Y_S = \left(\int_0^n y_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}$ and p_i is the price of the intermediate good i. The first order condition (FOC) implies

$$p = Y_S^{\frac{1}{\sigma}} y^{-\frac{1}{\sigma}}.$$

In a symmetric equilibrium where each firm uses the identical amount of capital $k_i = \bar{k}$, where $\bar{k}$ is the social level of capital, the aggregate demand in S-sector is given by

$$Y_S = n^{\frac{\sigma}{\sigma-1}} A_s \bar{k},$$

where $n^{\frac{\sigma}{\sigma-1}} A_s$ is the total factor productivity (TFP) in S-sector, which is increasing in n. Revenue of a firm in S-sector is

$$py = Y_S^{\frac{1}{\sigma}} y^{\frac{\sigma-1}{\sigma}} = n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{\frac{\sigma-1}{\sigma}} \equiv \pi(n, A_s, k).$$

In equilibrium where $k = \bar{k}$, the revenue is $\pi = n^{\frac{1}{\sigma-1}} A_s \bar{k}$.

Firms' exit decision: Given that the debt is restructured to $\hat{D}_i$, the Free Entry Condition (FEC) for firm i who choose whether to continue operations in S-sector or to exit can be written as

$$\pi(n, A_s, k) - \hat{D}_i \geq \varepsilon. \tag{1}$$

The firm continues to operate in S-sector and repay $\hat{D}_i$ if (1) is satisfied. If (1) is not satisfied, the firm is said to have debt overhang. The firm with debt overhang has two options, i.e., either to earn $\pi(n, A_s, k)$ in S-sector and repay $\min\{\pi(n, A_s, k),\ \hat{D}_i\}$ to the lender, or to move to C-sector to produce $A_L k$ $(< \hat{D}_i)$ units of consumer good and repay them to the lender. Since $\pi - \hat{D}_i < \varepsilon$, the firm obtains less than ε if it remains in S-sector.

Assumption 1 implies that the firm with debt overhang exits S-sector and goes to C-sector. In sum, the revenue of firm i can be given by $y(\hat{D}_i)$, where

$$y(\hat{D}_i) = \begin{cases} \pi(n, A_s, k) & \text{if} \quad \pi(n, A_s, k) \geq \hat{D}_i + \varepsilon, \\ A_L k & \text{if} \quad \pi(n, A_s, k) < \hat{D}_i + \varepsilon. \end{cases} \tag{2}$$

Let $N(n)$ is the measure of firms who satisfies (1), where n is given in $\pi(n, A_s, k)$. In equilibrium, the rational expectations, i.e., $N(n) = n$ must holds. Since $N(n) = n$ may have multiple solutions, the equilibrium values of n can be multiple. For example, $n = 0$ is always an equilibrium value, as $N(0) = 0$ because $\pi(0, A_s, k) = 0 < \hat{D}_i + \varepsilon$ for any $\hat{D}_i \geq 0$. We make the following assumption that agents are optimistic to eliminate the possibility of multiple equilibria due to pure coordination failure of expectations.

Assumption 2. When there exist multiple values of n, which satisfies $N(n) = n$, the expectations of households and firms are coordinated such that the largest value of n prevails as the commonly-held expectation in equilibrium.

This assumption says that the macroeconomic expectations are coordinated to be the most optimistic one among all feasible expectations.

Lenders' decision on debt restructuring: Taking n as given and anticipating firms' exit decision, the lender i solves the following debt restructuring problem to maximize her profit.

$$\max_{\hat{D}} \left[\min\{\hat{D}, y(\hat{D})\} - z_i(D - \hat{D})^{\phi} \right], \quad \text{s.t.} \quad \hat{D} \leq D. \tag{3}$$

The solution is given explicitly, as follows. Here we use π as the abbreviation of $\pi(n, A_s, k)$ flexibly. If $D \leq \pi(n, A_s, k) - \varepsilon$, then the lender chooses $\hat{D} = D$, and the firm earns $\pi(n, A_s, k)$ and repay D. In the case where $D > \pi(n, A_s, k) - \varepsilon$, consider the lender i whose z_i satisfies

$$\pi - \varepsilon - z_i(D - \pi + \varepsilon)^{\phi} \geq A_L k, \tag{4}$$

which is rewritten as

$$z_i \leq \frac{\pi - \varepsilon - A_L k}{(D - \pi + \varepsilon)^{\phi}}. \tag{5}$$

This lender i restructures the debt to $\hat{D} = \pi - \varepsilon$, and the firm i earns π to repay $\pi - \varepsilon$ to the lender. The lender with z_i that is larger than $\frac{\pi - \varepsilon - A_L k}{(D - \pi + \varepsilon)^{\phi}}$ does not restructure the debt, i.e., $\hat{D} = D$, and the firm goes to C-sector to earn $A_L k$ and repay all $A_L k$ to the lender. In sum, we have proven the following lemma.

Lemma 1. *When $D > \pi - \varepsilon$, the lenders choose $\hat{D}$ such that the borrowing firms obtain nothing (except for the compensation of utility cost ε).*

Debt overhang effect: Firm i's decision to exit S-sector when $\hat{D}_i$ is large is inefficient. This is because the exiting firm's capital cannot be used efficiently in S-sector with productivity A_H or A_M, but is used inefficiently in C-sector with productivity A_L. This individual inefficiency for an exiting firm can be called debt overhang effect, which is the inefficiency caused by the lack of commitment by the lenders in the following sense (Kobayashi, Nakajima and Takahashi 2022): When $\pi - \hat{D} \leq \varepsilon$, the firm would have chosen to continue operations in S-sector if the lender could promised to give ε to the firm to compensate the utility cost which is defined in Assumption 1; but, the lender cannot commit to give ε because the lender has the legitimate right to take $\hat{D}$ and leave $\pi - \hat{D}$ to the borrower, which is less than ε. The borrower precisely expects that the lender will leave less than ε, and chooses to exit S-sector to save the utility cost ε.

Aggregate demand externality: In addition to the inefficient use of capital for the exiting firm itself, the exit of the firm has a negative externality on the other firms. The exit of one firm reduces the other firms' expected revenues of operating in S-sector by reducing the aggregate demand Y_S, because the revenue of a firm π depends on Y_S: $\pi = py = Y_S^{\frac{1}{\sigma}} y^{\frac{\sigma-1}{\sigma}}$. Since $Y_S = n^{\frac{\sigma}{\sigma-1}} A_s \bar{k}$, we can also rephrase this result as debt overhang decreases the TFP of S-sector, $n^{\frac{\sigma}{\sigma-1}} A_s$, by decreasing the equilibrium value of n. As this negative effect works through reducing the aggregate demand Y_S, we call it the aggregate demand externality in this paper. This aggregate demand externality is similar to the spillover effect that Lamont (1995) pointed out in his argument of macroeconomic debt overhang.

2.3 Decision making in period 1

Firms promise to pay $D(k) = Qk$ units of consumer goods in period 2 in exchange for receiving k in period 1. The firms install k in period 1 for specialized production in period 2. There are two unknowns in period 1: k and Q, which are given by two conditions: the FOC with respect to k for the maximization of the firms' expected profit, and the participation condition (PC) for households' selling capital.

Borrower's problem: Firms know that the lenders' decision making in period 2 implies that a firm obtains zero if $\pi(n, A_s, k) - D(k) < \varepsilon$ in period 2, as shown in Lemma 1. Knowing this and taking n as given, the firms in period 1 solve

$$\max_k E[\max\{\pi(n, A_s, k) - \varepsilon - D(k),\ 0\}]. \tag{6}$$

The FOC with respect to k is

$$E\left[\left(\frac{\sigma-1}{\sigma}\right) n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{-\frac{1}{\sigma}} - Q \ \middle|\ \text{(No D.O.)}\right] = 0, \tag{7}$$

where $E[\ \cdot\ |\ \text{(No D.O.)}]$ is the expectation conditional on that debt overhang does not occur, i.e., $\pi(n, A_s, k) - D(k) \geq \varepsilon$.[4] The FOC must hold with equality since otherwise k goes to 0 or $+\infty$. In equilibrium where $k = \bar{k}$, this condition implies

$$Q = \left(\frac{\sigma - 1}{\sigma}\right) E[n_s^{\frac{1}{\sigma-1}} A_s \mid \text{(No D.O.)}]. \tag{8}$$

When the price Q is given by (8), the quantity of capital k is determined as $k = \bar{k}$ by (7).

Lender's problem: The households (or lenders) maximize the expected value of their consumption in period 2, given that their choice is either to sell capital K to the firms in exchange for the risky debt or to hold the capital and sell it in the next period for the use in C-sector. The households' choice is limited to the two options because they are subject to the technological constraint that they cannot produce output in S-sector nor C-sector. Thus, the households' decision-making in period 1 is degenerated such that they sell the capital to the firms if the following participation condition (PC) is satisfied, and they hold the capital until period 2 if the PC is not satisfied. The PC for households' selling capital is given as follows. On one hand, the household can obtain ρQ units of period-2 consumer good by selling one unit of capital in period 1 in exchange for the debt that matures in period 2, where ρ is the expected value of recovery rate of debt, which is given endogenously (see the next paragraph). On the other hand, when the household does not sell one unit of capital in period 1, she can obtain A_L units of period-2 consumer good by selling it in period 2 as an input to C-sector, because the capital is used in S-sector only if it is sold to a firm and is installed for specialization in period 1. Thus the PC is

$$\rho Q \geq A_L. \tag{9}$$

If the inequality in PC is strict ($>$), then all capital K is sold to the firms in period 1:

$$k = K.$$

If the PC holds with equality ($=$), then $k \leq K$.

---

[4]Condition that $\pi(n, A_s, k) - \varepsilon - D(k) > 0$ gives the threshold $A(Q, k)$ such that the debt overhang does not occur if and only if $A_s \geq A(Q, k)$. With our discrete setting that $A_s \in \{A_M, A_H\}$, it is easily shown that the FOC (7) can be rewritten as

$$E\left[\left(\frac{\sigma-1}{\sigma}\right) n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{-\frac{1}{\sigma}} - Q \;\middle|\; A_s \geq A(Q, k)\right] = 0,$$

with $A(Q, k) = A_M$ or $A(Q, k) = A_H$. In the case where the value of A_s distributes continuously, it can be easily shown that the FOC (7) is also given by the above equation, where $A(Q, k)$ is chosen from the continuous distribution. See Appendix A for the details.

Recovery rate of debt: In the case of no debt overhang, the recovery rate of debt is 1. In the case of debt overhang, i.e., $\pi - D < \varepsilon$, there emerges a threshold $\bar{z}$, such that the lenders with $z_i \leq \bar{z}$ reduce debt and recover $\pi - \varepsilon$, while the lenders with $z_i > \bar{z}$ does not reduce debt and recover $A_L k$. See (14) for $\bar{z}$ in the Debt Overhang Equilibrium (DOE). The expected value of recovery rate is

$$\rho = \Pr(\text{No D.O.}) + [1 - \Pr(\text{No D.O.})] \frac{E[R(z_i) - \Gamma(z_i) \ \mid \ (\text{D.O.})]}{D},$$

where Pr(No D.O.) is probability of no debt overhang, $E[\cdot|(\text{D.O.})]$ is the expectation conditional on that debt overhang occurs, $R(z_i)$ is the repayment recovered by household i, and $\Gamma(z_i)$ is the cost of debt restructuring for household i. We will see the value of ρ for the DOE in Section 3.2.

2.4 Social optimum

We can consider the problem for the social planner who chooses k, the amount of capital installed in period 1 for S-sector, and n, the number of remaining firms in period 2 in S-sector facing the realization of $A_s \in \{A_M, A_H\}$. We assume that the social planner chooses k in period 1 to maximize the social welfare $E[C]$, where C is the household consumption. We know $C = Y$. Since $A_L < A_M \ll A_H$ and the total production in S-sector is $Y_S = n^{\frac{\sigma}{\sigma-1}} A_s k$, production in S-sector is always more efficient than production in C-sector if $n = 1$. Thus, the socially optimal allocation is obviously $k = K$ and $n = 1$.

3 Equilibrium

In this paper, we focus on the equilibrium where all capital is sold to firms in period 1: $k = K$, by assuming the parameter region where PC holds with strict inequality in equilibrium: $\rho Q > A_L$. Since there are only two states ($s = M$ and $s = H$) in period 2, it is sufficient to check the existence of two possible equilibria: the Normal Equilibrium (NE), where debt overhang never occurs, and the Debt Overhang Equilibrium (DOE), where debt overhang occurs when $A_s = A_M$ and does not occur when $A_s = A_H$. We will see that the NE exists if A_H is not so large, while the DOE emerges and the NE ceases to exist if A_H is sufficiently large.[5]

[5] Both NE and DOE could coexist for moderately large values of A_H. We do not scrutinize the condition for the existence of multiple equilibria, though, because our focus in this paper is to analyze the equilibrium allocation and policy options for the equilibrium with asset-price boom, bust and debt overhang, i.e., the Debt Overhang Equilibrium.

3.1 Normal Equilibrium

We consider the conditions for existence of the Normal Equilibrium (NE) where debt overhang does not occur in any state, $A_s = A_H$ or $A_s = A_M$. Define $\xi = p_H(A_H/A_M) + 1 - p_H$. Suppose the NE with $k = K$ and $n = 1$ exists. Then, (8) implies

$$Q^N = \left(\frac{\sigma - 1}{\sigma}\right) \xi A_M,$$

and $D^N = Q^N K$, where the superscript N denotes the NE. The condition for no debt overhang, or the FEC (1), in period 2 at $n = 1$ and $A_s = A_M$, is

$$\left[1 - \left(\frac{\sigma - 1}{\sigma}\right) \xi\right] A_M K > \varepsilon,$$

which is rewritten in the limit of $\varepsilon \to 0$ as

$$A_H < \left(\frac{1}{(\sigma - 1)p_H} + 1\right) A_M. \tag{10}$$

The PC for selling capital is satisfied with strict inequality if $\rho Q^N = Q^N > A_L$, where $\rho = 1$ because no default occurs in the NE. This condition is satisfied if $\left(\frac{\sigma-1}{\sigma}\right) \xi A_M > A_L$. Since $\xi > 1$ the sufficient condition for $Q^N > A_L$ is

$$\left(\frac{\sigma - 1}{\sigma}\right) A_M > A_L. \tag{11}$$

We focus on the parameter region where (11) is satisfied.

Condition for no deviation: To complete the proof of existence of the NE, we need to show there is no deviation. In the NE, a firm could deviate in a way that it increases k to a certain value, k_d, such that it cannot repay $D_d = Q^N k_d$, when $A_s = A_M$, and it repays D_d only when $A_s = A_H$. For the existence of the NE, it is necessary to confirm this deviation is not profitable. The expected profits for a firm when it does not deviate is $E[\pi^N - \varepsilon - D^N] = \xi A_M K/\sigma - \varepsilon$. The expected profits for a deviating firm is $E[\pi_d - \varepsilon - D_d \mid \text{(No D.O.)}] = p_H\{A_H \bar{k}^{\frac{1}{\sigma}} k_d^{\frac{\sigma-1}{\sigma}} - \varepsilon - Q^N k_d\}$. It is maximized by $k_d = \left(\frac{A_H}{\xi A_M}\right)^{\sigma} K$ and the maximized value of profits from deviation is

$$E[\pi_d - \varepsilon - D_d \mid \text{(No D.O.)}] = p_H \frac{(\xi A_M)^{1-\sigma} A_H^{\sigma}}{\sigma} K - p_H \varepsilon$$

The condition for no deviation is $E[\pi^N - \varepsilon - D^N] > E[\pi_d - \varepsilon - D_d \mid \text{(No D.O.)}]$, which is, in the limit of $\varepsilon \to 0$,

$$\frac{A_H}{A_M} < \frac{(1 - p_H)}{(1 - p_H^{\frac{\sigma-1}{\sigma}}) p_H^{\frac{1}{\sigma}}}, \tag{12}$$

which is satisfied if A_H is not so large.[6] We have shown that the following proposition holds for A_L sufficiently small and A_H not too large:

Proposition 2. *In the limit of $\varepsilon \to 0$, suppose that* (10)*,* (11) *and* (12) *are satisfied. Then, there exists the Normal Equilibrium where $n = 1$ and $k = K$, and the debt is always repaid fully. The asset price is $Q^N = \left(\frac{\sigma-1}{\sigma}\right)\xi A_M$ and the debt is $D^N = Q^N K$.*

In the Normal Equilibrium, the TFP is either A_M or A_H, which is strictly bigger than A_L. As $k = K$ and $n = 1$ in all states, the Normal Equilibrium is socially optimal. The ex-ante social welfare is measured by $W = E[Y]$. In the NE, the welfare W^N is given by

$$W^N = [p_H A_H + (1 - p_H)A_M]K,$$

in the limit of $\varepsilon \to 0$. This is the first-best value of the social welfare.

3.2 Debt Overhang Equilibrium

First, in Section 3.2.1, we specify the nature of the Debt Overhang Equilibrium (DOE) where debt overhang occurs when $A_s = A_M$, and does not occur when $A_s = A_H$, on the premise that the DOE exists. Second, in Section 3.2.2, we then clarify the (sufficient) condition for its existence. We focus on the parameter region where $\rho Q > A_L$ so that $k = K$. The parameter region is to be specified later in Section 3.2.2.

3.2.1 Nature of Debt Overhang Equilibrium

Now, suppose that the DOE exists. Since it must be the case that $n = 1$ when debt overhang does not occur, i.e., $\pi - D \geq \varepsilon$, the FOC (8) implies that the asset price must be

$$Q^B = \left(\frac{\sigma - 1}{\sigma}\right) A_H,$$

where the superscript B denotes the DOE (i.e., asset boom). Since the expected value of the productivity of the capital is ξA_M and $Q^N = \left(\frac{\sigma-1}{\sigma}\right)\xi A_M$, the asset price in DOE, Q^B, is higher than the "fundamental" price Q^N. In other words, the firms bid up the price to Q^B because they are willing to buy the capital at a higher price as they only care about the state of no debt overhang, i.e., $A_s = A_H$, and they do not care about the lenders' loss from their default at $A_s = A_M$.

The number of firms in S-sector is $n = 1$ for $A_s = A_H$, and n is endogenously determined for $A_s = A_M$ by the lenders' decisions on debt restructuring in period 2.

[6] We could be interested in whether the deviated firm actually default on D_d when $A_s = A_M$, that is, whether $\pi(1, A_M, k_d) - Qk_d < \varepsilon$. But this inequality is not necessary for the existence of the NE. Suppose $\pi(1, A_M, k_d) - Qk_d < \varepsilon$ is satisfied. In this case, the deviation is feasible and is not profitable as long as (12) is satisfied. Suppose $\pi(1, A_M, k_d) - Qk_d \geq \varepsilon$. In this case, the optimal deviation with default is not feasible and therefore the NE can exist stably. As (12) is the sufficient condition for no deviation, we assume this condition is satisfied.

Equilibrium value of n when $A_s = A_M$: When A_M is realized, the firms default on D, and the lenders decide whether to restructure the debt. As we argued in Section 2.2, the lender i takes n as given and restructures the debt when the following condition, which is equivalent to (5), is satisfied in the DOE where $k = K$, $Q^B = [(\sigma-1)/\sigma]A_H$, and $\pi = n^{\frac{1}{\sigma-1}} A_M K$:

$$n^{\frac{1}{\sigma-1}} A_M K - \varepsilon - z_i \left[\left(\frac{\sigma-1}{\sigma} \right) A_H K - n^{\frac{1}{\sigma-1}} A_M K + \varepsilon \right]^{\phi} \geq A_L K. \tag{13}$$

This condition is rewritten as

$$z_i \leq \bar{z}, \tag{14}$$

where $\bar{z} = \hat{G}(n) \equiv \max\{0, \ \min\{z_{\max}, \ G(n)\}\}$ and

$$G(n) \equiv \frac{n^{\frac{1}{\sigma-1}} A_M - \varepsilon' - A_L}{\left[\left(\frac{\sigma-1}{\sigma} \right) A_H - n^{\frac{1}{\sigma-1}} A_M + \varepsilon' \right]^{\phi} K^{\phi-1}}, \tag{15}$$

where $\varepsilon' = \varepsilon/K$. Since lender i, with $z_i \leq \bar{z}$, restructures debt to $\hat{D} = n^{\frac{1}{\sigma-1}} A_M K - \varepsilon$ and the borrowing firm i continues operation in S-sector, the equilibrium value of n is given by

$$n = F(\bar{z}). \tag{16}$$

These two conditions $\bar{z} = \hat{G}(n)$ and (16) imply that the equilibrium value of n is determined by

$$n = F(\hat{G}(n)). \tag{17}$$

Note that there may exist multiple values of n that satisfy (17). Assumption 2 guarantees that the largest n among the solutions to (17) is selected as an equilibrium value of n.

Larger boom leads to deeper recession: We consider the graphs of $n = F(z)$ and $z = \hat{G}(n)$ in the (n, z) space of Figure 1, where the horizontal axis is n-axis and the vertical axis is z-axis. Suppose A_H is small enough such that $G(1) > z_{\max}$. In this case, Assumption 2 implies that $\bar{z} = z_{\max}$ and $n = 1$. All lenders restructure debt and socially optimal production in S-sector takes place. Suppose A_H is large such that $G(1) < z_{\max}$. In this case, there are two possibilities: (P1) The graph of $\bar{z} = G(n)$ and $n = F(\bar{z})$ have no intersections, or (P2) they have intersections.

- In the case (P1), no lenders reduce debt and $\bar{z}^e = n^e = 0$ in equilibrium.[7] All capital are used in C-sector and total production is $Y = A_L K$.

[7] The proof is the following. The graph of $n = F(z)$ is always above that of $z = G(n)$ in the case (P1), meaning that, for any given n, firms' exit decision implies that the number of firms remaining in S-sector is strictly smaller than n, except for the case of $n = 0$. Thus, $n = \bar{z} = 0$ is the only equilibrium.

- In the case (P2), the equilibrium value of n^e, which corresponds to the rightmost intersection of $n = F(z)$ and $z = G(n)$, is smaller than 1 and it is graphically shown that n^e is smaller for a larger A_H. See Figure 1.[8] The intuitive explanation is as follows. A larger A_H makes the debt D larger, implying that the debt restructuring cost is also larger. Condition (13) implies that the larger debt makes the threshold value $\bar{z}$ lower and the number of remaining firms, $n = F(\bar{z})$, smaller.

- Since $G(\underline{n}) = 0$ for any A_H, where $\underline{n} = \{(A_L + \varepsilon')/A_M\}^{\sigma-1}$, the following claim is shown graphically:

  Claim 1. Suppose that the case (P2) is realized for a certain value $A_H = A_H^c$. Then, there exists a threshold $\hat{A}_H$ that is larger than A_H^c such that (P2) is realized and $n^e > 0$ for any $A_H \in [A_H^c, \hat{A}_H]$, whereas, for $A_H > \hat{A}_H$, (P1) is realized and $n^e = 0$.

  This claim implies the following: If A_H exceeds $\hat{A}_H$ from below to above, then n^e jump down from a positive value to zero.

Both cases (P1) and (P2) imply that a larger A_H leads to a lower n^e. As a larger A_H can be interpreted as a larger asset boom, while a smaller n^e a deeper recession or lower productivity, we can interpret that a larger boom ex-ante leads to a deeper recession ex-post. Here we can confirm the following.

Lemma 3. *The total output in state M, i.e., $Y(A_M) = Y_S + Y_C$, decreases as n decreases.*

Proof. Given the equilibrium values of n and $\bar{z}$, that satisfy $n = F(\bar{z})$, the total output in S-sector is given by

$$\begin{aligned} Y_S &= n^{\frac{\sigma}{\sigma-1}} A_M K - \left[\left(\frac{\sigma - 1}{\sigma} \right) A_H - n^{\frac{1}{\sigma-1}} A_M + \varepsilon' \right]^{\phi} K^{\phi} \int_0^{\bar{z}} z dF(z) \\ &= \int_0^{\bar{z}} y_S(n, z) dF(z), \end{aligned} \tag{18}$$

where

$$y_S(n, z) = \left\{ n^{\frac{1}{\sigma-1}} A_M K - \left[\left(\frac{\sigma - 1}{\sigma} \right) A_H - n^{\frac{1}{\sigma-1}} A_M + \varepsilon' \right]^{\phi} K^{\phi} z \right\}. \tag{19}$$

By definition of $\bar{z}$, we have $y_S(n, \bar{z}) = A_L K$ and $y_S(n, z)$ is decreasing in z, implying $Y_S > n A_L K$. Thus, $Y(A_M) = Y_S + Y_C$, where $Y_C = (1 - n) A_L K$, satisfies $Y(A_M) > A_L K$.

[8] The proof is as follows. It is graphically confirmed in Figure 1 that $z = G(n)$ intersects $n = F(z)$ from above to below as n increases at the largest intersection n^e, because $G(1) < z_{\max}$. This means that when A_H increases the intersection n^e shifts to the left. This is because $z = G(n)$ shifts lower as A_H increases and the cumulative distribution function $F(z)$ is monotonically increasing in z. Therefore, we can conclude that n^e is smaller for a larger A_H.

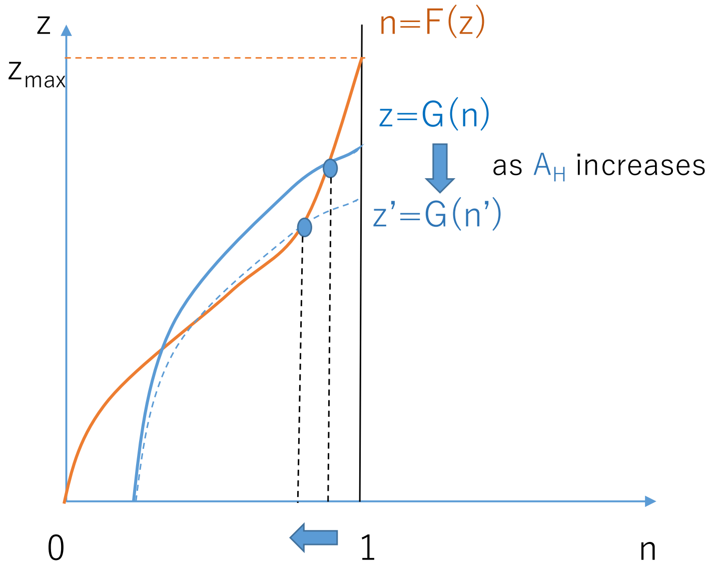

Figure 1: Larger boom (A_H) leads to smaller n

Noting $n = F(\bar{z})$ and $y_S(n, \bar{z}) = A_L K$, differentiate $Y(A_M)$ with $\bar{z}$ to get

$$\begin{aligned}\frac{dY(A_M)}{d\bar{z}} =& \frac{dY_S}{d\bar{z}} + \frac{dY_C}{d\bar{z}} \\ =& y_S(n, \bar{z}) f(\bar{z}) + \int_0^{\bar{z}} \left[\frac{\partial y_S(n, z)}{\partial n} \right] f(\bar{z}) dF(z) - A_L K f(\bar{z}) \\ =& \int_0^{\bar{z}} \left[\frac{\partial y_S(n, z)}{\partial n} \right] f(\bar{z}) dF(z).\end{aligned}$$

The definition (19) implies that $\frac{\partial y_S(n,z)}{\partial n} > 0$, and $\frac{dn}{d\bar{z}} > 0$. Thus we obtain

$$\frac{dY(A_M)}{dn} > 0.$$

Therefore, $Y(A_M)$ decreases as n decreases. □

A comparison with the literature: The result that output in the ex-post recession is lower for a larger ex-ante asset boom is also shown by Allen, Barlevy and Gale (2022). Comparing our result with theirs makes clear the difference. Their result is derived from the exogenous assumption that cost of default is increasing in the amount of defaulted debt. In our model, we also assume the exogenous cost of debt restructuring, and output Y_S in (18) is divided into the production ($n^{\frac{\sigma}{\sigma-1}} A_M K$) and the cost of debt restructuring ($-\left[\left(\frac{\sigma-1}{\sigma}\right) A_H - n^{\frac{1}{\sigma-1}} A_M + \varepsilon' \right]^{\phi} K^{\phi} \int_0^{\bar{z}} z dF(z)$). The decrease in output due to the cost of debt restructuring is the same effect that Allen, Barlevy and Gale (2022) point out, whereas a new finding in our model is that debt overhang causes the endogenous decrease in n, which leads to a decrease in the aggregate productivity and production $n^{\frac{\sigma}{\sigma-1}} A_M K$. This decrease in productivity and output is the adverse effect of the aggregate demand externality, which is unique to our result. This mechanism may underscore the linkage between ex-ante asset booms and ex-post declines in output and productivity, in the following reason. Although both Allen, Barlevy and Gale (2022) and our model assume that the cost of debt restructuring and the cost of default are dead-weight loss, it may be possible that these costs are not a loss but just a transfer of resources among economic agents. If the cost of default is a transfer, the model in Allen, Barlevy and Gale (2022) cannot predict that a larger boom in asset prices results in a bigger output loss, whereas our model can still have the same prediction of the decrease in productivity and output, even if the cost of debt restructuring is just a transfer of output, because the total output in this case is $n^{\frac{\sigma}{\sigma-1}} A_M K$, which decreases as n decreases.

3.2.2 Existence of Debt Overhang Equilibrium

In this subsection, we specify the sufficient conditions for the existence of the DOE. The following conditions must be satisfied:

$$\left[1-\left(\frac{\sigma-1}{\sigma}\right)\right]A_H K > \varepsilon, \tag{20}$$

$$\left[A_M-\left(\frac{\sigma-1}{\sigma}\right)A_H\right]K < \varepsilon, \tag{21}$$

where (20) says there is no default if $A_s = A_H$, and (21) says that a firm cannot fully repay the debt even if all other firms stay in S-sector, when $A_s = A_M$. The first condition is always satisfied as ε is infinitesimally small. The second condition is satisfied in the limit of $\varepsilon \to 0$ if A_H is so large that

$$\frac{A_H}{A_M} > \frac{\sigma}{\sigma-1}. \tag{22}$$

Another necessary condition for existence of DOE is that the firms have no incentive to deviate from the equilibrium. Now, we specify the condition for no deviation. The expected profit for a firm in the DOE is

$$p_H(A_H K - \varepsilon - D^B) = \frac{p_H A_H}{\sigma}K - p_H\varepsilon.$$

Suppose that a firm considers to deviate from the DOE by reducing k to k_d so that it does not default on $D_d = Q^B k_d$ when $A_s = A_M$. The optimization problem for a deviating firm is

$$\max_{k_d}\ [p_H A_H + (1-p_H)n^{\frac{1}{\sigma-1}}A_M]K^{\frac{1}{\sigma}}k_d^{\frac{\sigma}{\sigma-1}} - \left(\frac{\sigma-1}{\sigma}\right)A_H k_d - \varepsilon, \tag{23}$$

$$\text{s.t.}\ \ n^{\frac{1}{\sigma-1}}A_M K^{\frac{1}{\sigma}}k_d^{\frac{\sigma-1}{\sigma}} - \varepsilon - \left(\frac{\sigma-1}{\sigma}\right)A_H k_d \geq 0. \tag{24}$$

The condition (24) says that k_d is chosen such that the firm does not default on the debt when $A_s = A_M$. The solution to (23) on the premise that (24) is nonbinding is

$$k_d = \left[p_H + (1-p_H)n^{\frac{1}{\sigma-1}}\frac{A_M}{A_H}\right]^{\sigma}K. \tag{25}$$

Substituting (25) into (24), it is shown that (24) is equivalent to

$$\left[1-\left(\frac{\sigma-1}{\sigma}\right)(1-p_H)\right]n^{\frac{1}{\sigma-1}}A_M - \varepsilon'' \geq \left(\frac{\sigma-1}{\sigma}\right)p_H A_H, \tag{26}$$

at the solution (25), where $\varepsilon'' = \varepsilon K^{-1}[p_H + (1-p_H)n^{\frac{1}{\sigma-1}}(A_M/A_H)]^{1-\sigma}$. If (26) is violated, the profit of the firm at A_M is negative, implying that (23) at k_d that satisfies (25) is smaller than the profit when it does not deviate, i.e., $p_H A_H K/\sigma - p_H\varepsilon$. Thus, the sufficient condition for no deviation is

$$\left[1-\left(\frac{\sigma-1}{\sigma}\right)(1-p_H)\right]n^{\frac{1}{\sigma-1}}A_M - \varepsilon'' < \left(\frac{\sigma-1}{\sigma}\right)p_H A_H.$$

Since $\varepsilon'' > 0$ and $n \leq 1$, the sufficient condition for the above condition is $[1 - (\sigma - 1)\sigma^{-1}(1 - p_H)]A_M < (\sigma - 1)\sigma^{-1}p_H A_H$, which is equivalent to $A_M < Q^N$, and can be rewritten as

$$A_H > \left(\frac{1}{(\sigma - 1)p_H} + 1\right) A_M. \tag{27}$$

When this condition is satisfied, (22) is automatically satisfied, because $[\sigma/(\sigma - 1) - (1 - p_H)]p_H^{-1} > \sigma/(\sigma - 1)$ for any $p_H \in (0, 1)$ and $\sigma > 1$.

Participation constraint for lenders: What to be done finally is to specify the parameter region where $\rho Q^B > A_L$ is satisfied. Note that

$$\rho = p_H + (1 - p_H)\frac{Y(A_M) - \varepsilon F(\bar{z})}{Q^B K}.$$

Lender's optimal decision on debt restructuring means that $Y(A_M) - \varepsilon F(\bar{z}) > A_L K$, as shown in (4). Therefore, $\rho > p_H + (1 - p_H)\frac{A_L}{Q^B}$, and the sufficient condition for $\rho Q^B > A_L$ is given by $[p_H + (1 - p_H)\frac{A_L}{Q^B}]Q^B > A_L$, which can be rewritten as

$$\frac{A_H}{A_L} > \frac{\sigma}{\sigma - 1}, \tag{28}$$

which is automatically satisfied if (27) and (22) are satisfied. We have proven the following proposition.

Proposition 4. *The Debt Overhang Equilibrium exists if A_H is sufficiently large and satisfy* (27)*. In this equilibrium, $k = K$, $Q^B = (\frac{\sigma-1}{\sigma})A_H$, and $D^B = Q^B K$. The number of firms in S-sector is $n = 1$ if $A_s = A_H$, and it is n^e, which is the largest solution to* (17)*, if $A_s = A_M$.*

Note that condition (27) is not compatible with condition (10), and therefore when (27) holds the NE cannot exist. It may be possible that both the NE and the DOE coexist for A_H that satisfies (10). We do not further specify the condition for multiple equilibria, though, as our focus is on the analysis in the case where there is a large asset-price boom, which corresponds to the case where A_H is sufficiently large and the equilibrium is the DOE.

Welfare: In the DOE, the ex-ante welfare is

$$W^B = p_H A_H K + (1 - p_H)Y(A_M),$$

in the limit of $\varepsilon \to 0$. As we see that $Y(A_M) < A_M K$ and $Y(A_M)$ decreases as n decreases, it is obvious that $W^B < W^N$, where W^N is the first-best level of the social welfare. Whether or not W^B is decreasing in A_H is ambiguous because the first term

$(A_H K)$ is increasing in A_H, while the second term $(Y(A_M))$ is decreasing. However, as we see in Claim 1 an infinitesimal increase in A_H from $\hat{A}_H$ makes n^e jump down from a positive value to zero, meaning that an infinitesimal increase in A_H can lead to a big jump down of W^B from $p_H A_H K + (1 - p_H)Y(A_M)$ to $p_H A_H K + (1 - p_H)A_L K$. In the end, we can say that a sizable increase in A_H decreases the social welfare W^B in the neighborhood of $A_H = \hat{A}_H$. Therefore, it can be said that a larger asset-price boom impairs the ex-ante social welfare by making the ex-post recession deeper.

4 Policy responses

Our model enables us to assess ex-ante and ex-post policy interventions to the boom and bust of asset prices, followed by macroeconomic debt overhang. In this section, we consider the case where (27) is satisfied, so that the equilibrium is the DOE. In other words, we consider the case where there arrives a news shock in period 1 that the productivity of capital A_H can be extremely high in period 2. We analyze ex-ante macroprudential policy in the next subsection, and ex-post subsidy to debt restructuring in Section 4.2. Finally, we will argue about monetary policy in a modified model, in which nominal money is introduced as a unit of account.

The analysis in this section can be summarized in the following three points. First, ex-ante imposition of borrowing limit is the first best in our model, while setting the borrowing limits for individual firms is not likely to be feasible. Second, ex-post subsidy to lenders who reduce the debt overhang is welfare improving. In contrast to the existing literature (Bianchi 2016; Chari and Kehoe 2016; Green 2010; Keister 2016: Keister and Narasiman 2016), the ex-post policy does not cause time inconsistency in our model as long as the participation constraint for lenders $\rho Q > A_L$ is satisfied with strict inequality. Third, an ex-post inflation can be welfare improving as it reduces the burden of debt overhang.

4.1 Ex-ante macroprudential policy

It is easily shown that an appropriately designed macroprudential policy can modify the equilibrium in such a way that no default occurs when $A_s = A_M$. Suppose that the financial regulator imposes the borrowing constraint in period 1 that each firm's debt D cannot exceed $\bar{D}$, where

$$A_L K < \bar{D} \leq A_M K - \varepsilon. \tag{29}$$

In this case, the asset price in equilibrium becomes $Q = \bar{D}/K$, and the PC is satisfied: $\rho Q = Q > A_L$. Each firm buys K units of capital in period 1, and when A_s turns out to be A_M in period 2, the firms can pay the debt $\bar{D}$, because their earnings are $A_M K$, given

$n = 1$. There is no default and no exit from S-sector. The allocation, $k = K$ and $n = 1$, is socially optimal.[9]

Although implementing the ex-ante policy is the first best, it may be practically infeasible to find the appropriate level of $\bar{D}$ for individual firms. So the ex-post policy response is also very important.

4.2 Ex-post debt restructuring

The inefficiency of debt overhang emerges when the state turns out to be A_M, in the Debt Overhang Equilibrium. In this subsection, we focus on period 2 of the DOE, when $A_s = A_M$ is realized. There is a chance of government intervention at the beginning of period 2 after the aggregate shock $A_s = A_M$ is revealed and before production takes place.

Socially optimal debt restructuring: Given the debt overhang $D = Q^B K$, the social planner would maximize the total output, by solving the following optimization problem:

$$\max_{\bar{z}} \; n^{\frac{\sigma}{\sigma-1}} A_M K - n\varepsilon - \left[\left(\frac{\sigma-1}{\sigma}\right) A_H + \varepsilon' - n^{\frac{1}{\sigma-1}} A_M\right]^{\phi} K^{\phi} \int_0^{\bar{z}} z dF(z) + (1-n) A_L K, \tag{30}$$

$$\text{s.t.} \quad n = F(\bar{z}).$$

The optimal value $\bar{z}^o$ is given as the solution to the FOC of the above problem:

$$n^{\frac{1}{\sigma-1}} A_M K - \varepsilon - \bar{z} \left[\left(\frac{\sigma-1}{\sigma}\right) A_H + \varepsilon' - n^{\frac{1}{\sigma-1}} A_M\right]^{\phi} K^{\phi} + T(n, \bar{z}) \geq A_L K, \tag{31}$$

where

$$T(n, \bar{z}) = \frac{n^{\frac{1}{\sigma-1}} A_M K}{\sigma - 1} + \frac{\phi n^{\frac{2-\sigma}{\sigma-1}} A_M}{\sigma - 1} \left[\left(\frac{\sigma-1}{\sigma}\right) A_H + \varepsilon' - n^{\frac{1}{\sigma-1}} A_M\right]^{\phi-1} K^{\phi} \int_0^{\bar{z}} z dF(z).$$

The solution is $(\bar{z}^o, n^o) = (z_{\max}, 1)$ if the inequality of the FOC is strict ($>$), while $\bar{z}^o < z_{\max}$ and $n^o < 1$ if the FOC holds with equality.

Optimal ex-post policy: Notice that the value of $\bar{z}$ is determined in a competitive equilibrium without government interventions by (4), which is the condition for a lender to be better off by debt restructuring. The condition (4) can be rewritten as follows to determine $\bar{z}$ in the DOE:

$$n^{\frac{1}{\sigma-1}} A_M K - \varepsilon - \bar{z} \left[\left(\frac{\sigma-1}{\sigma}\right) A_H + \varepsilon' - n^{\frac{1}{\sigma-1}} A_M\right]^{\phi} K^{\phi} = A_L K, \tag{32}$$

[9] For some parameter values, both the NE and DOE coexist. In this case, for any $\bar{D}' \in (Q^N K, Q^B K)$, if we set the ex-ante borrowing limit at $\bar{D}'$, then the economy goes to the NE, leaving the borrowing constraint $D \leq \bar{D}'$ nonbinding.

where the left-hand side is the lender's profit of restructuring debt overhang, while the right-hand side is what the lender can get if she does not restructure the debt. This condition determines the equilibrium value $\bar{z}^e$ without policy intervention. Comparing this condition with (31), we can prove the following proposition.

Proposition 5. *The government can realize the optimal allocation $n^o = F(\bar{z}^o)$ by giving the subsidy, the schedule of which is $T(n, \bar{z})$, to the lenders who restructure the debt.*

Proof. Given the subsidy $T(n, \bar{z})$, the optimal exit decision by firms implies that the equilibrium $(n, \bar{z})$ is determined by (31) and $n = F(\bar{z})$. If there exist multiple solutions, Assumption 2 guarantees that the largest possible n (and $\bar{z}$) is realized in equilibrium. □

This ex-post subsidy for debt restructuring can improve social welfare by internalizing the aggregate demand externality in the monopolistic competition of S-sector. The aggregate demand externality can be seen as one example of externalities caused by the financial crisis, which can be resolved by debt restructuring, such as the counterparty risk among borrowing firms or the free-rider problem among lenders who have claims on the same borrower and want to free ride on the other lenders' debt restructuring. Our result demonstrates that an ex-post government intervention to debt overhang can improve welfare, as debt overhang is quite likely to cause serious externalities.

Equilibrium with anticipated ex-post interventions: What happens if the agents expect that government intervention $T(n, \bar{z})$ will take place when debt overhang occurs? The answer is that nothing changes except that n becomes n^o when debt overhang occurs at $A_s = A_M$. Given that the subsidy is for lenders, not borrowers, the firms obtain nothing when they have debt overhang, i.e., $\pi - \varepsilon < D$, as in the case without subsidy, which is shown in Lemma 1. We can show as follows that the equilibrium does not change with anticipation of ex-post policy intervention. First, the ex-post debt restructuring policy affects the allocation only in the state where debt overhang occurs. Second, as long as the participation condition for lenders, $\rho Q \geq A_L$, continues to hold with strict inequality, the decision making by firms in period 1 is irrelevant to the anticipation about what happens when debt overhang occurs in period 2 because the firms do not care about the debt-overhang state, where they obtain nothing anyway. The conditions for existence of the NE are not affected by the anticipation of the government intervention, and thus Proposition 2 still holds. Concerning the DOE, we have the following proposition that shows the DOE is identical in period 1 no matter whether the expectations of ex-post policy interventions exist or not.

Proposition 6. *We assume parameters satisfy* (27). *Suppose all agents expect that the government gives subsidy with the schedule $T(n, \bar{z})$ to the lenders, conditional on undertaking debt restructuring, if $D = D^B$ and $A_s = A_M$. Then, there is the Debt Overhang*

Equilibrium, where $k = K$, $Q^B = \left(\frac{\sigma-1}{\sigma}\right) A_H$, and $D^B = Q^B K$. These values are the same as those in Proposition 2.

Proof. The expectations of government intervention affects only ρ, which changes the participation condition (PC) for households' selling capital: $\rho Q > A_L$. Given our assumption on parameters (27) and (28), it is obvious that the PC holds with strict inequality, even when the government intervention is anticipated. Therefore, nothing changes in conditions for equilibrium. □

Agency problem: One would be concerned that anticipation of ex-post policy intervention may have adverse effect to aggravate agency problems. In this paper, we did not explicitly assume agency problems.[10] The risk shifting from the firms to the lending households in our model is possible due to the technological constraint that only firms can produce output, and the households cannot produce anything from capital. Even if we explicitly introduce private information and agency problems into our model, ex-post debt restructuring policy would have minimal adverse effects when it is conditional on the macroeconomic variables such as A_s, which would be observable and verifiable.

Partial subsidy to debt restructuring: The nature of the aggregate demand externality in our model implies that subsidy for a small fraction of firms, not for all firms, may be sufficient to attain the social optimum. This is trivially demonstrated in the case where z_i is observable.

Proposition 7. *Suppose z_i is observable. The optimal allocation $n^o = F(\bar{z}^o)$ is realized if the government gives the following subsidy $T_2(n, z_i)$ for debt restructuring to only lenders i whose z_i satisfy $z_i \in [\bar{z}^e, \bar{z}^o]$, where*

$$T_2(n, z_i) = A_L K - n^{\frac{1}{\sigma-1}} A_M K + \varepsilon + z_i \left[\left(\frac{\sigma - 1}{\sigma} \right) A_H + \varepsilon - n^{\frac{1}{\sigma-1}} A_M \right]^{\phi} K^{\phi},$$

Proof. If the subsidy $T_2(n, z_i)$ is given to the lenders, conditional on debt restructuring, the lender i with $z_i \in [\bar{z}^e, \bar{z}^o]$ is weakly better off by debt restructuring. As a result of the policy, all lenders with $z_i \in [0, \bar{z}^o]$ restructure debt and thus n becomes n^o in equilibrium. □

4.3 Monetary policy in a model with nominal variables

In this subsection, we modify our model by adding money. Money is just a unit of account used both in period 1 and period 2, and we assume that the quantity of money supplied is zero.

[10] In this paper, we implicitly assume that debt is optimally chosen contract, implying that there exist idiosyncratic shocks that are private information that causes agency problem.

Debt contract is made in terms of money. In period 1, a firm purchases k units of capital in exchange for debt $Q'k$, where Q' is the asset price in terms of money in period 1. Here the debt evolves at the loan rate $1+I$ and the firm is obliged to repay $D' = (1+I)Q'k$ in terms of money in period 2 to the lender household. We can define P_s as the price of period-2 consumer goods in terms of money in the state s, where $s \in \{M, H\}$. Then, the real burden of debt is $D_s = (1+I)Q'/P_s$ in terms of period-2 consumer goods.

We assume that the central bank can set the nominal rate I and the nominal price levels P_s. Setting the nominal rate I in period 1 is ex-ante monetary policy, whereas setting P_s for $s \in \{M, H\}$ is ex-post monetary policy. We assume that the values of P_s is anticipated by firms and households in period 1.[11] We will assess ex-ante and ex-post policies respectively.

Given I and P_s, a firm in period 1 maximizes the expected profit:

$$\max_k E[\max\{\pi - \varepsilon - D,\ 0\}],$$

where $\pi \equiv p(y)y = n^{\frac{1}{\sigma-1}} A_s \bar{k}^{\frac{1}{\sigma}} k^{\frac{\sigma-1}{\sigma}}$ and $D_s = (1+I)Q'k/P_s$. FOC wrt k at $k = \bar{k}$ decides $(1+I)Q'$ by

$$E\left[P_s^{-1} | (\text{No D.O.})\right]\ (1+I)Q' = E[n^{\frac{1}{\sigma-1}} A_s \mid (\text{No D.O.})] \left(\frac{\sigma-1}{\sigma}\right)$$

The real burden of debt overhang D_s at the state $s \in \{M, H\}$ is

$$D_s = \frac{(1+I)Q'k}{P_s} = \frac{E[n^{\frac{1}{\sigma-1}} A_s \mid (\text{No D.O.})]}{E\left[P_s^{-1} | (\text{No D.O.})\right]} \left(\frac{\sigma-1}{\sigma}\right) P_s^{-1} k.$$

In this modified model, we focus on the DOE where debt overhang ($\pi - \varepsilon - D < 0$) does not occur in the state H and debt overhang occurs in the state M. Thus, since $n_H = 1$ and $E[P^{-1} | (\text{No D.O.})] = P_H^{-1}$, we have

$$D_H = \left(\frac{\sigma-1}{\sigma}\right) A_H K, \tag{33}$$

$$D_M = \left(\frac{\sigma-1}{\sigma}\right) A_H \frac{P_H}{P_M} K. \tag{34}$$

Ex-ante monetary policy: We assume period-2 prices (P_H or P_M) are fixed, because they are control variables for ex-post monetary policy, not ex-ante monetary policy. Since $(1+I)Q' = \left(\frac{\sigma-1}{\sigma}\right) A_H P_H$ in the DOE, a change in I is exactly offset by the corresponding change in Q' so that $(1+I)Q'$ is unchanged. It is obvious from this that ex-ante monetary policy, i.e., a change in I, has no effect on equilibrium allocation. This is because the nominal rate I is irrelevant to the real debt burden D_s and to the decision-makings by lenders and firms in both period 1 and period 2.

[11] Our results in this subsection hold qualitatively unchanged, even if the central bank can set the totally unexpected values of P_s.

Ex-post monetary policy: Central bank decides period-2 prices, P_s for $s \in \{M, H\}$. We do not specify how central bank controls P_s, and just assume that central bank can decide P_s. This assumption is a shortcut for the description of monetary policy. We focus on the debt-overhang state $A_s = A_M$ in period 2, where lenders restructure debt to choose $\bar{z}$ and n^e. As (34) indicates, higher P_M for state A_M reduces real burden of debt $D_M = \frac{(1+I)Q'K}{P_M}$, and shifts the graph of $\bar{z} = G(n) = \frac{\pi(n)-A_L K}{(D-\varepsilon-\pi(n))^{\phi}}$ upward in Figure 1, increasing $\bar{z}$ and n^e in equilibrium. Higher P_M at state A_M is interpreted as ex-post monetary easing. Therefore, the ex-post monetary easing, whether anticipated or unanticipated, can reduce the real debt burden D_M and increase efficiency and output.[12] This policy implication is valid only to the extent that the central bank can control the price level. If P_M cannot be raised by monetary policy, then monetary policy is not effective to improve social welfare in the second period.

There may be also other policy interventions such as tax/subsidy on C-sector.[13]

5 Conclusion (to be revised)

We demonstrated that the model of risk-shifting booms of asset prices and ex-post debt overhang can replicate empirical regularities, i.e., credit-fueled asset boom usually ends up with the bust, followed by a deep and persistent recession, associated with productivity declines. The risk-shifting effect endogenously increases the probability of the occurrence of the ex-post inefficiency of debt overhang. Therefore, our theory implies that a credit-fueled asset-price boom may be intrinsically inefficient in terms of ex-ante welfare. It is also shown that a larger asset-price boom leads to a deeper recession ex-post. As the inefficiency of debt overhang is aggravated by aggregate demand externality, ex-post policy

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[12] If P_M is sufficiently large, it makes $D_M = (1 + I)Q'K/P_M$ so small that debt overhang never occurs. Then, the first best allocation is attained, given that the participation condition for lending households, $E[\rho_s(1 + I)Q'/P_s] > A_L$, be satisfied. To make policy analysis more realistic, we can assume exogenous nominal rigidity that P_M cannot exceed a certain upper limit, and therefore the firms default in the state A_M.

[13] I thank Tack Yun for pointing to the policy issues of monetary policy and the tax/subsidy in C-sector. Consider a business income tax on firms in C-sector: $\tau A_L k$ for producing $A_L k$. With this policy, the effective productivity in C-sector becomes $(1 - \tau)A_L$. An increase in τ increases n^e by shifting the graph of $\bar{z} = G(n)$ upward in Figure 1, where

$$\bar{z} = G(n) = \frac{n^{\frac{1}{\sigma-1}} A_M - (1-\tau)A_L}{\left[\left(\frac{\sigma-1}{\sigma}\right) A_H - n^{\frac{1}{\sigma-1}} A_M\right]^{\phi} K^{\phi-1}}.$$

The tax on C-sector, τ, may be welfare improving, given that tax revenue is transferred back to the households in a lump-sum fashion. The interpretation of the tax on C-sector is not straightforward, though, because $A_L k$ can be interpreted as a fire-sale value of the asset k. The above argument may imply that subsidy to facilitate the fire sale, i.e., a negative value of τ, is welfare reducing.

intervention that enhances debt restructuring improves welfare. In particular, the ex-post fiscal policy that subsidizes the lenders who restructure the debt overhang may increase the aggregate productivity and output. We also showed that time inconsistency typically associated with the bailout policies may disappear from the ex-post debt restructuring policies under some circumstances. These results may shed some light on the aspects of policy responses to financial crises that may be worth studying further in the literature.

Appendix A: Continuous distribution of A_s

We can modify the model such that the productivity parameter A_s is not a binary variable but a continuous variable. Suppose that $A_s \in [0,\ A_{\max}]$, and the distribution function is $H(A)$, i.e., $\Pr(A_s \le A) = H(A)$. The threshold $A(Q,k)$ is given by the solution to $\pi(n,A,k) = Qk + \varepsilon$. Then, given Q, the firm in period 1 solves

$$\max_k \int_{A(Q,k)}^{A_{\max}} \{\pi(n,A,k) - Qk - \varepsilon\} dH(A),$$

as the firm can default on the debt Qk when $\pi - Qk - \varepsilon < 0$. Noting that $\pi(n, A(Q,k), k) - Qk - \varepsilon = 0$, the FOC wrt k can be written as

$$\int_{A(Q,k)}^{A_{\max}} \left\{ \left(\frac{\sigma-1}{\sigma}\right) n^{\frac{1}{\sigma-1}} A \bar{k}^{\frac{1}{\sigma}} k^{-\frac{1}{\sigma}} - Q \right\} dH(A) = 0.$$

This condition decides Q:

$$Q = \left(\frac{\sigma-1}{\sigma}\right) E[A \mid A \ge A(Q,k)] \tag{35}$$

The condition $\pi(n, A(Q,k), k) - Qk - \varepsilon = 0$ can be written in the equilibrium where $n = 1$ and $k = K$ as

$$Q = A(Q,K) + \varepsilon', \tag{36}$$

where $\varepsilon' = \varepsilon/K$. The two variables Q and $A(Q,K)$ are determined by the above conditions. In what follows, we write $\underline{A} \equiv A(Q,K)$ and $\Psi(\underline{A}) = E[A|A \ge \underline{A}]$. The variables Q and $\underline{A}$ are determined by the above two conditions, which are rewritten as

$$Q = \left(\frac{\sigma-1}{\sigma}\right) \Psi(\underline{A}), \tag{37}$$

$$Q = \underline{A} + \varepsilon'. \tag{38}$$

Note that (37) decides Q from $\underline{A}$ and (38) decides $\underline{A}$ from Q. We consider the graphs of (37) and (38) in the $(\underline{A}, Q)$-space, where the horizontal axis is $\underline{A}$-axis and the vertical axis is Q-axis. Since $\lim_{A\to A_{\max}} \Psi(A) = A_{\max}$, the graph of (37) becomes asymptotically a line with a slope of $\frac{\sigma-1}{\sigma} < 1$, whereas the graph of (38) is a line with a slope of 1. Since $\Psi(0) > \varepsilon'$ as ε' is infinitesimally small, we can see graphically that there exists at least one intersection of (37) and (38), implying that there exists at least one equilibrium. The number of intersections can be multiple and in that case we have multiple equilibria.

In that case, it is shown as follows that the rightmost intersection in the $(\underline{A}, Q)$-space is a stable equilibrium in the following sense. The stability of equilibrium against a small perturbation can be evaluated by considering how $(\underline{A}, Q)$ are decided by (37) and (38). If, in the $(\underline{A}, Q)$-space, the graph of (37) intersects (38) from above to below as $\underline{A}$ increases, then the intersection is a stable equilibrium, because (37) decides Q from $\underline{A}$ and (38) decides $\underline{A}$ from Q. Therefore, the rightmost intersection is a stable equilibrium. In particular, if the intersection is unique, it is a stable and unique equilibrium.

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