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$MPK < g$ [*]

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Abstract

It is well known that land does not prevent dynamic inefficiency (a rate of capital accumulation higher than the Golden rule level, or, a return to capital, MPK, less than the economy's growth rate, g) if land is subject to a transaction tax. This paper attempts to evaluate quantitatively the potency of this argument, both positively as well as normatively. Using a heterogenous-agent, general equilibrium overlapping generations growth model calibrated to the U.S. economy, the main quantitative finding is that a 6.18% tax on the sale of land produces balanced growth paths that exhibit dynamic inefficiency. When an unfunded social security system is introduced, the economy moves toward dynamic efficiency, welfare improves, and the optimal replacement rate is 70%.

Keywords: Dynamic inefficiency, overlapping generations, transaction costs, social security.

JEL Classification: E6, H1.

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1 Introduction

It is well known since Kim and Lee (1997) that land does not prevent dynamic inefficiency (a rate of capital accumulation higher than the Golden rule level, or, a return to capital MPK less than the economy's growth rate g), if land is subject to transaction, property, or capital gains taxes.[1] This paper attempts to evaluate quantitatively the potency of this argument, and uses a growth model in order to look at dynamic inefficiency in the U.S. economy, both positively as well as normatively.

In particular, this paper develops a quantitative general equilibrium model populated with overlapping generations of individuals who face stochastic income streams, borrowing constraints and mortality risk, calibrates the model to U.S. data over the period 1960-2018, and explores the role of a transaction tax on land in yielding dynamic inefficiency and a tax-transfer scheme to increase long run welfare.

Following Koh, Santaeulàlia-Llopis, and Zheng (2020) I choose a labor share of 0.65, target a land income share of 5% (and therefore a capital share of 30%) by calibrating a land transaction tax using the model's balanced growth path conditions. Given the factor shares in the constant returns to scale Cobb-Douglas production function and a 56% replacement rate of social security (see Briggs and Springstead (2008)) I use data from national accounts, fixed asset tables, and flow of funds to match the wealth-output ratio in the data by calibrating the subjective discount factor. The idiosyncratic wage risk is represented by an $AR(1)$ process following Krueger, Mitman, and Perri (2016) and discretized by a five-state, first order discrete Markov process.

The main quantitative result is that there is dynamic inefficiency in the baseline laissez-faire economy, and in all balanced growth paths with a social security replacement rate less than 70%. An unfunded public pension system as a simple tax-transfer program moves he economy toward dynamic efficiency and raises welfare. The optimal replacement rate is 70%.

I conduct sensitivity analyses and find that any calibration target that raises private savings increases the scope for dynamic inefficiency and therefore admits a larger beneficial role for government intervention. For example, a larger transaction tax, or, a higher wealth-output ratio as a target (and hence a higher subjective discount factor), amplifies the size of the dynamic inefficiency. Furthermore, dynamic inefficiency exists for values of land's output share as low as 2%, alternative calibrations of the stochastic process representing the idiosyncratic wage risk, alternative calibration targets and inclusion of intangible capital in wealth. The quantitative findings suggest that some of the government interventions may be rationalized in the long run for helping restore dynamic efficiency.

The paper is organized as follows. Section 2 the related literature and describes the paper's contributions. Section 3 presents the model, Section 4 describes the calibration

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[1] The possibility to have dynamic inefficiency in an overlapping generations model with a transaction tax on capital was first argued by Kim and Lee (1997), and, later by Hellwig (2020) in a different but related two-period overlapping generations setting.

of the model, Section 5 contains the main quantitative findings and sensitivity analyses, and Section 6 concludes.

2 Related Literature

Recent low real yields, r, on government bonds relative to output growth rates, especially since the 1980s, have generated a large literature on studying economies where $r < g$. This inequality is important in discussions of debt rollover and the welfare effects of debt management, as Abel and Panageas (2022) point out. However, MPK vs g is more relevant in discussions of dynamic inefficiency.

Bloise and Reichlin (2023) develop an overlapping generations economy to understand which relations between safe interest rates, risky returns and output growth rates are important in evaluating dynamic efficiency. They conclude that in an overlapping generations model with stochastic growth, the condition for dynamic inefficiency is not a simple comparison of interest rates with average growth and that a social security scheme may not be welfare improving.[2]

This paper is more closely related to the dynamic inefficiency results using deterministic overlapping generations models. This literature goes back to at least Malinvaud (1953), Samuelson (1958), Diamond (1965), and Cass (1972). In the classic Diamond (1965) paper, conditions for the Golden rule capital stock are derived and a simple criterion is given to assess dynamic efficiency: if the (net) marginal return to capital exceeds the growth rate of the economy, then the economy is dynamically efficient.

In their seminal paper, Abel, Mankiw, Summers, and Zeckhauser (1989) use an extended version of the two-period overlapping generations model of Diamond (1965) to derive an alternative condition to test for dynamic efficiency. If capital income exceeds investment then the economy is dynamically efficient and if it is the other way around then the economy is dynamically inefficient. Abel, Mankiw, Summers, and Zeckhauser (1989) use data on national income accounts and find that all the G7 economies are dynamically efficient.

Geerolf (2018) revisits this issue using a recent harmonized national accounting data, attributes some of the ambiguous income to labor, estimates and separates pure profits and land rents from capital income, and finds that the sufficient conditions for dynamic efficiency are not verified for any of these economies and that Japan and South Korea satisfy the sufficient conditions for dynamic inefficiency. Using either condition, comparing the net return to capital to an economy's growth rate, or, comparing the income to owners of capital and investment, has its measurement challenges.

What is typically done in the literature is to measure the average return to capital

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[2] There is a related literature that uses the neoclassical growth model with idiosyncratic risk to study capital overaccummulation and policies that may improve welfare. See Huggett (1993), Aiyagari (1994), Aiyagari and McGrattan (1998), Davila, Hong, Krusell, and Rios-Rull (2012), Reis (2020) , and Aguiar, Amador, and Arellano (2021).

where as theory uses the marginal return to capital in the sufficient condition for dynamic efficiency. Similarly, measuring capital income is difficult since it also contains income to land, a non-produced asset.[3]

An additional measurement issue concerning capital income is emphasized by Kopczuk and Zwick (2020) who document the significant increase in business income of partnerships and pass-through entities over the last few decades. Clearly this has made it difficult to classify certain components of income as pure profits or returns to human capital as opposed to capital income. These empirical challenges have made it quite difficult to implement simple tests of dynamic inefficiency in actual economies.

This paper complements the literature by exploring the quantitative efficacy of Kim and Lee (1997) in an overlapping generations model with deterministic growth. 1) Can a large scale general equilibrium OG model, calibrated to the U.S. economy generate dynamic inefficiency? 2) Is this robust to different calibration targets for wealth output ratio, different factor income share estimates, different estimates of the idiosyncratic wage risk? 3) Can an unfunded social security system move the economy toward dynamic efficiency and improve welfare? The answers are yes to all of these questions. This paper does not address which government policies should be used to restore dynamic efficiency, nor how different individuals are affected by it. This is left for future research.

3 The Model Economy

The setup is a general equilibrium model populated with overlapping generations of individuals who face uninsurable income risk, borrowing constraints and uncertain lifetimes, a production function with capital, labor, and a fixed factor land, and a transaction tax on land.[4] Markets are incomplete so risks cannot be fully hedged. These features produce a stationary distribution of individuals that differ in wealth in addition to age.

Since agents have no access to private annuity markets, a publicly administered tax-transfer scheme such as a pay as you go social security system can provide partial insurance and increase long run welfare. In addition, when dynamic inefficiency exists, the same government intervention can pull the economy toward the Golden Rule and potentially raise welfare.

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[3] Davis and Heathcote (2007) estimate the price and quantity of residential land using the residual method to extract the value of land as the difference between the value of a property and its replacement cost starting from the flow of funds data and making adjustments for capital gains. Davis (2009) extends this to land owned by noncorporate and corporate businesses. Rognlie (2015) estimates a land share of income of 4% in his attempt to examine the secular movements in factor shares where as Rhee (1991)'s estimate is 5%.

[4] The model used in this paper has been a workhorse framework that casts an income fluctuations problem in heterogeneous-agent models with individuals facing uninsurable income streams and borrowing constraints to study various fiscal policy issues. See for example Huggett (1993), Aiyagari (1994), İmrohoroğlu, İmrohoroğlu, and Joines (1995), and more recent analyses of wealth distribution and fiscal policy in Benhabib, Bisin, and Zhu (2011), Bhandari, Evans, Golosov, and Sargent (2018), and Ma, Stachurski, and Toda (2020).

3.1 Firms

I assume that there is a representative firm that operates a constant returns to scale Cobb-Douglas production function

$$Y = BK^{\alpha}N^{\eta}L^{1-\alpha-\eta} \ , \tag{1}$$

where Y is output, $\alpha \in (0,1)$ is capital's share of output, $\eta \in (0,1)$ is labor's share, and K, N and L are aggregate inputs of capital, labor, and land, respectively. I will restrict attention to balanced growth paths.

$B > 0$ denotes total factor productivity (TFP) which grows at a constant, exogenously given rate ρ. The aggregate capital stock depreciates at the rate δ.

Land is a fixed factor and I normalize its quantity to unity, $L = 1$. The first order conditions for profit maximization of the firm are

$$r = \alpha BK^{\alpha-1}N^{\eta} - \delta \ , \quad w = \eta BK^{\alpha}N^{\eta-1}, \tag{2}$$

where r is the return to capital net of depreciation and w is the wage rate.[5] I assume that capital and land have identical returns.

3.2 Individuals

The economy is populated by overlapping generations of *ex ante* identical agents who maximize expected, discounted lifetime utility

$$E_0 \sum_{j=1}^{J} \beta^{j-1} \Big[\prod_{k=1}^{j} \psi_k \Big] U(c_j) \ , \tag{3}$$

where β is the subjective discount factor, ψ_j is the conditional probability of survival from age $j-1$ to age j, c_j is consumption of an age-j individual, J is the maximum possible life span, and E_0 is the expectations operator conditional on information at the beginning of age 1. By definition $\psi_1 = 1$ and $\psi_i = 0$ for $i > J$.

I assume that population grows at an exogenous, constant rate n. The share of age-j individuals in the population is given by the fraction $\mu_j, \ j = 1, 2, \ldots, J,$ where

$$\mu_{j+1} = \frac{\psi_{j+1}}{(1+\rho)} \mu_j$$

and

$$\sum_{j=1}^{J} \mu_j = 1.$$

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[5] In this paper, there is no government debt and I will use r and MPK interchangeably.

The period utility function is given by

$$U(c_j) = \begin{cases} \frac{c_j^{1-\gamma}}{1-\gamma} & \text{for } \gamma > 0, \gamma \neq 1, \\ \ln c_j & \text{for } \gamma = 1, \end{cases}$$

where γ is the coefficient of constant relative risk aversion.

There is an exogenously given mandatory retirement age, j_R. In each period, non-retirees inelastically supply one unit of labor, and earn labor income $w\varepsilon_j\phi_j$, where ε_j is the deterministic efficiency and ϕ_j is the stochastic efficiency of an age-j agent. In this paper, I follow Krueger, Mitman, and Perri (2016) and assume an $AR(1)$ process to represent the stochastic productivity of an agent:

$$\phi_{j+1} = \rho\phi_j + u_{j+1}, \quad u_{j+1} \sim \mathcal{N}(0, \sigma_u^2). \tag{4}$$

After retirement, the disposable income of a retiree is equal to the social security benefit, b, calculated to be a fraction $\theta \in \{0, 1\}$ of the average lifetime income, where $\bar{\phi}$ is the mean of the stochastic efficiency process.[6]

$$b = \begin{cases} 0 & \text{for } j = 1, 2, \ldots, j_R - 1, \\ \theta\frac{\sum_{i=1}^{j_R-1} w\varepsilon_j\bar{\phi}}{j_R-1} & \text{for } j = j_R, j_R + 1, \ldots, J. \end{cases} \tag{5}$$

The disposable income of an agent is given by:

$$q_j = \begin{cases} (1-\tau_s)w\epsilon_j\phi_j & \text{for } j = 1, 2, \ldots, j_R - 1, \\ b & \text{for } j = j_R, j_R + 1, \ldots, J. \end{cases} \tag{6}$$

In this economy, there are no private markets for insurance against income and longevity risks. Agents can accumulate assets to help smooth consumption over the life cycle, but they face borrowing constraints and may not have negative assets at any age:

$$a_j \geq 0 \quad , \ \forall j, \tag{7}$$

where a_j is the end-of-period asset holdings of an age-j individual. An implication of this and the assumption $\psi_j = 0$ for $j > J$ is that individuals who are alive at age J will choose not to carry over any assets to the next period in the absence of a bequest motive: $a_J = 0$.

I assume that there is a zero-profit firm that operates a mutual fund and individuals purchase shares in this fund. Under this interpretation a_j is the end-of period holdings in this mutual fund for an age-j individual. It is assumed that capital and land are perfect substitutes and therefore command identical returns in equilibrium.

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[6] I abstract from the detailed social security rules in the United States in order to introduce a simple tax-transfer scheme and focus on dynamic inefficiency in the model with land and transaction costs.

Although the individuals' portfolios are indeterminate, the aggregate quantities of capital and land are given by a resource constraint that forces the sum of these two aggregate assets to equal the aggregate asset holdings of the individuals, namely the total mutual fund. In particular, given the price of land P (equation 13) and the normalization $L = 1$, the aggregate capital stock is given by the difference between aggregate asset holdings (or the total mutual fund) and P.

As some agents die before age J, I assume that each period the government distributes all accidental bequests equally among the members of all generations in the amount ξ, in a lump-sum fashion.

The budget constraint of an individual is given by

$$c_j + a_j = (1 + r)a_{j-1} + q_j + \xi + \sigma \ , \ a_0 \text{ given}, \tag{8}$$

where σ is a lump-sum redistribution of transaction taxes on land.

I will now describe the recursive problem of the individual. Given any beginning-of-period asset holding and productivity state (a, ϕ) define the constraint set of an age-j agent $\Omega_j(a, \phi) \in R^2_+$ as all pairs (c_j, a_j) such that

$$c_j \geq 0, \tag{9}$$

and constraints (5), (6), (7) and (8) are satisfied.

Now the consumer's problem can be written as a finite-state, finite-horizon discounted dynamic program. Let $V_j(a, \phi)$ be the value of the objective function of an age-j agent with beginning-of-period asset holdings and productivity (a, ϕ) given by

$$V_j(a, \phi) = \max_{(c,a')\in\Omega_j(a,\phi)} \Big\{U(c) + \beta\psi_{j+1}E_{\phi'}V_{j+1}(a', \phi')\Big\}, j = 1, 2, \cdots, J, \tag{10}$$

subject to (4), (5), (6), (7), (8) and (9).

3.3 Land and Capital Accumulation in the Long Run

Given the constant growth rates of the labor input, n, and TFP, ρ, I assume that there is a balanced growth path along which the capital-output ratio is constant. With per capita output growth rate given by g', the balanced growth rate of output is $g = (1+g')(1+n)-1$.

Each period, a unit of land produces its marginal product and its owner also gets capital gains. In equilibrium, the following arbitrage condition requires that the net returns on capital and land are identical:

$$r = \frac{(1 - \kappa)P_{t+1} + (1 - \alpha - \eta)Y_{t+1} - P_t}{P_t}, \tag{11}$$

where $P_t \geq 0$ denotes the price of land at the end of period t, and $0 \leq \kappa < 1$ is a transaction tax driving a wedge between the buying price of a unit of land and the selling price. Solving for the price of land gives

$$P_t = \frac{(1-\alpha-\eta)Y_{t+1} + (1-\kappa)P_{t+1}}{1+r}. \tag{12}$$

Substituting recursively for future land prices gives the current land price as $(1-\alpha-\eta)$ times the discounted present value of all future output of the economy:

$$P = \frac{(1-\alpha-\eta)(1+g)}{r-g+\kappa(1+g)} Y > 0. \tag{13}$$

With transaction tax the denominator contains κ and now $MPK - g$ can be negative (dynamic inefficiency) as long as the denominator is positive.

If $MPK \equiv r > g$ the economy is on a dynamically efficient balanced growth path, whereas $MPK < g$ indicates overaccumulation of capital. If $\kappa = 0$ then dynamic inefficiency is ruled out by the mere presence of land as a fixed factor of production; for $P > 0$ in equation 13 we must have $r > g$. In this case, any increase in the private saving of the individuals would be absorbed by a higher price of land and hence there would not be any capital overaccumulation.

However, for $\kappa > 0$ it is possible to have a long run equilibrium with $P > 0$ and $MPK < g$. With a $\kappa > 0$, the return to land is less than when $\kappa = 0$. Since capital is a perfect substitute to land, a positive κ also means a lower return to capital. This is only possible in equilibrium if there is more capital accumulation than the case of $\kappa = 0$, and the model exhibits dynamic inefficiency.

In the next section, I will calibrate $\kappa > 0$ so that the model produces a land share of output that is consistent with recent estimates. Then I will analyze if the resulting quantitative stationary equilibria produce dynamic inefficiency and if so whether a tax-transfer scheme like an unfunded social security system restores dynamic efficiency.[7]

4 Calibration

The model described in the previous section is an off-the-shelf overlapping generations model with incomplete markets that researchers have used to study macroeconomic issues, and in particular wealth inequality and taxation. Following on this theme, the calibration exercise will map this model to U.S. data in the spirit of Cooley and Prescott (1995).

4.1 Social Security Replacement Rate

The tax-transfer scheme used in this paper is a simple pay as you go social security system in which a fraction of average lifetime income is received upon retirement which is financed

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[7] The definition of the stationary equilibrium is given in appendix A.

by a flat tax rate imposed on working age individuals. This replacement rate is calculated as a portion of pre-retirement income.

There are different ways of calculating the notion of pre-retirement income and as s result the replacement rate differs depending on which method is used. Briggs and Springstead (2008) describe four alternative measures of replacement rates.

First, one could use labor earnings in the final year before retirement. Many defined benefits programs use this method or something like it. It is very simple to calculate and this measure was used in the early 1980s by the Greenspan Commission and appeared in the Trustees Reports. However, this measure has several drawbacks as it can be volatile as many individuals scale back their labor supply toward the eventual retirement age and this measure may understate true earnings. In general, this or any simple average of last few years would not be representative of life time earnings.

Second, one could use the present value payment method which is calculated as a constant payment based on the present value of lifetime earnings. However, since individuals vary greatly with respect to their tax liabilities and other costs, this measure is rarely used in practice and certainly not by the Social Security Administration (SSA).

Third, in computing the principal insurance amount, the SSA relies on the average indexed monthly earnings (AIME) which is reported in the Trustees Reports in recent years. As it only includes the highest 35 years of earnings, it may restrict the full value of lifetime earnings. One could calculate this average over all earnings but this measure would still overstate recent real earnings mechanically as the averaging stops at age 60 where as individual retire much later in the life cycle. As a result, this measure would not be representative of lifetime earnings.

Finally, inflation-adjusted average lifetime earnings has great appeal in capturing real resources available to an individual over the life cycle and this measure directly corresponds to the model's measure of the replacement rate.

Using 2005 administrative data, Briggs and Springstead (2008) calculate this measure to be 56% for the median retiree. According to Iams and Purcell (2013) the difference between the mean and the median retirement benefits for a 65 year or older worker is about 0.4% and since the measure of replacement rate in my model corresponds closer to the inflation-adjusted average lifetime earnings and I use 56% in the baseline economy.[8]

4.2 Demographics

A model period is one year. Economically active life starts at age 21 ($j = 1$) and ends with certainty at age 100 ($J = 80$). Exogenous retirement occurs at age 65 ($j = 45$). I take the conditional survival probabilities from the Social Security Adminstration's 2000 Cohort Life Tables. The population growth rate is 1%.

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[8] The AIME measure yields 47% which is another replacement rate estimate used in quantitative studies of social security. In an earlier version of this paper, this value was used with very similar quantitative results.

4.3 Income Process

The deterministic life cycle efficiency process ε_j is the hump-shaped process from Hansen (1993).

There is a sizable literature on the stochastic process governing individual wage uncertainty. What is agreed to is that there is significant persistence and heterogeneity of wage shocks over the life cycle. However, there is ongoing research on the precise nature of how this persistence and heterogeneity is best represented in a parsimonious manner as an input to quantitative macro models.

I use a stochastic income process is given by $AR(1)$ process:

$$\phi_{j+1} = \rho\phi_j + u_{j+1}, \quad u_{j+1} \sim \mathcal{N}(0, \sigma_u^2).$$

I approximate this continuous process with a 5-state first-order discrete Markov chain $\varphi = \{\varphi_1, \ldots, \varphi_5\}$ with a transition matrix Π using $\rho = 0.94$ and $\sigma_u^2 = 0.02$, which are in the range of estimates using the PSID data.[9]

4.4 Technology

First, I choose a labor share of income of $\alpha = 0.65$ following Koh, Santaeulàlia-Llopis, and Zheng (2020) who use NIPA data over 1960-2018 to calculate this value and argue that the observed decline in the labor share is entirely accounted for by the capitalization of intellectual property products (IPP) in the Bureau of Economic Analysis' National Income and Product Accounts (NIPA).[10]

Second, I use the same NIPA data over 1960-2018 to calculate the rate of depreciation of capital, δ, to be 0.064 and the average capital stock to GDP ratio, K/Y, of 3.24. Using the Flow of Funds (FFUS) data from the Board of Governors of the Federal Reserve System and NIPA data over the same time period, I calculate the average land to GDP ratio, P/Y, to be 0.83, giving a wealth to GDP ratio of 4.07.

It is difficult to use NIPA and other data sets to directly compute factor shares in our constant returns to scale production function. There are significant conceptual and measurement issues to separate income flows to property into capital and land income. İmrohoroğlu, İmrohoroğlu, and Joines (1999) organize the NIPA data for the U.S. economy as a three sector (business, government, and household) and three factor (capital, land, labor) economy and estimate a land share of 0.033. If one abstracts from labor income in the household sector, which is the case in this paper, then the share of land becomes 0.045 in their approach.

Rognlie (2015), who uses a different, nested production function, calculates the income share of land as 4%, and Rhee (1991) estimates the land share to be 5%. More recently,

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[9] In Section 5.2.5 I report very similar quantitative findings from using the discretized versions of the estimates in Chang et al. (2019), Floden and Linde (2001), French (2005), and Krueger, Mitman, and Perri (2016).

[10] In their attempt to explore the decline in the labor share globally, Gutiérrez and Piton (2019) estimate a labor share of 0.6323, which leads to very similar quantitative findings in the current context.

Reis (2020) deducts a land's share of 5% GDP from measured capital income (see their Figure 1).

In this paper, I use a land share of 5% and conduct a sensitivity analysis for 2, 3, and 4%.[11] Given a labor share of income of 0.65 and a land share of 0.05, the capital share is given by $1 - 0.65 - 0.05 = 0.3$.

Then, I use equation 13 and the balanced growth condition $r = \alpha Y/K - \delta$, with long run (1960-2018) averages of capital, land, and output to obtain an expression for the income share of capital

$$\alpha = \frac{(1+g)(1-\eta) + [\delta + g - \kappa(1+g)]P_t/Y_t}{(1+g) + P_t/K_t}, \tag{14}$$

to back out the transaction tax on land κ as 0.0618. Boerma (2019) uses Dutch administrative data and calibrates a similar parameter to obtain a 6% transaction tax.[12]

The growth rate of per person GDP over 1960-2018 is 2% using the NIPA data and the TFP parameter B is taken as 0.8 so that the model output is close to unity in the baseline equilibrium.

4.5 Preferences

The coefficient of relative risk aversion (CRRA), γ, is the inverse of the intertemporal elasticity of substitution (IES) in consumption in the current context and I will rely on the estimates of IES to calibrate this preference parameter. However, these estimates vary substantially, from 0.17-0.36 by Cashin and Unayama (2016) and 0.2 by Yogo (2004) and Hall (1988) at the low end, to 0.668 by Attanasio and Weber (1993, 1995), 0.3-1.0 by Vissing-Jørgensen (2002), and 2 by Gruber (2013), at the high end. Given the dispersion in these estimates, I choose $1/\gamma = 0.5$ or $\gamma = 2$ for the baseline results and then conduct a sensitivity analysis with $\gamma = 1$ and $\gamma = 3$.

Adding the capital-GDP ratio 3.24 and the land-GDP ratio 0.83 gives the wealth-GDP ratio $W/Y = 4.07$. Given a replacement of 56%, a subjective discount factor $\beta = 1.0201$ allows the model to match this observed wealth-GDP ratio.

Table 1 summarizes the key calibration choices for this section.

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[11] I do not report results from using a land share of 6% or higher because in these cases the scope for dynamic inefficiency is larger than that in the baseline case.

[12] In real estate transactions, agent or broker fees are typically about 5-6% and the closing costs (loan origination, inspection, surveying, appraisal, title insurance, transfer tax, and other costs) are about 2-4%, making the total transaction costs about 7-10% of the purchase price.

Table 1: Calibration

Technology		
per capita growth rate of output	$g = 0.02$	NIPA data
depreciation rate of capital	$\delta = 0.064$	NIPA data
labor share	$\eta = 0.65$	Koh et al. (2020)
capital share	$\alpha = 0.30$	BGP conditions, K/Y and P/Y (see text)
land share	0.05	(see text)
Demographics		
conditional survival probabilities	$\{\psi_j\}_{j=1}^{J}$	SSA Cohort Life Tables 2000
population growth rate	$n = 0.01$	CPS data
Preferences		
CRRA	$\gamma = 2$	literature on IES (see text)
subjective discount factor	$\beta = 1.0201$	Target $(K + P)/Y = 4.07$
Transaction Tax		
transaction tax	$\kappa = 0.0618$	Target land share 5%

5 Quantitative Findings

The baseline economy is calibrated to produce the observed wealth-output ratio of 4.07 for a replacement rate of 56%, given all the other parameters. The thought experiment is to examine the economy under different replacement rates to see if dynamic inefficiency exists, especially at the zero replacement rate economy (the laissez faire case), and whether an unfunded social security system restores dynamic efficiency and raises long run welfare.

5.1 Main Results

Table 2 displays detailed numerical findings from the baseline economy and shows that dynamic inefficiency exists in all balanced growth paths until the $\theta = 0.7$ economy. Fiscal intervention in the form of an unfunded social security system is optimal with a replacement rate of 70%. In other words, the introduction of an unfunded social security from $\theta = 0$ to $\theta > 0$ starts to bring the economy back to the Golden Rule and welfare rises.

In balanced growth paths with $\theta < 0.7$, the individuals accummulate capital in excess of the Golden Rule level and as a result $MPK < g$. This occurs because increased wealth holdings with a smaller tax-transfer scheme (driven by the retirement saving motive) cannot be entirely absorbed by a higher price of land. The existence of a land transaction tax leads to a reduction in the return to land and therefore that of capital as the two assets are perfect substitutes. Such a reduction in the return to capital is only possible if

Table 2: Economy with land and transaction costs: Main Findings

θ	K/Q	P/Q	W/Q	MPK	V_1
0.0	4.051	1.182	5.234	0.0101	-98.982
0.1	3.870	1.095	4.965	0.0136	-96.582
0.2	3.714	1.024	4.737	0.0169	-94.891
0.3	3.560	0.959	4.519	0.0203	-93.612
0.4	3.430	0.905	4.335	0.0235	-92.703
0.5	3.313	0.858	4.171	0.0266	-92.113
0.6	3.196	0.813	4.010	0.0299	-91.827
($g =$	0.0302)				
0.7	3.099	0.777	3.876	0.0328	-91.742
0.8	3.004	0.743	3.748	0.0359	-91.896
0.9	2.918	0.713	3.631	0.0388	-92.229
1.0	2.849	0.685	3.534	0.0417	-92.751

capital is overaccummulated.[13]

5.2 Sensitivity Analysis

In this section, I explore the sensitivity of the main quantitative findings to various calibration choices. First, the role of the CRRA is studied. Next, balanced growth paths are computed and displayed for various targeted land share parameter, yielding different transaction tax values. Third, the role of the subjective discount factor is quantitatively examined. Fourth, I present alternative calculations that rely on a range of calibration targets that researchers have used in similar general equilibrium overlapping generations models. These include i) using the more recent 2000-2018 subperiod in the baseline calibration, ii) McGrattan and Prescott (2017), iii) Davila, Hong, Krusell, and Rios-Rull (2012), and iv) adding an estimate of intangible capital (1.16 GDP) to my baseline wealth to output target to calibrate the subjective discount factor. Finally, I use different estimates of the $AR(1)$ process used in the literature to check for the robustness of the main quantitative findings.

5.2.1 The Role of CRRA

In the baseline results, the value of the CRRA is taken as 2 corresponding to an estimate of the IES of 0.5 that seems to be in the middle of the range of estimates in the literature. In this subsection, two additional values of γ will be used. Note that with each value

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[13] The Golden Rule balanced growth path in the baseline economy is characterized with $K/Q = 3.184, K = 3.351$ and $Q = 1.053$.

of γ, a different subjective discount factor β has to be chosen in order for the model to produce a wealth output ratio of 4.07 with $\theta = 0.56$.[14]

Table 3: Sensitivity to CRRA

$\gamma = 1.0$ ($\beta = 0.9977$)			$\gamma = 2.0$ ($\beta = 1.0148$)			$\gamma = 3.0$ ($\beta = 1.0300$)		
θ	MPK	V_1	θ	MPK	V_1	θ	MPK	V_1
0.0	0.0175	17.619	0.0	0.0101	-98.982	0.0	0.0058	-71.845
0.1	0.0198	18.805	0.1	0.0136	-96.582	0.1	0.0097	-68.428
0.2	0.0219	19.715	0.2	0.0169	-94.891	0.2	0.0138	-66.217
0.3	0.0238	20.534	0.3	0.0203	-93.612	0.3	0.0180	-64.927
0.4	0.0257	21.205	0.4	0.0235	-92.703	0.4	0.0220	-64.327
0.5	0.0274	21.727	0.5	0.0266	-92.113	0.5	0.0264	-64.540
0.6	0.0293	22.169	0.6	0.0299	-91.827	0.6	0.0306	-65.241
0.7	0.0310	22.528	0.7	0.0328	-91.742	0.7	0.0346	-66.437
0.8	0.0327	22.785	0.8	0.0359	-91.896	0.8	0.0389	-68.182
0.9	0.0344	22.967	0.9	0.0388	-92.229	0.9	0.0430	-70.320
1.0	0.0360	23.096	1.0	0.0417	-92.751	1.0	0.0468	-72.735

Table 3 presents the numerical results using three values of γ. The three columns in the middle replicate the replacement rate θ, the return to capital MPK, and the expected lifetime utility V_1 in the baseline calculations with $\gamma = 2.0$ shown in Table 2, where dynamic inefficiency exists until $\theta = 0.7$ and the optimal replacement rate is $\theta = 0.7$.

For $\gamma = 1$, the range of θ for dynamic inefficiency is the same as the baseline case: $\theta \in \{0.0, 0.6\}$. For any balanced growth path for $\theta < 0.7$, $MPK < g = 0.0302$. The optimal replacement rate is 100%.

With a higher relative risk aversion coefficient $\gamma = 3$, dynamic inefficiency exists for all balanced growth paths with $\theta < 0.6$. The optimal replacement rate is now $\theta = 0.4$.

Table 3 shows that the presence of dynamic inefficiency is fairly robust to the CRRA parameter as long as the calibration targets the observed wealth output ratio of 4.07 at a replacement rate of 56%. The optimality of the unfunded social security system depends somewhat on the assumed risk aversion.[15]

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[14] I keep the income shares of labor, capital and land the same as in the baseline case as well as the transaction tax rate κ.

[15] The wealth output ratio in this model is positively correlated with the subjective discount factor and negatively correlated with the CRRA. In order to match the observed wealth output ratio of 4.07 with a higher CRRA, we need a higher β and capital stock and wealth are more sensitive to changes in $theta$. As a result, as θ is raised from 0, MPK can more easily or quickly exceed g. There is less scope for dynamic inefficiency with a higher CRRA.

5.2.2 Sensitivity to Land's Share of Income

In the baseline economy, I use 5% as a target for the land share of output leading to a transaction tax of 6.18%. In this section I present numerical findings for different targets for the land share parameter. In particular, I use 4%, 3%, and 2% as alternative targets and calibrate the transaction tax κ to be consistent with these targets. In each case, I re-calibrate the subjective discount factor to match the same wealth to output ratio of 4.07 as before.

A lower target for land's share of output (that produces a lower transaction tax) leads to a smaller number of balanced growth paths that displays dynamic inefficiency. According to Table 4, dynamic inefficiency exists until $\theta = 60\%$ for a land share parameter 0.04 and the optimal replacement rate is $\theta = 60\%$.

With a land share target of 3%, dynamic inefficiency exists for values of $\theta \in \{0.0, 0.4\}$ and the optimal replacement rate is 0.5. For a 2% land share parameter, we get dynamically inefficient balanced growth paths until $\theta = 0.3$, which is the optimal replacement rate.

Table 4: Sensitivity to Land's Income Share

0.04			0.03			0.02		
$(\kappa = 0.0468, \beta = 1.0174)$			$(\kappa = 0.0317, \beta = 1.0147)$			$(\kappa = 0.0166, \beta = 1.0118)$		
θ	MPK	V_1	θ	MPK	V_1	θ	MPK	V_1
0.0	0.0138	-87.658	0.0	0.0181	-78.223	0.0	0.0242	-69.550
0.1	0.0171	-86.070	0.1	0.0211	-77.227	0.1	0.0262	-69.332
0.2	0.0201	-84.800	0.2	0.0238	-76.658	0.2	0.0284	-69.187
0.3	0.0234	-83.965	0.3	0.0268	-76.252	0.3	0.0308	-69.187
0.4	0.0265	-83.545	0.4	0.0297	-76.119	0.4	0.0333	-69.292
0.5	0.0297	-83.484	0.5	0.0329	-76.100	0.5	0.0363	-69.557
0.6	0.0329	-83.476	0.6	0.0359	-76.295	0.6	0.0389	-69.945
0.7	0.0358	-83.509	0.7	0.0388	-76.645	0.7	0.0420	-70.529
0.8	0.0390	-84.048	0.8	0.0419	-77.378	0.8	0.0448	-71.091
0.9	0.0419	-84.572	0.9	0.0448	-78.039	0.9	0.0477	-71.929
1.0	0.0448	-85.246	1.0	0.0478	-78.869	1.0	0.0505	-72.827

Table 4 shows that even for a land share parameter as small as 2%, one obtains dynamically inefficient balanced growth paths.[16]

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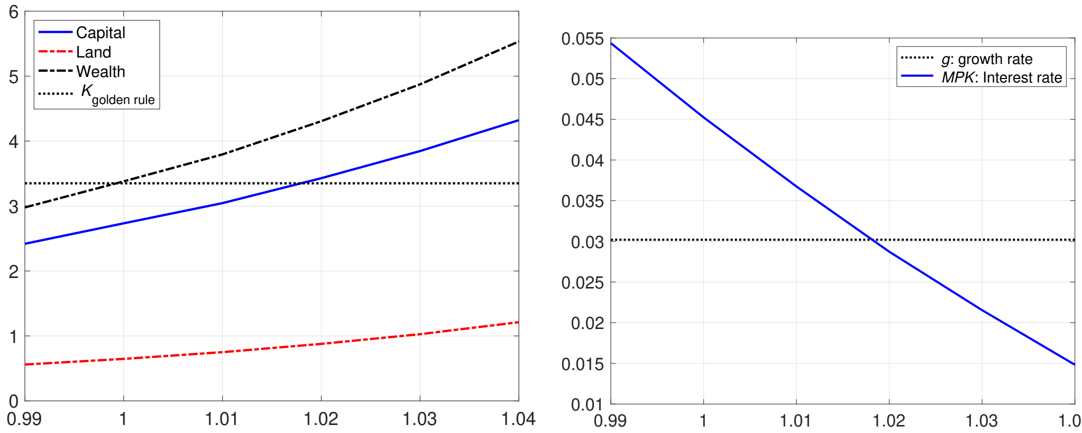
[16] A different exercise is to consider varying the transaction tax as the policy instrument instead of the size of the social security system. In these calculations, for each $\kappa \in \{0.01, 0.05\}$, I calculate the income shares of capital and land using equation 14 given a labor share of 0.65 and calibrate β to match a wealth-output ratio of 4.07. This alternative approach produces similar findings with the range for dynamic inefficiency rising from $\theta \in \{0.0, 0.1\}$ for $\kappa = 0.01$ to $\theta \in \{0.1, 0.5\}$ for $\kappa = 0.05$. In addition, searching over the optimal transaction tax given $\theta = 0.56$ yields $\kappa_{opt} = 0.035$ and for $\theta = 0.0$, I get

5.2.3 The Role of the Subjective Discount Factor Parameter β

In this subsection, I use the baseline model with $\kappa = 0.0618$ and $\theta = 0.56$ and vary the subjective discount factor to document balanced growth paths that exhibit dynamic inefficiency.



(a) Capital, Land, and Wealth

(b) Return to Capital

Figure 1: Subjective Discount Factor and Dynamic Inefficiency

Figure 1 displays balanced growth paths for various values of the subjective discount factor along the horizontal axis. Figure 1a shows the balanced growth path values of capital, land and wealth, where as Figure 1b depicts the balanced growth path MPKs, as β is varied between 0.99 and 1.04.

With a transaction cost of 6.18%, an increase in private savings (with higher β) raises both the capital stock and the price of land and there are balanced growth paths that exhibit dynamic inefficiency for a sufficiently high β. In other words, as the capital stock rises above the golden rule capital stock, defined here as the steady state capital stock at which $MPK = g$, the MPK falls below the exogenous growth rate of the economy which is 3.02%. Hence, depending on the strength of the private saving motives, institutional arrangements in the economic environment, and the details of the calibration of the model, it is quantitatively quite possible for dynamic inefficiency to exist and for fiscal policy to restore dynamic efficiency and improve long run welfare.

5.2.4 Alternative Calibrations

In this subsection I will present the findings from six alternative calibrations. Table 5 displays i) the baseline calculations, ii) the baseline calibration but using the more recent 2000-2018 period to generate data moments to match, iii) following the McGrattan and

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$\kappa_{opt} = 0.065$.

Prescott (2017) (MP2017) calibration, iv) the approach in Davila, Hong, Krusell, and Rios-Rull (2012) (DHKR2012), and v) adding an estimate 1.16 GDP of intangible capital (following MP2017) to the baseline calibration target for the wealth to output ratio.

In these alternative calibrations, I take the labor share of income and the depreciation rate in these papers as given, reduce the capital share of income in these papers by five percentage points so that the land share of income is 0.05 as in the baseline case. Given these targets, I then re-calibrate κ accordingly.

In DHKR2012 which abstracts from land, I add 0.83 (my estimate of the land to GDP ratio) to the DHKR2012 target for the capital output ratio to arrive at the target for the wealth output ratio. For all these calculations, I re-calibrate the subjective discount factors β to hit the targeted wealth-output ratios in these alternative calibrations.

Table 5 summarizes the results that show dynamic inefficiency in the laissez-faire steady states in all of alternative calibration exercises. In other words, MPK is less than g for $\theta = 0$ in all cases. With the exception of MP2017, dynamic inefficiency exists for θ at 70% or higher.

Table 5: Alternative Calibration Targets (land share = 0.05)

	Baseline	2000-2018	MP2017	DHKR2012	$K_{intangible}$
K/Q	3.24	3.302	4.915	3.0	4.4
P/Q	0.83	0.974	0.885	0.83	0.83
W/Q	4.07	4.276	5.8	3.83	5.27
labor share ρ	0.65	0.65	0.585	0.64	0.65
capital share α	0.3	0.3	0.365	0.31	0.3
δ	0.064	0.0655	0.0319	0.08	0.064
κ	0.0618	0.0561	0.0447	0.0669	0.0855
β	1.0202	1.0240	1.0273	1.0231	1.055
MPK	0.0287	0.0264	0.0419	0.0234	0.0037
$MPK - g$	-0.0015	-0.0038	0.0117	-0.0068	-0.0265
$(1+g)\kappa$	0.0637	0.0578	0.0491	0.0689	0.0881
$MPK < g$ for $\theta \in$	$\{0.0, 0.7\}$	$\{0.0, 0.8\}$	$\{0.0\}$	$\{0.0, 0.7\}$	$\{0.0, 1.0\}$
θ_{opt}	0.7	0.8	0.2	0.9	1.0

Using the 2000-2018 Period: When the more recent period of 2000-2018 is used for calibration instead of the longer 1960-2018, the average capital-output ratio is slightly higher but the land-output ratio is markedly higher leading to a wealth to output ratio of 4.276 as a target for the model to match. With a slightly higher depreciation rate at $\delta = 0.0655$, the model needs a slightly higher subjective discount factor $\beta = 1.0240$ to match the average wealth to output ratio of 4.276 over 2000-2018. As a result, the quantitative findings are very similar to the baseline case but the scope for dynamic inefficiency is a bit larger. Now, $MPK < g$ for $\theta \in \{0.0, 0.8\}$ and the optimal replacement rate is 80%.

McGrattan and Prescott (2017) Calibration: Given the labor and capital shares of 0.585 and 0.365, respectively, a $\kappa = 0.0477$ is needed to obtain a land share of 0.05. The third column is Table 5 uses a much lower depreciation rate (0.0319) but a much higher capital stock which includes intangible capital. The target wealth to output ratio is 5.8. Matching this in the model requires a subjective discount factor of 1.0273.

In this calibration, the MPK at $\theta = 0.0$, the laissez-faire case, is 0.0297 and this steady state is dynamically inefficient. The optimal replacement rate is $\theta = 0.2$.

Davila, Hong, Krusell, and Rios-Rull (2012) Calibration: Davila, Hong, Krusell, and Rios-Rull (2012) target a capital-output ratio of 3 and with the 0.83 land-output ratio that I calculate, the target wealth-output ratio becomes 3.83. Labor share of output is taken as 0.64 and the depreciation rate is 8%. The factor shares in this case produce a land transaction tax rate of 6.69% (using equation 14) and a $\beta = 1.0231$ produces a model wealth-output ratio equal to that in this calibration target of 3.83. In this case, the balanced growth paths for $\theta < 0.8$ are dynamically inefficient and the optimal replacement rate is 50%.

Baseline Calibration with Intangible Capital Added: The last row of Table 5 presents the results from adding intangible capital estimated by McGrattan and Prescott (2017), 1.2 (as a fraction of output), to the capital-output ratio of 3.24 of the baseline calibration to obtain the new capital-output ratio of 4.44. The target wealth-output ratio now becomes 5.27. A subjective discount factor of 1.055 is required to match this target. Now, the optimal replacement rate is 100%.[17]

These alternative calculations demonstrate that obtaining dynamic inefficiency in the laissez version of the model with land and a transaction tax is quantitatively robust to different calibrations. In addition, in these equilibria, adding a tax-transfer scheme in the form of a simple unfunded social security system moves the economy toward dynamic efficiency and raises steady state welfare.

5.2.5 Alternative AR(1) Processes

So far, I have used an AR(1) process (discretized with a 5-state, 1st-order Markov chain) using a persistence parameter of 0.94 and a shock variance of 0.02.

$$\phi_{j+1} = \rho \phi_j + u_{j+1}, \quad u_{j+1} \sim \mathcal{N}(0, \sigma_u^2). \tag{15}$$

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[17] Note that I have capped the maximum replacement rate at 100% which is binding in this case.

In this section, I summarize the main findings on dynamic inefficiency using four alternative estimates of the same process using a similar 5-state, first-order Markov discretization. For each case, I use the income shares of capital, labor and land that are identical to those in the baseline case in addition to the transaction tax on land. However, a different subjective discount factor β is needed for each case to match the 4.07 wealth to output ratio target of the baseline calibration.

Table 6: Alternative AR(1) Processes

	(ρ, σ_u^2)	β	$MPK < g$	θ_{opt}
Chang et al. (2019)	$(0.975, 0.06)$	0.9915	$\{0.0, 0.6\}$	0.4
Floden and Linde (2001)	$(0.9136, 0.0426)$	1.0069	$\{0.0, 0.6\}$	0.3
French (2005)	$(0.977, 0.0141)$	0.9915	$\{0.0, 0.6\}$	0.5
Krueger, Mitman, and Perri (2016)	$(0.9, 0.05)$	0.9915	$\{0.0, 0.6\}$	0.3

According to the numerical findings reported in Table 6, dynamic inefficiency is present in all balanced growth paths up to and including a replacement rate of 60%, similar to the baseline quantitative findings. In addition, an unfunded social security reduces the magnitude of this dynamic inefficiency and raises welfare.

6 Concluding Remarks

An earlier literature shows that introducing a non-produced fixed asset such as land in an overlapping generations model that is otherwise capable of generating dynamic inefficiency eliminates this dynamic inefficiency. The price of the fixed factor absorbs the increased savings and the capital stock asymptotes to the Golden Rule level.[18]

However, Kim and Lee (1997) show that when a property tax on land is introduced, an overlapping generations model with land can generate dynamic inefficiency. They use a simple 2-period overlapping generations model with capital and land as productive factors and show that an arbitrage condition for the returns to these two assets rules out dynamic inefficiency in the absence of a transaction tax, but with a transaction tax on land, the same condition allows for the net return to capital to be less than the growth rate of output, creating dynamic inefficiency. As a result, a government intervention such as a tax-transfer scheme can potentially improve long run welfare by moving the economy toward the golden rule capital stock at which the return to capital (and land) equals the growth rate of output.

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[18] İmrohoroğlu, İmrohoroğlu, and Joines (1999) implement this idea in a general equilibrium model populated with overlapping generations of individuals, calibrate the model to certain features of the U.S. economy, and generate balanced growth paths in which dynamic ineffiiciency is eliminated, canceling any role for an unfunded social security system that helps remove dynamic inefficiency in the absence of land in the model.

This paper implements Kim and Lee (1997)'s idea in a large-scale heterogenous-agent, general equilibrium model populated with overlapping generations, calibrates the model to U.S. flow and stock data from 1960-2018, and shows quantitatively that a transaction tax produces balanced growth paths that exhibit dynamic inefficiency despite the presence of land as a fixed factor. Furthermore, an unfunded social security system is optimal in the long run in part because it eliminates this dynamic inefficiency.

In the baseline model, a 6.18% tax on land transactions, corresponding to a land's output share of 5%, produces dynamic inefficiency in the laissez faire economy and other balanced growth paths with a replacement rate less than 70% which is the optimal size of the tax-transfer scheme considered in the paper. A sensitivity analysis using different calibration targets for the wealth-output ratio or different stochastic processes for the idiosyncratic wage risk yields similar numerical findings. Furthermore, different relative risk aversion parameters or a land's share parameter as low as 2% produce balanced growth paths with dynamic inefficiency and an unfunded social security system can be optimal in the long run. With a rising income share of land in most advanced economies, these findings suggest incorporating land in quantitative models to explore the role of government policy may have different policy implications.

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A Definition of Stationary Equilibrium

Consider a discrete set of asset holding choices on the grid $D = \{d_1, d_2, \ldots, d_m\}$. A *Stationary Equilibrium* with land for a given set of government policy parameters $\{\theta, \tau_s, \xi, \kappa, \sigma, \}$ and stochastic income process $\{\varphi, \Pi\}$ is a list of value functions $V_j(a, \varphi)$, policy rules $C_j : D \times \Lambda \to R_+$, and $A_j : D \times \Lambda \to D$, measures of agent types $\lambda_j(a, \varphi)$ for each age $j = 1, 2, \ldots, J$, a wage rate $w > 0$, return to capital r, and a price of land $P > 0$, such that

i. aggregate wealth and labor input are given by:

$$K + P = \sum_j \sum_a \sum_\varphi \mu_j \lambda_j(a, \varphi) A_{j-1}(a, \varphi) \text{ and } N = \sum_{j=1}^{j_R-1} \sum_a \mu_j \lambda_j(a, \varphi) \varepsilon_j,$$

ii. the wage rate and the return to capital $\{w, r\}$ solve the firm's profit maximization problem by satisfying equation (2),

iii. the price of land $P > 0$ is given by equation (12) in the text, and the transaction costs are rebated back to the individuals in a lump-sum fashion $\sigma = \kappa P$,

iv. given factor prices $\{w, r, P\}$, government policy $\{\theta, \tau_s, \xi, \kappa, \sigma, \}$ stochastic income process $\{\varphi, \Pi\}$, and lump-sum transfers ξ and σ, the policy rules $C_j(a, \varphi), A_j(a, \varphi)$ solve the agents' dynamic program (10),

v. the commodity market clears,

$$\begin{aligned} &\sum_j \sum_a \sum_\varphi \mu_j \lambda_j(a, \varphi) \Big[C_j(a, \varphi) + A_j(a, \varphi) \Big] \\ &= Y + (1 - \delta) \sum_j \sum_a \sum_\varphi \mu_j \lambda_j(a, \varphi) A_{j-1}(a, \varphi), \end{aligned}$$

where the initial wealth of agents, A_0, is taken as given,

vi. the set of age-dependent, time-invariant measures $\lambda_j(a, \varphi)$ for $j = 1, 2, \ldots, J$, satisfies

$$\lambda_j(a', \varphi') = \sum_\varphi \sum_{a : a' = A_j(a, \varphi)} \Pi(\varphi', \varphi) \lambda_{j-1}(a, \varphi),$$

where the initial measure of agents at birth, λ_1, is taken as given,

vii. the social security system is self-financing:

$$\tau_s = \frac{\sum_{j=j_R}^{J} \sum_a \mu_j \lambda_j(a, \varphi) b}{\sum_{j=1}^{j_R-1} \sum_a \mu_j \lambda_j(a, \varphi) w \varepsilon_j \bar{\varphi}},$$

ix. the lump-sum distribution of accidental bequests is given by

$$\xi = \sum_j \sum_a \sum_\varphi \mu_j \lambda_j(a, \varphi)(1 - \psi_{j+1}) A_j(a, \varphi).$$

B Calculating Flows and Stocks

In order to calculate the labor share η, the rate of depreciation of capital δ, and the capital-output K/Y and wealth-output $(K+P)/Y$ ratios, I follow Cooley and Prescott (1995) and Koh, Santaeulàlia-Llopis, and Zheng (2020). I use data from 1960-2018, and split ambiguous income such as proprietor's income (PI) and taxes (less subsidies) on production and imports (TP) into labor versus capital according to the income share:

$$\eta = 1 - [\text{GOS} - \eta(\text{PI} + \text{TP})]/Y,$$

where Y is real Gross Domestic Product (GDP). This yields $\eta = 0.65$. For details and the data used, see Koh, Santaeulàlia-Llopis, and Zheng (2020) and their Stata file which performs all the calculations in their paper.

BEA reports values of the U.S. Capital stock in their Fixed Asset Tables and I take K from their Table 1.1 Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods.

To calculate the value of land, I use the Financial Accounts of the U.S. from the Board of Governors of the Federal Reserve System. In particular, the Balance Sheets tables 101 (for households and nonprofit organizations), 103 (for nonfinancial corporate business), and 104 (for nonfinancial noncorporate business) show the real estate holdings of these entities at market value. Using the residual mehod, I subtract the value of the structures to obtain an estimate of the value of land.[19]

Table B.1 reports the values of stocks of capital and land, and their sum, wealth.

Table B.1: U.S. Data Adjustments to Compare to Model, stocks averaged 1960-2018

Capital	3.240
Fixed assets, private	2.182
Fixed assets, public	0.738
Consumer durables	0.320
Land	0.830
Households and nonprofit organizations	0.416
Nonfinancial corporate business	0.131
Nonfinancial noncorporate business	0.283
Wealth	4.070

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[19] Davis (2009) estimates the value of non-government land to be about 11 trillion; with the 2008 GDP at about \$14.7 trillion, this gives a land to GDP ratio of about 75%.