



The Canon Institute for Global Studies

CIGS Working Paper Series No. 19-005E

Optimal taxation with directed search and moral hazard

Tomoyuki Nakajima
The University of Tokyo / The Canon Institute for Global Studies

July 3, 2019

※Opinions expressed or implied in the CIGS Working Paper Series are solely those of the author, and do not necessarily represent the views of the CIGS or its sponsor.
※CIGS Working Paper Series is circulated in order to stimulate lively discussion and comments.
※Copyright belongs to the author(s) of each paper unless stated otherwise.

General Incorporated Foundation

The Canon Institute for Global Studies

一般財団法人 キヤノングローバル戦略研究所

Phone: +81-3-6213-0550 <https://www.canon-igs.org>

Optimal taxation with directed search and moral hazard

Tomoyuki Nakajima*

July 3, 2019

Abstract

I consider an optimal taxation problem in a directed search model with moral hazard. I show how a constrained efficient allocation can be achieved as an equilibrium allocation with unemployment benefits, income taxes, and subsidies for job creation. In particular, optimal income taxes are lump sum when the social welfare function is utilitarian, but they take the form of nonlinear income taxes when it is non-utilitarian. Furthermore, the income tax function has a simple form, with a clear relationship with the social welfare function, which makes a sharp contrast with the formula obtained in the standard Mirrleesian taxation literature.

Keywords: optimal taxation; moral hazard; directed search; competitive search.

JEL Classification numbers: D82; D86; H2; J64; J65.

*Department of Economics, University of Tokyo, and the Canon Institute for Global Studies. Email: tomoyuki.nakajima@gmail.com. I thank seminar participants at Nagoya University, Doshisha University, and Kyoto University for helpful comments. Financial support from the Inamori Foundation, and the JSPS Grant-in-Aid for Scientific Research is gratefully acknowledged (16H02026, 18H03638).

I Introduction

In this paper I augment the directed search model of Moen (1997) and Acemoglu and Shimer (1999) with moral hazard and examine how to design a tax policy in order to achieve a socially desirable allocation of resources.

Using this framework, I consider the following kinds of heterogeneity of individuals (workers) as potential motives for redistribution. First, workers are heterogeneous in productivity. This is viewed as *ex-ante* heterogeneity in the sense that it exists before workers are employed. The productivity of each worker is also called his/her type. To keep the model simple, I assume that each worker's type is observable. The second source of heterogeneity is employment status, *i.e.*, employed or unemployed.¹ It arises because the matching technology is stochastic. Some workers are successfully matched with firms, but others fail and become unemployed. The third source of heterogeneity is the level of output produced by each employed worker. It is determined by the worker's productivity, his/her effort, and exogenous shocks. I assume that workers' effort is their private information, thus creating a moral hazard problem between the firm and the worker.

Workers and firms are matched through a directed search process. In the search market, when creating a vacancy, each firm offers a wage contract that describes the wage payment as a function of output produced by the worker. It is designed so as to provide the worker with incentives and insurance in the profit maximizing way. The fact that firms provide incentives and insurance has important implications for the way income should be taxed. Indeed, the optimal tax formula derived in this paper is very different from that found in the standard Mirrleesian literature,² where the wage rate is simply given by the marginal product of labor.

The optimal taxation problem is examined using the mechanism design approach, that is, a constrained Pareto problem is defined, and then taxes are constructed so as to implement the solution to the Pareto problem. I assume that the planner in the Pareto problem is constrained in that he/she is subject to the same agency problem (moral hazard problem) as the firms in the competitive economy. Regarding the social welfare function (the planner's objective function), I consider two specifications: One is referred to as the "utilitarian" case, in which the social welfare function is given by a weighted average of (*ex-ante*) expected utilities of workers; the other is "non-utilitarian," in which the planner cares about the *ex-post* distribution of consumption, in addition to the *ex-ante* distribution of expected utilities.

¹I focus on equilibria where all workers participate the labor market.

²Initiated by Mirrlees (1971), this literature is very large. Relatively recent work includes Saez (2001), Farhi and Werning (2013), Golosov, Troskin, and Tsyvinski (2016), Saez and Stantcheva (2016), among many others. A survey is given by Piketty and Saez (2013).

In the utilitarian case, the constrained efficient allocation can be implemented by a combination of lump-sum income taxes, subsidies to create vacancies, and unemployment benefits. Income taxes should be lump-sum, because, otherwise, there would be inefficient distortions in the incentives/insurance that firms provide to workers. The need of unemployment benefits is clear. Probably less obvious is the need to subsidize vacancy creations. The level of vacancy creation in a competitive equilibrium tends to be inefficiently low because its value to the government budget is not taken into account (fiscal externality). Additional employment improves the government budget by reducing the spending on unemployment benefits, and increasing the revenue from income taxes.

In the non-utilitarian case, the constrained efficient allocation can be achieved as a competitive equilibrium with the same set of taxes/subsidies, except that income taxes are no longer lump-sum, and given by a (non-linear) function of the income level. In the Mirrleesian literature, the optimal tax function typically takes a very complicated form. In my model, the optimal income tax function has a very transparent and intuitive relationship with the social welfare function. This makes a sharp contrast with the existing literature.

Related literatures: This paper is related to several literatures. First, it is related to the literatures on directed (or competitive) search and optimal taxation. Both literatures are large, but, to my knowledge, there are not many papers which examine the optimal taxation in the directed search model.

First, Acemoglu and Shimer (1999) consider an optimal pair of lump-sum income taxes and unemployment benefits. But their tax policy does not have vacancy subsidies, and thus, there is a room for welfare improvement.

Second, Golosov, Maziero, and Menzio (2013) study an optimal taxation problem in a directed search model. They find that the constrained efficient allocation under the utilitarian social welfare function can be implemented by a pair of unemployment benefits and non-linear income taxes, which are increasing but regressive in income. Key differences between their model and mine are that in their model, (i) the number (measure) of firms is limited so that the free entry condition for firms does not bind in equilibrium; (ii) firms are heterogeneous in productivity, which is observable; (iii) workers are identical and no uncertainty remains after matches are formed; and (iv) workers' job applications are their private information. Our result that income taxes should be lump-sum (under the utilitarian social welfare function) makes a sharp contrast to their result, and indicates the importance of the source of income differences to determine the income tax function.

The literature on directed search with informational frictions includes, among others, Guerrieri, Shimer, and Wright (2010) and Moen and Rosen (2011). Guerrieri, Shimer, and Wright (2010) analyze a directed search equilibrium with adverse selection. Moen and Rosen (2011) study a directed search equilibrium with moral hazard. Their focus is on the characterization of competitive equilibrium and they do not consider tax policy. In addition, they assume risk-neutral workers.

Concerning moral hazard, I use the continuous-time approach initiated by Holmstrom and Milgrom (1987), and further developed by Schättler and Sung (1993); DeMarzo and Sannikov (2006); Sannikov (2008); Cvitanic, Wan, and Zhang (2009); Williams (2011); Cvitanic and Zhang (2013); Williams (2015), among others. Among these, my model is closely related to Cvitanic, Wan, and Zhang (2009). Their model allows me to obtain the profit-maximizing wage contract offered by the firm almost in closed form, even though the utility function of workers are a general concave function of consumption (not exponential).

My paper is also related to the literature on optimal taxation which considers the interaction of private markets and tax policy. For instance, Golosov and Tsyvinski (2007), Chetty and Saez (2010), and Krueger and Perri (2011) examine (optimal) tax policies when private insurance firms provide insurance to workers, at least to some extent. Stantcheva (2014) considers an optimal taxation problem in a Miyazaki–Wilson–Spence model of adverse selection.

The rest of the paper is organized as follows. Section 2 describes the baseline setup of the model economy and define the laissez-faire competitive equilibrium. Section 3 analyzes the constrained efficient allocation with the utilitarian planner. Section 4 demonstrates how to implement the constrained efficient allocation obtained in section 3 as a competitive equilibrium with taxes. Section 5 shows nonlinear income taxes can be optimal if the planner is non-utilitarian. Section 6 gives conclusion. All the proofs are given in the appendix.

2 Laissez-faire economy

We consider an economy that starts at time 0 and ends at time 1. There is a continuum of workers with measure one. There are I types of workers, indexed by $i \in \mathbb{I} \equiv \{1, \dots, I\}$. Let ϕ_i be the fraction of type i workers: $\phi_i > 0$, for all $i \in \mathbb{I}$, and $\sum_i \phi_i = 1$. Types are observable. Every agent is endowed with A units of initial wealth at time 0. It can be stored until time 1 with gross rate of return of unity, and then is transformed one-to-one into consumption goods. Alternatively, it can be invested in mutual funds, which, in turn, invest in firms. The initial wealth, A , is common across all types.

There is a continuum of potential firms. Potential firms are identical. The number (measure) of

potential firms is large enough that the free entry condition always binds in equilibrium.

Workers may or may not be employed. They are matched with firms through a directed search process as in Moen (1997) and Acemoglu and Shimer (1999). Each worker (firm) can be matched with at most one firm (worker). After a match is formed, output is produced continuously over time for $t \in [0, 1]$. The production of output is affected by stochastic shocks, as well as effort of the worker. The effort process is unobservable, so that the relationship between the firm and the worker involves moral hazard. We first describe this moral hazard problem, and then the equilibrium in the search market.

2.1 Moral hazard

Consider a matched pair of a firm and a worker of some type $i \in \mathbb{I}$. Our formulation of the moral hazard problem builds on the model discussed in Cvitanic, Wan, and Zhang (2009) and Cvitanic and Zhang (2013).

Let $Y_{i,1}$ denote the total amount of output produced by the pair from time 0 to time 1. It is a random variable and determined as the time 1 value of a stochastic process $Y_i = (Y_{i,t})_{t \in [0,1]}$, which is simply referred to as the “output process.” The output process is observable by both parties. Its distribution is affected by the worker’s effort as well as his/her type (ability). The worker’s ability is public information, and determines the initial value $Y_{i,0} = y_i \in \mathbb{R}_{++}$. Since the worker’s ability matters only for the initial value of the output process, we identify y_i as the worker’s type. The worker provides effort continuously over time, which is described by a stochastic process $\epsilon = (\epsilon_t)_{t \in [0,1]}$. Here, $\epsilon_t \in \mathbb{R}_+$ denotes the level of effort, which is the worker’s private information and not observed by the firm.

The dependence of the distribution of output on the worker’s effort is modeled using the “weak formulation.”³ For this, we first determine the set of sample paths for the output process Y_i . Specifically, let the output process be given as the (strong) solution to the stochastic differential equation:

$$dY_{i,t} = \sigma Y_{i,t} dB_t, \quad Y_{i,0} = y_i, \quad (1)$$

where B_t is a standard Brownian motion in some probability space (Ω, \mathcal{F}, P) , and $\sigma \in \mathbb{R}_{++}$ is a constant parameter. Let $\mathbf{F}^B = (\mathcal{F}_t^B)_{t \in [0,1]}$ be the augmented filtration generated by B .

Here, the probability measure P is interpreted as the one corresponding to the benchmark case

³See, for instance, Sannikov (2008), Cvitanic, Wan, and Zhang (2009), Williams (2011), Williams (2015), among others. The weak formulation is a natural extension of the approach taken in the static moral hazard models, where the worker’s effort determines the probability distribution of output.

where the worker provides no effort at all: $\epsilon_t = 0$, for all $t \in [0, 1]$. More generally, let the effort process, $\epsilon = (\epsilon_t)_{t \in [0,1]}$, be an \mathbf{F}^B -adapted stochastic process. We provide in the appendix the technical assumptions imposed on the effort process (and the contract offered by the firm).

Since the Girsanov Theorem holds,⁴ define

$$B_t^\epsilon \equiv B_t - \int_0^t \epsilon_s ds; \quad (2)$$

$$M_t^\epsilon \equiv \exp \left(\int_0^t \epsilon_s dB_s - \frac{1}{2} \int_0^t \epsilon_s^2 ds \right); \quad (3)$$

and a new probability measure P^ϵ by

$$\frac{dP^\epsilon}{dP} \equiv M_1^\epsilon. \quad (4)$$

Then, B^ϵ is a standard Brownian motion under P^ϵ , and

$$\begin{aligned} dY_{i,t} &= \sigma Y_{i,t} dB_t, \\ &= \sigma \epsilon_t Y_{i,t} dt + \sigma Y_{i,t} dB_t^\epsilon. \end{aligned}$$

Intuitively, by providing effort ϵ , the worker controls the probability measure P^ϵ of the output process Y . Formally, $(Y_i, B^\epsilon, P^\epsilon)$ is a weak solution to the stochastic differential equation: $dY_t = \sigma \epsilon_t Y_t dt + \sigma Y_t dW_t$.

Let \mathbb{E} and \mathbb{E}^ϵ denote the expectation operators corresponding to the probability measures P and P^ϵ , respectively. Then, given an effort process ϵ , the expected amount of output at time 1 is

$$\mathbb{E}^\epsilon[Y_{i,1}] = \mathbb{E}[M_1^\epsilon Y_{i,1}].$$

In particular, if $\epsilon_t = 0$, for all $t \in [0, 1]$, then we have $M_1^\epsilon \equiv 1$ and the expected amount of output and its variance are given, respectively, by

$$\mathbb{E}[Y_{i,1}] = y_i, \quad \text{and} \quad \text{Var}[Y_{i,1}] = y_i^2 (e^{\sigma^2} - 1).$$

2.1.1 Problem of the worker

Let w_i be the wage payment that the worker receives from the firm at time 1. It depends on the history of her output, $Y = (Y_t)_{t \in [0,1]}$, and thus it is specified as a random variable defined on the measurable space (Ω, \mathcal{F}_1) . The amount of consumption of the worker at date 1 is given by a random variable $C_i = A + w_i$, from which she receives utility $u(C_i)$. The function $u(\cdot)$ satisfies

⁴Lemma 7.1 in Cvitanic, Wan, and Zhang (2009). For the Girsanov Theorem, see, for instance, section 3.5 of Karatzas and Shreve (1998).

the usual conditions: it is twice continuously differentiable with $u' > 0$ and $u'' < 0$, and satisfies the Inada condition, $\lim_{c \rightarrow 0} u'(c) = +\infty$ and $\lim_{c \rightarrow \infty} u'(c) = 0$.

Supplying effort yields disutility. For the sake of tractability, we assume that the disutility of providing effort ϵ_t at each $t \in [0, 1]$ is quadratic, and given by $\frac{\chi}{2}\epsilon_t^2$, where $\chi \in \mathbb{R}_{++}$ is a constant parameter.

Then the type- i worker's expected utility evaluated at time 0, $V_{i,E}$, is given by

$$V_{i,E} \equiv \mathbb{E}^\epsilon \left[u(C_i) - \int_0^1 \frac{\chi}{2} \epsilon_t^2 dt \right] = \mathbb{E} \left[M_1^\epsilon \left\{ u(C_i) - \int_0^1 \frac{\chi}{2} \epsilon_t^2 dt \right\} \right].$$

Similarly, for $t \in [0, 1]$, define the “remaining” utility process, $V_{i,E,t}$ by

$$V_{i,E,t} \equiv \mathbb{E}_t^\epsilon \left[u(C_i) - \int_t^1 \frac{\chi}{2} \epsilon_s^2 ds \right] = \mathbb{E}_t \left[M_1^\epsilon \left\{ u(C_i) - \int_t^1 \frac{\chi}{2} \epsilon_s^2 ds \right\} \right].$$

Thus, $V_{i,E} = V_{i,E,0}$.

We can apply the extended Martingale Representation Theorem⁵ to show that there exists a P^ϵ -square integrable \mathbf{F}^B -adapted process $Z_i^\epsilon = (Z_{i,t}^\epsilon)_{t \in [0,1]}$ such that the worker's remaining utility is expressed as

$$\begin{aligned} V_{i,E,t} &= u(C_i) - \int_t^1 \frac{\chi}{2} \epsilon_s^2 ds - \int_t^1 Z_{i,s}^\epsilon dB_s^\epsilon, \\ &= u(C_i) - \int_t^1 \left(\frac{\chi}{2} \epsilon_s^2 - Z_{i,s}^\epsilon \epsilon_s \right) ds - \int_t^1 Z_{i,s}^\epsilon dB_s. \end{aligned}$$

Let $\epsilon_i = (\epsilon_{i,t})_{t \in [0,1]}$ denote the optimal effort process of the worker. Given the wage payment w_i , the type- i worker chooses ϵ_i as⁶

$$\epsilon_{i,t} = \frac{1}{\chi} Z_{i,t}^\epsilon. \quad (5)$$

The optimal effort process ϵ_i implies that the worker's remaining utility process is given by

$$dV_{i,E,t} = -\frac{\chi}{2} \epsilon_{i,t}^2 dt + \chi \epsilon_{i,t} dB_t$$

It follows that⁷

$$d \exp \left(\frac{V_{i,E,t}}{\chi} \right) = \exp \left(\frac{V_{i,E,t}}{\chi} \right) \epsilon_{i,t} dB_t. \quad (6)$$

⁵Lemma 3.1 in Cvitanic, Wan, and Zhang (2009), Lemma 10.4.6 in Cvitanic and Zhang (2013).

⁶Theorem 3.1 in Cvitanic, Wan, and Zhang (2009).

⁷Applying Itô's formula,

$$d \exp \left(\frac{V_{i,E,t}}{\chi} \right) = \exp \left(\frac{V_{i,E,t}}{\chi} \right) \left\{ \frac{1}{\chi} dV_{i,E,t} + \frac{1}{2\chi^2} (dV_{i,E,t})^2 \right\} = \exp \left(\frac{V_{i,E,t}}{\chi} \right) \epsilon_{i,t} dB_t.$$

Let P^{ϵ_i} denote the probability measure associated with the optimal effort process ϵ_i , and $M_1^{\epsilon_i}$ the corresponding density. Comparing (3) and (6), we obtain

$$M_t^{\epsilon_i} = \frac{\exp\left(\frac{V_{i,E,t}}{\chi}\right)}{\exp\left(\frac{V_{i,E}}{\chi}\right)} = \exp\left(-\frac{V_{i,E}}{\chi}\right) \exp\left(\frac{V_{i,E,t}}{\chi}\right)$$

By definition, $V_{i,E,1} = u(C_i) = u(A + w_i)$, and thus, the density $M_1^{\epsilon_i}$ is written as a function of $u(A + w_i)$ and $V_{i,E}$:

$$M_1^{\epsilon_i} = \exp\left(-\frac{V_{i,E}}{\chi}\right) \exp\left(\frac{u(A + w_i)}{\chi}\right) \quad (7)$$

This equation explicitly tells us how the contract (payment), w_i , affects the probability distribution (density) of output, Y , through its influence on the worker's choice of effort, ϵ_i .

2.1.2 Problem of the firm

The firm's profit realized at time 1 is $Y_{i,1} - w_i$. Let $V_{i,F}$ be its expected value evaluated at $t = 0$, given the worker's effort process, $\epsilon_i = (\epsilon_{i,t})_{t \in [0,1]}$:

$$V_{i,F} = \mathbb{E}^{\epsilon_i} [Y_{i,1} - w_i] = \mathbb{E} [M_1^{\epsilon_i} (Y_{i,1} - w_i)],$$

where the density $M_1^{\epsilon_i}$ depends on the payment w_i as shown in equation (7).

Let $\mathcal{U} \equiv [u(A), u(\infty))$. Given that the firm has to guarantee the expected utility, $V_{i,E} = v \in \mathcal{U}$, to the worker, the firm chooses w_i so as to maximize its expected profit $V_{i,F}$.⁸ That is,

$$V_{i,F}(v) = \sup_w \mathbb{E} \left[\exp\left(-\frac{v}{\chi}\right) \exp\left(\frac{u(A + w)}{\chi}\right) (Y_{i,1} - w) \right] \quad (8)$$

$$\text{s.t. } \mathbb{E} \left[\exp\left(-\frac{v}{\chi}\right) \exp\left(\frac{u(A + w)}{\chi}\right) \right] = 1. \quad (9)$$

Here, $V_{i,F}(v)$ is the maximized profit of the firm given the worker's expected utility $V_{i,E} = v$.

Letting λ_i be the Lagrange multiplier for (9), the Lagrangian is constructed as

$$\begin{aligned} \mathcal{L} = & \mathbb{E} \left[\exp\left(-\frac{v}{\chi}\right) \exp\left(\frac{u(A + w)}{\chi}\right) (Y_{i,1} - w) \right] \\ & + \lambda_i \left\{ \mathbb{E} \left[\exp\left(-\frac{v}{\chi}\right) \exp\left(\frac{u(A + w)}{\chi}\right) \right] - 1 \right\} \end{aligned}$$

The first-order condition is

$$\frac{1}{\chi} u' [A + w_i(Y_{i,1})] [Y_{i,1} - w_i(Y_{i,1}) + \lambda_i] = 1, \quad \forall Y_{i,1} > 0. \quad (10)$$

⁸See the appendix for the assumptions that feasible wage contracts must satisfy.

The multiplier λ_i is determined so that the solution to the above equation $w_i(\cdot)$ satisfies

$$\mathbb{E}_i \left[\exp \left(-\frac{v}{\chi} \right) \exp \left(\frac{u[A + w_i(Y_{i,1})]}{\chi} \right) \right] = 1.$$

The following assumption is made throughout the paper.

Assumption 1. For each $i \in \mathbb{I}$ and $x \in \mathbb{R}_+$,

$$\mathbb{E} \left[\exp \left(\frac{u(Y_{i,1} + x)}{\chi} \right) \right] < \infty.$$

Lemma 1. Consider the maximization problem given by (8). Assume that $V_{i,F}(\underline{u}) < +\infty$. Then, there is a unique solution w_i for each $i \in \mathbb{I}$ and $v \in \mathcal{U}$. The profit function $V_{i,F} : \mathcal{U} \rightarrow \mathbb{R}$ is a continuous and strictly decreasing function.

2.2 Directed search

A match between a firm and a worker is formed through a directed search process (Moen (1997); Acemoglu and Shimer (1999)). The funds which are required to create a vacancy are provided by mutual funds. Firms' profits are distributed as dividends to the mutual funds. Assuming that the law of large number applies, each firm makes a decision to maximize the expected profit.

If a firm decides to create a vacancy, it has to pay a fixed (and sunk) cost, k . A vacancy specifies the type of the worker, $i \in \mathbb{I}$, that the firm is looking for, and the promised utility level, $v \in \mathcal{U}$. That is, the labor market is segmented into submarkets indexed by i and v . Firms can commit themselves to the contract they offer. As we have seen above, for each $i \in \mathbb{I}$ and $v \in \mathcal{U}$, there is a profit maximizing wage contract $w_i(\cdot)$ and a utility maximizing effort process $\epsilon_i = (\epsilon_{i,t})_{t \in [0,1]}$.

For each $i \in \mathbb{I}$ and $v \in \mathcal{U}$, let $\theta_i(v)$ be the market tightness in the submarket (i, v) . That is, $\theta_i(v)$ is the ratio of the size (measure) of vacancies to the size (measure) of applicants in submarket (i, v) . Both firms and workers take the function $\theta_i(\cdot)$, $i \in \mathbb{I}$, as given.

A match is formed stochastically via a linearly homogeneous matching function. If a worker applies for a job with tightness θ , the probability that he/she is successfully matched with a firm is $\mu(\theta)$, where $\mu : \mathbb{R}_+ \cup \{+\infty\} \rightarrow [0, 1]$ is a strictly increasing function of θ with boundary conditions $\mu(0) = 0$ and $\mu(\infty) = 1$. Similarly, if a firm posts a vacancy in a submarket with tightness θ , its probability to be matched with a worker is $\eta(\theta)$, where $\eta : \mathbb{R}_+ \cup \{+\infty\} \rightarrow [0, 1]$ is a strictly decreasing function of θ with boundary conditions $\eta(0) = 1$ and $\eta(\infty) = 0$. Since the matching technology is linearly homogeneous, the functions μ and η satisfy the condition $\mu(\theta) = \theta\eta(\theta)$ for all θ . Unmatched workers or firms do not produce any output. We assume that the matching probability functions, μ and η , are common across different types of workers.

2.3 Equilibrium

To describe an equilibrium in the search market, we define an allocation by a tuple $(\gamma_i, \mathcal{V}_i, \Theta_i, \bar{U}_i)_{i \in \mathbb{I}}$, where γ_i is a measure on \mathcal{U} with support \mathcal{V}_i ; $\Theta_i : \mathcal{U} \rightarrow \mathbb{R}_+ \cup \{+\infty\}$, with $\Theta_i(v)$ denoting the market tightness in each submarket $i \in \mathbb{I}$ and $v \in \mathcal{U}$; and $\bar{U}_i \in \mathcal{U}$ is the type- i worker's overall utility level (taking into account the possibility of unemployment). The measure γ_i describes the distribution of vacancies targeted at type- i workers, and its support, \mathcal{V}_i , is the set of utility levels that are offered in the given allocation. Associated with each $v \in \mathcal{U}$, there exists a profit-maximizing wage contract $w_i(\cdot)$ and a utility-maximizing effort process $\epsilon_i = (\epsilon_{i,t})_{t \in [0,1]}$. The firm's profit is given by the function $V_{i,F}(v)$. A competitive equilibrium in the search market is defined in the standard way.⁹

Definition 1. Let $\underline{u} = u(A)$ and $\mathcal{U} = [\underline{u}, u^{-1}(\infty))$. A competitive equilibrium is an allocation $(\gamma_i, \mathcal{V}_i, \Theta_i, \bar{U}_i)_{i \in \mathbb{I}}$ that satisfies the following conditions:

- (i) Profit maximization and free entry: For all $i \in \mathbb{I}$ and $v \in \mathcal{U}$,

$$\eta(\Theta_i(v))V_{i,F}(v) \leq k,$$

with equality if $v \in \mathcal{V}_i$. Here, $V_{i,F}(v)$ is the profit function of a firm matched with a worker of type i given the promised utility level $v \in \mathcal{U}$, as defined in (8).

- (ii) Workers' optimal search: For each $i \in \mathbb{I}$, \bar{U}_i satisfies

$$\bar{U}_i = \max \left\{ \underline{u}, \max_{v \in \mathcal{V}_i} \mu(\Theta_i(v))v + [1 - \mu(\Theta_i(v))] \underline{u} \right\},$$

where $\bar{U}_i = \underline{u}$ if $\mathcal{V}_i = \emptyset$. Furthermore, for each $i \in \mathbb{I}$ and $v \in \mathcal{U}$,

$$\bar{U}_i \geq \mu(\Theta_i(v))v + [1 - \mu(\Theta_i(v))] \underline{u}$$

with equality if $\Theta_i(v) < \infty$ and $\gamma_i(v) > 0$.

- (iii) Market clearing: For each i ,

$$\int_{\mathcal{V}_i} \frac{1}{\Theta_i(v)} d\gamma_i(v) \leq \phi_i,$$

with equality if $\bar{U}_i > \underline{u}$.

⁹See, for instance, Acemoglu and Shimer (1999), Guerrieri, Shimer, and Wright (2010), Golosov, Maziero, and Menzio (2013), among many others.

Our first result is that the equilibrium allocations in this model are characterized as solutions to maximization problems.¹⁰ For each type $i \in \mathbb{I}$, consider the problem:¹¹

$$\begin{aligned} \max_{\theta \in [0, \infty), v \in \mathcal{U}} \quad & \mu(\theta)v + [1 - \mu(\theta)]\underline{u} \\ \text{s.t.} \quad & \eta(\theta)V_{i,F}(v) \geq k. \end{aligned} \tag{P}_i$$

Then we say that a tuple $(\mathcal{I}, (\bar{U}_i)_{i \in \mathbb{I}}, (\theta_i)_{i \in \mathcal{I}}, (V_{i,E})_{i \in \mathcal{I}})$ solves problem (P) if the following conditions hold: (i) $\mathcal{I} \subset \mathbb{I}$ denotes the set of i such that the constraint set of $(P)_i$ is nonempty and the maximized value is strictly greater than \underline{u} ; (ii) for each $i \in \mathcal{I}$, the pair $(\theta_i, V_{i,E})$ solves the problem $(P)_i$ and $\bar{U}_i = \mu(\theta_i)V_{i,E} + [1 - \mu(\theta_i)]\underline{u}$; and (iii) for any $i \notin \mathcal{I}$, $\bar{U}_i = \underline{u}$. To further simplify the argument, we make the following assumption in what follows.

Assumption 2. For all $i \in \mathbb{I}$, $y_i > k$, and there exists $\bar{v}_i \in \mathcal{U}$ such that $V_{i,F}(v) < k$ for all $v > \bar{v}_i$.

Lemma 2. *Problem (P) has a solution, $(\mathcal{I}, (\bar{U}_i)_{i \in \mathbb{I}}, (\theta_i)_{i \in \mathcal{I}}, (V_{i,E})_{i \in \mathcal{I}})$. It has properties such that (i) $\bar{U}_i > u(A)$ for all $i \in \mathbb{I}$, and thus $\mathcal{I} = \mathbb{I}$; (ii) $\theta_i \in (0, \infty)$ for all $i \in \mathcal{I}$; (iii) $\eta(\theta_i)V_{i,F}(V_{i,E}) = k$ for all $i \in \mathcal{I}$.*

Proposition 1.

(i) *Suppose $(\mathcal{I}, (\bar{U}_i)_{i \in \mathbb{I}}, (\theta_i)_{i \in \mathcal{I}}, (V_{i,E})_{i \in \mathcal{I}})$ solves Problem (P), where $\mathcal{I} = \mathbb{I}$. Then there exists a competitive equilibrium $(\gamma_i, \mathcal{V}_i, \Theta_i, \bar{U}_i)_{i \in \mathbb{I}}$ such that $\mathcal{V}_i = \{V_{i,E}\}$, $\gamma_i(\{V_{i,E}\}) = \phi_i \theta_i$, $\Theta_i(V_{i,E}) = \theta_i$.*

(ii) *Let $(\gamma_i, \mathcal{V}_i, \Theta_i, \bar{U}_i)_{i \in \mathbb{I}}$ be a competitive equilibrium. Then, for all $i \in \mathbb{I}$, $\bar{U}_i > \underline{u}$, and $0 < \Theta_i(V_{i,E}) < \infty$ for any $V_{i,E} \in \mathcal{V}_i$. Moreover, take any $(V_{i,E})_{i \in \mathbb{I}}$ and $(\theta_i)_{i \in \mathbb{I}}$ with $V_{i,E} \in \mathcal{V}_i$ and $\theta_i = \Theta_i(V_{i,E})$. Then $(\mathbb{I}, (\bar{U}_i)_{i \in \mathbb{I}}, (\theta_i)_{i \in \mathbb{I}}, (V_{i,E})_{i \in \mathbb{I}})$ solves problem (P).*

(iii) *A competitive equilibrium exists, and the equilibrium value of $(\bar{U}_i)_{i \in \mathbb{I}}$ is unique.*

To characterize the equilibrium, consider program $(P)_i$. Given the definition of $V_{i,F}$ in (8), it can

¹⁰This is a standard result in the literature on directed search. In particular, the statement of the result here follows Guerrieri, Shimer, and Wright (2010), although types in our model are not private information.

¹¹Since $\eta(\infty) = 0$, $\theta = \infty$ will never be a solution to this problem. We thus restrict the set of values for θ to $[0, \infty)$.

be restated as

$$\begin{aligned} \max_{w_i(\cdot), \theta_i, V_{i,E}} \quad & \mu(\theta_i)V_{i,E} + [1 - \mu(\theta_i)]u(A) \\ \text{s.t.} \quad & \eta(\theta_i) \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{1}{\chi} u[A + w_i(Y_{i,1})] \right\} \{Y_{i,1} - w_i(Y_{i,1})\} \right] \geq k, \\ & \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{1}{\chi} u[A + w_i(Y_{i,1})] \right\} \right] = 1. \end{aligned}$$

Letting $\zeta_{i,1}$ and $\zeta_{i,2}$ be the multipliers for the two constraints, the Lagrangian is constructed as

$$\begin{aligned} \mathcal{L} = & \mu(\theta_i)V_{i,E} + [1 - \mu(\theta_i)]u(A) \\ & + \zeta_{i,1} \left\{ \eta(\theta_i) \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{1}{\chi} u[A + w_i(Y_{i,1})] \right\} \{Y_{i,1} - w_i(Y_{i,1})\} \right] - k \right\} \\ & + \zeta_{i,2} \left\{ \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{1}{\chi} u[A + w_i(Y_{i,1})] \right\} \right] - 1 \right\} \end{aligned}$$

The profit of a firm matched with a type- i worker, $V_{i,F}$, is defined by

$$V_{i,F} = \mathbb{E}_i \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{1}{\chi} u[A + w_i(Y_1)] \right\} \{Y_1 - w_i(Y_1)\} \right]. \quad (\text{I1})$$

Then the first-order condition with respect to θ_i is given by

$$0 = \mu'(\theta_i)[V_{i,E} - u(A)] + \zeta_{i,1}\eta'(\theta_i)V_{i,F}. \quad (\text{I2})$$

By Lemma 2, we know that $\theta_i \in (0, \infty)$. Hence condition (I2) implies that $\zeta_{i,1} > 0$.

Define $\lambda_i \equiv \zeta_{i,2}/(\zeta_{i,1}\eta(\theta_i))$. Then, just as we have seen in (I0), the first-order condition for $w_i(Y_{i,1})$ is expressed as:

$$0 = \frac{1}{\chi} u'[A + w_i(Y_{i,1})] \{Y_{i,1} - w_i(Y_{i,1}) + \lambda_i\} - 1, \quad \forall Y_{i,1} > 0. \quad (\text{I3})$$

Using the definition of $V_{i,F}$, the first-order condition with respect to $V_{i,E}$ can be written as

$$0 = \mu(\theta_i) - \frac{1}{\chi} \zeta_{i,1} \eta(\theta_i) (V_{i,F} + \lambda_i) \quad (\text{I4})$$

Using equations (I2) and (I4), we can eliminate $\zeta_{i,1}$ and obtain

$$0 = \mu'(\theta_i)[V_{i,E} - u(A)] + \mu(\theta_i) \frac{\eta'(\theta_i)}{\eta(\theta_i)} \frac{V_{i,F}}{(V_{i,F} + \lambda_i)/\chi}. \quad (\text{I5})$$

The free entry condition binds so that

$$\eta(\theta_i)V_{i,F} = k. \quad (\text{I6})$$

Finally, the promised utility levels, $V_{i,E}$, must satisfy

$$\mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{1}{\chi} u[A + w_i(Y_{i,1})] \right\} \right] = 1. \quad (\text{I7})$$

To sum, the competitive equilibrium for the laissez-faire economy is given by $(V_{i,F}, V_{i,E}, \theta_i, \lambda_i, w_i(\cdot))_{i \in \mathbb{I}}$ that satisfies (I1), (I3), (I5), (I6), (I7).

3 Constrained efficient allocation

Here, we consider constrained efficient allocations. The planner can allocate resources so as to maximize the social welfare, but her ability to do so is constrained by the market. Specifically, we assume that the planner can create firms (vacancies) by paying the fixed cost k , and offer a type-dependent contract $(V_{i,E}, C_i)$ to workers, where $V_{i,E}$ is the promised level of utility for a type- i worker, and $C_i : Y_{i,1} \mapsto C_i(Y_{i,1})$ specifies the amount of consumption depending on the date-1 output. The planner cannot directly control the effort level of workers, and is also subject to the same matching technology $(\mu(\cdot), \eta(\cdot))$ as in the previous section. In addition, the planner makes transfers to unemployed individuals for each type, z_i . We assume that whether or not a worker applies for a job is public information, and that the eligibility for unemployment benefits is contingent on the participation in the job market.¹²

Formally, let $\alpha = (\alpha_1, \dots, \alpha_I)$ be the Pareto weights for workers of different types. As a normalization, we require $\alpha_i > 0$ for all i and $\sum_i \alpha_i = 1$. The planner chooses $(\theta_i, V_{i,E}, z_i, C_i(\cdot))_{i \in \mathbb{I}}$ so as to solve the following program (P*):

$$\max \sum_{i=1}^I \alpha_i \left\{ \mu(\theta_i) V_{i,E} + [1 - \mu(\theta_i)] u(A + z_i) \right\} \quad (\text{P}^*)$$

subject to

$$\begin{aligned} & \sum_{i=1}^I \phi_i \left\{ [1 - \mu(\theta_i)] (A + z_i) + \mu(\theta_i) \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[C_i(Y_{i,1})]}{\chi} \right\} C_i(Y_{i,1}) \right] + \theta_i k \right\} \\ & \leq A + \sum_{i=1}^I \phi_i \mu(\theta_i) \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[C_i(Y_{i,1})]}{\chi} \right\} Y_{i,1} \right], \\ & \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[C_i(Y_{i,1})]}{\chi} \right\} \right] = 1, \end{aligned} \quad (\text{I8})$$

$$\mu(\theta_i) V_{i,E} + [1 - \mu(\theta_i)] u(A + z_i) \geq u(A). \quad (\text{I9})$$

¹²It is straightforward to modify this assumption so that job application activity is private information of each worker. In that case, the participation constraints (I9) for workers in the planner's problem should be changed to $V_{i,E} \geq u(A + z_i)$ for all i .

We denote the solution to this program by $*$, which is also referred to as the “constrained efficient allocation.”

The first constraint in the planner’s problem is the resource constraint. For a later use, let us rewrite it as

$$\begin{aligned} & \sum_{i=1}^I \phi_i \mu(\theta_i) \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[C_i(Y_{i,1})]}{\chi} \right\} \left\{ A + Y_{i,1} - C_i(Y_{i,1}) \right\} \right] \\ & - \sum_{i=1}^I \phi_i \left([1 - \mu(\theta_i)] z_i + \theta_i k \right) \geq 0. \end{aligned} \quad (20)$$

The second constraint is the consistency between $V_{i,E}$ and $C_i(\cdot)$. The third one is the participation constraints for workers.

We have seen in Lemma 2 that $\mu(\theta_i) > 0$ holds for all $i \in \mathbb{I}$ in the laissez-faire equilibrium, that is, there are at least some employed workers for all types. For the ease of exposition, we want the same property to hold for the constrained efficient allocation. For this purpose, we add the following assumption, which may or may not be stronger than Assumption 2.

Assumption 3. For all $i \in \mathbb{I}$, $\mu'(0)y_i > k$.

Lemma 3. *Problem (P*) has a solution, $(\theta_i^*, V_{i,E}^*, z_i^*, C_i^*(\cdot))_{i \in \mathbb{I}}$, and $\theta_i^* \in (0, \infty)$ for all $i \in \mathbb{I}$.*

To solve (P*), construct the Lagrangian as

$$\begin{aligned} \mathcal{L} = & \sum_{i=1}^I \alpha_i \left\{ \mu(\theta_i) V_{i,E} + [1 - \mu(\theta_i)] u(A + z_i) \right\} \\ & + \xi_1 \left\{ \sum_{i=1}^I \phi_i \mu(\theta_i) \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[C_i(Y_{i,1})]}{\chi} \right\} \left\{ A + Y_{i,1} - C_i(Y_{i,1}) \right\} \right] \right. \\ & \quad \left. - \sum_{i=1}^I \phi_i \left([1 - \mu(\theta_i)] z_i + \theta_i k \right) \right\} \\ & + \sum_{i=1}^I \xi_{i,2} \left\{ \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[C_i(Y_{i,1})]}{\chi} \right\} \right] - 1 \right\} \\ & + \sum_{i=1}^I \xi_{i,3} \left\{ \mu(\theta_i) V_{i,E} + [1 - \mu(\theta_i)] u(A + z_i) - u(A) \right\}, \end{aligned}$$

which can be rewritten as

$$\begin{aligned} \mathcal{L} = & \sum_{i=1}^I (\alpha_i + \xi_{i,3}) \left\{ \mu(\theta_i) V_{i,E} + [1 - \mu(\theta_i)] u(A + z_i) \right\} \\ & + \xi_1 \left\{ \sum_{i=1}^I \phi_i \mu(\theta_i) \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[C_i(Y_{i,1})]}{\chi} \right\} \left\{ A + Y_{i,1} - C_i(Y_{i,1}) \right\} \right] \right. \\ & \quad \left. - \sum_{i=1}^I \phi_i \left([1 - \mu(\theta_i)] z_i + \theta_i k \right) \right\} \\ & + \sum_{i=1}^I \xi_{i,2} \left\{ \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[C_i(Y_{i,1})]}{\chi} \right\} \right] - 1 \right\} - \sum_{i=1}^I \xi_{i,3} u(A). \end{aligned}$$

From this, we can see that there is no loss of generality by assuming that the participation constraints (i9) in the planner's problem never bind.¹³ So, in what follows, we assume that $\xi_{i,3}^* = 0$ for all $i \in \mathbb{I}$.

The first-order conditions with respect to z_i , $V_{i,E}$, θ_i , and $C_i(Y_{i,1})$ for each $Y_{i,1}$ are given, respectively, by

$$\begin{aligned} 0 &= \alpha_i [1 - \mu(\theta_i^*)] u'(A + z_i^*) - \xi_1^* \phi_i [1 - \mu(\theta_i^*)], \\ 0 &= \alpha_i \mu(\theta_i^*) - \xi_1^* \phi_i \mu(\theta_i^*) \frac{1}{\chi} \mathbb{E} \left[\exp \left(-\frac{V_{i,E}^*}{\chi} \right) \exp \left\{ \frac{u[C_i^*(Y_{i,1})]}{\chi} \right\} \left\{ A + Y_{i,1} - C_i^*(Y_{i,1}) \right\} \right] \\ & \quad - \xi_{i,2}^* \frac{1}{\chi} \mathbb{E} \left[\exp \left(-\frac{V_{i,E}^*}{\chi} \right) \exp \left\{ \frac{u[C_i^*(Y_{i,1})]}{\chi} \right\} \right], \\ 0 &= \alpha_i \mu'(\theta_i^*) [V_{i,E}^* - u(A + z_i^*)] \\ & \quad + \xi_1^* \phi_i \left\{ \mu'(\theta_i^*) \mathbb{E} \left[\exp \left(-\frac{V_{i,E}^*}{\chi} \right) \exp \left\{ \frac{u[C_i^*(Y_{i,1})]}{\chi} \right\} \left\{ A + Y_{i,1} - C_i^*(Y_{i,1}) \right\} \right] + \mu'(\theta_i^*) z_i^* - k \right\}, \\ 0 &= \xi_1^* \phi_i \mu(\theta_i^*) \exp \left(-\frac{V_{i,E}^*}{\chi} \right) \exp \left\{ \frac{u[C_i^*(Y_{i,1})]}{\chi} \right\} \left[\frac{u'[C_i^*(Y_{i,1})]}{\chi} \left\{ A + Y_{i,1} - C_i^*(Y_{i,1}) \right\} - 1 \right] \\ & \quad + \xi_{i,2}^* \exp \left(-\frac{V_{i,E}^*}{\chi} \right) \exp \left\{ \frac{u[C_i^*(Y_{i,1})]}{\chi} \right\} \frac{u'[C_i^*(Y_{i,1})]}{\chi}. \end{aligned}$$

Define for each $i \in \mathbb{I}$

$$\lambda_i^* \equiv \frac{\xi_{i,2}^*}{\xi_1^* \phi_i \mu(\theta_i^*)},$$

which are well defined because $\xi_1^* > 0$ and $\theta_i^* > 0$ for all $i \in \mathbb{I}$. Define also $V_{i,F}^*$ and $\bar{\tau}_i^*$ by

$$V_{i,F}^* \equiv \frac{k}{\eta(\theta_i^*)}, \quad (21)$$

$$\bar{\tau}_i^* \equiv \mathbb{E} \left[\exp \left(-\frac{V_{i,E}^*}{\chi} \right) \exp \left\{ \frac{u[C_i^*(Y_{i,1})]}{\chi} \right\} \left\{ A + Y_{i,1} - C_i^*(Y_{i,1}) \right\} \right] - V_{i,F}^*. \quad (22)$$

¹³If $\xi_{i,3}^* > 0$ for some i in some constrained efficient allocation, then we can always redefine α 's so that the resulting solution remains the same but the participation constraints do not bind for any i (i.e., $\xi_{i,3}^* = 0$ for all i).

Then the first-order condition with respect to z_i implies that

$$u'(A + z_i^*) = \frac{\phi_i}{\alpha_i} \xi_1^*. \quad (23)$$

That is, the marginal utility of the unemployed, normalized by α_i/ϕ_i , is equated across all types.

The first-order conditions with respect to $V_{i,E}$, θ_i , and $C_i(Y_{i,1})$ are simplified, respectively, as

$$1 = u'(A + z_i^*) \frac{1}{\chi} (V_{i,F}^* + \bar{\tau}_i^* + \lambda_i^*), \quad (24)$$

$$0 = V_{i,E}^* - u(A + z_i^*) + u'(A + z_i^*) \left\{ \frac{\eta'(\theta_i^*)}{\eta(\theta_i^*)} \frac{\mu(\theta_i^*)}{\mu'(\theta_i^*)} V_{i,F}^* + \bar{\tau}_i^* + z_i^* \right\}, \quad (25)$$

$$1 = \frac{u'[C_i^*(Y_{i,1})]}{\chi} \left\{ A + Y_{i,1} + \lambda_i^* - C_i^*(Y_{i,1}) \right\}. \quad (26)$$

To summarize, the solution to the constrained efficient program (\mathbf{P}^*) is given by $(\xi_1^*, (\theta_i^*, V_{i,E}^*, C_i^*, V_{i,F}^*, \bar{\tau}_i^*, z_i^*, \lambda_i^*)_{i \in \mathbb{I}})$ satisfying (18), (21), (22), (23), (24), (25), (26) for $i \in \mathcal{I}^*$, and (20).

4 Implementation through taxes

In this section we take an efficient allocation, $(\xi_1^*, (\theta_i^*, V_{i,E}^*, C_i^*(\cdot), V_{i,F}^*, \bar{\tau}_i^*, z_i^*, \lambda_i^*)_{i \in \mathbb{I}})$, as given, and consider how to implement it as a competitive equilibrium with taxes and subsidies. Specifically, we consider unemployment benefits, income taxes, and subsidies for creating vacancies, which are all type dependent and lump sum. As in the previous section, unemployed workers can receive unemployment benefits only if they have participated in the job market.

Let z_i be the transfer to an unemployed worker of type i ; $\tau_{i,E}$ be the tax on the income created by a firm matched with a type- i worker; and $\tau_{i,v}$ be the subsidy for creating a vacancy targeted at type- i workers. We call $(z_i, \tau_{i,E}, \tau_{i,v})_{i \in \mathbb{I}}$ a fiscal policy.

Consider a match between a firm and a worker of type i . Given the promised value for the worker, v , the profit maximization problem of the firm is formulated as

$$V_{i,F}(v) = \max_{w_i(\cdot)} \mathbb{E} \left[\exp \left(-\frac{v}{\chi} \right) \exp \left\{ \frac{1}{\chi} u[A + w_i(Y_{i,1})] \right\} \left\{ Y_{i,1} - w_i(Y_{i,1}) \right\} \right] - \tau_{i,E} \quad (27)$$

$$\text{s.t. } \mathbb{E} \left[\exp \left(-\frac{v}{\chi} \right) \exp \left\{ \frac{1}{\chi} u[A + w_i(Y_{i,1})] \right\} \right] = 1. \quad (28)$$

As in the laissez-faire case, define $\underline{u} \equiv u(A)$ and $\mathcal{U} \equiv [\underline{u}, u^{-1}(\infty))$. Then a competitive equilibrium is defined as follows.

Definition 2. Let a fiscal policy be given by $(z_i, \tau_{i,E}, \tau_{i,v})_{i \in \mathbb{I}}$. A competitive equilibrium is an allocation $(\gamma_i, \mathcal{V}_i, \Theta_i, \bar{U}_i)_{i \in \mathbb{I}}$ that satisfies the following conditions:

(i) Profit maximization and free entry: For all $i \in \mathbb{I}$ and $v \in \mathcal{U}$,

$$\eta(\Theta_i(v))V_{i,F}(v) \leq k - \tau_{i,v},$$

with equality if $v \in \mathcal{V}_i$. Here, $V_{i,F}(v)$ is the profit function of a firm matched with a worker of type i when the promised utility level is $v \in \mathcal{U}$, as defined in (27).

(ii) Workers' optimal search: For each $i \in \mathbb{I}$, \bar{U}_i satisfies

$$\bar{U}_i = \max \left\{ \underline{u}, \max_{v \in \mathcal{V}_i} \mu(\Theta_i(v))v + [1 - \mu(\Theta_i(v))]u(A + z_i) \right\},$$

where $\bar{U}_i = \underline{u}$ if $\mathcal{V}_i = \emptyset$. Furthermore, for each $i \in \mathbb{I}$ and $v \in \mathcal{U}$,

$$\bar{U}_i \geq \mu(\Theta_i(v))v + [1 - \mu(\Theta_i(v))]u(A + z_i),$$

with equality if $\Theta_i(v) < \infty$ and $\gamma_i(v) > 0$.

(iii) Market clearing: For each i ,

$$\int_{\mathcal{V}_i} \frac{1}{\Theta_i(v)} d\gamma_i(v) \leq \phi_i,$$

with equality if $\bar{U}_i > \underline{u}$.

(iv) Government budget:

$$\sum_{i=1}^I \int_{\mathcal{V}_i} \left\{ \eta[\Theta_i(v)]\tau_{i,E} - \tau_{i,v} \right\} d\gamma_i(v) = \sum_{i=1}^I \left\{ \phi_i - \int_{\mathcal{V}_i} \eta[\Theta_i(v)] d\gamma_i(v) \right\} z_i.$$

We first discuss how to construct a candidate for the optimal taxes $(\hat{z}_i, \hat{\tau}_{i,E}, \hat{\tau}_{i,v})_{i \in \mathbb{I}}$, and then verify next that a competitive equilibrium associated with it is indeed constrained efficient.

For each $i \in \mathbb{I}$, consider the maximization problem (P_i) as in the laissez-faire economy:

$$\begin{aligned} & \max_{(\theta_i, V_{i,E}, w_i(\cdot))} \mu(\theta_i)V_{i,E} + [1 - \mu(\theta_i)]u(A + \hat{z}_i), & (P_i) \\ & \text{s.t.} \quad \eta(\theta_i)V_{i,F} \geq k - \hat{\tau}_{i,v}, \\ & \quad \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[A + w_i(Y_{i,1})]}{\chi} \right\} \right] = 1, \end{aligned}$$

where

$$V_{i,F} = \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[A + w_i(Y_{i,1})]}{\chi} \right\} \{Y_{i,1} - w_i(Y_{i,1})\} \right] - \hat{\tau}_{i,E}.$$

Suppose that the constraint set each (P_i) is nonempty. We use a hat ($\hat{\cdot}$) to denote the solution to this problem.

The Lagrangian for this problem is given by

$$\begin{aligned} \mathcal{L} = & \mu(\theta_i)V_{i,E} + [1 - \mu(\theta_i)]u(A + \hat{z}_i) \\ & + \zeta_{i,1} \left(\eta(\theta_i) \left\{ \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[A + w_i(Y_{i,1})]}{\chi} \right\} \{Y_{i,1} - w_i(Y_{i,1})\} \right] - \hat{\tau}_{i,E} \right\} - k + \hat{\tau}_{i,v} \right) \\ & + \zeta_{i,2} \left\{ \mathbb{E} \left[\exp \left(-\frac{V_{i,E}}{\chi} \right) \exp \left\{ \frac{u[A + w_i(Y_{i,1})]}{\chi} \right\} \right] - 1 \right\}. \end{aligned}$$

Define

$$\hat{\lambda}_i \equiv \frac{\hat{\zeta}_{i,2}}{\hat{\zeta}_{i,1}\eta(\hat{\theta}_i)},$$

which is well defined because $\hat{\zeta}_{i,1} > 0$ and $\eta(\hat{\theta}_i) > 0$ (given that we are implementing $\theta_i^* \in (0, \infty)$).

Then the first-order conditions with respect to $V_{i,E}$ and θ_i lead to

$$\begin{aligned} 0 &= \mu(\hat{\theta}_i) - \frac{1}{\chi}\hat{\zeta}_{i,1}\eta(\hat{\theta}_i)(\hat{V}_{i,F} + \hat{\tau}_{i,E} + \hat{\lambda}_i), \\ 0 &= \mu'(\hat{\theta}_i)[\hat{V}_{i,E} - u(A + \hat{z}_i)] + \hat{\zeta}_{i,1}\eta'(\hat{\theta}_i)\hat{V}_{i,F}. \end{aligned}$$

Eliminating $\hat{\zeta}_{i,1}$, we obtain

$$0 = \hat{V}_{i,E} - u(A + \hat{z}_i) + \chi \frac{\mu(\hat{\theta}_i) \eta'(\hat{\theta}_i)}{\mu'(\hat{\theta}_i) \eta(\hat{\theta}_i)} \frac{\hat{V}_{i,F}}{\hat{V}_{i,F} + \hat{\lambda}_i + \hat{\tau}_{i,E}}. \quad (29)$$

The first-order condition with respect to $w_i(Y_{i,t})$ for each $Y_{i,1}$ is written as

$$0 = \frac{1}{\chi}u'[A + \hat{w}_i(Y_{i,1})] \{Y_{i,1} - \hat{w}_i(Y_{i,1}) + \hat{\lambda}_i\} - 1. \quad (30)$$

Summarizing, the solution to program (P_i) , $(\hat{\theta}_i, \hat{V}_{i,E}, \hat{V}_{i,F}, \hat{\lambda}_i, \hat{w}_i)$, is determined by (29), (30), and

$$\eta(\hat{\theta}_i)\hat{V}_{i,F} = k - \hat{\tau}_{i,v} \quad (31)$$

$$\mathbb{E} \left[\exp \left(-\frac{\hat{V}_{i,E}}{\chi} \right) \exp \left\{ \frac{u[A + \hat{w}_i(Y_{i,1})]}{\chi} \right\} \right] = 1, \quad (32)$$

$$\hat{V}_{i,F} = \mathbb{E} \left[\exp \left(-\frac{\hat{V}_{i,E}}{\chi} \right) \exp \left\{ \frac{u[A + \hat{w}_i(Y_{i,1})]}{\chi} \right\} \{Y_{i,1} - \hat{w}_i(Y_{i,1})\} \right] - \hat{\tau}_{i,E}. \quad (33)$$

Note that

$$\begin{aligned} \hat{V}_{i,F} + \hat{\lambda}_i + \hat{\tau}_{i,E} &= \mathbb{E} \left[\exp \left(-\frac{\hat{V}_{i,E}}{\chi} \right) \exp \left\{ \frac{u[A + \hat{w}_i(Y_{i,1})]}{\chi} \right\} \{Y_{i,1} - \hat{w}_i(Y_{i,1}) + \hat{\lambda}_i\} \right] \\ &= E \left[\exp \left(-\frac{\hat{V}_{i,E}}{\chi} \right) \exp \left\{ \frac{u[A + \hat{w}_i(Y_{i,1})]}{\chi} \right\} \frac{\chi}{u'[A + \hat{w}_i(Y_{i,1})]} \right] > 0. \end{aligned}$$

It then follows from (29) that

$$\hat{V}_{i,E} - u(A + \hat{z}_i) = -\chi \frac{\mu(\hat{\theta}_i) \eta'(\hat{\theta}_i)}{\mu'(\hat{\theta}_i) \eta(\hat{\theta}_i)} \frac{\hat{V}_{i,F}}{\hat{V}_{i,F} + \hat{\lambda}_i + \hat{\tau}_{i,E}} > 0, \quad (34)$$

because $\mu' > 0$ and $\eta' < 0$.

In Proposition 2 below, we show that $(\hat{\theta}_i, \hat{V}_{i,E}, \hat{V}_{i,F}, \hat{\lambda}_i, \hat{w}_i)$ is attained as a competitive equilibrium. But, before doing so, let us see that we can choose a fiscal policy $(\hat{z}_i, \hat{\tau}_{i,E}, \hat{\tau}_{i,v})_{i \in \mathbb{I}}$ such that the collection of solutions to (P_i) , $i \in \mathbb{I}$, corresponds to the constrained efficient allocation.

Lemma 4. *Let a constrained efficient allocation, $(\xi_1^*, (\theta_i^*, V_{i,E}^*, C_i^*(\cdot), V_{i,F}^*, \bar{\tau}_i^*, z_i^*, \lambda_i^*)_{i \in \mathbb{I}})$, be given. Define $(\hat{z}_i, \hat{\tau}_{i,E}, \hat{\tau}_{i,v})_{i \in \mathbb{I}}$ by*

$$\hat{z}_i = z_i^*, \quad (35)$$

$$\hat{\tau}_{i,E} = \bar{\tau}_i^* - \frac{\mu'(\theta_i^*) \eta(\theta_i^*)}{\mu(\theta_i^*) \eta'(\theta_i^*)} (\bar{\tau}_i^* + z_i^*), \quad (36)$$

$$\hat{\tau}_{i,v} = \eta(\theta_i^*) (\hat{\tau}_{i,E} - \bar{\tau}_i^*). \quad (37)$$

Then, the solution to program (P_i) is given by $\hat{\theta}_i = \theta_i^*$, $\hat{V}_{i,E} = V_{i,E}^*$, $\hat{V}_{i,F} = V_{i,F}^* + \bar{\tau}_i^* - \hat{\tau}_{i,E}$, $\hat{\lambda}_i = \lambda_i^*$, and $\hat{w}_i = C_i^* - A$.

Finally, the next proposition shows that a competitive equilibrium associated with the fiscal policy $(\hat{z}_i, \hat{\tau}_{i,E}, \hat{\tau}_{i,v})_{i \in \mathbb{I}}$ constructed in Lemma 4 achieves the constrained efficient allocation.

Proposition 2. *Given the constrained efficient allocation, $(\xi_1^*, (\theta_i^*, V_{i,E}^*, C_i^*(\cdot), V_{i,F}^*, \bar{\tau}_i^*, z_i^*, \lambda_i^*)_{i \in \mathbb{I}})$, define the fiscal policy $(\hat{z}_i, \hat{\tau}_{i,E}, \hat{\tau}_{i,v})_{i \in \mathbb{I}}$ as in Lemma 4. Then there exists a competitive equilibrium $(\hat{\gamma}_i, \hat{V}_i, \hat{\Theta}_i, \hat{U}_i)_{i \in \mathbb{I}}$ such that $\hat{V}_i = \{V_{i,E}^*\}$, $\hat{\gamma}_i(\{V_{i,E}^*\}) = \phi_i \theta_i^*$, $\hat{\Theta}_i(V_{i,E}^*) = \theta_i^*$.*

5 Non-utilitarian planner and non-linear taxes

In the previous section, we have seen that lump-sum taxes and subsidies can be used to achieve the constrained efficient allocation defined in Section 3. This reflects our assumption that the social welfare function is utilitarian, that is, it is defined as a weighted sum of expected utility of workers. In this section, we shall see that nonlinear income taxes are necessary if the social welfare function is non-utilitarian. But we also show that even in this case, the optimal income tax function has a very simple form with a clear relationship with the social welfare function. To simplify the exposition, we assume $I = 1$ in this section so that there is only one type of workers.

5.1 Taxes that are nonlinear in output

Let us begin with the case where the optima taxation is nonlinear in output, Y_1 . Since there is only one type, the representative worker's expected utility is given by $\mu(\theta)V_E + [1 - \mu(\theta)]u(A + z)$. We now assume that the planner's objective function is different from the worker's expected utility. Instead, it is given by

$$\mu(\theta)\gamma_E V_E + [1 - \mu(\theta)]\gamma_U u(A + z) \quad (38)$$

where $\gamma_U > 0$ is a given constant, but γ_E is determined (endogenously) as

$$\gamma_E \equiv \mathbb{E} \left[\exp \left(-\frac{V_E}{\chi} \right) \exp \left(\frac{u(C(Y_1))}{\chi} \right) \gamma(Y_1) \right] \quad (39)$$

with $\gamma : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ being a given function.

Here, γ_U is the social weight on the unemployed workers, and $\gamma(Y_1)$ the weight on an employed worker who has produced Y_1 at time 1. The expected value of $\gamma(Y_1)$, γ_E , represents the weight on the employed workers. The utilitarian case is the one with $\gamma_U = \gamma(Y)$ for all Y . To simplify the exposition, unless otherwise stated, we focus on the case with $V_E > 0$ in the rest of this section. Then the planner's desire for redistribution is captured by a decreasing social weight function, that is, by assuming $\gamma'(Y_1) < 0$.

The feasibility constraint for the planner is given in the same way as in Section 3:

$$\mu(\theta) \mathbb{E} \left[\exp \left(-\frac{V_E}{\chi} \right) \exp \left(\frac{u(C(Y_1))}{\chi} \right) \{Y_1 + A - C(Y_1)\} \right] - [1 - \mu(\theta)]z - \theta k \geq 0, \quad (40)$$

$$\mathbb{E} \left[\exp \left(-\frac{V_E}{\chi} \right) \exp \left(\frac{u(C(Y_1))}{\chi} \right) \right] = 1, \quad (41)$$

$$\mu(\theta)V_E + [1 - \mu(\theta)]u(A + z) \geq u(A). \quad (42)$$

The planner maximizes the social welfare given by (38) subject to these constraints.

Proceeding in the same way as in Section 3, the constrained efficient allocation is characterized

by the solution to the following system of equations , $(z^*, \bar{\tau}^*, \theta^*, \gamma_E^*, V_E^*, V_F^*, C^*, \lambda^*)$:

$$\mu(\theta^*)\bar{\tau}^* = [1 - \mu(\theta^*)]z^*, \quad (43)$$

$$\mathbb{E} \left[\exp \left(-\frac{V_E^*}{\chi} \right) \exp \left(\frac{u[C^*(Y_1)]}{\chi} \right) \right] = 1, \quad (44)$$

$$\gamma_E^* = \mathbb{E} \left[\exp \left(-\frac{V_E^*}{\chi} \right) \exp \left(\frac{u[C^*(Y_1)]}{\chi} \right) \gamma(Y_1) \right], \quad (45)$$

$$\eta(\theta^*)V_F^* = k, \quad (46)$$

$$\bar{\tau}^* = \left[\exp \left(-\frac{V_E^*}{\chi} \right) \exp \left(\frac{u[C^*(Y_1)]}{\chi} \right) \{Y_1 + A - C^*(Y_1)\} \right] - V_F^*, \quad (47)$$

$$0 = \frac{\gamma_E^*V_E^* - \gamma_U u(A + z^*)}{\gamma_U u'(A + z^*)} + \frac{\eta'(\theta^*) \mu(\theta^*)}{\eta(\theta^*) \mu'(\theta^*)} V_F^* + \bar{\tau}^* + z^*, \quad (48)$$

$$0 = \frac{\gamma_E^*}{\gamma_U u'(A + z^*)} - \frac{1}{\chi} \left\{ \frac{\gamma_E^*V_E^*}{\gamma_U u'(A + z^*)} + V_F^* + \bar{\tau}^* + \lambda^* \right\}, \quad (49)$$

$$1 = \frac{u'[C^*(Y_1)]}{\chi} \left\{ Y_1 + A - C^*(Y_1) + \frac{V_E^*}{\gamma_U u'(A + z^*)} \gamma(Y_1) + \lambda^* \right\}. \quad (50)$$

Now let us consider how to implement the constrained efficient allocation by taxes. We use $(z, \tau_v, \tau(\cdot))$, where z is unemployment benefits, τ_v is the lump-sum subsidy for vacancy creation, and $\tau(Y_1)$ is the income tax for each level of output Y_1 . A competitive equilibrium is defined as in Definition 2 in Section 4, with a slight modification. Define the profit function of a firm:

$$\begin{aligned} V_F(v) &= \max_{w(\cdot)} \mathbb{E} \left[\exp \left(-\frac{v}{\chi} \right) \left(\frac{u[A + w(Y_1)]}{\chi} \right) \{Y_1 - w(Y_1) - \tau(Y_1)\} \right] \\ \text{s.t. } &\mathbb{E} \left[\exp \left(-\frac{v}{\chi} \right) \left(\frac{u[A + w(Y_1)]}{\chi} \right) \right] = 1. \end{aligned}$$

Then define the average income tax, τ_E , by

$$\tau_E \equiv \mathbb{E} \left[\exp \left(-\frac{v}{\chi} \right) \left(\frac{u[A + w(Y_1)]}{\chi} \right) \tau(Y_1) \right]$$

Then Definition 2 applies to the model with fiscal policy $(z, \tau_v, \tau(\cdot))$.

As before, a competitive equilibrium allocation is obtained as a solution to the maximization problem:

$$\begin{aligned} \max_{(\theta, V_E, w(\cdot))} &\mu(\theta)V_E + [1 - \mu(\theta)]u(A + z) \\ \text{s.t. } &\eta(\theta) \mathbb{E} \left[\exp \left(-\frac{V_E}{\chi} \right) \left(\frac{u[A + w(Y_1)]}{\chi} \right) \{Y_1 - w(Y_1) - \tau(Y_1)\} \right] \geq k - \tau_v, \\ &\mathbb{E} \left[\exp \left(-\frac{V_E}{\chi} \right) \left(\frac{u[A + w(Y_1)]}{\chi} \right) \right] = 1, \end{aligned}$$

where the fiscal policy satisfies the government budget constraint: $\mu(\theta)\tau_E - \theta\tau_v = [1 - \mu(\theta)]z$.

Then we can show that a competitive equilibrium allocation is given by $((\hat{z}, \hat{\tau}_v, \hat{\tau}(\cdot)), (\hat{\theta}, \hat{V}_E, \hat{V}_F, \hat{w}(\cdot), \hat{\lambda}, \hat{\tau}_E))$ which satisfies

$$\hat{\tau}_E \equiv \mathbb{E} \left[\exp \left(-\frac{\hat{V}_E}{\chi} \right) \left(\frac{u[A + \hat{w}(Y_1)]}{\chi} \right) \hat{\tau}(Y_1) \right], \quad (51)$$

$$\mu(\hat{\theta})\hat{\tau}_E - \hat{\theta}\hat{\tau}_v = [1 - \mu(\hat{\theta})]\hat{z}, \quad (52)$$

$$\mathbb{E} \left[\exp \left(-\frac{\hat{V}_E}{\chi} \right) \left(\frac{u[A + \hat{w}(Y_1)]}{\chi} \right) \right] = 1, \quad (53)$$

$$\hat{V}_F = \mathbb{E} \left[\exp \left(-\frac{\hat{V}_E}{\chi} \right) \left(\frac{u[A + \hat{w}(Y_1)]}{\chi} \right) \{Y_1 - \hat{w}(Y_1) - \hat{\tau}(Y_1)\} \right], \quad (54)$$

$$\eta(\hat{\theta})\hat{V}_F = k - \hat{\tau}_v, \quad (55)$$

$$0 = \hat{V}_E - u(A + \hat{z}) + \frac{\eta'(\hat{\theta})}{\eta(\hat{\theta})} \frac{\mu(\hat{\theta})}{\mu'(\hat{\theta})} \frac{\chi \hat{V}_F}{\hat{V}_F + \hat{\lambda}}, \quad (56)$$

$$1 = \frac{u'[A + \hat{w}(Y_1)]}{\chi} \{Y_1 - \hat{w}(Y_1) - \hat{\tau}(Y_1) + \hat{\lambda}\}. \quad (57)$$

The following proposition describes how to set the fiscal policy in order to implement a given constrained efficient allocation.

Proposition 3. *Let a constrained efficient allocation, $(z^*, \bar{\tau}^*, \theta^*, \gamma_E^*, V_E^*, V_F^*, C^*, \lambda^*)$, be given. Define $(\hat{z}, \hat{\tau}_v, \hat{\tau}(\cdot))$ as*

$$\hat{z} = z^*, \quad (58)$$

$$\hat{\tau}_v = \eta(\theta^*)(\hat{\tau}_E - \bar{\tau}^*), \quad (59)$$

$$\hat{\tau}(Y_1) = \hat{\tau}_E - \frac{V_E^*}{\gamma_U u'(A + z^*)} [\gamma(Y_1) - \gamma_E^*], \quad (60)$$

where

$$\hat{\tau}_E = \bar{\tau}^* + \frac{\mu'(\theta^*)}{\mu(\theta^*)} \frac{\eta(\theta^*)}{\eta'(\theta^*)} \left\{ \frac{\gamma_U - \gamma_E^*}{\gamma_U} \frac{u(A + z^*)}{u'(A + z^*)} - (\bar{\tau}^* + z^*) \right\}. \quad (61)$$

Then there exists a competitive equilibrium $(\hat{\gamma}, \hat{V}, \hat{\Theta}, \hat{U})$ such that $\hat{V} = \{V_E^*\}$, $\hat{\gamma}(\{V_E^*\}) = \theta^*$, $\hat{\Theta}(V_E^*) = \theta^*$.

5.2 Taxes that are nonlinear in wage

In the model in the previous subsection, the optimal income tax is a function of the level of output, Y_1 , rather than the wage paid to the worker, $w(Y_1)$. It is probably more empirically appealing if the optimal income tax is specified as a function of the wage payment. In this subsection, we modify the social welfare function in such a way.

We continue to assume that the planner's preferences are given by (38), but now the endogenously

determined social weight on employed workers, γ_E , is defined by

$$\gamma_E \equiv \mathbb{E} \left[\exp \left(-\frac{V_E}{\chi} \right) \exp \left(\frac{u(C(Y_1))}{\chi} \right) \gamma(C(Y_1)) \right] \quad (62)$$

In the previous subsection, the social weight function γ depends on the level of output, Y_1 , but now it reflects the level of consumption, $C(Y_1)$, of each worker. To motivate redistribution, we assume $\gamma'(c) < 0$.

The planner maximizes (38) subject to (40)-(42), and (62). Then the constrained efficient allocation is given by $(z^*, \bar{\tau}^*, \theta^*, \gamma_E^*, V_E^*, V_F^*, C^*, \lambda^*)$ that solves the following system of equations:

$$\mu(\theta^*)\bar{\tau}^* = [1 - \mu(\theta^*)]z^*, \quad (63)$$

$$\mathbb{E} \left[\exp \left(-\frac{V_E^*}{\chi} \right) \exp \left(\frac{u[C^*(Y_1)]}{\chi} \right) \right] = 1, \quad (64)$$

$$\gamma_E^* = \mathbb{E} \left[\exp \left(-\frac{V_E^*}{\chi} \right) \exp \left(\frac{u[C^*(Y_1)]}{\chi} \right) \gamma(C^*(Y_1)) \right], \quad (65)$$

$$\eta(\theta^*)V_F^* = k, \quad (66)$$

$$\bar{\tau}^* = \left[\exp \left(-\frac{V_E^*}{\chi} \right) \exp \left(\frac{u[C^*(Y_1)]}{\chi} \right) \{Y_1 + A - C^*(Y_1)\} \right] - V_F^*, \quad (67)$$

$$0 = \frac{\gamma_E^*V_E^* - \gamma_U u(A + z^*)}{\gamma_U u'(A + z^*)} + \frac{\eta'(\theta^*)\mu(\theta^*)}{\eta(\theta^*)\mu'(\theta^*)} V_F^* + \bar{\tau}^* + z^*, \quad (68)$$

$$0 = \frac{\gamma_E^*}{\gamma_U u'(A + z^*)} - \frac{1}{\chi} \left\{ \frac{\gamma_E^*V_E^*}{\gamma_U u'(A + z^*)} + V_F^* + \bar{\tau}^* + \lambda^* \right\}, \quad (69)$$

$$\begin{aligned} 1 - \frac{V_E^*}{\gamma_U u'(A + z^*)} \gamma'(C^*(Y_1)) & \quad (70) \\ & = \frac{u'[C^*(Y_1)]}{\chi} \left\{ Y_1 + A - C^*(Y_1) + \frac{V_E^*}{\gamma_U u'(A + z^*)} \gamma(C^*(Y_1)) + \lambda^* \right\}. \end{aligned}$$

Next turn to implementation. We consider $(z, \tau_v, \tau(\cdot))$, where the income tax function τ is now a function of the (before-tax) wage rate, w , rather than the level of output, Y_1 . The profit function of a firm is written as

$$\begin{aligned} V_F(v) &= \max_{w(\cdot)} \mathbb{E} \left[\exp \left(-\frac{v}{\chi} \right) \left(\frac{u[A + w(Y_1) - \tau(w(Y_1))]}{\chi} \right) \{Y_1 - w(Y_1)\} \right] \\ \text{s.t. } & \mathbb{E} \left[\exp \left(-\frac{v}{\chi} \right) \left(\frac{u[A + w(Y_1) - \tau(w(Y_1))]}{\chi} \right) \right] = 1. \end{aligned}$$

The (endogenously determined) average income tax is

$$\tau_E \equiv \mathbb{E} \left[\exp \left(-\frac{V_E}{\chi} \right) \left(\frac{u[A + w(Y_1) - \tau(w(Y_1))]}{\chi} \right) \tau(w(Y_1)) \right]$$

Then Definition 2 applies to the model with fiscal policy $(z, \tau_v, \tau(\cdot))$.

The associated maximization problem is

$$\begin{aligned} \max_{(\theta, V_E, w(\cdot))} \quad & \mu(\theta)V_E + [1 - \mu(\theta)]u(A + z) \\ \text{s.t.} \quad & \eta(\theta) \mathbb{E} \left[\exp \left(-\frac{V_E}{\chi} \right) \left(\frac{u[A + w(Y_1) - \tau(w(Y_1))]}{\chi} \right) \{Y_1 - w(Y_1)\} \right] \geq k - \tau_v, \\ & \mathbb{E} \left[\exp \left(-\frac{V_E}{\chi} \right) \left(\frac{u[A + w(Y_1) - \tau(w(Y_1))]}{\chi} \right) \right] = 1, \end{aligned}$$

where the fiscal policy satisfies the government budget constraint: $\mu(\theta)\tau_E - \theta\tau_v = [1 - \mu(\theta)]z$.

Then a competitive equilibrium allocation is given by $((\hat{z}, \hat{\tau}_v, \hat{\tau}(\cdot)), (\hat{\theta}, \hat{V}_E, \hat{V}_F, \hat{w}(\cdot), \hat{\lambda}, \hat{\tau}_E))$ which satisfies

$$\hat{\tau}_E \equiv \mathbb{E} \left[\exp \left(-\frac{\hat{V}_E}{\chi} \right) \left(\frac{u[A + \hat{w}(Y_1) - \hat{\tau}(\hat{w}(Y_1))]}{\chi} \right) \hat{\tau}(\hat{w}(Y_1)) \right], \quad (71)$$

$$\mu(\hat{\theta})\hat{\tau}_E - \hat{\theta}\hat{\tau}_v = [1 - \mu(\hat{\theta})]\hat{z}, \quad (72)$$

$$\mathbb{E} \left[\exp \left(-\frac{\hat{V}_E}{\chi} \right) \left(\frac{u[A + \hat{w}(Y_1) - \hat{\tau}(\hat{w}(Y_1))]}{\chi} \right) \right] = 1, \quad (73)$$

$$\hat{V}_F = \mathbb{E} \left[\exp \left(-\frac{\hat{V}_E}{\chi} \right) \left(\frac{u[A + \hat{w}(Y_1) - \hat{\tau}(\hat{w}(Y_1))]}{\chi} \right) \{Y_1 - \hat{w}(Y_1)\} \right], \quad (74)$$

$$\eta(\hat{\theta})\hat{V}_F = k - \hat{\tau}_v, \quad (75)$$

$$0 = \hat{V}_E - u(A + \hat{z}) + \frac{\eta'(\hat{\theta})}{\eta(\hat{\theta})} \frac{\mu(\hat{\theta})}{\mu'(\hat{\theta})} \frac{\chi\hat{V}_F}{\hat{V}_F + \hat{\lambda}}, \quad (76)$$

$$[1 - \hat{\tau}'(\hat{w}(Y_1))] = \frac{u'[A + \hat{w}(Y_1) - \hat{\tau}(\hat{w}(Y_1))]}{\chi} \{Y_1 - \hat{w}(Y_1) + \hat{\lambda}\}. \quad (77)$$

The following proposition describes how to set the fiscal policy in order to implement a given constrained efficient allocation.

Proposition 4. *Let a constrained efficient allocation, $(z^*, \bar{\tau}^*, \theta^*, \gamma_E^*, V_E^*, V_F^*, C^*, \lambda^*)$, be given. Define $(\hat{z}, \hat{\tau}_v, \hat{\tau}(\cdot))$ by the following equations:*

$$\hat{z} = z^*, \quad (78)$$

$$\hat{\tau}_v = \eta(\theta^*)(\hat{\tau}_E - \bar{\tau}^*), \quad (79)$$

$$\hat{\tau}(w) = \hat{\tau}_E - \frac{V_E^*}{\gamma_U u'(A + z^*)} [\gamma(A + w - \hat{\tau}(w)) - \gamma_E^*], \quad (80)$$

where

$$\hat{\tau}_E = \bar{\tau}^* + \frac{\mu'(\theta^*)}{\mu(\theta^*)} \frac{\eta(\theta^*)}{\eta'(\theta^*)} \left\{ \frac{\gamma_U - \gamma_E^*}{\gamma_U} \frac{u(A + z^*)}{u'(A + z^*)} - (\bar{\tau}^* + z^*) \right\}. \quad (81)$$

Then there exists a competitive equilibrium $(\hat{\gamma}, \hat{V}, \hat{\theta}, \hat{U})$ such that $\hat{V} = \{V_E^*\}$, $\hat{\gamma}(\{V_E^*\}) = \theta^*$, $\hat{\theta}(V_E^*) = \theta^*$.

6 Conclusion

In this paper, I consider an optimal taxation problem in a model with labor search and moral hazard. Workers and firms are matched through a directed search process. Moral hazard arises from the assumption that effort of workers is unobservable to their employers. The (constrained) efficient allocation is defined for two kinds of social welfare functions, one “utilitarian” and the other “non-utilitarian.” The utilitarian welfare function is given by a weighted average of (ex-ante) expected utilities of individuals. In the non-utilitarian case, the level of social welfare depends on ex-post distribution of consumption, in addition to expected utilities. I show that in the utilitarian case, the constrained efficient allocation is implemented by a combination of lump-sum taxes on income, subsidies for vacancy creation, and subsidies for the unemployed. In the non-utilitarian case, the constrained efficient allocation implemented similarly, with the exception that income taxes must be a non-linear function.

References

- Acemoglu, D. and R. Shimer (1999): “Efficient Unemployment Insurance,” *Journal of Political Economy*, 107, 893–928.
- Chetty, R. and E. Saez (2010): “Optimal Taxation and Social Insurance with Endogenous Private Insurance,” *American Economic Journal: Economic Policy*, 2, 85–116.
- Cvitanic, J., X. Wan, and J. Zhang (2009): “Optimal Compensation with Hidden Action and Lump-Sum Payment in a Continuous-Time Model,” *Applied Mathematics and Optimization*, 59, 99–146.
- Cvitanic, J. and J. Zhang (2013): *Contract Theory in Continuous-Time Models*, Springer, 2013 ed.
- DeMarzo, P. M. and Y. Sannikov (2006): “Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model,” *Journal of Finance*, 61, 2681–2724.
- Farhi, E. and I. Werning (2013): “Insurance and Taxation over the Life Cycle,” *Review of Economic Studies*, 80, 596–635.
- Golosov, M., P. Maziero, and G. Menzio (2013): “Taxation and Redistribution of Residual Income Inequality,” *Journal of Political Economy*, 121, 1160–1204.
- Golosov, M., M. Troshkin, and A. Tsyvinski (2016): “Redistribution and Social Insurance,” *American Economic Review*, 106, 359–86.

REFERENCES

- Golosov, M. and A. Tsyvinski (2007): “Optimal Taxation with Endogenous Insurance Markets,” *Quarterly Journal of Economics*, 122, 487–534.
- Guerrieri, V., R. Shimer, and R. Wright (2010): “Adverse Selection in Competitive Search Equilibrium,” *Econometrica*, 78, 1823–1862.
- Holmstrom, B. and P. Milgrom (1987): “Aggregation and Linearity in the Provision of Intertemporal Incentives,” *Econometrica*, 55, 303–328.
- Karatzas, I. and S. Shreve (1998): *Brownian Motion and Stochastic Calculus*, Graduate Texts in Mathematics, Springer-Verlag, 2 ed.
- Krueger, D. and F. Perri (2011): “Public versus private risk sharing,” *Journal of Economic Theory*, 146, 920 – 956, incompleteness and Uncertainty in Economics.
- Mirrlees, J. A. (1971): “An Exploration in the Theory of Optimum Income Taxation,” *Review of Economic Studies*, 38, 175–208.
- Moen, E. R. (1997): “Competitive Search Equilibrium,” *Journal of Political Economy*, 105, 385–411.
- Moen, E. R. and A. Rosen (2011): “Incentives in Competitive Search Equilibrium,” *Review of Economic Studies*, 78, 733–761.
- Piketty, T. and E. Saez (2013): “Chapter 7 - Optimal Labor Income Taxation,” in *Handbook of Public Economics*, Vol. 5, ed. by A. J. Auerbach, R. Chetty, M. Feldstein, and E. Saez, Elsevier, 391–474.
- Saez, E. (2001): “Using Elasticities to Derive Optimal Income Tax Rates,” *Review of Economic Studies*, 68, 205–229.
- Saez, E. and S. Stantcheva (2016): “Generalized Social Marginal Welfare Weights for Optimal Tax Theory,” *American Economic Review*, 106, 24–45.
- Sannikov, Y. (2008): “A Continuous-Time Version of the Principal–Agent Problem,” *Review of Economic Studies*, 75, 957–984.
- Schättler, H. and J. Sung (1993): “The First-Order Approach to the Continuous-Time Principal–Agent Problem with Exponential Utility,” *Journal of Economic Theory*, 61, 331 – 371.
- Stantcheva, S. (2014): “Optimal Income Taxation with Adverse Selection in the Labour Market,” *Review of Economic Studies*, 81, 1296–1329.

REFERENCES

Williams, N. (2011): “Persistent Private Information,” *Econometrica*, 79, 1233–1275.

——— (2015): “A Solvable Continuous Time Dynamic Principal–Agent Model,” *Journal of Economic Theory*, 159, 989–1015.