CIGS The Canon Institute for Global Studies

CIGS Working Paper Series No. 26-015E

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September 10, 2026

General Incorporated Foundation
The Canon Institute for Global Studies
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Phone: +81-3-6213-0550　　https://cigs.canon/

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Abstract

This paper develops a theoretical model of financial crisis cycles where credit-driven asset price booms are followed by busts and protracted recessions. The accumulation of private debt during booms generates a debt overhang that discourages borrowers from enhancing productivity in the post-crash period. When this debt burden is substantial, coordination failures among creditors delay necessary debt restructuring, leading to a persistent decline in aggregate productivity. We show that bank recapitalization or subsidies, conditional on debt restructuring, are more efficient than unconditional subsidies to borrowing firms, achieving economic recovery at a lower fiscal cost. Furthermore, we demonstrate that borrower subsidies, or expansionary fiscal policy more broadly, can inadvertently prolong stagnation by discouraging banks from restructuring debt.

1 Introduction

We construct and analyze a simple theoretical model of financial crises that accounts for key empirical regularities and provides a new perspective on both the crisis mechanism and its associated policy implications. A defining empirical regularity of financial crises is that they are typically preceded by booms in asset prices and credit; these booms subsequently burst, leading to deep and persistent declines in productivity (see, e.g., Jordà, Schularick, and Taylor, 2015).

The core mechanism of a financial crisis in our model unfolds as follows: First, an asset price boom emerges from optimistic expectations regarding productivity of the assets. During this period, firms finance asset purchases through increased leverage, which further bids up asset prices. Once these expectations prove unfounded, asset prices collapse sharply, leaving borrowers with substantial debt overhang. Debt overhang, in turn,

*We thank the seminar participants at the 2026 CIGS Conference on Macroeconomic Theory and Policy for their valuable comments.

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impairs productive efficiency of the assets by discouraging borrowers from exerting effort. Although banks recognize that debt restructuring is essential to restore the productivity of the assets owned by distressed borrowers, they fail to implement such measures in a timely manner due to coordination failures when the debt is sufficiently large. This is because the restructuring of larger debt necessitates coordination among a larger number of stakeholders on the lender side. This delay in restructuring ultimately results in protracted stagnation. We formalize this mechanism by integrating risk-shifting bubbles (Allen and Gale, 2000), borrower debt overhang (Lamont, 1995), and creditor coordination frictions. A notable contribution of our framework is that it accounts for post-crisis stagnation as emerging from borrowers' balance sheet problems, rather than from standard financial frictions, such as borrowing constraints or pecuniary externalities that tighten those constraints. This distinction is important because the existing literature, which emphasizes these standard frictions, tends to focus on problems and policy measures aimed at the financial sector—such as macroprudential regulation or monetary policy—often overlooking the balance sheet problems of borrowers. Our model demonstrates that resolving borrower debt overhang is the key to economic recovery in the aftermath of a crisis.

Therefore, our framework yields distinct policy implications for ex-post crisis management. In our model, a debt overhang distorts borrower incentives and reduces productivity after a crash. To restore productivity through debt resolution, we find that bank recapitalization or subsidies contingent on restructuring are more cost-effective than direct bailouts of borrowers. Furthermore, borrower subsidies can be counterproductive: if the subsidy amounts are insufficient, they may actually discourage debt restructuring by exacerbating distortions in bank managers' decision-making. This is because borrower subsidies increase bank managers' gains from postponing debt restructuring relatively more than those from implementing it right away. Moreover, if we interpret borrower subsidies as a form of expansionary fiscal policy aimed at boosting the incomes of borrowers in general, this result implies that such policy may inadvertently impede productivity recovery in the aftermath of a financial crisis. In other words, fiscal expansion can hinder economic recovery should the fundamental cause of the productivity decline be a debt overhang. These results provide critical insights for the design of crisis management policies.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 develops the baseline model under the assumption that banks are prohibited from restructuring nonperforming loans. Section 4 demonstrates that the equilibrium path exhibits persistent stagnation due to debt overhang in the aftermath of a financial crisis. Section 5 extends the model to incorporate debt restructuring through costly internal coordination on the lender side. Section 6 discusses the policy implications, and Section 7 concludes.

2 Literature

2.1 Empirical Regularities

Our model provides an integrated account of the empirical regularities characterizing the financial crisis. Jordà, Schularick, and Taylor (2015), analyzing 140 years of data across 17 countries, show that asset-price booms accompanied by credit expansions tend to culminate in financial crises, followed by deep and protracted recessions. Similarly, Greenwood, Hanson, Shleifer, and Sørensen (2022) find that rapid growth in private credit and asset prices serves as a robust predictor of financial crises. Several studies demonstrate that credit booms alone can predict crises.[1] Schularick and Taylor (2012) and Verner (2019) report that credit booms frequently precede financial crises. Furthermore, Giroud and Mueller (2021) find that firm leverage booms predict boom-bust cycles in employment. Krishnamurthy and Muir (2024) document that pre-crisis periods are characterized by high credit growth and abnormally low credit spreads, typically followed by severe and protracted recessions. Corporate debt appears to be a crucial driver of these dynamics. While much of the literature since the Global Financial Crisis (GFC) has focused on household debt, recent empirical work emphasizes the role of corporate debt buildups (Greenwood et al., 2022; Jordà et al., 2022; Sever, 2023; Ivashina et al., 2024). For instance, Ivashina et al. (2024) find that corporate debt accounts for the majority of nonperforming loans post-crisis and predicts slower recovery in the average country. They argue that their findings differ from Jordà et al. (2022), who find corporate debt to be a weaker predictor, due to differences in data coverage; specifically, Ivashina et al. (2024) focus on bank loans, whereas Jordà et al. (2022) include corporate bonds, which appear less correlated with crises. The institutional environment also plays a pivotal role. Kornejew, Lian, Ma, Ottonello, and Perez (2024) document that business credit booms lead to severe output declines in environments with inefficient bankruptcy systems. While they also develop a model of non-fundamental credit booms, our framework differs in that we explicitly incorporate asset prices and analyze policy interventions aimed at enhancing debt restructuring. Similarly, Müller and Verner (2023) distinguish between "good" and "bad" credit booms, noting that the latter are often concentrated in the non-tradable sector.[2]

Finally, our work relates to the literature on post-crisis productivity. Financial crises

[1] While long-run credit deepening is associated with higher long-term economic growth (King and Levine, 1993), short-run credit booms often lead to crises.

[2] Regarding the drivers of these booms, Verner (2019) and Justiniano, Primiceri, and Tambalotti (2019) argue that pre-crisis expansions are primarily driven by credit-supply shocks rather than credit-demand shocks. Mian, Sufi, and Verner (2017) also show that credit-supply shocks induce consumption booms and persistently lower subsequent GDP growth.

are well-known to be followed by persistent productivity slowdowns (Adler et al., 2017). Duval, Hong, and Timmer (2020) argue that financial frictions were a primary driver of the productivity slump during the Great Recession. This is consistent with the literature on decade-long recessions in the 20th century, where deep and persistent productivity declines are identified as the major cause (Hayashi and Prescott, 2002; Kehoe and Prescott, 2002).

2.2 Theoretical ingredients

We integrate the theories of risk-shifting asset booms and debt overhang to characterize the key features of financial crises.

Risk-shifting asset booms: This study relates to the literature on risk-shifting asset bubbles, as theoretically analyzed by Allen and Gale (2000) and Allen, Barlevy, and Gale (2022). These studies demonstrate that asset-price booms can be driven by risk-shifting incentives among investors who leverage their purchases. In their models, however, the costs of default are exogenous, serving as the primary source of ex-post inefficiency. Consequently, there are no policy measures available to mitigate such inefficiency. In contrast, in our model, the inefficiency is endogenously determined, and ex-post debt reduction can effectively alleviate it.

Debt overhang: Our analysis also contributes to the broad literature on debt overhang. Following Kobayashi, Nakajima, and Takahashi (2023), debt overhang can be categorized into two distinct types: the first arising from a lack of borrower commitment, and the second from a lack of lender commitment. This paper focuses on the latter, which focuses on inefficiencies arising when lenders cannot credibly commit to rewarding borrower effort. When debt levels exceed borrower revenues, lenders possess the legal right to seize all proceeds. Anticipating this, borrowers refrain from exerting effort, leading to productive inefficiency. We emphasize the second type because it aligns with the experiences of Japan's "lost decades," which were characterized by a persistent contraction in demand. The first type of debt overhang represents a form of borrowing constraint, as analyzed by Albuquerque and Hopenhayn (2004), Kovrijnykh and Szentes (2007), and Aguiar, Amador, and Gopinath (2009). The second type is discussed in the macroeconomic literature by Sachs (1988), Krugman (1988), Lamont (1995), Occhino and Pescatori (2015), and Kobayashi et al. (2023). Empirical evidence for debt overhang is provided by Honda, Ono, Uesugi, and Yasuda (2024), who study the turnaround of small and medium-sized enterprises in Japan.

2.3 Theoretical studies on financial crises and policy responses

This paper is related to the extensive literature on financial crises and policy interventions. We distinguish our model from existing studies across three key dimensions: the source of inefficiencies, the propagation of inefficiencies, and the design of policy interventions. First, regarding the source of inefficiency, the literature primarily emphasizes pecuniary externalities arising from borrowing constraints (e.g., Benigno et al., 2023; Bianchi, 2011, 2016; Bianchi and Mendoza, 2010; Gertler, Kiyotaki, and Queralto, 2012; Jeanne and Korinek, 2020; Lorenzoni, 2008) or coordination failures, such as bank runs (Diamond and Dybvig, 1983; Gertler and Kiyotaki, 2015; Keister, 2016; Keister and Narasiman, 2016). This paper emphasizes debt overhang as the source of inefficiency. Second, concerning the propagation of inefficiencies, many existing models feature consumption misallocations (Bianchi, 2011; Chari and Kehoe, 2016; Farhi, Golosov, and Tsyvinski, 2009; Keister, 2016) or inefficient production due to a credit crunch (Bianchi, 2016; Bianchi and Mendoza, 2010; Gertler, Kiyotaki, and Queralto, 2012; Lorenzoni, 2008), whereas we focus on the coordination game that delays debt resolution as a mechanism of persistence. Third, regarding policy interventions, the existing literature primarily focuses on time inconsistency—specifically, the trade-off between ex-ante incentives and ex-post efficiency induced by bailout policies (Bianchi, 2016; Chari and Kehoe, 2016; Farhi and Tirole 2012; Green, 2010; Keister, 2016; Keister and Narasiman, 2016). In our model, we compare bank recapitalization and direct bailout of the borrowers as crisis management policies to resolve debt problems. Haavio, Ripatti, and Takalo (2025) and Homar and van Wijnbergen (2017) also compare bank recapitalization and borrower bailouts, while their analyses focus on resolving credit crunch and banks' risk-shifting to the government, whereas our model focuses on borrowers' debt overhang. Finally, our paper relates to the literature on secular stagnation (e.g., Rachel and Summers, 2019; Eggertsson, Mehrotra, and Robbins, 2019).

2.4 Zombie Lending

Our theory is closely related to the growing literature on zombie lending and evergreening in the aftermath of financial crises. Zombie lending refers to credit extension to non-viable or distressed firms driven by distorted bank incentives. The pioneering work of Peek and Rosengren (2005) and Caballero, Hoshi, and Kashyap (2008) documented the proliferation of zombie lending in Japan during the 1990s (see also Sekine, Kobayashi, and Saita (2003) for an early account of "forbearance lending"). More recently, Acharya, Lenzu, and Wang (2023) and the references therein analyze models of zombie lending, emphasizing that accommodative government policies can distort bank incentives and induce lending practices that lead to persistent stagnation. Our model complements the existing literature in three key respects. First, while much of the literature focuses on distortionary policies, in our model, internal coordination failure in a bank is sufficient to generate persistent debt overhang and protracted stagnation. Second, whereas Acharya et al. (2021) imply that the removal of distortionary policy is welfare-improving, our framework suggests that active policy intervention is necessary to resolve debt overhang. Third, while most studies assume that zombie firms are intrinsically inefficient and that their exit improves productivity by mitigating congestion, our model implies that such firms can restore efficiency if their debt burden is reduced. The empirical findings of Nakamura and Fukuda (2013) provide support for our theory; they report that a significant portion of Japanese zombie firms in the non-tradable sector eventually recovered and became productive in the 2000s, suggesting they were not intrinsically unproductive. Furthermore, Becker and Ivashina (2022) and Kornejew et al. (2024) demonstrate, both empirically and theoretically, that inefficient bankruptcy procedures exacerbate the costs of zombie lending. These results imply that debt reduction through restructuring or bankruptcy can enhance efficiency, further supporting our argument regarding the welfare-improving effects of debt resolution.

3 Model without debt restructuring

Time is discrete and continues from 0 to infinity: $t = 0, 1, 2, \cdots$. There are a unit mass of banks and a unit mass of firms. There exist two assets in the economy: land k_t and safe assets x_t. Land is a fixed-supply asset, the total amount of which is a unit mass ($k_t = 1$). In the initial period 0, each bank owns one unit of land so that all land is owned by banks. Safe assets are variable supply, the total amount of which is x_t, where x_t is an endogenous variable. In every period, banks can generate safe assets from consumption goods on a one-to-one basis. The safe assets represent the general capital in the economy as in Allen and Gale (2000). While firms produce consumption goods using safe assets and land as detailed below, banks lack the necessary productive technologies to utilize these inputs. Consequently, banks are limited to selling their holdings of safe assets and land to firms on credit. The following assumption formalizes this setup:

Assumption 1. Banks cannot produce goods from safe assets or land. Rather, banks can only sell these assets to firms on credit or hold them passively yielding zero output. Safe assets depreciate completely in each period, whereas land is not subject to depreciation.

Production using safe assets: Similarly to Allen and Gale (2000), safe assets can be interpreted as capital goods leased to the general corporate sector, operating independently of the specific banks and firms analyzed in this paper. In period t, a firm purchases x_{t+1} units of the safe assets from a bank on credit. Thus, the firm borrows $D^S_{t+1} = x_{t+1}$ to buy x_{t+1} units of the assets. The loan rate r^L_{t+1} is determined in equilibrium as described below. The firm that has acquired the safe assets x_{t+1} immediately leases them out to the general corporate sector where the consumption goods are produced using a constant-returns-to-scale (CRS) technology, $F(x,l)$, where l is the labor input. We assume for simplicity that the labor supply in the general corporate sector is unity, $l=1$, which is provided by a unit mass of the hand-to-mouth workers. The firm who leased x_{t+1} will have the return $r^S_{t+1}x_{t+1}$ in period $t+1$, where the rate of return r^S_{t+1} is determined in the market as described below. The general corporate sector maximizes the profit:

$$F(x_{t+1},l_{t+1})-r^S_{t+1}x_{t+1}-w_{t+1}l_{t+1},$$

where w_{t+1} and l_{t+1} are the wage rate and the labor input in period $t+1$. Define $f(x)\equiv F(x,1)$. In the competitive equilibrium where $l=1$, x and w are given by

$$r^S_{t+1}=f'(x_{t+1})$$

and $w=f(x)-f'(x)x$. The wage $wl=w$ is consumed by the hand-to-mouth workers in the general corporate sector. Equation $r^S_{t+1}=f'(x_{t+1})$ can be interpreted as an equation that determines r^S_{t+1} from the aggregate amount of x_{t+1} in the market. The production function $f(x)$ satisfies $f'(x)>0$ and $f''(x)<0$ for all $x>0$, $\lim_{x\to 0}f'(x)=+\infty$, and $\lim_{x\to\infty}f'(x)=0$.

Production using land: Each bank owns one unit of land $k=1$ in period 0. Banks sell land k to firms on credit so that a firm purchases k units of land by borrowing the loan $D_1=Qk$ from a bank, where Q is the price of the land in period 0 in terms of consumer goods. As firms pay the debt D_1 gradually over time, there may be remaining debt D_{t+1} in period t that evolves at the loan rate r^L_t. Firms can produce consumption goods $y(e_t,\tilde{A},k)$ using land k, where e_t is additional input of effort, where the firms can choose whether $e_t=0$ or $e_t=\varepsilon>0$. The value of e_t, which is either 0 or ε, is the consumption equivalence of disutility that the firm suffers from expending the effort.

$$y(e_t,\tilde{A},k)=\begin{cases}A_Lk^{\psi} & \text{for } e_t=0,\\ \tilde{A}k^{\psi} & \text{for } e_t=\varepsilon,\end{cases}\tag{1}$$

where $0<\psi<1$ and $\tilde{A}$ is a macroeconomic random variable in period 0, which is revealed in period 1. The value of $\tilde{A}$ is the following, which is common for all firms and time-invariant from period 1 on:

$$\tilde{A}=\begin{cases}B & \text{with probability } p,\\ A_H & \text{with probability } 1-p.\end{cases}$$

We assume

$$A_L<A_H\ll B,$$
$$A_L<A_H-\varepsilon.$$

Incorporating production technology and market structure of Dixit-Stiglitz monopolistic competition allows for an analysis of aggregate demand externalities arising from debt overhang, as in Lamont (1995). For expositional simplicity, however, we defer this analysis to Appendix A.

Purchase of safe assets: Banks produce safe assets x_{t+1} in period t. We assume that the consumption goods produced in period t are all consumed or rotten when the safe assets x_{t+1} put up for sale. So firms cannot pay the unconsumed goods for x_{t+1}, but need to borrow a bank loan $D^S_{t+1} = x_{t+1}$ to purchase x_{t+1} from the bank.[3]

Loan rates: We assume that banks cannot distinguish between loans intended for the purchase of land D_{t+1}, and those for safe assets D^S_{t+1}.[4] This setup implies the following assumption regarding the loan rates:

Assumption 2. The loan rates for D_{t+1} and D^S_{t+1} are identical and denoted by r^L_{t+1}.

Firm's problem: Denote the firm's value in period t by $V(D_t, D^S_t, x_t; \tilde{A}, k)$. Note that the land input k is constant from period 0, while its productivity $\tilde{A}$ is constant from period 1 when $\tilde{A} = B$ or $\tilde{A} = A_H$ is revealed. The firm purchases land in period 0 and operates it in producing the consumer goods from period 1 on. We formulate the following assumption concerning the firm's participation constraint.

Assumption 3. The firm can cease operations and default on its debt if its anticipated value is negative; in contrast, it fulfills its repayment obligations specified by the bank as long as its value remains non-negative. In the event of default, the firm incurs no penalty and the bank recovers zero value.

In period 0, a firm solves

$$\begin{aligned} V_0 = & \max_{D_1, D_1^S, x_1, k} \beta \mathbb{E}_{\tilde{A}}[V(D_1, D_1^S, x_1; \tilde{A}, k)], \\ \text{s.t. } & D_1 = Qk, \\ & D_1^S = x_1. \end{aligned}$$

[3]The constraint that $D^S_{t+1} = x_{t+1}$ is equivalent to imposing the following constraint

$$c = y(e, A, k) + \max\{r^S x - r^L D^S,\ 0\} - b(e)$$

on the firm's problem, which says that all remaining consumption goods after paying for input costs (the RHS of the above equation) must be consumed and cannot be used to purchase safe assets, x_{t+1}.

[4]Specifically, the bank provides a single loan, which the firm allocates between D_{t+1} and D^S_{t+1} after borrowing.

In period t, where $t \geq 1$ and $\tilde{A}$ has been revealed to be $A \in \{A_H, B\}$ in period 1, a firm solves the following problem, given the expectation $V^e(\cdot)$:

$$
\begin{aligned}
V(D, D^S, x; A, k) = \max_{D^S_{+1}, x_{+1}, e, c} \; & c^f - e + \beta V^e(D_{+1}, D^S_{+1}, x_{+1}; A, k), \\
\text{s.t. } & c^f + b(e, D, D^S, x; A, k) = y(e, A, k) + \max\{r^S x - r^L D^S,\ 0\}, \\
& D_{+1} = r^L D - b(e, D, D^S, x; A, k) + \max\{r^L D^S - r^S x,\ 0\}, \\
& D^S_{+1} = x_{+1},
\end{aligned}
$$

where c^f is consumption, which can be interpreted as the dividend to the firm owner. The equilibrium condition is

$$V(\cdot) = V^e(\cdot).$$

The first constraint in the above problem is the resource constraint in which the profits from safe assets $(r^S x - r^L D^S)$ is added to the available resources when it is positive. When it is negative, the second constraint, which is the law of motion for debt D_t, states that the unpaid portion of $r^L_t D^S_t$, i.e., $r^L_t D^S_t - r^S_t x_t$, is capitalized into the subsequent period's debt D_{t+1}. The repayment $b(e, D, D^S, x; A, k)$ is taken as given by the firm and is chosen as the policy function by the bank that solves the following problem.

Bank's problem: The value of the bank is denoted by W. Here we make a crucial assumption:

Assumption 4. In every period t (≥ 1), the bank decides the amount to be repaid b after observing the borrower's choice of production $e \in \{0, \varepsilon\}$ in that period.

In period 0, a bank solves

$$
\begin{aligned}
\max_{k \in [0,1], x_1 \geq 0, D_1, D^S_1,} \; & \beta \mathbb{E}_{\tilde{A}}[W(D_1, D^S_1, x_1, e_1; \tilde{A}, k)] - x_1, \\
\text{s.t. } \; & D_1 = Qk, \\
& D^S_1 = x_1, \\
& e_1 = e(D_1, D^S_1, x_1; \tilde{A}, k),
\end{aligned}
$$

where $e(D, D^S, x; \tilde{A}, k)$ is the equilibrium policy function for the firm. In period t, where $t \geq 1$ and $\tilde{A}$ has been revealed to be $A \in \{A_H, B\}$ in period 1, the bank solves the following problem, given the expectations $W^e(\cdot)$:

$$\begin{aligned}
W(D, D^S, x, e; A, k) = \max_{b, x_{+1} \geq 0} \ & b + \min\{r^L D^S, r^S x\} - x_{+1} + \beta W^e(D_{+1}, D^S_{+1}, x_{+1}, e_{+1}; A, k), \\
\text{s.t.} \ & 0 \leq b \leq y(e, A, k) + \max\{r^S x - r^L D^S,\ 0\}, \\
& 0 \leq b \leq r^L D + \max\{r^L D^S - r^S x,\ 0\}, \\
& D_{+1} = r^L D - b + \max\{r^L D^S - r^S x,\ 0\}, \\
& D^S_{+1} = x_{t+1}, \\
& e_{+1} = e(D_{+1}, D^S_{+1}, x_{+1}; A, k).
\end{aligned}$$

The equilibrium condition is

$$W(\cdot) = W^e(\cdot).$$

The solution $b = b(e, D, D^S, x; A, k)$ is taken as given by the firms. Note that b is bounded from above.[5] [6]

Resource constraint: Total production of the consumption goods, $y(e, A, k) + f(x)$, is divided by the firms, the banks and the hand-to-mouth workers. And the consumption goods are consumed and invested in the safe assets. Thus, the resource constraint can be written as

$$y(e, A, k) + f(x) = c^f + b + \min\{r^L x, r^S x\} + w = c^f + c^b + w + x_{+1}, \tag{2}$$

where c^b is the consumption (or dividend) for the banks.

Determination of the interest rates $r^S_t = r^L_t$: We can show that $r^S_t = r^L_t = \frac{1}{\beta}$ with reasoning similar to that of Allen and Gale (2000). First, we show that $r^S_t = r^L_t$ for $t \geq 1$ by analyzing the firm's problem. Since $D^S = x$, the firm's problem can be rewritten as follows. To ease exposition, we omit the variables (A, k) when there is no risk of confusion, since they are invariant from period 1 onward.

$$\begin{aligned}
V(D, x) = \max_{x_{+1}, e, c^f} \ & c^f - e + \beta V^e(D_{+1}, x_{+1})], \\
\text{s.t.} \ & c + b(e, D, x) = y(e) + \max\{(r^S - r^L)x, 0\}, \\
& D_{+1} = r^L D - b(e, D, x) + \max\{(r^L - r^S)x, 0\}, .
\end{aligned}$$

Suppose that $r^S > r^L$. Then, the firm can make $c = +\infty$ by choosing $x = +\infty$ in period $t-1$ for $t \geq 1$, because $b(e)$ and $y(e, k)$ are bounded from above and x is not bounded. But

[5] If $r^S \geq r^L$, $b \leq r^L D$ as $x = D^S$. If $r^L \geq r^S$, $b \leq y(e, A, k)$. Thus, it is the case that $b \leq \max\{r^L D, y(e, k)\}$.

[6] Note also that firms choose the demand for the safe and risky assets (x^d_{+1} and k^d), banks choose the supply of them (x^s_{+1} and k^s), and they are equal in equilibrium ($x_{+1} = x^d_{+1} = x^s_{+1}$ and $k = k^d = k^s$).

$x = +\infty$ and $r^S > r^L \geq 0$ contradict the fact that $r^S = f'(x)$ and $f'(\infty) = 0$. Therefore, $r^S \leq r^L$ must hold. Now suppose that $r^S < r^L$. In this case, the firm can minimize D by setting $x = 0$ in period $t-1$. The firms choose optimally $x = 0$, while the result that $r^S < r^L$ and $x = 0$ contradicts the fact that $r^S = f'(x)$ and $f'(0) = +\infty$. Therefore, $r^S = r^L$. In what follows, we denote

$$r_t \equiv r_t^L = r_t^S.$$

Note that firms are indifferent to the choice of x_t when $r_t^S = r_t^L$ for all t, as the firm value remains independent of this decision.

Determination of the interest rate $r_t = \frac{1}{\beta}$: Define $d(e, D, x) \equiv W - rx$. Since $r^L = r^S$ and $x = D^S$, the bank's problem can be rewritten as follows.

$$\begin{aligned} d(e, D, x; A, k) = &\max_{b, x_{+1} \geq 0} b + (\beta r_{+1} - 1)x_{+1} + \beta d^e(e_{+1}, D_{+1}, x_{+1}; A, k), \\ \text{s.t. } & b \leq y(e), \\ & 0 \leq b \leq rD, \\ & D_{+1} = rD - b, \\ & e_{+1} = e(D_{+1}, x_{+1}), \end{aligned}$$

with the equilibrium condition

$$d(\cdot) = d^e(\cdot).$$

The bank's problem in period 0 is $\max_{k, x_1, D_1} \beta \mathbb{E}_{\tilde{A}}[d(e_1, Qk, x_1; \tilde{A}, k)] + (\beta r_1 - 1)x_1$. Now, suppose that $\beta r_{+1} > 1$. Then, the bank could make $d(e, D, x; A, k) = +\infty$ by setting $x_{+1} = +\infty$. But since $r_{+1} = f'(x_{+1})$, $x_{+1} = \infty$ implies that $r_{+1} = f'(\infty) = 0$, which contradicts $\beta r > 1$. Therefore, $\beta r_{+1} \leq 1$ must hold. Now suppose that $\beta r_{+1} < 1$. Then, the bank would optimally choose $x_{+1} = 0$. In this case, $r_{+1} = f'(x_{+1}) = f'(0) = +\infty$, which contradicts $\beta r_{+1} < 1$. Thus, it must be the case that $r_t (= r_t^S = r_t^L) = \frac{1}{\beta}$ for all $t \geq 1$.

Given that $r_t = \frac{1}{\beta}$ for all t, both firms and banks are indifferent to the choice of x_t, as values of firms and banks are independent of x_t. Thus, the value of x_t is given as $x_t = x^*$, where x^* is the solution to $f'(x) = \frac{1}{\beta}$. Except that the choice of x_t determines the interest rate r_t, it is irrelevant to the equilibrium path of the other variables. Thus, given that $x_t = x^*$ and $r_t = \frac{1}{\beta}$, the problems for firms and banks can be rewritten as follows.

Simplified firm's problem: In period 0, a firm solves

$$\begin{aligned} V_0 = &\max_{D_1, k} \beta \mathbb{E}_{\tilde{A}}[V(D_1; \tilde{A}, k)], \\ \text{s.t. } & D_1 = Qk. \end{aligned} \tag{3}$$

In period t, where $t \geq 1$ and $\tilde{A}$ has been revealed to be $A \in \{A_H, B\}$ in period 1, a firm solves

$$\begin{aligned} V(D) = & \max_e y(e) - e - b(e, D) + \beta V^e(D_{+1}), \quad (4) \\ & \text{s.t. } D_{+1} = rD - b(e, D), \end{aligned}$$

with the equilibrium condition $V(\cdot) = V^e(\cdot)$, where $b(e, D)$ is the solution to (6). Regarding the choice of e, we make the following tie-breaking assumption to avoid ambiguity in the firm's behavior:

Assumption 5. The firm chooses $e = \varepsilon$ when it is indifferent between $e = \varepsilon$ and $e = 0$.

Simplified bank's problem: In period 0, a bank solves

$$\begin{aligned} & \max_{k \in [0,1]} \beta \mathbb{E}_{\tilde{A}}[d(D_1, e_1; \tilde{A}, k)], \quad (5) \\ & \text{s.t. } D_1 = Qk, \\ & \quad e_1 = e(D_1; \tilde{A}, k), \end{aligned}$$

where $e(D; A, k)$ is the equilibrium policy function for the firm. In period t (≥ 1), a bank solves the following problem, given that e is already decided by the firm and $\tilde{A}$ has been revealed to be $A \in \{A_H, B\}$ in period 1:

$$\begin{aligned} d(e, D) = & \max_b b + \beta d^e(e_{+1}, D_{+1}), \quad (6) \\ & \text{s.t. } 0 \leq b \leq y(e), \\ & \quad 0 \leq b \leq rD, \\ & \quad D_{+1} = rD - b, \\ & \quad e_{+1} = e(D_{+1}; A, k), \end{aligned}$$

with the equilibrium condition $d(\cdot) = d^e(\cdot)$.

4 Equilibrium without debt restructuring

In period 0, firms and banks expect that $\tilde{A}$ will be B in period 1 with probability p and A_H with probability $1 - p$. We are interested in the case where $\tilde{A}$ turns out to be A_H in period 1, and we will show that the overly optimistic expectation $B(\gg A_H)$ generates asset-price boom due to the risk-shifting effect demonstrated by Allen and Gale (2000).

Definition of equilibrium: The equilibrium concept in this paper is the Markov perfect equilibrium. The equilibrium is the set of price Q, quantity k, debt D, value functions $\{V(D; A, k),\ d(D; A, k)\}$, the policy function for firms $e(D; A, k)$ and the policy function for banks $b(e, D; A, k)$ that satisfies the following conditions:

- Given price Q, the allocation k and debt D_1 solve (3) and (5), and the market clears, $k = 1$.
- Given that $\tilde{A}$ is revealed to be A, the policy functions solve (4) and (6), respectively, yielding the corresponding value functions. Note that $d(D; A, k) = d(e(D; A, k), D; A, k)$, where $d(e, D; A, k)$ is the solution to (6).
- Debt D evolves according to $D_{+1} = rD - b(e(D; A, k), D; A, k)$.

The following proposition states that there exists a debt overhang equilibrium from period 1 onward. We will specify the equilibrium in period 0 later, where the price Q and the initial debt D_1 are given by (10).

Proposition 1. *There exists a debt-overhang equilibrium, which is the following Markov perfect equilibrium:*

$$V(D; A, k) = \begin{cases} V_{\max} - rD, & \text{if } rD \leq V_{\max}, \\ 0, & \text{if } rD > V_{\max}, \end{cases} \tag{7}$$

$$d(D; A, k) = \begin{cases} rD, & \text{if } rD \leq V_{\max}, \\ d_L, & \text{if } rD > V_{\max}, \end{cases} \tag{8}$$

$$e(D; A, k) = \begin{cases} \varepsilon, & \text{if } rD \leq V_{\max}, \\ 0, & \text{if } rD > V_{\max}, \end{cases} \tag{9}$$

$k = 1$ and $r = \beta^{-1}$, where

$$\begin{aligned} V_{\max} &= \frac{y_H - \varepsilon}{1 - \beta}, \\ y_H &= Ak^{\psi}, \\ d_L &= \frac{y_L}{1 - \beta}, \\ y_L &= A_L k^{\psi}. \end{aligned}$$

In the case where $rD \leq V_{\max}$, the value of $b(e, D; A, k) \in [0, \ \min\{rD, \ y(e, A, k)\}]$ is determined arbitrarily, on the condition that $rD_{+1} \leq V_{\max}$. If $rD_{+1} \leq V_{\max}$ is not feasible, then $b(e, D; A, k) = y(e, A, k)$. The law of motion for D is given by

$$D_{+1} = rD - b(e(D; A, k), D; A, k).$$

Note that D_t grows explosively once rD_t exceeds $V_{\max}$. In this case, the state variable D_t is no longer a payoff relevant state variable, once it satisfies $rD_t > V_{\max}$.

Proof. To ease the exposition, we omit the arguments (A, k) whenever there is no risk of confusion, since they remain constant from period 1 onward. It remains to verify the following optimality conditions for the bank and the firm.

- Bank's optimization: Suppose that the continuation value $d^e(e(D), D)$ satisfies (8) with the firm's effort policy $e(D)$ given by (9), and take the firm's current effort e as fixed. Then, when $e = e(D)$, the solution to (6) also satisfies (8). Any b with $0 \leq b \leq \min\{rD, y(e)\}$ can serve as the policy function $b(e, D)$ as long as $rD_{+1} \leq V_{\max}$, where $D_{+1} = rD - b$; if $rD_{+1} \leq V_{\max}$ is infeasible, then $b = y(e)$.

- Firm's optimization: Suppose that the bank's policy function $b(e, D)$ satisfies $0 \leq b(e, D) \leq \min\{rD, y(e)\}$, with $b(e, D) = y(e)$ whenever $rD_{+1} \leq V_{\max}$ is infeasible, and that the continuation value $V^e(D)$ satisfies (7). Then the solution to (4) also satisfies (7), with the policy function $e(D) = \varepsilon$ for $rD \leq V_{\max}$ and $e(D) = 0$ for $rD > V_{\max}$.

Bank's optimization: Since $d^e(e(D), D)$ satisfies (8), the bank's problem (6) can be rewritten as

$$d(e, D) = \begin{cases} \max_{0 \leq b \leq \min\{rD, y(e)\}} rD & \text{if } rD_{+1} \leq V_{\max}, \\ \max_{0 \leq b \leq \min\{rD, y(e)\}} b + \beta d_L & \text{if } rD_{+1} > V_{\max}, \end{cases}$$

where $D_{+1} = rD - b$. Note that rD is the contractual amount of debt, which equals the maximum the bank can collect from the borrower.

Bank's optimization when $e = \varepsilon$: Suppose that the firm chooses $e = \varepsilon$ in the current period. We distinguish the cases $rD \leq V_{\max}$ and $rD > V_{\max}$.

- Consider the case $rD \leq V_{\max}$. The bank can choose b so that $rD_{+1} \leq V_{\max}$, for the following reason. The condition $rD_{+1} \leq V_{\max}$ is equivalent to $b \geq rD - \beta V_{\max}$, a sufficient condition for which is $b \geq (1 - \beta)V_{\max} = y_H - \varepsilon$; and $b \geq y_H - \varepsilon$ is feasible when $y_H - \varepsilon < rD \leq V_{\max}$, because $e = \varepsilon$ implies $y(e) = y_H$. If instead $rD \leq y_H - \varepsilon$, the bank can attain $rD_{+1} = 0 \leq V_{\max}$ by setting $b = rD$. Therefore, the bank optimally chooses b so as to make $rD_{+1} \leq V_{\max}$, thereby obtaining the maximum value $d(D) = rD$.

- Next consider the case $rD > V_{\max}$, which we split into $V_{\max} < rD \leq \beta V_{\max} + y_H$ and $rD > \beta V_{\max} + y_H$. (Note that the case $e = \varepsilon$ with $rD > V_{\max}$ does not arise in equilibrium; it is an off-equilibrium path.)
  - Consider the case $V_{\max} < rD \leq \beta V_{\max} + y_H$. Here the bank can choose b with $rD - \beta V_{\max} \leq b \leq y_H$, which yields $rD_{+1} \leq V_{\max}$. Choosing such b optimally, the bank obtains the maximum value $d(D) = rD$.
  - Consider the case $rD > \beta V_{\max} + y_H$. Here $rD_{+1} \leq V_{\max}$ is infeasible, because $rD - \beta V_{\max} > y_H \geq b$ implies $rD_{+1} > V_{\max}$. Since $rD_{+1} > V_{\max}$, the bank's value becomes $d(D) = \max_b b + \beta d_L$, which is maximized at $b = y(e) = y_H$.

Bank's optimization when $e = 0$: Suppose that the firm chooses $e = 0$ in the current period.

- Consider the case $rD \leq V_{\max}$, which we split into $rD \leq \beta V_{\max} + y_L$ and $\beta V_{\max} + y_L < rD \leq V_{\max}$. (Note that the case $e = 0$ with $rD \leq V_{\max}$ does not arise in equilibrium; it is an off-equilibrium path.)
  - Consider the case $rD \leq \beta V_{\max} + y_L$. Here the bank can choose b with $rD - \beta V_{\max} \leq b \leq y_L$, which yields $rD_{+1} \leq V_{\max}$, and thereby obtains the maximum value $d(D) = rD$.
  - Consider the case $\beta V_{\max} + y_L < rD \leq V_{\max}$. Here $rD_{+1} \leq V_{\max}$ is infeasible, because $rD - \beta V_{\max} > y_L \geq b$ implies $rD_{+1} > V_{\max}$. Therefore $d(D) = \max_b b + \beta d_L$, which the bank maximizes by choosing $b = y(e) = y_L$.
- Consider the case $rD > V_{\max}$. A necessary condition for $rD_{+1} \leq V_{\max}$ is $b \geq rD - \beta V_{\max} > (1-\beta)V_{\max} = y_H - \varepsilon > y_L$. Since $b \leq y_L$, attaining $rD_{+1} \leq V_{\max}$ is infeasible. Therefore $d(D) = \max_b b + \beta d_L$, which the bank maximizes by choosing $b = y_L$, giving $d(D) = d_L$.

The above arguments yield the following lemma for the solution to the bank's problem.

Lemma 2. *The bank sets the repayment b at any value satisfying $0 \leq b \leq \min\{rD, y(e, A, k)\}$ as long as $rD_{+1} \leq V_{\max}$ can be attained, where $D_{+1} = rD - b$. If $rD_{+1} \leq V_{\max}$ is infeasible, the bank sets $b = y(e, A, k)$.*

Firm's optimization: Since $V^e(D)$ satisfies (7), the firm's problem (4) can be rewritten as

$$V(D) = \begin{cases} \max_e y(e) - e + \beta V_{\max} - rD & \text{if } rD_{+1} \leq V_{\max}, \\ \max_e y(e) - e - b(e) & \text{if } rD_{+1} > V_{\max}. \end{cases}$$

We consider the cases $rD \leq V_{\max}$ and $rD > V_{\max}$ in turn.

- Consider the case $rD \leq V_{\max}$. The bank chooses $b(e) \in [0, \min\{rD, y(e)\}]$ so that $rD_{+1} \leq V_{\max}$, setting $b(e) = y(e)$ if $rD_{+1} \leq V_{\max}$ cannot be attained for any $b \in [0, \min\{rD, y(e)\}]$. Define $V(e, D)$ by

$$V(e, D) = \begin{cases} y(e) - e + \beta V_{\max} - rD & \text{if } rD - b(e) \leq \beta V_{\max}, \\ y(e) - e - b(e) & \text{if } rD - b(e) > \beta V_{\max}. \end{cases}$$

  (Note that $y(e) - e + \beta V_{\max} - rD \geq y(e) - e - b(e)$ if and only if $rD - b(e) \leq \beta V_{\max}$.)

  - Suppose the firm chooses $e = \varepsilon$. Then the bank chooses $b(\varepsilon)$ so that $rD_{+1} \leq V_{\max}$, giving $V(\varepsilon, D) = y_H - \varepsilon + \beta V_{\max} - rD = V_{\max} - rD \geq 0$.

- Suppose the firm chooses $e = 0$, and distinguish the cases $rD \leq y_L + \beta V_{\max}$ and $y_L + \beta V_{\max} < rD \leq V_{\max}$.
  - If $rD \leq y_L + \beta V_{\max}$, the bank chooses $b(0)$ so that $rD_{+1} \leq V_{\max}$, since this is feasible. Thus $V(0, D) = y(0) - 0 + \beta V_{\max} - rD = y_L + \beta V_{\max} - rD < V_{\max} - rD$.
  - If $y_L + \beta V_{\max} < rD \leq V_{\max}$, then $rD_{+1} > V_{\max}$ for all $b \in [0, \min\{rD, y_L\}]$, so $b(0) = y(0) = y_L$. In this case $V(0, D) = y(0) - 0 - b(0) = 0$.

These comparisons show that $V(\varepsilon, D) \geq V(0, D)$ for $rD \leq V_{\max}$. The firm therefore optimally chooses $e = \varepsilon$ and obtains $V(D) = V(\varepsilon, D) = V_{\max} - rD$ for $rD \leq V_{\max}$.

- Consider the case $rD > V_{\max}$. If $rD \geq \frac{y_H}{1-\beta}$, then $rD_{+1} \geq \frac{y_H}{1-\beta}$ for all $b \leq y_H$. In this case the bank chooses $b(e) = y(e)$, and $V(D) = \max_e y(e) - e - b(e) + \beta \cdot 0 = \max_e(-e) = 0$, so the firm optimally chooses $e = 0$. Next consider the case $V_{\max} < rD < \frac{y_H}{1-\beta}$, examining $e = \varepsilon$ and $e = 0$ in turn.
  - Suppose the firm chooses $e = \varepsilon$. If $rD \leq y_H + \beta V_{\max}$, the bank can choose $b \in [rD - \beta V_{\max}, y_H]$, so that $rD_{+1} \leq V_{\max}$; then $V(\varepsilon, D) = y(\varepsilon) - \varepsilon + \beta V_{\max} - rD = V_{\max} - rD < 0$. If $rD > y_H + \beta V_{\max}$, then $rD_{+1} > V_{\max}$ necessarily holds, so the bank chooses $b(\varepsilon) = y(\varepsilon)$ and $V(\varepsilon, D) = y(\varepsilon) - \varepsilon - y(\varepsilon) + \beta \cdot 0 = -\varepsilon < 0$. In either case, $V(\varepsilon, D) < 0$ when $rD > V_{\max}$.
  - Suppose the firm chooses $e = 0$. Then $rD_{+1} > V_{\max}$ necessarily holds: attaining $rD_{+1} \leq V_{\max}$ would require $b \geq rD - \beta V_{\max} > (1 - \beta)V_{\max} = y_H - \varepsilon > y_L$, whereas $b \leq y(0) = y_L$. In this case $V(0, D) = y(0) - 0 - y(0) + \beta \cdot 0 = 0$.

Comparing the two cases shows that the firm optimally chooses $e = 0$ and obtains $V(D) = V(0, D) = 0$ for $rD > V_{\max}$.

The above arguments yield the following lemma for the solution to the firm's problem.

Lemma 3. *The firm chooses $e(D) = \varepsilon$ if $rD \leq V_{\max}$ and $e(D) = 0$ if $rD > V_{\max}$. The firm value is then $V(D) = V_{\max} - rD$ if $rD \leq V_{\max}$, and $V(D) = 0$ if $rD > V_{\max}$.*

Lemmas 2 and 3 imply that if $rD \leq V_{\max}$, then $e = \varepsilon$ and $rD_{+1} \leq V_{\max}$, in which case the bank's optimization above gives $d(D) = rD$. Similarly, if $rD > V_{\max}$, then $e = 0$ and $rD_{+1} > V_{\max}$, in which case the bank's optimization above gives $d(D) = d_L$. $\square$

Note on the policy function $b(e, D)$: Proposition 1 establishes the existence of the debt-overhang equilibrium, while allowing for the multiplicity of the bank's policy function $b(e, D)$. Crucially, the value functions remain invariant to any $b \in [0, \min\{rD, y(e)\}]$ provided that $rD_{+1} = r(rD - b) \leq V_{\max}$. This invariance property implying that $b =$

$\min\{rD, y(e)\}$ weakly dominates any other optimal choice $b' < [0, \min\{rD, y(e)\}]$, we introduce the following rule:

Assumption 6. Whenever the bank's optimization problem yields multiple optimal solutions, the bank selects the largest permissible value of $b \in [0, \min\{rD, y(e)\}]$.

This assumption uniquely pins down the policy function $b(e, D) = \min\{rD, y(e)\}$ in the debt-overhang equilibrium.

Note on the uniqueness of the equilibrium: The debt-overhang equilibrium is a unique equilibrium in the following sense (see Appendix B for the details):

- It is straightforward to show that the equilibrium is unique for $rD \leq V_{\max}$, under which $V(D) = V_{\max} - rD$, $d(D) = rD$, $b(e, D) = \min\{rD, y(e)\}$, and $e(D) = \varepsilon$.
- Although multiple equilibria may potentially arise for $rD > V_{\max}$, only the debt-overhang equilibrium characterized in the above proposition survives as the unique equilibrium when the infinite-horizon economy is defined as the limit of its finite-horizon counterpart. This selection mechanism operates because the equilibrium is uniquely determined in any finite-horizon economy. The proof is provided in Appendix B.

Alternatively, the debt-overhang equilibrium can be justified as the unique equilibrium by refinement of Markov perfect equilibria (Maskin and Tirole, 2001). This resolution of multiplicity stems from the fact that D ceases to be a payoff-relevant state variable once $rD > V_{\max}$, and no other state variables exist in this economy.

Intuition of debt overhang: In the above proof of the proposition, we have shown that the productivity is low ($e = 0$) for all D that satisfies $rD > V_{\max}$. This is due to debt overhang effect, the intuition of which is given by the following one-period example.

- Consider a one-period economy with a single firm that has an outstanding debt D.
- By exerting an additional effort level e, the firm generates output y according to the following production technology:

$$y = \begin{cases} y_L & \text{if } e = 0, \\ y_H & \text{if } e = \varepsilon, \end{cases}$$

  where $y_L < y_H < D$.
- The firm chooses its effort level e, where e also represents the disutility of effort. Since the firm can default on its remaining debt obligations after transferring all realized output to the bank, its net payoff V is determined as follows:

– If $e = 0$, then $V = \max\{0, y_L - D\} - e = 0$, as the bank seizes all output y_L for debt repayment under limited liability.

– If $e = \varepsilon$, then $V = \max\{0, y_H - D\} - e = -\varepsilon < 0$, since the bank similarly appropriates all output y_H, leaving the firm to bear only the disutility of effort.

- Therefore, in equilibrium, the firm optimally chooses $e = 0$, resulting in the low output level y_L. In summary, the expectation that the bank will appropriate all ex-post output—due to the debt burden exceeding the firm's maximum revenue—disincentivizes the firm from exerting effort. This mechanism captures the debt overhang effect.

Risk-shifting bubble of asset price in period 0: Firm's problem in period 0 is written as

$$\max_{D_1=Qk} \beta \mathbb{E}_{\tilde{A}}[V(D_1; \tilde{A}, k)] = \max_{D_1=Qk} \mathbb{E}_{\tilde{A}}[\max\{\beta V_{\max} - \beta r D,\ 0\}] = \max_k p \left[\beta \frac{Bk^{\psi} - \varepsilon}{1-\beta} - Qk\right].$$

The last equality is due to the assumption that the equilibrium price of land is so high that the firm falls into the debt overhang state ($rD_1 > V_{\max}$) where $V = 0$, if $\tilde{A} = A_H$. The FOC with respect to k at $k = 1$ implies that

$$D_1 = Q = \frac{\beta\psi}{1-\beta} B, \tag{10}$$

whereas the fundamental price of the asset should be $Q^F = \frac{\beta\psi}{1-\beta}\{pB + (1-p)A_H\}$. Thus the market price of the asset Q is much higher than the fundamental price Q^F, due to the risk shifting effect (as in Allen and Gale (2000)). We assume that $\psi B < B - \varepsilon$ to ensure that D_1 is repayable when the productivity parameter $\tilde{A}$ turns out to be B. We also assume that $\psi B > A_H - \varepsilon$ to ensure that $rD_1 > V_{\max}$ when $\tilde{A}$ turns out to be A_H.

5 Model with debt restructuring

If $D > D_{\max} \equiv \frac{1}{r} V_{\max} = \frac{\beta(y_H - \varepsilon)}{1-\beta}$, the social surplus is $d + V = d_L = \frac{y_L}{1-\beta}$, while if $D \leq D_{\max}$, it is $V_{\max} = \frac{y_H - \varepsilon}{1-\beta}$. Since $V_{\max} > d_L$, it is clear that when $D > D_{\max}$, debt restructuring (DR)—which writes down the contractual debt from D to $D_{\max}$—improves social surplus and increases the value of the bank from d_L to $V_{\max}$.[7] While DR by banks was prohibited in the baseline model of the previous section, this section modifies the framework to allow banks to implement debt restructuring.

We assume that at the beginning of each period t, the bank has the opportunity to implement DR, conditional on DR not having been implemented in any prior period. As

[7] We could assume that the bank can choose any reduced debt level $D' \leq D$. However, because the bank optimally chooses $D' = D_{\max}$ to maximize its payoff, we assume for simplicity that the bank reduces D exactly to $D_{\max}$ in the event of DR.

explained in the next subsection, when deciding whether to implement DR, the bank faces internal conflicts regarding the allocation of the restructuring surplus.

5.1 Bank decision-making on debt restructuring

When $D_t > D_{\max}$, the bank is better off reducing D_t to $D_{\max}$. However, the bank's decision-making regarding debt restructuring is subject to the following frictions: first, to implement DR, the bank manager must make a concession by accepting a lower share of the bank's payoff; and second, she must pay fixed compensation to internal stakeholders. We introduce a state variable ω_t that governs the status of DR in period t.

Definition 1. The state variable ω_t takes a value in $\{-1, 0, 1\}$, where

- $\omega_t = -1$ if the bank decides not to implement DR in period t;
- $\omega_t = 0$ if the bank decides to implement DR in period t; and
- $\omega_t = 1$ if DR has already been implemented in a previous period $t' \leq t-1$.

The law of motion for ω_t is as follows: if $\omega_{t-1} = -1$, the bank can choose $\omega_t \in \{-1, 0\}$; if $\omega_{t-1} \in \{0, 1\}$, then $\omega_t = 1$.

Timeline. We outline the timeline of events within period t, which is divided into three subperiods: *morning*, *afternoon*, and *evening*.

- **Morning:** The firm begins the period owing D_t to the bank. The state variable ω_t realizes: the bank manager chooses $\omega_t \in \{-1, 0\}$ if $\omega_{t-1} = -1$, and $\omega_t = 1$ if $\omega_{t-1} \in \{0, 1\}$. The effective contractual debt $\tilde{D}_t \equiv \tilde{D}(\omega_t, D_t)$ is determined as follows: $\tilde{D}(-1, D_t) = \tilde{D}(1, D_t) = D_t$ and $\tilde{D}(0, D_t) = \min\{D_t, D_{\max}\}$.
- **Afternoon:** Conditional on ω_t, the firm chooses its effort level $e_t = e(\omega_t, \tilde{D}_t)$, and production $y(e_t)$ takes place.
- **Evening:** Given ω_t and e_t, the bank determines the repayment amount $b_t = b(\omega_t, e_t, \tilde{D}_t)$. The debt level then evolves according to $D_{t+1} = r\tilde{D}_t - b_t$.

Internal friction 1: Concession. To implement debt restructuring, the bank manager (BM) must make a concession to opponents within the bank. Specifically, the BM's compensation structure is determined as follows:

- **When $\omega_t = -1$ (DR is not implemented):** The BM's compensation is proportional to the bank's revenue b_t.

- **When $\omega_t \in \{0,1\}$ (DR is or has been implemented):** The BM's compensation is proportional to αb_t, where $0 < \alpha < 1$. Accepting this lower payoff share α represents the BM's concession.

This compensation structure serves as a reduced form for internal conflicts within the bank, which can be characterized by a war of attrition. Specifically, opponents resist DR unless the BM makes a concession, thereby leading to the inefficient continuation of debt overhang. Alternatively, this concession can be interpreted as a cost paid to external parties; for instance, the BM must pay a fraction $1-\alpha$ of the bank's revenue to a financial advisor to facilitate the restructuring process.

Internal friction 2: Coordination cost. When the BM implements DR in period t (i.e., $\omega_t = 0$), she must pay a coordination cost ξ to each of the n stakeholders involved in the debt project. The total coordination cost incurred in period t is $n\xi$, where the number of stakeholders n is given by

$$n = \phi D_1,$$

and D_1 denotes the initial loan amount originated in period 0. We assume that the number of stakeholders increases with the size of the initial loan D_1, reflecting the natural premise that larger loans involve more decision-makers within the bank.

In what follows we distinguish between the payoff for the bank manager, denoted by $d^{(BM)}(\omega, e, D)$, and the payoff for the bank as a whole, denoted by $d(\omega, e, D)$. Since the BM is the sole decision-maker regarding debt restructuring, our analysis primarily focuses on the BM's problem and $d^{(BM)}(\cdot)$.

5.2 Optimization problems

We characterize the optimization problems within period t backward from the evening to the morning subperiods.

Evening: Bank manager's problem (BME). Given the realized states $(\omega_t, e_t, \tilde{D}_t)$ and the expectations regarding future variables $(d^{(BM)e}(\cdot), \omega^e(\cdot), e^e(\cdot))$, the BM solves the following problem in the evening subperiod:

$$\begin{aligned} d^{(BM)}(\omega, e, \tilde{D}) = &\max_b \Pi(b, \omega) + \beta d^{(BM)e}(\omega_{+1}, e_{+1}, D_{+1}), \\ \text{s.t. } &0 \leq b \leq \min\{r\tilde{D}, y(e)\}, \\ &D_{+1} = r\tilde{D} - b, \\ &\omega_{+1} = \omega^e(\omega, D_{+1}), \\ &e_{+1} = e^e(\omega_{+1}, D_{+1}), \end{aligned} \tag{11}$$

where the BM's current payoff function $\Pi(b, \omega)$ is given by

$$\Pi(b, \omega) = \begin{cases} b & \text{if } \omega = -1, \\ \alpha b - n\xi & \text{if } \omega = 0, \\ \alpha b & \text{if } \omega = 1. \end{cases} \tag{12}$$

We denote the optimal policy function for this problem by $b_t = b(\omega, e, \tilde{D})$.

Afternoon: Firm's problem (FPA). Given the realized states $(\omega_t, \tilde{D}_t)$ and expectations regarding future variables $(\omega^e(\cdot), b^e(\cdot), V^e(\cdot))$, the firm solves the following problem in the afternoon subperiod:

$$\begin{aligned} V(\omega, \tilde{D}) =& \max_e \left\{ y(e) - e - b^e(\omega, e, \tilde{D}) + \beta V^e(\omega_{+1}, D_{+1}) \right\}, \\ \text{s.t. } & D_{+1} = r\tilde{D} - b^e(\omega, e, \tilde{D}), \\ & \omega_{+1} = \omega^e(\omega, D_{+1}). \end{aligned} \tag{13}$$

We denote the optimal policy function for this problem by $e_t = e(\omega, \tilde{D})$.

Morning: Bank manager's problem (BMM). If and only if debt restructuring has not been implemented in any prior period (i.e., $\omega_{t-1} = -1$), the BM chooses whether to implement DR ($\omega_t = 0$) or defer it ($\omega_t = -1$) in the morning subperiod. Let $d^{(BM)}(\omega, D)$ be the reduced-form payoff function for the BM, defined as

$$d^{(BM)}(\omega, D) \equiv d^{(BM)}(\omega, e(\omega, \tilde{D}), \tilde{D}),$$

where $\tilde{D} = \tilde{D}(\omega, D)$, and $d^{(BM)}(\omega, e, \tilde{D})$ is the value function obtained from (11). Given $\omega_{t-1} = -1$ and the initial debt D_t, the BM solves the following problem:

$$\max_{\omega \in \{-1, 0\}} d^{(BM)}(\omega, D_t). \tag{14}$$

The optimal policy function is denoted by $\omega_t = \omega(\omega_{t-1}, D_t)$, where $\omega(-1, D_t)$ solves (14), and $\omega(0, D_t) = \omega(1, D_t) = 1$ by definition.

Equilibrium conditions: A rational expectations equilibrium requires that the subjective expectations of all economic agents coincide with the actual realizations of the policy and value functions:

$$\omega^e(\cdot) = \omega(\cdot), \quad e^e(\cdot) = e(\cdot), \quad b^e(\cdot) = b(\cdot), \quad V^e(\cdot) = V(\cdot), \quad d^{(BM)e}(\cdot) = d^{(BM)}(\cdot).$$

5.3 Equilibrium with debt restructuring

This subsection characterizes the Markov perfect equilibrium (MPE), where the state space is defined by the state variables (ω_{t-1}, D_t). We define the thresholds D^* and n^* as follows:

$$\begin{aligned} D^* &\equiv \frac{1}{\phi\xi}(\alpha V_{\max} - d_L), \\ n^* &\equiv \phi D^*. \end{aligned}$$

As shown in the subsequent proposition, debt restructuring never occurs if the initial debt satisfies $D_1 > D^*$. Conversely, it occurs immediately in period 1 if $D_{\max} < D_1 \leq D^*$.

Relationship between D^* and production payoffs. Before presenting the equilibrium analysis, we specify the parameter space for the threshold D^*. Throughout this section, we restrict our attention to cases where the initial debt D_1 satisfies

$$0 < D_1 < \frac{\alpha V_{\max}}{\phi\xi}, \tag{15}$$

which ensures that the BM's net payoff from debt restructuring, $\alpha V_{\max} - n\xi$, remains strictly positive. Under this parameter constraint, we establish the following relationship between the threshold D^* and the production payoffs:

Lemma 4. *For a parameter $x \in [0, 1]$, $D^* = (1-x)\frac{\alpha V_{\max}}{\phi\xi}$ if and only if $x = \frac{y_L}{\alpha(y_H - \varepsilon)}$.*

Recall that the effective contractual debt $\tilde{D} = \tilde{D}(\omega, D)$ is defined by $\tilde{D}(-1, D) = \tilde{D}(1, D) = D$, and $\tilde{D}(0, D) = D_{\max}$ for $D > D_{\max}$.[8] The equilibrium is summarized in the following proposition.

Proposition 5. *There exists a Markov perfect equilibrium with the following features, which we refer to as the debt-restructuring equilibrium. The bank manager's restructuring choice $\omega(\omega_{t-1}, D)$ is characterized by*

$$\omega(-1, D) = \begin{cases} -1 & \text{for } n > n^* \text{ or } D \leq D_{\max}, \\ 0 & \text{for } n \leq n^* \text{ and } D > D_{\max}, \end{cases} \tag{16}$$

where $n = \phi D_1$ (implying that $n \leq n^*$ is equivalent to $D_1 \leq D^*$), and $\omega(0, D) = \omega(1, D) =$ 1. In this equilibrium, the firm's optimal effort is given by

$$e(\omega, \tilde{D}) = \begin{cases} \varepsilon & \text{for } \tilde{D} \leq D_{\max}, \\ 0 & \text{for } \tilde{D} > D_{\max}, \end{cases} \tag{17}$$

[8] The BM will never optimally choose $\omega = 0$ when $D \leq D_{\max}$. However, for completeness, we can define $\tilde{D}(0, D) = D$ for $D \leq D_{\max}$.

where $\tilde{D}(\omega, D) = D$ *for* $\omega \in \{-1, 1\}$ *and* $\tilde{D}(\omega, D) = D_{\max}$ *for* $\omega = 0$*, and the optimal repayment policy* $b \in [0, \min\{r\tilde{D}, y(e)\}]$ *satisfies* $D_{+1} = r\tilde{D} - b \leq D_{\max}$ *whenever feasible; otherwise,* $b(\omega, e, \tilde{D}) = y(e)$*. The firm's value function* $V(\omega, \tilde{D})$ *is given by*

$$V(\omega, \tilde{D}) = \begin{cases} V_{\max} - r\tilde{D} & \text{for } \tilde{D} \leq D_{\max}, \\ 0 & \text{for } \tilde{D} > D_{\max}, \end{cases} \tag{18}$$

and the BM's value functions are given by

$$d^{(BM)}(1, D) = \begin{cases} \alpha r D & \text{for } D \leq D_{\max}, \\ \alpha d_L & \text{for } D > D_{\max}, \end{cases} \tag{19}$$

$$d^{(BM)}(0, D) = \alpha V_{\max} - n\xi \quad \text{for } D > D_{\max}, \tag{20}$$

while $d^{(BM)}(-1, D)$ *satisfies:*

- *For* $n > n^*$ *,*

$$d^{(BM)}(-1, D) = \begin{cases} rD & \text{for } D \leq D_{\max}, \\ d_L & \text{for } D > D_{\max}, \end{cases} \tag{21}$$

- *For* $n \leq n^*$ *,*

$$d^{(BM)}(-1, D) = \begin{cases} rD & \text{for } D \leq D_{\max}, \\ y_L + \beta(\alpha V_{\max} - n\xi) & \text{for } D > D_{\max}. \end{cases} \tag{22}$$

The equilibrium path (16) *of* ω_t *indicates that debt restructuring never occurs if* $D_1 > D^*$*, whereas it occurs immediately in period 1 if* $D_1 \leq D^*$*. Moreover,* (17) *implies that, along the equilibrium path,* $b(\omega, e, \tilde{D}) = y(e) = y_L$ *whenever* $\tilde{D} > D_{\max}$*.*[9]

[9] For reference, we provide the payoff function for the bank as a whole, denoted by $d(\omega, D)$. The BM receives $d^{(BM)}(\omega, D)$ as compensation, which constitutes a portion of $d(\omega, D)$:

$$d(1, D) = \begin{cases} rD & \text{for } D \leq D_{\max}, \\ d_L & \text{for } D > D_{\max}, \end{cases}$$

$$d(0, D) = V_{\max} \quad \text{for } D > D_{\max},$$

while $d(-1, D)$ is determined as follows:

- For $n > n^*$,

$$d(-1, D) = \begin{cases} rD & \text{for } D \leq D_{\max}, \\ d_L & \text{for } D > D_{\max}, \end{cases}$$

- For $n \leq n^*$,

$$d(-1, D) = \begin{cases} rD & \text{for } D \leq D_{\max}, \\ y_L + \beta V_{\max} & \text{for } D > D_{\max}. \end{cases}$$

Proof. We establish the existence of the debt-restructuring equilibrium by verifying the following three optimality conditions for the bank and the firm.

- **Bank's optimization in choosing b_t:** Suppose that in the bank's evening problem (BME), the expectation $d^{(BM)e}(\cdot)$ satisfies (19)–(22), $\omega^e(\cdot)$ satisfies (16), and $e^e(\cdot)$ satisfies (17). Given the current restructuring decision ω and the firm's effort e, the value function $d^{(BM)}(\omega, D)$ that is defined as $d^{(BM)}(\omega, e(\omega, D), D)$, where $d^{(BM)}(\omega, e, D)$ solves (11), also satisfies (19)–(22). Any repayment $b \in [0, \min\{r\tilde{D}, y(e)\}]$ characterizes the optimal policy $b(\omega, e, D)$ as long as $rD_{+1} \leq V_{\max}$, where $D_{+1} = r\tilde{D} - b$. If $rD_{+1} \leq V_{\max}$ is infeasible, the optimal policy requires $b = y(e)$.
- **Firm's optimization in choosing e_t:** Suppose that the bank's repayment policy $b(\omega, e, D)$ satisfies $0 \leq b(\omega, e, D) \leq \min\{r\tilde{D}, y(e)\}$, with $b(\omega, e, D) = y(e)$ whenever $rD_{+1} \leq V_{\max}$ is infeasible, and that the firm's subjective expectation $V^e(\omega, D)$ satisfies (18). Then, the value function solving (13) also satisfies (18), and the optimal effort policy is given by (17).
- **Bank's optimization in choosing ω_t:** Suppose that in the bank's morning problem (BMM), the expectation $d^{(BM)e}(\cdot)$ satisfies (19)–(22), $\omega^e(\cdot)$ satisfies (16), and $e^e(\cdot)$ satisfies (17). Then, the optimal restructuring choice $\omega(\omega_{t-1}, D)$ also satisfies (16).

Bank's optimization in choosing b_t: Since $d^{(BM)e}(\omega, D)$ is independent of D for $rD > V_{\max}$, analogous arguments to those in the proof of Proposition 1 apply, from which this optimality condition straightforwardly follows.

Firm's optimization in choosing e_t: Given that $V^e(\cdot)$ satisfies (18), the state variable ω affects the firm's problem exclusively through the restructured contractual debt $\tilde{D}$. Consequently, the firm's optimization problem becomes identical to that in Proposition 1. Therefore, the same arguments apply, ensuring that the optimal effort policy $e(\cdot)$ is characterized by (17).

Bank's optimization in choosing ω_t: We focus on the case where $D > D_{\max}$. If the bank manager chooses $\omega = 0$, her payoff simplifies to

$$d^{(BM)}(0, D) = \alpha V_{\max} - n\xi.$$

Alternatively, if she chooses $\omega = -1$, given that expectations $(d^{(BM)e}, \omega^e, e^e)$ satisfy (19)–(22), (16), and (17), her payoff is determined as follows:

- For $n > n^*$, because $\omega^e(\cdot) = -1$ along the equilibrium path, we have

$$d^{(BM)}(-1, D) = y_L + \beta d^{(BM)e}(\omega_{+1}, D_{+1}) = y_L + \beta d^{(BM)e}(-1, D_{+1}) = y_L + \beta d_L = d_L.$$

- For $n \leq n^*$, because $\omega^e(\cdot) = 0$ along the equilibrium path, we have

$$d^{(BM)}(-1, D) = y_L + \beta d^{(BM)e}(\omega_{+1}, D_{+1}) = y_L + \beta d^{(BM)e}(0, D_{+1}) = y_L + \beta(\alpha V_{\max} - n\xi).$$

Recall that the number of stakeholders satisfies $n = \phi D_1$; hence, the conditions $n > n^*$ and $n \leq n^*$ are equivalent to $D_1 > D^*$ and $D_1 \leq D^*$, respectively. Since n is time-invariant, the bank manager solves the following static comparison for a given n:

$$\max_{\omega \in \{-1,0\}} d^{(BM)}(\omega, D).$$

For $n > n^*$, the condition for $d^{(BM)}(-1, D) > d^{(BM)}(0, D)$ requires $\alpha V_{\max} - n\xi < d_L$, which simplifies to $n > n^*$. Conversely, for $n \leq n^*$, the condition for $d^{(BM)}(-1, D) \leq d^{(BM)}(0, D)$ requires $\alpha V_{\max} - n\xi \geq y_L + \beta(\alpha V_{\max} - n\xi)$, which simplifies to $\alpha V_{\max} - n\xi \geq d_L$, which is equal to $n \leq n^*$. This verifies that the optimal restructuring policy $\omega(\cdot)$ satisfies (16).

We have verified the optimality of all decision-making rules for both the bank and the firm: the value functions and policy functions in equilibrium perfectly coincide with those described in Proposition 5, given that subjective expectations are rational. This completes the proof of the existence of the debt-restructuring equilibrium. □

Note on uniqueness and externalities. Having established the existence of the debt-restructuring equilibrium—where restructuring occurs immediately in period 1 if $D_1 \in (D_{\max}, D^*]$ and is indefinitely precluded if $D_1 > D^*$—an analytical caveat regarding equilibrium uniqueness analogous to that for the debt-overhang equilibrium (Proposition 1) applies here. We are therefore confident that focusing on the debt-restructuring equilibrium is well-grounded, leaving other potential equilibria outside the scope of our primary analysis. Regarding externalities, embedding monopolistic competition into this framework generates a demand externality that yields the following intuitive macroeconomic predictions:

- **Idiosyncratic shocks:** When a credit-fueled asset price bubble bursts and the resulting debt overhang is confined to narrow market segments, banks tend to implement debt restructuring promptly, rendering the ensuing economic stagnation short-lived.

- **Aggregate shocks:** Conversely, when the bust is economy-wide and debt overhang pervades the entire market, banks lack the incentive to restructure. This coordination failure can trap the economy in a state of protracted stagnation.

See Appendix A for further details.

6 Policy implications

The analysis in the preceding section implies that a policy intervention aimed at reducing the borrower's debt burden from D ($> D^*$) to D^* would induce voluntary debt restructuring and restore efficient production ($y_H - \varepsilon$). Given our earlier finding that banks spontaneously restructure debt to $D_{\max}$ as long as $D_1 \leq D^*$, this section focuses exclusively on the parameter region where

$$D_1 > D^*,$$

and compares the following two policy options:

- **Firm subsidy:** A direct subsidy paid to the borrowing firms, denoted by S_t^F.
- **Bank subsidy:** A conditional subsidy paid to banks, denoted by S_t^B, triggered only upon the restructuring of nonperforming loans from D_t to $D_{\max}$.

This section demonstrates that bank subsidies strictly dominate firm subsidies in terms of fiscal efficiency: a smaller fiscal outlay directed toward banks can more effectively alleviate debt overhang and restore firm productivity. Conversely, we show that firm subsidies carry a potential pitfall, where an insufficient subsidy may inadvertently hinder the bank's incentive to restructure.

V-shaped recovery versus L-shaped recession: We provide a brief note on the conventional tradeoff debated in the literature between a sharp V-shaped recovery and a protracted L-shaped recession (or secular stagnation). In policy debates following financial crises, it is frequently argued that governments face a choice between the rapid liquidation of over-indebted zombie firms and their gradual, drawn-out resolution. The underlying premise is that a swift liquidation of a large mass of zombie firms induces a severe short-term contraction followed by a rapid rebound (a V-shaped recovery), whereas a gradual resolution mitigates the initial downturn at the cost of an extended period of low growth (an L-shaped stagnation); see, for example, Acharya, Lenzu, and Wang (2023). While both a V-shaped recovery and an L-shaped stagnation entail substantial economic distress due to the inevitable liquidation of over-indebted borrowers, our model suggests that debt restructuring facilitates a macroeconomic recovery entirely devoid of such inefficiencies. A debt write-down from D_t ($> D_{\max}$) to $D_{\max}$ allows production to immediately recover from y_L to $y_H - \varepsilon$ without incurring any real costs associated with firm liquidation.[10]

[10] Although a debt reduction of $D_t - D_{\max}$ is recognized as an extraordinary loss in corporate and bank accounting, it does not reduce actual physical production. Gross Domestic Product (GDP) does not decline with the write-down; rather, it increases as firms shift production from y_L to $y_H - \varepsilon$. Furthermore, the reduction in the BM's payoff due to intra-bank conflicts represents a distributional transfer within the financial institution rather than an aggregate deadweight loss. Thus, debt restructuring entails no real costs for the economy.

6.1 Subsidy to borrowers

In a state where $D_1 > D_{\max}$ and production is inefficient (y_L), a direct subsidy to borrowing firms seems to be a natural policy option to restore the efficient production level $y_H - \varepsilon$. Suppose that the government injects a subsidy S_1^F into the firm in period 1, immediately after the productivity A_H is realized. The intra-period timeline within period 1 unfolds as follows:

- **Government announcement:** The government publicly commits to providing a subsidy S^F to the firm.
- **Bank's restructuring decision:** The bank decides whether to restructure the face value of debt from D to $\tilde{D} \in \{D, D_{\max} + \beta S^F\}$ by choosing $\omega \in \{-1, 0\}$.
- **Firm's production and subsidy payout:** The firm chooses its effort level e and production takes place. Concurrently, the government disperses the subsidy S^F to the firm.
- **Repayment and debt evolution:** The bank determines the repayment amount $b \leq \min\{r\tilde{D}, y(e) + S^F\}$. The debt then evolves according to $D_{+1} = r\tilde{D} - b$.

We compare the bank manager's payoffs under $\omega = 0$ and $\omega = -1$. Before analyzing the region where debt restructuring is necessary, we briefly characterize the benchmark case where the subsidy is large enough to make the initial debt fully repayable.

When S^F makes debt fully repayable: Suppose that the subsidy S^F is sufficiently large such that

$$S^F \geq r(D - D_{\max}),$$

which can be rewritten as $rD \leq V_{\max} + S^F$. This condition implies that the existing debt can be fully serviced via the combination of the fiscal subsidy S^F and the underlying firm value $V_{\max}$. In this regime, the firm internalizes the full marginal return of its actions and chooses the efficient effort level $e_1 = \varepsilon$ (yielding high productivity A_H). Consequently, explicit debt restructuring is unnecessary since the contractual amount of debt becomes fully repayable with the fiscal injection.

When S^F is insufficient: Next, we turn to the more realistic case where the subsidy is small relative to the debt overhang:

$$S^F < r(D - D_{\max}).$$

If the bank chooses to implement debt restructuring ($\omega = 0$), the BM's payoff—denoted with the superscript SF to indicate the presence of the subsidy—is given by

$$d^{(BM)SF}(0, D) = \alpha(V_{\max} + S^F) - n\xi.$$

Alternatively, if the bank defers restructuring ($\omega = -1$), the contractual debt remains at $\tilde{D} = D > D_{\max}$, leading the firm to exert zero effort ($e = 0$) and produce the low output y_L. Since $rD - S^F > rD_{\max}$, the next-period debt remains above the overhang threshold ($D_{+1} > D_{\max}$) because $D_{+1} = rD - S^F - y_L > rD_{\max} - y_L > rD_{\max} - (y_H - \varepsilon) = D_{\max}$. This implies that the bank extracts the entire cash flow $b = y_L + S^F$, and agents expect no restructuring in the next period ($\omega^e(-1, D_{+1}) = -1$). These equilibrium transitions yield the following payoff for the BM:

$$d^{(BM)SF}(-1, D) = y_L + S^F + \beta d^{(BM)e}(-1, D_{+1}) = S^F + d_L.$$

Recall that $\alpha V_{\max} - n\xi - d_L < 0$ holds for any $D > D^*$. Since the government's objective is to induce debt restructuring to restore efficient production ($e = \varepsilon$), the policy is effective only if it satisfies

$$d^{(BM)SF}(0, D) - d^{(BM)SF}(-1, D) = (\alpha - 1)S^F + \underbrace{\alpha V_{\max} - n\xi - d_L}_{<0} > 0.$$

Crucially, because $\alpha < 1$, this inequality can never be satisfied for any non-negative subsidy $S^F \geq 0$. Instead, the firm subsidy counterproductively diminishes the BM's incentives to restructure:

$$d^{(BM)SF}(0, D) - d^{(BM)SF}(-1, D) < d^{(BM)}(0, D) - d^{(BM)}(-1, D) < 0.$$

The economic intuition behind this perverse result is straightforward. While the borrower subsidy S^F expands the total cash flow available to the bank manager under both strategies, it disproportionately benefits the status quo ($\omega = -1$) by a factor of S^F, whereas it only increases the restructuring payoff ($\omega = 0$) by αS^F. This asymmetric rent extraction defeats the policy's intended goal, which requires making the payoff of $\omega = 0$ larger than that of $\omega = -1$. This formalizes our argument that direct subsidies to borrowers disincentivize bank managers from choosing $\omega = 0$, thereby impeding debt restructuring. We summarize this mechanism in the following lemma.

Lemma 6. *A firm subsidy S^F introduced in period 1 successfully restores the high-productivity state A_H if and only if it satisfies $S^F \geq S^F(D_1)$, where the threshold is defined as:*

$$S^F(D_1) \equiv r(D_1 - D_{\max}). \tag{23}$$

Conversely, any subsidy below this threshold ($S^F < S^F(D_1)$) is counterproductive: it distorts bank incentives, precludes debt restructuring, and impedes productivity recovery.

This highlights a critical non-monotonicity in firm subsidies: they are either costly or detrimental, supporting the relative efficiency of bank subsidies, which we analyze below. One caveat deserves mention. We assume that subsidies are financed via lump-sum taxes to neutralize any distortionary effects of taxation. This simplification is justified because our analysis focuses on the relative scale of S^F versus S^B. Insofar as tax distortions apply equally to both schemes, omitting them does not alter the fundamental comparison between the two policies.

Interpretation of fiscal policy: If we interpret borrower subsidies as a form of expansionary fiscal policy aimed at boosting the incomes of borrowers in general, this result implies that such expansionary policy may inadvertently impede productivity recovery in the aftermath of a financial crisis. In other words, fiscal expansion can hinder economic recovery if the fundamental cause of the productivity decline is a debt overhang.

6.2 Subsidy to banks

In this subsection, we analyze a bank subsidy, denoted by S_t^B, which the government disperses in period 1 to banks that restructures its outstanding loan from D_1 down to $D_{\max}$. By assumption, the government cannot directly subsidize the bank manager or individual internal stakeholders. This restriction captures the informational friction where intra-bank coordination and rent allocation take place within the bank, rendering them neither observable nor verifiable to the government.

The intra-period timeline within period 1 unfolds as follows:

- **Government announcement:** The government publicly commits to providing a conditional subsidy S^B to the bank.
- **Bank's restructuring decision:** The bank decides whether to restructure the face value of debt to $\tilde{D} \in \{D, D_{\max}\}$ by choosing $\omega \in \{-1, 0\}$.
- **Firm's production and subsidy payout:** The firm chooses its effort level e and production takes place. The government disperses the subsidy S^B to the bank if and only if debt restructuring has been implemented ($\omega = 0$).
- **Repayment and debt evolution:** The bank determines the repayment amount $b \leq \min\{r\tilde{D}, y(e)\}$. The debt then evolves according to $D_{+1} = r\tilde{D} - b$.

We evaluate the bank manager's payoffs under $\omega = 0$ and $\omega = -1$. If the bank implements debt restructuring ($\omega = 0$), the BM's post-subsidy payoff evaluates to

$$d^{(BM)SB}(0, D) = \alpha(V_{\max} + S^B) - n\xi,$$

where we denote the BM's payoff with the subsidy S^B by $d^{(BM)SB}(\cdot)$. Conversely, if the bank defers restructuring ($\omega = -1$), the contractual debt remains at $\tilde{D} = D > D_{\max}$. Arguments analogous to the proof of Proposition 1 ensure that the firm exerts zero effort ($e = 0$) and next-period debt remains in the overhang zone ($D_{+1} > D_{\max}$). Consequently, the bank collects $b = y_L$ and agents expect persistent inaction ($\omega^e(-1, D_{+1}) = -1$), yielding the following payoff for the BM:

$$d^{(BM)SB}(-1, D) = y_L + \beta d^{(BM)e}(-1, D_{+1}) = y_L + \beta d_L = d_L.$$

To incentivize debt restructuring, the government must ensure that the policy satisfies the participation constraint:

$$d^{(BM)SB}(0, D) - d^{(BM)SB}(-1, D) \geq 0,$$

which holds if and only if $S^B \geq S^B(D_1)$, where the threshold is defined as

$$S^B(D_1) \equiv \frac{\phi\xi}{\alpha}(D_1 - D^*). \tag{24}$$

To focus on the economically interesting region where initial debt satisfies $D_1 > D_{\max}$ and given the upper bound $D_1 < \alpha V_{\max}/(\phi\xi)$ imposed in condition (15), we require the following parameter restriction:

$$\beta < \frac{\alpha}{\phi\xi}. \tag{25}$$

Under condition (25) and given $D_{\max} < D^*$, it straightforwardly follows that

$$S^B(D_1) = \frac{\phi\xi}{\alpha}(D_1 - D^*) < r(D_1 - D_{\max}) = S^F(D_1).$$

We summarize these findings in the following proposition.

Proposition 7. *The minimum bank subsidy required to induce productivity recovery is given by (24). Furthermore, bank subsidies strictly dominate borrower subsidies in terms of fiscal efficiency, as the required bank subsidy $S^B(D_1)$ is strictly smaller than the corresponding firm subsidy $S^F(D_1)$.*

The bank subsidy in our framework can be naturally interpreted as *bank recapitalization* contingent on the write-down of nonperforming loans. Our findings thus suggest that bank recapitalization designed to align financial incentives constitutes a more efficient policy response to financial crises than direct bailouts of over-indebted borrowers.

Intuition for the efficiency of bank subsidies: The relative efficiency of bank subsidies can be understood as follows. Since the objective of these subsidies is to facilitate debt restructuring and restore productivity, their effectiveness relies on being contingent upon the restructuring process. In our framework, bank subsidies can be directly conditioned on debt restructuring because banks are the primary agents responsible for deciding and implementing these operations. In contrast, subsidies to firms cannot be tied to restructuring; more importantly, such unconditional transfers exacerbate distortions in bank managers' incentives, thereby hindering the restructuring process.

Bank subsidy does not induce moral hazard of borrowers: We can show that the bank subsidy does not induce the ex-ante moral hazard of the borrowers.

Lemma 8. *The announcement in period 0 of the bank subsidy provided in period 1 or later does not affect the equilibrium prices and allocations in period 0.*

The economic intuition for the reason why the bank subsidy does not intrinsically induce borrowers' moral hazard is the following: The bank subsidy is paid in the state where $V = 0$. It affects only the ex-post payoffs of banks, leaving the ex-post payoffs of firms unchanged ($V = 0$). On the other hand, the ex-ante equilibrium prices and allocations are determined by the firms' actions. Since the firms' payoffs remain unchanged at $V = 0$ regardless of whether the bank subsidy is anticipated, their actions in period 0 are unaffected by the bank subsidy.

7 Conclusion

This paper has demonstrated that a framework incorporating risk-shifting asset bubbles, borrower debt overhang, and costly creditor coordination can replicate key empirical regularities of financial crises—specifically, booms in asset prices and credit followed by a bust and persistent productivity declines. Our analysis reveals that when debt levels are modest, voluntary restructuring occurs, and the social optimum is achieved without intervention. In contrast, when debt is elevated, debt restructuring is delayed due to coordination failure within the banks. This finding is consistent with empirical evidence that larger credit-driven asset booms tend to precede more protracted recessions. A notable contribution of our model is that it explains post-crisis stagnation without invoking standard financial frictions, such as credit crunches or tightening of borrowing constraints. This point is noteworthy because the existing literature that focuses on credit frictions primarily emphasizes policies targeting the supply side of credit, such as monetary policy or prudential regulations, whereas our model underscores the necessity of policies aimed at repairing borrower balance sheets.

Therefore, the policy implications of our framework diverge from the existing literature. We demonstrate that to mitigate the inefficiencies induced by debt overhang, bank recapitalization or conditional subsidies tied to debt restructuring is more efficient than direct bailouts of borrowers. This advantage arises because bank subsidies can be made contingent on the implementation of debt restructuring, whereas direct firm subsidies lack this conditioning mechanism; thus, bank subsidies achieve macroeconomic recovery at a lower fiscal cost. Conversely, we find that unconditional borrower subsidies can be counterproductive: unless the subsidy amount is exceptionally large, they impede debt restructuring by exacerbating distortions in bank decision-making. Consequently, as borrower subsidies can be interpreted as a form of expansionary fiscal policy aimed at boosting the incomes of firms and households, this result implies that such expansionary policy may inadvertently impede productivity recovery in the aftermath of a financial crisis. In other words, unconditional fiscal expansion can hinder economic recovery if the fundamental cause of productivity decline is a debt overhang. These insights offer critical guidelines for the design of future crisis management policies.

Appendix A: Demand externality

As noted in the paragraph of "Production using land" in Section 3, we can extend the baseline model to incorporate monopolistic competition following the Dixit-Stiglitz framework. In this extended model, an aggregate demand externality emerges that hampers debt restructuring and amplifies persistent stagnation. We contrast the case of an aggregate shock, where all firms face debt overhang simultaneously, with the case of an idiosyncratic shock, where financial distress is confined to a single atomistic firm.

Production Technology: We introduce the following assumptions regarding the production technology and market structure.

- The risky asset is heterogeneous. Each bank $i \in [0, 1]$ initially owns one unit of asset i, whose productivity parameter A_i may differ from A_j for $j \neq i$. The price of asset i, denoted by Q_i, can also vary across assets. Competition among buyers (firms) bids up the asset price Q_i, and the buyer of asset i is referred to as firm i.

- The realized productivity parameter A_i takes a value of either A_H or B (where $A_H < B$), and B captures the initial optimism.

- The output of asset i (produced by firm i) is y_i, which aggregates into final production Y via the following technology:

$$Y = \left(\int_0^1 y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

  where $y_i = A_i k = A_i$ given $k = 1$, and $\sigma > 1$ denotes the elasticity of substitution. Normalizing the final good price to unity yields the revenue function for firm i as

$p_i y_i = Y^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}}$. In the symmetric case where $A_j = \bar{A}$ for all $j \neq i$, the revenue of firm i simplifies to $\bar{A}^{\frac{1}{\sigma}} A_i^{\frac{\sigma-1}{\sigma}}$.

- Given the realized productivity $A_i \in \{A_H, B\}$, firm i produces A_i if it expends effort $e = \varepsilon$ and it produces A_L if it does not expend effort, $e = 0$, where $A_L < A_H$.

Productivity shocks: We consider two distinct shock configurations:

- **Aggregate shock:** All assets experience perfectly correlated productivity shocks. In period 0, productivity is uncertain, and in period 1, it is publicly revealed to be either $A_i = A_H$ for all i or $A_i = B$ for all i.
- **Idiosyncratic shock:** A single atomistic asset i experiences a stochastic productivity shock $A_i \in \{A_H, B\}$. All other assets $j \neq i$ face non-stochastic productivity, where it is common knowledge from period 0 onward that $A_j = A_H$ for all $j \neq i$.

Idiosyncratic shock case: Consider the case where asset i's productivity is revealed to be A_H, leaving firm i as the sole entity facing a debt overhang. Because the remaining mass of firms is unaffected, they do not suffer from debt overhang and operate at the efficient frontier, producing $y_H = A_H$ from period 1 onward. Consequently, aggregate output evaluates to $Y = A_H$. Under this regime, only bank i faces the choice of whether to implement debt restructuring. Given $Y = A_H$ in the idiosyncratic shock case, if debt restructuring is implemented, firm i expends effort $e = \varepsilon$ and its revenue $p_i y_i$ is y_H; if the debt overhang continues, the firm expends zero effort ($e = 0$) and its revenue reduces to y_L, where

$$
\begin{aligned}
y_H &= A_H, \\
y_L &= A_H^{\frac{1}{\sigma}} A_L^{\frac{\sigma-1}{\sigma}}.
\end{aligned}
$$

Aggregate shock case: Next, consider the systemic case where $A_i = A_H$ realizes for all i, and all firms suffer from debt overhang. We analyze bank i's restructuring decision under the conjecture that all other banks defer debt restructuring. If bank i unilaterally deviates by implementing debt restructuring, firm i expends effort $e = \varepsilon$ and its revenue $p_i y_i$ is given by $\tilde{y}_H$. Conversely, if bank i also defers restructuring, firm i expends zero effort ($e = 0$) and its revenue is $\tilde{y}_L$. Since aggregate output collapses to $Y = A_L$ when all other banks maintain their debt overhang, these revenues evaluate to

$$
\begin{aligned}
\tilde{y}_H &= A_L^{\frac{1}{\sigma}} A_H^{\frac{\sigma-1}{\sigma}} = \eta y_H, \\
\tilde{y}_L &= A_L = \eta y_L, \\
\eta &\equiv (A_L/A_H)^{\frac{1}{\sigma}} < 1.
\end{aligned}
$$

Thresholds of debt restructuring: As established in the proof of Proposition 5 in Section 5.3, debt restructuring occurs immediately in period 1 if $D_1 \leq D^*$, whereas the debt overhang persists indefinitely if $D_1 > D^*$. In this extended framework, we characterize separate thresholds for the idiosyncratic and aggregate shock cases. Let D^* and $\tilde{D}^*$ denote the boundaries under the idiosyncratic and aggregate regimes, respectively. Applying the same arguments as in Section 5.3 yields

$$D^* = \frac{\alpha(y_H - \varepsilon) - y_L}{\phi\xi(1-\beta)},$$
$$\tilde{D}^* = \frac{\alpha(\eta y_H - \varepsilon) - \eta y_L}{\phi\xi(1-\beta)}.$$

Because $\eta < 1$ and $y_H > y_L$, it immediately follows that $\tilde{D}^* < D^*$.

Suppose the initial debt burden satisfies $\tilde{D}^* < D_1 < D^*$. In this intermediate parameter region:

- If the debt overhang D_1 is generated by an idiosyncratic shock, debt restructuring ensues, thereby restoring the firm's productivity A_H because $D_1 < D^*$.
- If the debt overhang D_1 arises from an aggregate shock, debt restructuring is not implemented, and inefficient production $\tilde{y}_L$ continues because $D_1 > \tilde{D}^*$.

We reiterate that the aggregate-shock case above evaluates the restructuring decision of a single bank under the assumption that all other banks do not restructure. The divergence between the idiosyncratic and aggregate shock regimes stems from the aggregate demand externality. This wedge provides a theoretical explanation for why the resolution of nonperforming loans tends to be significantly more protracted when they originate from a systemic financial crisis rather than an isolated idiosyncratic shock.

Appendix B: Convergence of finite-horizon truncations

In this appendix, we prove the uniqueness of the equilibrium in (any) finite-horizon truncations of our model economy.

In what follows, we use x instead of D as the state variable:

$$x := rD, \quad \text{so that} \quad x_{+1} = r(x-b), \quad b \in [0, \min\{x, y(e)\}]$$

We work with the reduced game that governs the economy from period 1 onward. From that point the productivity–capital pair (A, k) is invariant with $k = 1$, so we suppress it and write $y(e) \in \{y_H, y_L\}$ for output. Furthermore, in the truncated games considered below, we call the initial period "period 0," rather than period 1.

We assume that

$$\varepsilon \leq \beta y_H, \quad \text{and} \quad y_H - \varepsilon > y_L \tag{26}$$

(The first assumption is equivalent to $V_{\max} \geq y_H$, which implies that $\min\{x, y(e)\} = y(e)$ for all $x > V_{\max}$.)

Consider the T-period truncation, with terminal values $V_T \equiv 0$, $d_T \equiv 0$. Its (unique) subgame perfect equilibrium (SPE) can be solved by backward induction. Write $k = T - t$ for the periods remaining and $V^{(k)}$, $d^{(k)}$, $e^{(k)}$ for the resulting objects, where the stage at k has the firm choose e, the bank observe it and choose $b \leq \min\{x, y(e)\}$, and continuation values $V^{(k-1)}, d^{(k-1)}$.

Explicitly, starting from the terminal values $V^{(0)} \equiv 0$, $d^{(0)} \equiv 0$, the stage-k problems at claim x are written as follows. Having observed the firm's effort e, the bank chooses its repayment to maximize its own discounted collection,

$$d^{(k)}(x, e) = \max_{b \in [0, \min\{x, y(e)\}]} \Big\{ b + \beta\, d^{(k-1)}\big(r(x - b)\big) \Big\},$$

with maximizer $b^{(k)}(x, e)$; anticipating this repayment rule, the firm chooses its effort to maximize its own continuation value,

$$V^{(k)}(x) = \max_{e \in \{0, \varepsilon\}} \Big\{ y(e) - e - b^{(k)}(x, e) + \beta\, V^{(k-1)}\big(r(x - b^{(k)}(x, e))\big) \Big\},$$

with maximizer $e^{(k)}(x)$; and the realized bank value is $d^{(k)}(x) = d^{(k)}\big(x, e^{(k)}(x)\big)$. The debt then evolves by $x_{+1} = r(x - b)$.

Lemma 9 (Backward induction). *Consider a T-period truncation. It has a unique subgame-perfect equilibrium with the following properties. For every $k = 0, \dots, T$:*

(a) the bank seizes the whole income at every state: $b^{(k)}(x, e) = \min\{x, y(e)\}$;

(b) $V^{(k)}$ is nonincreasing, convex, and 1-Lipschitz. It vanishes for $x > V_{\max}$, and $0 \leq V^{(k)}(x) \leq V_{\max} - x$ for all $x \leq V_{\max}$; and

(c) the effort policy is a single threshold,

$$e^{(k)}(x) = \begin{cases} \varepsilon, & \text{for } x \leq x^{(k)}, \\ 0, & \text{for } x > x^{(k)}, \end{cases} \tag{27}$$

with $x^{(k)} \leq V_{\max}$.

Proof. We can solve the firm's and the bank's problem using backward induction. Their values and optimal strategies are uniquely determined.

The claim holds vacuously at $k = 0$, where $V^{(0)} \equiv 0$ and $d^{(0)} \equiv 0$. Assume it at $k - 1$.

(a) Bank seizes.

For any repayment path the collection telescopes,

$$\sum_{t=0}^{k-1} \beta^t b_t = \sum_{t=0}^{k-1} (\beta^t x_t - \beta^{t+1} x_{t+1}) = x - \beta^k x_k$$

With terminal recovery $d^{(0)} \equiv 0$ the bank's total value from x over the remaining k periods is exactly $x - \beta^k x_k$, where x_k is the terminal claim.

Consider the initial-period problem for the bank in the k-truncation. The bank chooses $b_0 \in [0, \min\{x, y(e)\}]$ so as to minimize the above object. Given b_0, $x_1 = r(x - b_0)$. By assumption, j-truncations satisfy the stated properties in the lemma for all $j \leq k-1$. Thus, for $t \geq 1$, the bank chooses full seizure:

$$b_t = b^{(k-t)}(x_t) = \min\left\{x_t, y\left[e^{(k-t)}(x_t)\right]\right\},$$

so that

$$x_{t+1} = r\big(x_t - y\left[e^{(k-t)}(x_t)\right]\big), \qquad 1 \leq t \leq k-1$$

Given $x_1 = r(x - b_0)$, we can use this recursion to define

$$x_t = x_t(r(x - b_0)), \qquad 2 \leq t \leq k$$

Then the stage-k bank's problem can be written as

$$\max_{b_0}\{b_0 + \beta d^{(k-1)}(r(x - b_0))\} = \max_{b_0}\{x - \beta^k x_k(r(x - b_0))\}$$

(As already seen above) the bank minimizes the terminal claim.

Note that the transition under complete seizure, $x_t \mapsto r\big(x_t - y\left[e^{(k-t)}(x_t)\right]\big)$, is non-decreasing; therefore, $x_k(\cdot)$ is non-decreasing. Since $r(x - b_0)$ is decreasing in b_0 and we invoke Assumption 6 in the main text, we conclude that the minimizer is the largest b_0, i.e. full seizure, $b_0 = b^{(k)}(x, e) = \min\{x, y(e)\}$.

(b) Firm value. With the bank seizing, the successor is

$$x_{+1} = r\left(x - y(e)\right)^+,$$

and the repayment is

$$b = \min\{x, y(e)\} = x - (x - y(e))^+.$$

Thus, the firm's value is written as

$$\begin{aligned} V^{(k)} &= \max_{e \in \{0, \varepsilon\}} y(e) - e - x + (x - y(e))^+ + \beta V^{(k-1)}\big(r\left(x - y(e)\right)^+\big) \\ &= \max_{e \in \{0, \varepsilon\}} \phi_e^{(k)}(x), \end{aligned}$$

where

$$\phi_e^{(k)}(x) := y(e) - e - x + H_{k-1}\big((x - y(e))^+\big), \tag{28}$$

$$H_{k-1}(w) := w + \beta V^{(k-1)}(rw) \tag{29}$$

Since $V^{(k-1)}$ is convex, $H_{k-1}(w) = w + \beta V^{(k-1)}(rw)$ is also convex in w. In addition, H_{k-1} is non-decreasing, since, where differentiable,

$$H'_{k-1}(w) = 1 + \beta r\, V^{(k-1)\prime}(rw) = 1 + V^{(k-1)\prime}(rw) \in [0, 1]$$

by the 1-Lipschitz, non-increasing hypothesis $V^{(k-1)\prime} \in [-1, 0]$.

As $(x - y(e))^+$ is convex and non-decreasing, $H_{k-1}((x - y(e))^+)$ is convex; adding the affine term, each $\phi_e^{(k)}$ is convex, so $V^{(k)} = \max_e \phi_e^{(k)}$ is convex.

Moreover, the derivative of $\phi_e^{(k)}$ is given by

$$\phi_e^{(k)\prime}(x) = -1 + H'_{k-1}((x - y(e))^+)\, \mathbf{1}[x > y(e)] \in [-1, 0],$$

so $V^{(k)}$ is non-increasing and 1-Lipschitz.

On the overhang region, $x > V_{\max} \geq y_H$, the shirk value is

$$\phi_0^{(k)}(x) = \beta V^{(k-1)}(r(x - y_L)) = 0,$$

as $r(x - y_L) > x > V_{\max}$. The work value is

$$\phi_\varepsilon^{(k)}(x) = -\varepsilon + \beta V^{(k-1)}(r(x - y_H))$$

If $r(x - y_H) \geq V_{\max}$,

$$\phi_\varepsilon^{(k)}(x) = -\varepsilon < 0,$$

and if $r(x - y_H) < V_{\max}$, by (b) at $k - 1$,

$$\phi_\varepsilon^{(k)}(x) \leq -\varepsilon + \beta(V_{\max} - r(x - y_H)) = -\varepsilon + \beta V_{\max} + y_H - x < -\varepsilon + y_H - (1 - \beta)V_{\max} = 0$$

Hence $V^{(k)} = 0$ on $\{x > V_{\max}\}$. Being 1-Lipschitz, non-increasing, and zero at $x = V_{\max}$ gives

$$0 \leq V^{(k)}(x) \leq V_{\max} - x$$

on $[0, V_{\max}]$.

(c) Effort threshold. The firm works iff

$$\Delta^{(k)}(x) := \phi_\varepsilon^{(k)}(x) - \phi_0^{(k)}(x) \geq 0$$

It can be rewritten as

$$\Delta^{(k)}(x) = (y_H - \varepsilon - y_L) - G(x), \qquad G(x) := H_{k-1}\big((x - y_L)^+\big) - H_{k-1}\big((x - y_H)^+\big) \geq 0,$$

where the sign follows from the fact that H_{k-1} is non-decreasing and $(x - y_L)^+ \geq (x - y_H)^+$.

The function G is non-decreasing: for $x \geq y_H$ it is the increment of the convex H_{k-1} over the interval $[x - y_H,\, x - y_L]$ of fixed width $y_H - y_L$ sliding rightward, which increases by convexity; for $y_L \le x < y_H$ it equals $H_{k-1}(x-y_L) - H_{k-1}(0)$, non-decreasing; and for $x < y_L$ it is 0.

Hence the work region

$$\{\Delta^{(k)} \ge 0\} = \{G(x) \le y_H - \varepsilon - y_L\}$$

is a threshold set $[0, x^{(k)}]$, with $x^{(k)} \le V_{\max}$ because $\Delta^{(k)} < 0$ on the overhang region as shown in (b) (In fact, the optimality of shirking for the firm in the overhang region has been shown there). Thus $e^{(k)}$ is a single threshold, completing the induction. □

Lemma 10 (Closed form solution)**.** *For every $k \ge 1$, let*

$$x^{(k)} = V_{\max}(1-\beta^k).$$

Then the equilibrium of the truncated game is given by

$$V^{(k)}(x) = \left(x^{(k)} - x\right)^+,$$

$$e^{(k)}(x) = \begin{cases} \varepsilon, & x \le x^{(k)}, \\ 0, & x > x^{(k)}, \end{cases}$$

$$d^{(k)}(x) = \begin{cases} x, & x \le x^{(k)}, \\ d_L\left(1-\beta^k\right), & x > x^{(k)}, \end{cases}$$

and $b^{(k)}(x,e) = \min\{x, y(e)\}$.

Proof. The bank seizes the whole income at every node (Lemma 9 (a)), so the firm's value is

$$V^{(k)} = \max_{e \in \{0,\varepsilon\}} \phi_e^{(k)}$$

with

$$\phi_e^{(k)}(x) = y(e) - e - x + H_{k-1}\left((x-y(e))^+\right),$$
$$H_{k-1}(w) = w + \beta V^{(k-1)}(rw),$$

and $x_1 = r(x-y(e))^+$.

Firm's value and threshold. Consider induction on k. At $k=1$,

$$x^{(1)} = (1-\beta)V_{\max} = y_H - \varepsilon$$

Since $H_0(w) = w$,

$$\phi_\varepsilon^{(1)}(x) = y_H - \varepsilon - x + (x-y_H)^+ = \begin{cases} -\varepsilon, & x > y_H, \\ y_H - \varepsilon - x, & x \le y_H, \end{cases}$$

$$\phi_0^{(1)}(x) = y_L - x + (x-y_L)^+ = \begin{cases} 0, & x > y_L, \\ y_L - x, & x \le y_L. \end{cases}$$

Since $x^{(1)} = y_H - \varepsilon$, we obtain

$$e^{(1)}(x) = \begin{cases} \varepsilon, & x \le x^{(1)}, \\ 0, & x > x^{(1)}. \end{cases}$$

Then

$$\begin{aligned} V^{(1)}(x) &= \max_e \phi_e^{(1)}(x) = (y_H - \varepsilon - x)^+ \\ &= (x^{(1)} - x)^+ \end{aligned}$$

Now assume $V^{(k-1)}(y) = (x^{(k-1)} - y)^+$. Since $\beta r = 1$,

$$H_{k-1}(w) = w + \beta\big(x^{(k-1)} - rw\big)^+ = \max\{w,\ \beta x^{(k-1)}\},$$

Since $x^{(k)} = (1-\beta^k)V_{\max}$, we have

$$\beta x^{(k-1)} = (\beta - \beta^k)V_{\max} = (1-\beta^k)V_{\max} - (1-\beta)V_{\max} = x^{(k)} - (y_H - \varepsilon)$$

by $(1-\beta)V_{\max} = y_H - \varepsilon$.

Since $H_{k-1}(w) = \max\{w,\ \beta x^{(k-1)}\}$,

$$\begin{aligned} \phi_\varepsilon^{(k)}(x) &= y_H - \varepsilon - x + \max\{(x-y_H)^+, \beta x^{(k-1)}\} \\ &= \begin{cases} y_H - \varepsilon - x + \beta x^{(k-1)}, & x - y_H \le \beta x^{(k-1)}, \\ -\varepsilon, & x - y_H > \beta x^{(k-1)}, \end{cases} \\ &= \begin{cases} x^{(k)} - x, & x \le x^{(k)} + \varepsilon, \\ -\varepsilon, & x > x^{(k)} + \varepsilon; \end{cases} \end{aligned}$$

$$\begin{aligned} \phi_0^{(k)}(x) &= y_L - x + \max\{(x-y_L)^+, \beta x^{(k-1)}\} \\ &= \begin{cases} y_L - x + \beta x^{(k-1)}, & x \le y_L, \\ \max\{0, y_L - x + \beta x^{(k-1)}\}, & x > y_L, \end{cases} \\ &= (y_L - x + \beta x^{(k-1)})^+ \\ &= (x^{(k)} - x + \underbrace{y_L - (y_H - \varepsilon)}_{<0})^+ \end{aligned}$$

Comparing $\phi_\varepsilon^{(k)}(x)$ and $\phi_0^{(k)}(x)$: (i) for $x \le x^{(k)}$, we have

$$\begin{aligned} &\phi_\varepsilon^{(k)}(x) = x^{(k)} - x \ge 0 \\ &\phi_0^{(k)}(x) = (x^{(k)} - x + y_L - (y_H - \varepsilon))^+ \le \phi_\varepsilon^{(k)}(x) \end{aligned}$$

and

$$V^{(k)}(x) = \phi_\varepsilon^{(k)}(x) = x^{(k)} - x$$

(ii) for $x \geq x^{(k)}$, we have

$$\phi_{\varepsilon}^{(k)}(x) = \max\{x^{(k)} - x, -\varepsilon\} < 0, \quad \text{and} \quad \phi_0^{(k)}(x) = 0$$

so that

$$V^{(k)}(x) = 0.$$

Therefore

$$V^{(k)}(x) = (x^{(k)} - x)^+,$$

attained by

$$e^{(k)} = \begin{cases} \varepsilon, & x \leq x^{(k)}, \\ 0, & x > x^{(k)} \end{cases}$$

Remember that $x^{(k-1)} < x^{(k)}$, and

$$r(x^{(k)} - y_H)^+ < x^{(k-1)},$$

since $x^{(k)} - y_H < x^{(k)} - (y_H - \varepsilon) = \beta x^{(k-1)}$.

Bank's value. Consider induction on k. For $k = 1$, as $d^{(0)} \equiv 0$,

$$d^{(1)}(x) = b_0 = \min\{x, y(e)\}$$

Since $y(e) = y_H$ if $x \leq x^{(1)} = y_H - \varepsilon$ and $y(e) = y_L$ if $x > x^{(1)} = y_H - \varepsilon$, we have

$$d^{(1)}(x) = \begin{cases} x, & x \leq x^{(1)}, \\ y_L = (1-\beta)d_L, & x > x^{(1)}, \end{cases}$$

as claimed in the lemma.

Suppose $d^{(k-1)}$ is given as in the lemma. Full seizure gives

$$d^{(k)}(x) = b_0 + \beta d^{(k-1)}(x_1), \quad \text{where} \quad b_0 = \min\{x, y(e^{(k)}(x))\}, \quad \text{and} \quad x_1 = r(x - b_0).$$

For $x \leq x^{(k)}$: $e^{(k)}(x) = \varepsilon$ and $y(e) = y_H$.

- if $x \leq y_H$ then

  $$b_0 = x, \quad x_1 = 0, \quad d^{(k-1)}(0) = 0,$$

  so

  $$d^{(k)}(x) = x;$$

- if $y_H < x \leq x^{(k)}$ then

  $$b_0 = y_H, \quad x_1 = r(x - y_H) \leq r(x^{(k)} - y_H) < x^{(k-1)}, \quad d^{(k-1)}(x_1) = x_1,$$

  so

  $$d^{(k)}(x) = y_H + \beta r(x - y_H) = x.$$

For $x > x^{(k)}$: $e^{(k)}(x) = 0$ and $y(e) = y_L$. As $x^{(k)} \geq x^{(1)} = y_H - \varepsilon > y_L$ we have

$$b_0 = y_L, \quad x_1 = r(x - y_L) > r(x^{(k)} - y_L) > x^{(k-1)},$$

Here, the last inequality follows from

$$(1-\beta^k)V_{\max} - y_L = (1-\beta)V_{\max} + (\beta - \beta^k)V_{\max} - y_L = (y_H - \varepsilon - y_L) + \beta x^{(k-1)} > \beta x^{(k-1)}$$

So $d^{(k-1)}(x_1) = d_L(1 - \beta^{k-1})$ and

$$d^{(k)}(x) = y_L + \beta d_L(1 - \beta^{k-1}) = (1 - \beta^k)d_L,$$

since $y_L + \beta d_L = d_L$. This completes the proof. □

Proposition 11 (Finite-horizon selection)**.** *As $T \to \infty$, the truncated equilibria converge to the debt-overhang equilibrium of Proposition 1 in the paper.*

Proof. Follows from the above lemmas. □

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