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Money Illusion and Business Cycle Fluctuations: Evidence from Japan*

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Abstract

We investigate the role of money illusion in business cycle fluctuations by modeling households as misperceiving the inflation rates used to convert nominal variables into real variables. These two forms of misperception operate through distinct channels, affecting labor supply and intertemporal demand, respectively. We estimate a medium-scale DSGE model with money illusion using Japanese macroeconomic data for 1995Q1–2019Q4, measuring the monetary policy stance with a shadow rate. Bayesian estimation shows that the model with money illusion outperforms the rational expectations model, and both current and future inflation misperceptions are well supported by the data. Counterfactual exercises and variance decompositions show that the two forms of inflation misperceptions affect shock propagation through distinct channels.

Keywords: Money illusion; inflation misperceptions; business cycles;

JEL Classifications: E31, E32, E52, D84

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1 Introduction

Does money illusion matter for aggregate fluctuations in a modern advanced economy? We show that it does. Estimating a medium-scale New Keynesian DSGE model with Japanese quarterly data from 1995Q1 to 2019Q4, we find that allowing households to exhibit money illusion improves the fit of the model relative to a rational expectations specification. The results indicate that money illusion is empirically relevant for business cycle analysis. While a few existing studies incorporate money illusion into macroeconomic models, they mainly focus on theoretical mechanisms and equilibrium properties. This paper quantitatively evaluates the empirical relevance of money illusion using Bayesian estimation of a medium-scale DSGE model.

We model money illusion as households' misperception of the inflation rates used to convert nominal variables into real variables. Households may misperceive current inflation when evaluating real wages, which affects labor supply, and expected future inflation when evaluating real interest rates, which affects intertemporal consumption and saving decisions. These two forms of inflation misperception operate through different equilibrium conditions and therefore imply distinct macroeconomic effects.

Our findings are as follows. First, the money illusion model is strongly favored over a nested rational expectations model in terms of marginal likelihood. Second, the estimated degree of money illusion with respect to current inflation is around 0.5, while the degree of money illusion with respect to future inflation is substantially larger, at around 0.8 in the posterior estimates. Finally, money illusion operates through distinct mechanisms: current inflation misperception primarily affects labor supply, while future inflation misperception primarily affects intertemporal demand and perceived real interest rates.

We use Japanese data for 1995Q1–2019Q4 because Japan provides an important empirical setting for studying business cycle fluctuations in a modern advanced economy with a prolonged low-interest-rate environment. The sample includes the zero lower bound period, during which conventional short-term nominal interest rates no longer provide a complete measure of the monetary policy stance. To capture monetary policy during this period, we use a shadow interest rate. This allows us to evaluate the empirical relevance of inflation misperceptions in a standard linearized DSGE framework.

To clarify the distinct roles of the two forms of money illusion, we also develop a simple theoretical framework. A tractable New Keynesian model delivers analytical equilibrium determinacy conditions and an extended Taylor principle under inflation misperceptions. Money illusion with respect to future inflation enlarges the determinacy region by weakening the response of intertemporal demand to perceived real interest rates, whereas money illusion with respect to current inflation mainly affects labor supply and marginal costs. The model also has implications for the forward guidance puzzle, since weaker responses to future inflation dampen the effects of anticipated monetary policy changes through the Euler equation. Impulse response analysis illustrates that current and future inflation misperceptions affect macroeconomic dynamics through different transmission channels.

Related literature This paper relates to several strands of literature on money illusion and behavioral macroeconomics.

The classic idea of money illusion dates back at least to Fisher (1928), who argued that economic agents may confuse nominal and real magnitudes. Empirical evidence supports the relevance of this mechanism. Shafir et al. (1997) show that individuals often respond more strongly to nominal than real values when evaluating economic outcomes. Fehr and Tyran (2001, 2007) provide experimental evidence that nominal framing can generate sluggish adjustment and persistent real effects. Brunnermeier and Julliard (2008) argue that misperceptions of future inflation help explain housing market dynamics. In labor markets, Akerlof et al. (1996, 2000) show that money illusion in wage setting can have substantial macroeconomic consequences.

Several studies incorporate money illusion into macroeconomic models. Vaona (2013) studies money illusion arising from confusion between nominal and real wages, while Miao and Xie (2013) introduce nominal consumption into utility in a growth framework. Tamegawa (2024) considers money illusion through direct nominal-real confusion in a New Keynesian model. Studies using current and future inflation misperceptions, as in the present paper, include Nutahara (2026a,b). Nutahara (2026a) shows that several common formulations of money illusion, such as nominal consumption utility or level-based wage misperception, may violate balanced-growth consistency in standard New Keynesian models. The present paper therefore adopts the inflation-misperception framework proposed there and evaluates its empirical relevance. Nutahara (2026b) studies monetary policy that responds to stock prices under inflation misperception and its implications for equilibrium determinacy. Building on this line of research, the present paper embeds the inflation-misperception framework in a medium-scale estimated DSGE model and evaluates its empirical relevance for the Japanese economy.

This paper is also related to the literature on behavioral expectations. Gabaix (2020) develops the cognitive discounting framework, in which expected future inflation is discounted in a manner similar to the future inflation misperception considered in this paper, but the same discounting is applied uniformly to all other forward-looking variables as well. Meggiorini (2023) extends this framework to a medium-scale New Keynesian DSGE model and finds quantitatively important departures from rational expectations in U.S. data. For Japan, Hirose et al. (2024) estimate a discounting parameter around 0.85, while Hirose and Yoo (2026) obtain estimates above 0.9. Although the parameters are not directly comparable, our estimates suggest a substantially stronger attenuation of expected inflation than that implied by the cognitive-discounting estimates reported for Japan. Meggiorini and Milani (2021) show that cognitive discounting can fit the data less well than learning-based alternatives.

Organization of the paper The remainder of the paper is organized as follows. Section 2 develops a simple model of money illusion and analyzes equilibrium determinacy and macroeconomic dynamics. Section 3 incorporates these mechanisms into a medium-scale DSGE model. Section 4 describes the data and discusses the main empirical results. Finally, Section 5 provides concluding remarks.

2 A Simple Model of Money Illusion

In this section, we develop a simple New Keynesian model of money illusion.

2.1 Main Idea: Money illusion as inflation misperceptions

We model money illusion through two distinct channels: misperceptions of current inflation and misperceptions of expected future inflation.[1]

Households are assumed to observe the previous-period price level P_{t-1} whereas they misperceive the current gross inflation rate used to deflate nominal variables,

$$\Pi_t \equiv \frac{P_t}{P_{t-1}}, \tag{1}$$

according to

$$\Pi_t^{MI} = (\Pi^*)^{\psi_c} \Pi_t^{1-\psi_c}, \tag{2}$$

where $\psi_c \in [0, 1]$ measures the degree of misperception of current inflation and Π^* denotes steady-state gross inflation.

Given this distortion, the perceived current price level P_t^{MI} is

$$P_t^{MI} = P_{t-1}\Pi_t^{MI}. \tag{3}$$

Hence, the perceived real wage is given by

$$\left(\frac{W_t}{P_t}\right)^{MI} \equiv \frac{W_t}{P_t^{MI}} = \frac{W_t}{P_t}\left(\frac{\Pi_t}{\Pi^*}\right)^{\psi_c}, \tag{4}$$

where W_t denotes the nominal wage.

Households also misperceive expected future inflation according to

$$E_t\Pi_{t+1}^{MI} = (\Pi^*)^{\psi_f}\left(E_t\Pi_{t+1}\right)^{1-\psi_f}, \tag{5}$$

[1] We use an inflation-misperception specification, which provides a convenient and theoretically coherent way to introduce money illusion into standard New Keynesian models. See Nutahara (2026a) for further discussion.

where $\psi_f \in [0, 1]$ captures the degree of misperception of expected future inflation. Hence, the perceived ex ante real gross interest rate is given by

$$\left(\frac{R_t^n}{E_t\Pi_{t+1}}\right)^{MI} \equiv \frac{R_t^n}{E_t\Pi_{t+1}^{MI}} = \frac{R_t^n}{E_t\Pi_{t+1}}\left(\frac{E_t\Pi_{t+1}}{\Pi^*}\right)^{\psi_f}, \tag{6}$$

where R_t^n denotes the gross nominal interest rate.

Current inflation misperception operates through the perceived real wage, whereas expected future inflation misperception operates through the perceived real interest rate.[2] The former affects labor supply decisions, while the latter affects intertemporal demand.

2.2 Household Problem

Households make decisions on the basis of perceived inflation. Actual transactions, however, are settled at market prices. Accordingly, the realized budget constraint is

$$P_tC_t + B_t = W_t\ell_t + R_{t-1}^n B_{t-1} + D_t, \tag{7}$$

where C_t denotes consumption, B_t end-of-period nominal bond holdings, ℓ_t labor supply, R_t^n the gross nominal interest rate, and D_t firm dividends.

The subjective budget constraint perceived by households is

$$P_t^{MI}C_t + B_t = W_t\ell_t + R_{t-1}^n B_{t-1} + D_t + \Omega_t, \tag{8}$$

where P_t^{MI} is the perceived price level and Ω_t is a valuation adjustment term interpreted as an exogenous lump-sum transfer. Households treat Ω_t as independent of their own choices, so that it does not affect

[2]Our specifications can be rationalized in several ways. One possible interpretation is that they arise as reduced-form implications of signal extraction for log inflation. We first consider current inflation misperception. Let $x_t \equiv \log\Pi_t - \log\Pi^*$. Suppose that households have the subjective prior $x_t \sim N(0, \sigma_{x,c}^2)$ and observe a noisy signal $y_t^c = x_t + \varepsilon_t^c$, where $\varepsilon_t^c \sim N(0, \sigma_{\varepsilon,c}^2)$. Bayesian updating yields the posterior mean $E_t^H[x_t \mid y_t^c] = K_c y_t^c$, where $E_t^H[\cdot]$ denotes the household's subjective expectation and $K_c \equiv \sigma_{x,c}^2/(\sigma_{x,c}^2 + \sigma_{\varepsilon,c}^2)$ is the updating weight on the signal. If perceived gross inflation is defined as the exponential of the posterior mean of log inflation and the aggregate reduced-form signal is evaluated at $y_t^c = x_t$, then

$$\begin{aligned}\Pi_t^{MI} &= \exp\{\log\Pi^* + E_t^H[x_t|y_t^c]\}\\ &= \exp\{\log\Pi^* + K_c(\log\Pi_t - \log\Pi^*)\}\\ &= (\Pi^*)^{1-K_c}\Pi_t^{K_c}.\end{aligned}$$

Setting $\psi_c \equiv 1 - K_c$ gives (2). The expected-inflation specification can be interpreted analogously with $\psi_f \equiv 1 - K_f$ in (5). Within this signal-extraction interpretation, costly information acquisition or rational inattention can be viewed as determining the precision of the inflation signal, the updating weight K_j, or, equivalently, the misperception parameter ψ_j. A related but broader interpretation of the expected-inflation component is cognitive discounting, as in Gabaix (2020). In contrast to models that attenuate forward-looking variables more generally, our formulation restricts the friction to inflation information because money illusion concerns the conversion of nominal variables into real variables.

the first-order conditions. In equilibrium, Ω_t is endogenously determined so as to reconcile perceived values with realized transactions.

Household preferences are given by

$$U = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{\ell_t^{1+\chi}}{1+\chi} \right], \tag{9}$$

where $\beta \in (0, 1)$ denotes the discount factor, $\sigma > 0$ the relative risk aversion, and χ the inverse of Frisch elasticity of labor supply. The optimal labor supply condition is

$$\ell_t^{\chi} = C_t^{-\sigma} \frac{W_t}{P_t} \left(\frac{\Pi_t}{\Pi^*} \right)^{\psi_c}. \tag{10}$$

Log-linearization yields

$$\chi \tilde{\ell}_t + \sigma \tilde{c}_t = \tilde{\omega}_t + \psi_c \tilde{\pi}_t, \tag{11}$$

where variables with tildes denote log-deviations from steady state, and $\tilde{\omega}_t$ denotes the log-deviation of the true real wage (W_t/P_t) from its steady-state value. (11) shows that current inflation misperception acts as a wedge in perceived real wages and therefore primarily affects intratemporal labor supply decisions.

The Euler equation is

$$C_t^{-\sigma} = \beta R_t^n E_t \left[\frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}^{MI}} \right]. \tag{12}$$

Log-linear approximation yields

$$\tilde{c}_t = E_t \tilde{c}_{t+1} - \frac{1}{\sigma} \left[\tilde{r}_t^n - (1 - \psi_f) E_t \tilde{\pi}_{t+1} \right]. \tag{13}$$

Equation (13) shows that expected future inflation misperception acts as a wedge in perceived real interest rates and therefore directly affects intertemporal demand decisions.

2.3 Other Conditions

Firms are standard and do not suffer from inflation misperceptions. Production is linear in labor:

$$Y_t = A_t \ell_t. \tag{14}$$

Real marginal cost of firms MC_t is given by

$$MC_t = \frac{W_t/P_t}{A_t}. \tag{15}$$

Under standard Calvo pricing, the New Keynesian Phillips curve is given by

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \lambda \widetilde{mc}_t + z_t^p, \tag{16}$$

where z_t^p denotes a price markup shock. The price markup shock follows

$$z_t^p = \rho_p z_{t-1}^p + \varepsilon_t^p, \tag{17}$$

where ε_t^p is an i.i.d. innovation.

The resource constraint is

$$Y_t = C_t. \tag{18}$$

The central bank sets the nominal interest rate according to a standard Taylor rule:

$$\log R_t^n = \log \bar{R}^n + \phi_\pi \log \Pi_t + \phi_y \log Y_t + z_t^r, \tag{19}$$

$$z_t^r = \rho_r z_{t-1}^r + \varepsilon_t^r, \tag{20}$$

where ε_t^r denotes a monetary policy shock. The parameters $\phi_\pi \geq 0$ and $\phi_y \geq 0$ denote the monetary policy sensitivities to inflation and output, respectively.

The technology process is given by

$$A_t = A^* \exp(z_t^a), \tag{21}$$

$$z_t^a = \rho_a z_{t-1}^a + \varepsilon_t^a, \tag{22}$$

where ε_t^a denotes a technology shock.

2.4 Equilibrium determinacy

For the determinacy analysis, we abstract from exogenous shocks and set $z_t^a = z_t^r = z_t^p = 0$. The log-linearized equilibrium conditions are

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left[\tilde{r}_t^n - (1-\psi_f) E_t \tilde{\pi}_{t+1}\right], \tag{23}$$

$$\tilde{\pi}_t = \frac{\beta}{1+\lambda\psi_c} E_t \tilde{\pi}_{t+1} + \frac{\lambda(\chi+\sigma)}{1+\lambda\psi_c}\tilde{y}_t, \tag{24}$$

$$\tilde{r}_t^n = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t. \tag{25}$$

This equilibrium system is reduced to a two-variable system of $\tilde{y}_t$ and $\tilde{\pi}_t$:

$$\mathbb{A}\begin{pmatrix}\tilde{y}_t \\ \tilde{\pi}_t\end{pmatrix} = \mathbb{B} E_t \begin{pmatrix}\tilde{y}_{t+1} \\ \tilde{\pi}_{t+1}\end{pmatrix}, \tag{26}$$

where

$$\mathbb{A} = \begin{pmatrix} 1+\phi_y/\sigma & \phi_\pi/\sigma \\ -\lambda(\chi+\sigma) & 1+\lambda\psi_c \end{pmatrix}, \qquad \mathbb{B} = \begin{pmatrix} 1 & (1-\psi_f)/\sigma \\ 0 & \beta \end{pmatrix}. \tag{27}$$

Let

$$\mathbb{M} \equiv \mathbb{A}^{-1}\mathbb{B}.$$

The variables $\tilde{y}_t$ and $\tilde{\pi}_t$ are both jump variables. Hence, equilibrium determinacy requires that both eigenvalues of $\mathbb{M}$ lie inside the unit circle. A necessary and sufficient condition for equilibrium determinacy can be derived analytically.

Proposition 1. *A necessary and sufficient condition for equilibrium determinacy is*

$$\phi_y(1-\beta+\lambda\psi_c)+\lambda(\chi+\sigma)(\phi_\pi+\psi_f-1)>0. \tag{28}$$

If $\phi_y = 0$, this condition reduces to

$$\phi_\pi > 1-\psi_f. \tag{29}$$

Proof. See Appendix A. □

This proposition indicates that the parameter ψ_f directly enlarges the region of equilibrium determinacy. By contrast, ψ_c contributes to determinacy only when monetary policy responds to output, i.e., when $\phi_y > 0$.

The determinacy condition obtained in Proposition 1 admits a more intuitive interpretation in terms of the sensitivity of output to inflation. Consider the New Keynesian Phillips curve and suppose that inflation rises permanently by 1%, so that $\tilde{\pi}_t = E_t\tilde{\pi}_{t+1} = 1$. Let S denote the corresponding response of output $\tilde{y}_t$. Then,

$$S \equiv \frac{\partial y_t}{\partial \pi_t} = \frac{1-\beta+\lambda\psi_c}{\lambda(\chi+\sigma)}. \tag{30}$$

A larger value of current inflation misperception, ψ_c, increases S, as money illusion stimulates labor supply and thereby amplifies the expansionary effect of inflation on output. Using this expression, condition (28) can be rewritten as

$$\phi_\pi + \phi_y S + \psi_f > 1. \tag{31}$$

This inequality can be interpreted as an extended Taylor principle. It implies that for the equilibrium to be determinate, a permanent 1% increase in inflation must induce a more-than-proportionate total stabilization effect of over 1%, accounting for both the central bank's policy rule and the private sector's misperception of inflation. The individual components of the left-hand side of (31) clarify the underlying mechanisms. The first term, ϕ_π, represents the direct monetary policy response to inflation, while the second term, $\phi_y S$, captures an indirect policy channel. In this channel, higher inflation stimulates output through the current misperception of inflation, to which the central bank then reacts via its output-gap

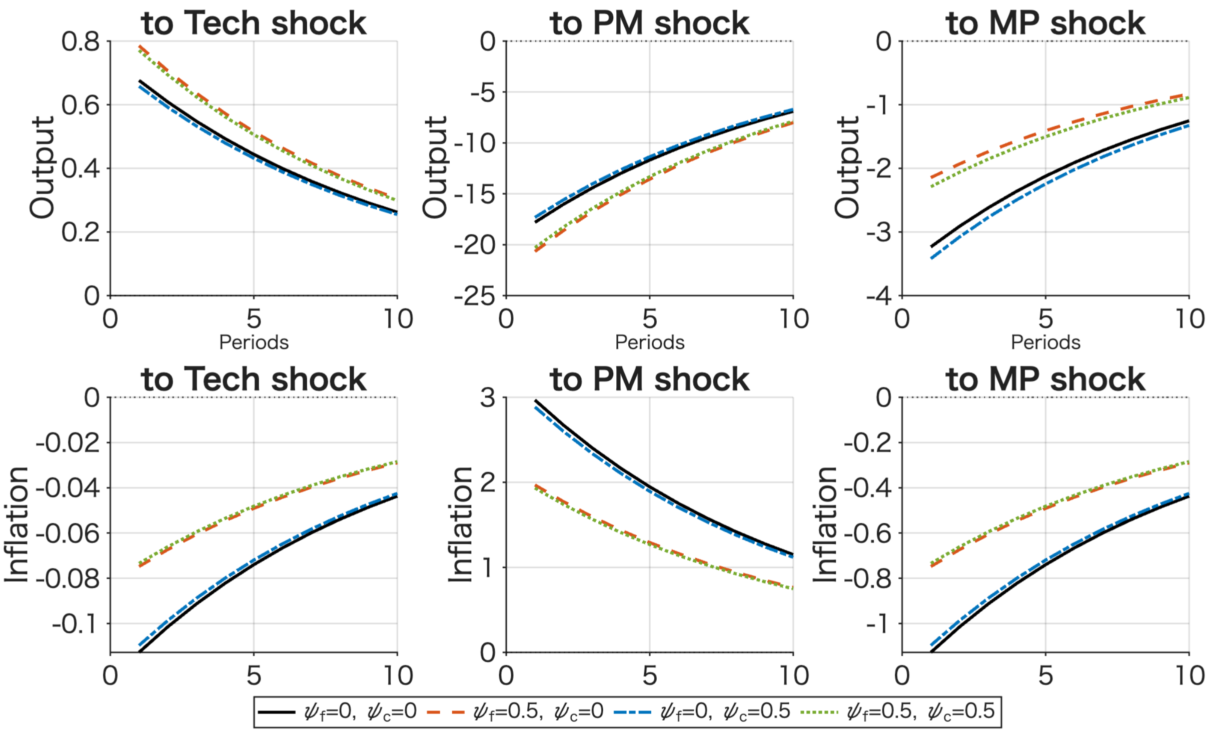

Figure 1: Impulse response functions: Simple model with money illusion

coefficient ϕ_y. Finally, the third term, ψ_f, reflects a self-stabilizing effect inherent in the private sector. It represents the automatic rise in the household's perceived real interest rate caused by the misperception of future inflation, a mechanism that operates independently of any nominal interest rate adjustment.

This result is related to the determinacy effects of cognitive discounting emphasized by Gabaix (2020). In both cases, attenuating forward-looking expectations can stabilize equilibrium dynamics. However, unlike cognitive discounting, the present mechanism operates specifically through misperceptions of expected future inflation rather than through a uniform discounting of all future variables.

2.5 Impulse response functions

To evaluate the implications of money illusion, we examine the impulse response functions to three structural shocks: a technology shock (Tech), a price markup shock (PM), and a monetary policy shock (MP). The baseline parameters are set as follows: the discount factor $\beta = 0.99$, the relative risk aversion $\sigma = 1$, the inverse of the Frisch elasticity $\chi = 1$, the slope of the Phillips curve $\lambda = 0.019$, and the monetary policy reaction coefficients $\phi_\pi = 1.5$ and $\phi_y = 0.5$. The persistence of each shock is set at $\rho_a = \rho_p = \rho_R = 0.9$.

Figure 1 displays the impulse responses of output and inflation to one-percent shocks in technology, price markup, and monetary policy shocks, respectively. Future inflation misperception (ψ_f) has large effects on the impulse responses. Higher values of ψ_f amplify the output response to technology shocks, while dampening the responses of output and inflation to monetary policy shocks. For price markup shocks, higher values of ψ_f increase the decline in output and reduce the inflation response. These responses arise through the perceived real interest rate channel and intertemporal demand adjustment.

Current inflation misperception (ψ_c) mainly affects inflation dynamics through labor supply and marginal cost movements. Its quantitative effects are smaller than those of ψ_f. Higher values of ψ_c slightly weaken inflation responses to all shocks, whereas the effects on output responses remain modest.

The impulse responses suggest that future inflation misperception plays a larger role in shock propagation.

2.6 Implications for the Forward Guidance Puzzle

The forward guidance puzzle refers to the finding that, in standard New Keynesian models with the zero lower bound on nominal interest rates binding, anticipated future monetary policy can have excessively large effects on current economic activity. In such an environment, expectations of future policy become central in intertemporal decisions.

In our framework, expected future inflation misperception weakens the response of intertemporal demand to expected future real interest rates. As a result, the model dampens the transmission of anticipated policy changes through the Euler equation channel. This mechanism shares a similar spirit with the cognitive discounting approach in Gabaix (2020), in the sense that future inflation is effectively discounted in intertemporal decisions, and may therefore contribute to mitigating the forward guidance puzzle.

A formal quantitative analysis of the forward guidance puzzle is beyond the scope of this paper, but the mechanism suggests that future inflation misperception may dampen the effects of anticipated monetary policy through the Euler equation channel.

3 A medium-scale DSGE model with money illusion

In this section, we incorporate money illusion into a standard medium-scale DSGE model à la Christiano et al. (2005) and Smets and Wouters (2007). Our model includes habit persistence, adjustment costs of investment, and variable capital utilization, along with Calvo-type nominal price and wage rigidities with partial inflation indexation. The central bank follows a Taylor-type nominal interest rate rule. There are many structural shocks: technology, preference (a shock to the discount factor), labor supply (a shock to the weight of disutility from labor supply), investment adjustment cost, price markup, wage markup, government purchases, and monetary policy shocks.[3]

Unlike the simple model in the previous section, which assumes a transitory technology shock, the medium-scale model allows for a permanent technology shock. It is useful in the empirical analysis because it allows the model to capture changes in the trend component of the data.

Final-good firms: The final-good firms are perfectly competitive, and they produce a homogeneous final-good Y_t using an intermediate-good $Y_t(f)$. The production function is given by

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{1}{1+\lambda_t^p}} df \right]^{1+\lambda_t^p}, \tag{32}$$

where λ_t^p is a time-varying parameter for elasticity of substitution among intermediate-good $\theta_t^p > 1$, which is defined by $\lambda_t^p = 1/(\theta_t^p - 1) > 0$.

Profit maximization implies the demand function of intermediate-good $Y_t(f)$:

$$Y_t(f) = \left[\frac{P_t(f)}{P_t} \right]^{-\frac{1+\lambda_t^p}{\lambda_t^p}} Y_t, \tag{33}$$

where P_t is the price of final-good Y_t and $P_t(f)$ is the price of intermediate-good $Y_t(f)$.

Intermediate-good firms: The intermediate-good firms are monopolistically competitive. The intermediate-good firm indexed by $f \in [0, 1]$ produces differentiated intermediate-good $Y_t(f)$ using labor input $\ell_t(f)$ and capital service $K_t^S(f)$. Then, the production function is given by

$$Y_t(f) = [K_t^S(f)]^\alpha [Z_t \ell_t(f)]^{1-\alpha} - \Phi Z_t, \tag{34}$$

where $\alpha \in (0, 1)$ is the cost share of capital; Φ is the fixed cost of production; $\phi \equiv \Phi/Y$ denotes the steady-state fixed cost-output ratio; and Z_t denotes the technology level, which evolves according to

$$\log Z_t = z^* + \log Z_{t-1} + z_t^z, \tag{35}$$

where z^* is the steady-state growth rate of technology and z_t^z is the technology growth rate shock.

The last term in the production function, ΦZ_t, is multiplied by Z_t to guarantee the existence of the balanced growth path. Then, the cost minimization of intermediate-good firms implies

$$R_t^k = mc_t \, \alpha \left[\frac{Z_t \ell_t(f)}{K_t^S(f)} \right]^{1-\alpha}, \tag{36}$$

$$W_t = mc_t \, (1-\alpha) Z_t \left[\frac{Z_t \ell_t(f)}{K_t^S(f)} \right]^{-\alpha}, \tag{37}$$

where mc_t is the real marginal cost, R_t^k is the rental rate of capital, and W_t is the real wage rate.

[3]In our model, the investment-specific technology shock is eliminated from the model of Hirose and Kurozumi (2012).

Next, we introduce the Calvo-type sticky prices. In every period, a fraction $1-\xi_p \in [0,1]$ of intermediate-good firms can reoptimize their prices. The other firms index their prices to the weighted average of past gross inflation (Π_{t-1}) and steady-state gross inflation (Π^*): $\Pi_{t-1}^{\gamma_p}(\Pi^*)^{1-\gamma_p}$, where $\gamma_p \in [0,1]$ is the relative weight of past inflation. The objective function of the intermediate-good firms that reoptimize their prices at period t is

$$E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \left(\frac{\Lambda_{t+j}}{\Lambda_t}\right) \left[\frac{P_t(f)}{P_{t+j}} \prod_{k=1}^{j} \left(\Pi_{t+k-1}^{\gamma_p}(\Pi^*)^{1-\gamma_p}\right) - mc_{t+j}\right] Y_{t+j}(j), \tag{38}$$

where Λ_t is the marginal utility of consumption of households and $\beta^j \frac{\Lambda_{t+j}}{\Lambda_t}$ is the stochastic discount factor. The demand function for $Y_{t+j}(f)$ is given by

$$Y_{t+j}(f) = \left[\frac{P_t(f)}{P_{t+j}} \prod_{k=1}^{j} \left(\Pi_{t+k-1}^{\gamma_p}(\Pi^*)^{1-\gamma_p}\right)\right]^{-\frac{1+\lambda_{t+j}^p}{\lambda_{t+j}^p}} Y_{t+j}. \tag{39}$$

The reoptimized price P_t^o is the same for all intermediate-good firms. The first-order condition for reoptimized price P_t^o is

$$1 = \frac{E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \frac{(1+\lambda_{t+j}^p) mc_{t+j} \Lambda_{t+j} Y_{t+j}}{\lambda_{t+j}^p} \left[\frac{P_t^0}{P_t} \prod_{k=1}^{j} \left(\frac{\Pi_{t+k-1}}{\Pi^*}\right)^{\gamma_p} \frac{\Pi^*}{\Pi_{t+k}}\right]^{-\frac{1+\lambda_{t+j}^p}{\lambda_{t+j}^p}}}{E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \frac{\Lambda_{t+j} Y_{t+j}}{\lambda_{t+j}^p} \left[\frac{P_t^0}{P_t} \prod_{k=1}^{j} \left(\frac{\Pi_{t+k-1}}{\Pi^*}\right)^{\gamma_p} \frac{\Pi^*}{\Pi_{t+k}}\right]^{-\frac{1}{\lambda_{t+j}^p}}}. \tag{40}$$

Households: The household indexed by $h \in [0,1]$ consumes $C_t(h)$, invests $I_t(h)$, holds safe asset $B_t(h)$ and capital stock $K_t(h)$, and supplies differentiated labor service $\ell_t(h)$. The households are subject to money illusion regarding inflation. Specifically, they misperceive current inflation as $\Pi_t^{MI} = (\Pi^*)^{\psi_c} \Pi_t^{1-\psi_c}$ and expected future inflation as $E_t \Pi_{t+1}^{MI} = (\Pi^*)^{\psi_f} (E_t \Pi_{t+1})^{1-\psi_f}$.

The utility function is then given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \exp(z_t^b) \left[\frac{[C_t(h) - \theta C_{t-1}(h)]^{1-\sigma}}{1-\sigma} - \frac{Z_t^{1-\sigma} \exp(z_t^\ell) \ell_t(h)^{1+\chi}}{1+\chi}\right], \tag{41}$$

where $\beta \in (0,1)$ denotes the discount factor; $\sigma > 0$ is the relative risk aversion; $\theta \in (0,1)$ is the degree of habit persistence; $\chi > 0$ denotes the inverse of the labor supply elasticity; and z_t^b and z_t^ℓ are the structural shocks.

The realized budget constraint of the household is

$$C_t(h) + I_t(h) + \frac{B_t(h)}{P_t} = W_t(h)\ell_t(h) + R_t^k u_t(h) K_{t-1}(h) + \frac{R_{t-1}^n B_{t-1}(h)}{P_t} + T_t(h), \tag{42}$$

where $W_t(h)$ is the real wage rate. Households make decisions based on the subjective budget constraint where the current price level is perceived as $P_t^{MI} = P_{t-1}\Pi_t^{MI}$.

The capital stock evolves as follows:

$$K_t(h) = [1 - \delta(u_t(h))]K_{t-1}(h) + \left[1 - S\left(\frac{I_t(h)}{I_{t-1}(h)}\frac{\exp(z_t^i)}{z^*}\right)\right] I_t(h), \tag{43}$$

where the investment adjustment cost function is specified as

$$S(x) = \frac{(x-1)^2}{2\zeta}, \tag{44}$$

where $\zeta > 0$. The capital utilization cost function satisfies

$$\mu = \frac{\delta'(u^*)}{\delta''(u^*)}, \tag{45}$$

where u^* denotes the steady-state capital utilization rate.

Under the existence of complete insurance markets, the first-order conditions are given by:

$$\Lambda_t = \exp(z_t^b)\,[C_t - \theta C_{t-1}]^{-\sigma} - \beta\theta E_t\left(\exp(z_{t+1}^b)\,[C_{t+1} - \theta C_t]^{-\sigma}\right), \tag{46}$$

$$\Lambda_t = \beta E_t\left[\Lambda_{t+1}\cdot\frac{R_t^n}{\Pi_{t+1}^{MI}}\right], \tag{47}$$

$$R_t^k = Q_t\delta'(u_t), \tag{48}$$

$$\begin{aligned} 1 = Q_t &\left[1 - S\left(\frac{I_t}{I_{t-1}}\frac{\exp(z_t^i)}{z^*}\right) - S'\left(\frac{I_t}{I_{t-1}}\frac{\exp(z_t^i)}{z^*}\right)\frac{I_t}{I_{t-1}}\frac{\exp(z_t^i)}{z^*}\right] \\ &+ \beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}Q_{t+1}S'\left(\frac{I_{t+1}}{I_t}\frac{\exp(z_{t+1}^i)}{z^*}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\frac{\exp(z_{t+1}^i)}{z^*}\right], \end{aligned} \tag{49}$$

$$Q_t = \beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_{t+1}^k u_{t+1} + Q_{t+1}(1 - \delta(u_{t+1}))}{\Pi_{t+1}^{MI}/\pi_{t+1}}\right], \tag{50}$$

where Λ_t is the marginal utility of consumption and Q_t is the real price of capital. Note that in eq. (50), the real return on capital is deflated by the perceived inflation.

Wage setting: Each household supplies a differentiated labor service $\ell_t(h)$ to the labor market. Households are subject to money illusion when evaluating the real wage. A labor union representing labor type h sets the nominal wage for that type under monopolistic competition and Calvo wage rigidity. The union itself does not have an independent inflation misperception. Current inflation misperception affects wage setting through the effective marginal labor supply cost implied by households' perceived real wage.

The household's intratemporal labor supply condition is

$$MRS_t(\ell_t(h); h) = \frac{W_t(h)}{P_t}\left(\frac{\Pi_t}{\Pi^*}\right)^{\psi_c}, \tag{51}$$

where $MRS_t(x; h)$ denotes the marginal rate of substitution between consumption and differentiated labor service x for household h:

$$MRS_t(x; h) \equiv \frac{\exp(z_t^b)Z_t^{1-\sigma}\exp(z_t^\ell)x^\chi}{\Lambda_t}. \tag{52}$$

The effective marginal labor supply cost relevant for wage setting is

$$\mathcal{M}_t(x;h) \equiv MRS_t(x;h)\left(\frac{\Pi_t}{\Pi^*}\right)^{-\psi_c}. \tag{53}$$

This object represents the effective real wage cost consistent with households' labor supply under current inflation misperception. The corresponding effective total labor supply cost relevant for wage setting is

$$\int_0^{\ell_t(h)} \mathcal{M}_t(x;h)dx. \tag{54}$$

When the union can reoptimize the wage for labor type h, it chooses $W_t^o(h)$ to maximize

$$E_t\sum_{j=0}^{\infty}(\beta\xi_w)^j\Lambda_{t+j}\left[\ell_{t+j}(h)\left(\frac{P_tW_t^o(h)}{P_{t+j}}\right)\prod_{k=1}^{j}\left(z^*\Pi_{t+k-1}^{\gamma_w}(\Pi^*)^{1-\gamma_w}\right)-\int_0^{\ell_{t+j}(h)}\mathcal{M}_{t+j}(x;h)dx\right], \tag{55}$$

subject to the demand for differentiated labor,

$$\ell_{t+j}(h)=\ell_{t+j}\left[\frac{W_t^o(h)(z^*)^j}{W_{t+j}}\prod_{k=1}^{j}\left(\frac{\Pi_{t+k-1}}{\Pi^*}\right)^{\gamma_w}\frac{\Pi^*}{\Pi_{t+k}}\right]^{-\frac{1+\lambda_{t+j}^w}{\lambda_{t+j}^w}}. \tag{56}$$

Here $1-\xi_w\in[0,1]$ is the wage reoptimization probability and $\gamma_w\in[0,1]$ is the wage indexation parameter.

The first-order condition for the reoptimized wage W_t^o is

$$1=\frac{E_t\sum_{j=0}^{\infty}(\beta\xi_w)^j\frac{1+\lambda_{t+j}^w}{\lambda_{t+j}^w}\Lambda_{t+j}\mathcal{M}_{t+j}(\ell_{t+j}(h);h)\,\ell_{t+j}\left[\frac{W_t^o(z^*)^j}{W_{t+j}}\prod_{k=1}^{j}\left(\frac{\Pi_{t+k-1}}{\Pi^*}\right)^{\gamma_w}\frac{\Pi^*}{\Pi_{t+k}}\right]^{-\frac{1+\lambda_{t+j}^w}{\lambda_{t+j}^w}}}{E_t\sum_{j=0}^{\infty}(\beta\xi_w)^j\frac{\Lambda_{t+j}W_{t+j}}{\lambda_{t+j}^w}\ell_{t+j}\left[\frac{W_t^o(z^*)^j}{W_{t+j}}\prod_{k=1}^{j}\left(\frac{\Pi_{t+k-1}}{\Pi^*}\right)^{\gamma_w}\frac{\Pi^*}{\Pi_{t+k}}\right]^{-\frac{1}{\lambda_{t+j}^w}}}. \tag{57}$$

Current inflation misperception does not enter the union's dynamic problem as an independent misperception by the union. It enters only through the effective marginal labor supply cost $\mathcal{M}_t(x;h)$ implied by households' perceived real wage.

Central bank: The central bank follows a Taylor-type nominal interest rate rule. That is,

$$\log R_t^n=\phi_r\log R_{t-1}^n+(1-\phi_r)\left[\log\bar{R}^n+\phi_\pi\frac{1}{4}\sum_{j=0}^{3}\log\left(\frac{\Pi_{t-j}}{\Pi^*}\right)+\phi_y\log\left(\frac{Y_t}{Y_t^P}\right)\right]+z_t^r, \tag{58}$$

where Π^* is the steady-state gross inflation rate, Y_t^P is the potential output, and z_t^r is the monetary policy shock. Parameter $\phi_r\in[0,1)$ represents the degree of interest rate smoothing, and $\phi_\pi>1$ and $\phi_y\geq 0$ are the monetary policy responses to inflation and output, respectively. Then, the potential output Y_t^P is defined by

$$Y_t^P=[u^*k^*Z_{t-1}]^{\alpha}[Z_t\ell^*]^{1-\alpha}-\Phi Z_t, \tag{59}$$

where u^* is the steady-state of capital utilization, k^* is the steady-state detrended capital stock (K_t/Z_t), and ℓ^* is the steady-state hours worked. This specification of the potential output is similar to the estimates of Hara et al. (2006). In this specification, only the permanent (technology) shock is considered as a driving force of potential output. This setup is similar to the estimates of Fueki et al. (2016).

Aggregations and market clearing conditions: Because the decisions on $u_t(h)$, $I_t(h)$, and $K_t(h)$ are the same for all households, the evolution of the capital stock (43) is given by

$$K_t = [1 - \delta(u_t)]K_{t-1} + \left[1 - S\left(\frac{I_t}{I_{t-1}}\frac{\exp(z_t^i)}{z^*}\right)\right] I_t. \tag{60}$$

and the capital market-clearing conditions are given by

$$u_t K_{t-1} = \int_0^1 K_t^S(f)df. \tag{61}$$

Combining the cost-minimization conditions of intermediate-good firms (36) and (37) and aggregation over intermediate-good firms yields

$$\frac{1-\alpha}{\alpha} = \frac{W_t \ell_t}{R_t^k u_t K_{t-1}}. \tag{62}$$

The real marginal cost is then given by

$$mc_t = \left(\frac{W_t}{(1-\alpha)Z_t}\right)^{1-\alpha} \left(\frac{R_t^k}{\alpha}\right)^{\alpha}. \tag{63}$$

Aggregating the production function (34) over intermediate-good firms yields

$$Y_t s_t = [u_t K_{t-1}]^{\alpha} [Z_t \ell_t]^{1-\alpha} - \Phi Z_t, \tag{64}$$

where $s_t = \int_0^1 [P_t(f)/P_t]^{-(1+\lambda_t^p)/\lambda_t^p} df$ is the price dispersion of the intermediate-good price. This price dispersion can be ignored in the linearized system around the steady state, where the steady-state value is one.

Finally, the resource constraint is

$$C_t + I_t + g^* Z_t \exp(z_t^g) = Y_t, \tag{65}$$

where g^* is the steady-state ratio of government purchases to output and z_t^g is a government shock.

Log-linearized equilibrium conditions and exogenous structural shocks: The endogenous variables, except for labor supply and the marginal utility of wealth Λ_t, are detrended by the technology level Z_t as $x_t = X_t/Z_t$. The marginal utility Λ_t is detrended as $\lambda_t = \Lambda_t/Z_t^{-\sigma}$. Labor supply ℓ_t is stationary. The equilibrium conditions are log-linearized around a steady state, and the resulting log-linearized equilibrium system is described in Appendix B.

The model contains seven independent exogenous shocks. These shocks follow AR(1) processes as follows:

$$\text{technology:} \quad z_t^z = \rho_z z_{t-1}^z + \sigma_z \varepsilon_t^z \tag{66}$$

$$\text{preference:} \quad z_t^b = \rho_b z_{t-1}^b + \sigma_b \varepsilon_t^b \tag{67}$$

$$\text{government purchases:} \quad z_t^g = \rho_g z_{t-1}^g + \sigma_g \varepsilon_t^g \tag{68}$$

$$\text{investment adjustment cost:} \quad z_t^i = \rho_i z_{t-1}^i + \sigma_i \varepsilon_t^i \tag{69}$$

$$\text{price markup:} \quad z_t^p = \rho_p z_{t-1}^p + \sigma_p \varepsilon_t^p \tag{70}$$

$$\text{wage:} \quad z_t^w = \rho_w z_{t-1}^w + \sigma_w \varepsilon_t^w \tag{71}$$

$$\text{monetary policy:} \quad z_t^r = \rho_r z_{t-1}^r + \sigma_r \varepsilon_t^r \tag{72}$$

where $\sigma_x \varepsilon_t^x$ is a structural shock to z_t^x for x in the set of indices $z, b, g, i, p, w,$ and r. The term ε_t^x is assumed to be independently and identically distributed with a mean of zero and unit standard deviation.

Regarding the markup shocks, the price markup shock z_t^p is defined by

$$z_t^p = \frac{(1-\xi_p)(1-\beta\xi_p(z^*)^{1-\sigma})}{\xi_p}\tilde{\lambda}_t^p.$$

Similarly, the wage shock z_t^w is defined as

$$z_t^w = \frac{1-\xi_w}{\xi_w} \times \frac{(1-\beta\xi_w(z^*)^{1-\sigma})\lambda^w}{\lambda^w + \xi_w(1+\lambda^w)} \left(\tilde{\lambda}_t^w + z_t^\ell\right).$$

In the log-linearized equilibrium system, the wage markup shock $\tilde{\lambda}_t^w$ and the labor supply shock z_t^ℓ are not separately identified. Therefore, the wage shock z_t^w is defined as a linear combination of the log-linearized wage markup shock and the labor supply shock, where λ^w denotes the steady-state value of the wage markup. Detailed derivations of these conditions are provided in Appendix B.

4 Empirical analysis of the Japanese economy using a medium-scale DSGE model with money illusion

4.1 Data and Estimation Strategy

We use seven quarterly Japanese series as observable variables: real GDP per capita (Y_t), real consumption per capita (C_t), real investment per capita (I_t), real wages (W_t), hours worked (ℓ_t), the consumer price index (P_t), and the nominal interest rate (R_t^n).

The estimation sample spans from 1995Q1 to 2019Q4, ending prior to the COVID-19 pandemic. The Japanese economy experienced a prolonged period at the zero lower bound during this sample. Since the model does not explicitly incorporate the non-linearity associated with the zero lower bound, we use the shadow rate as the measure of the monetary policy stance. Specifically, we use the shadow rate i_t^{shadow} estimated by Nakajima (2025), which applies the shadow-rate term structure framework developed by Ichiue and Ueno (2013). The sample period begins in 1995Q1 to match the availability of the shadow rate series provided by Nakajima (2025). We also utilize the series by Krippner (2013, 2015) for robustness checks in Appendix C, as both datasets are available from this date onward.

The overall data construction follows the definitions in Hirose and Kurozumi (2012), with certain modifications and specific data sources. The series for GDP, private consumption, and private investment are obtained from the nominal seasonally adjusted series in the *System of National Accounts*, published by the Cabinet Office of Japan. These are deflated by the core consumer price index (CPI) excluding fresh food, obtained from the *Consumer Price Index* published by the Statistics Bureau of Japan, to obtain real values, and then divided by the population aged 15 and over to construct per capita measures.

While the baseline follows Hirose and Kurozumi (2012), we introduce the following specific definitions for I_t, W_t, and ℓ_t. Following Kobayashi and Inaba (2006), I_t is defined as per capita gross fixed capital formation by the private sector. Real wages W_t are constructed as the ratio of average monthly scheduled cash earnings per employee to total monthly hours worked per employee, both drawn from the *Monthly Labor Survey* published by the Ministry of Health, Labour and Welfare, and deflated by the core CPI. Following Hayashi and Prescott (2002), Kobayashi and Inaba (2006), and Inaba et al. (2022), hours worked ℓ_t is constructed as:

$$\ell_t = \frac{\text{Average hours worked per employed person} \times \text{Employed persons}}{\text{Labor force}}.$$

The data for average hours worked are obtained from the *Monthly Labor Survey*, while employed persons and labor force data are from the *Labour Force Survey* published by the Ministry of Internal Affairs and Communications.

We independently seasonally adjust total hours worked, real wages, and the core CPI using the X-13ARIMA-SEATS procedure, and convert them to a quarterly frequency by taking three-month averages. The shadow rate series provided by Nakajima (2025) and Krippner (2013, 2015) are reported as annualized percentage rates. Accordingly, the quarterly nominal gross interest rate is defined as $R_t^n = 1 + i_t^{shadow}/400$. For hours worked, following Sugo and Ueda (2008) and Hirose and Kurozumi (2012), the observable is denoted by the label "$100 \log \ell_t$," which is defined as $100(\log \ell_t - \overline{\log \ell})$, where $\overline{\log \ell}$ is the sample average of log hours worked.

The observation equations are

$$\begin{bmatrix} 100\Delta\log Y_t \\ 100\Delta\log C_t \\ 100\Delta\log I_t \\ 100\Delta\log W_t \\ 100\log \ell_t \\ 100\Delta\log P_t \\ 100\log R_t^n \end{bmatrix} = \begin{bmatrix} z^* \\ z^* \\ z^* \\ z^* \\ \bar{\ell} \\ \pi^* \\ r^* + \pi^* \end{bmatrix} + \begin{bmatrix} \tilde{y}_t - \tilde{y}_{t-1} + z_t^z \\ \tilde{c}_t - \tilde{c}_{t-1} + z_t^z \\ \tilde{i}_t - \tilde{i}_{t-1} + z_t^z \\ \tilde{w}_t - \tilde{w}_{t-1} + z_t^z \\ \tilde{\ell}_t \\ \tilde{\pi}_t \\ \tilde{r}_t^n \end{bmatrix}, \tag{73}$$

where z^*, $\bar{\ell}$, π^*, and r^* denote the steady-state growth rate, hours worked, net inflation rate, and real interest rate expressed in percentage terms, respectively. Note that $\bar{\ell}$ is different from ℓ^* in (59) since the observable for hours worked is demeaned. Variables with tildes denote log deviations from the steady state.

Most of the model parameters, including the constant terms in the observation equations, are estimated. However, the following parameters are fixed based on Sugo and Ueda (2008): the steady-state depreciation rate of capital stock $\delta = 0.015$, the cost share of capital $\alpha = 0.37$, and the steady-state wage markup $\lambda^w = 0.20$. The steady-state ratio of government purchases to output is set to $g = 0.29$, corresponding to the sample mean.

The model is estimated using Bayesian methods, and the prior distributions of the parameters are summarized in Table 1. The degrees of current and future inflation misperception (ψ_c and ψ_f) are assigned Beta distributions with a mean of 0.5 and a standard deviation of 0.2. This reflects a neutral prior stance, allowing the data to determine the extent of money illusion within the unit interval. For the structural parameters, including the relative risk aversion σ, habit persistence θ, inverse of Frisch elasticity χ, investment adjustment cost $1/\zeta$, capital utilization cost μ, fixed cost ϕ, nominal rigidity parameters $(\xi_w, \xi_p, \gamma_w, \gamma_p)$, and monetary policy rule parameters $(\phi_r, \phi_\pi, \phi_y)$, we set the prior means and distributions following Hirose and Kurozumi (2012). For the price markup λ^p, the prior mean and standard deviation are taken from Justiniano et al. (2010), but a Gamma distribution is employed to satisfy the theoretical requirements. The steady-state parameters z^*, $\bar{\ell}$, π^*, and r^* are assumed to follow Normal prior distributions with a standard deviation of 0.05. The prior means are set to the corresponding sample averages over the estimation period.

For the persistence parameters of the structural shocks, we use Beta distributions with a mean of 0.5 and a standard deviation of 0.2. The standard deviations of the structural shocks σ_j follow Inverse-Gamma distributions with a mean of 0.5 and an infinite standard deviation. It should be noted that, in contrast to some existing studies, our model does not incorporate measurement errors; all observed fluctuations are explained by the structural shocks.

Table 1: Prior Distributions

	Parameter	Distribution	Mean	SD
ψ_c	Current inflation misperception	Beta	0.5000	0.2000
ψ_f	Expected inflation misperception	Beta	0.5000	0.2000
σ	Relative risk aversion	Gamma	1.0000	0.3750
θ	Habit persistence	Beta	0.7000	0.1500
χ	Inverse of Frisch elasticity	Gamma	2.0000	0.7500
$1/\zeta$	Adjustment cost of investment	Gamma	4.0000	1.5000
μ	Utilization adjustment cost	Gamma	1.0000	1.0000
ϕ	Fixed cost	Gamma	0.0750	0.0125
γ_w	Wage indexation	Beta	0.5000	0.2500
ξ_w	Wage stickiness	Beta	0.3750	0.1000
γ_p	Price indexation	Beta	0.5000	0.2500
ξ_p	Price stickiness	Beta	0.3750	0.1000
λ^p	Steady-state price markup	Gamma	0.1500	0.0500
z^*	Steady-state growth rate	Normal	-0.0220	0.0500
$\bar{\ell}$	Steady-state hours worked	Normal	0.0000	0.0500
π^*	Steady-state inflation rate	Normal	0.0270	0.0500
r^*	Steady-state real interest rate	Normal	-0.3040	0.0500
ϕ_r	Interest rate smoothing	Beta	0.8000	0.1000
ϕ_π	Policy response to inflation	Gamma	1.7000	0.1000
ϕ_y	Policy response to output	Gamma	0.1250	0.0500
ρ_z	Technology shock persistence	Beta	0.5000	0.2000
ρ_b	Preference shock persistence	Beta	0.5000	0.2000
ρ_i	Investment shock persistence	Beta	0.5000	0.2000
ρ_g	Government shock persistence	Beta	0.5000	0.2000
ρ_w	Wage shock persistence	Beta	0.5000	0.2000
ρ_p	Price markup shock persistence	Beta	0.5000	0.2000
ρ_r	Monetary policy persistence	Beta	0.5000	0.2000
σ_z	Std. dev. of technology shock	Inv-Gamma	0.5000	∞
σ_b	Std. dev. of preference shock	Inv-Gamma	0.5000	∞
σ_i	Std. dev. of investment shock	Inv-Gamma	0.5000	∞
σ_g	Std. dev. of government shock	Inv-Gamma	0.5000	∞
σ_w	Std. dev. of wage shock	Inv-Gamma	0.5000	∞
σ_p	Std. dev. of price markup shock	Inv-Gamma	0.5000	∞
σ_r	Std. dev. of monetary shock	Inv-Gamma	0.5000	∞

Table 2: Marginal Likelihood Comparison

	Log Marginal Likelihood		Bayes Factor
Shadow Rate Measure	MI	RE	(MI/RE)
Nakajima (2025)	-729.42	-737.85	4609
Krippner (2013, 2015)	-781.05	-788.84	2396

Notes: The log marginal likelihoods are computed using the modified harmonic mean estimator. The Bayes factor is computed as the exponential of the difference in log marginal likelihoods between the money illusion (MI) model and the rational expectations (RE) model, using the unrounded log marginal likelihoods. A Bayes factor greater than one indicates that the MI model is preferred to the RE model.

The model is estimated using Dynare. Following standard Bayesian approaches, we employ the Kalman filter to evaluate the likelihood function of the log-linearized equilibrium system. To generate draws from the posterior distribution of the structural parameters, we use the random-walk Metropolis–Hastings algorithm with two parallel chains. We generate 1,000,000 draws for each chain, discarding the first half as a burn-in period.

To evaluate the empirical relevance of money illusion, we also estimate a Rational Expectations (RE) model. This RE model is a nested version of our general framework, obtained by imposing the restriction $\psi_c = \psi_f = 0$. By comparing the marginal likelihoods of the MI and RE models, we can formally assess whether the introduction of inflation misperception significantly improves the model's ability to fit the Japanese macroeconomic data.

4.2 Marginal Likelihood Comparison

Table 2 reports the log marginal likelihoods of the money illusion (MI) model and the rational expectations (RE) model for alternative shadow rate measures.

For both shadow rate series, the MI model attains a higher marginal likelihood than the RE model. Using the shadow rate constructed by Nakajima (2025), the log marginal likelihood is -729.42 for the MI model and -737.85 for the RE model, implying a Bayes factor of 4609 in favor of the MI model. Using the shadow rate proposed by Krippner (2013, 2015), the corresponding values are -781.05 and -788.84, yielding a Bayes factor of 2396.

These results indicate strong empirical support for the MI model relative to the RE model. The finding is robust across the two alternative measures of the shadow rate, although the magnitude of the marginal likelihood differs between them. In particular, the superiority of the MI model is somewhat stronger when the shadow rate by Nakajima (2025) is employed.

In what follows, we focus on the estimation results using the shadow rate constructed by Nakajima (2025). This measure extends the shadow rate framework developed by Ichiue and Ueno (2013) and is estimated specifically for the Japanese yield curve environment, including the prolonged low interest rate period and the yield curve control regime. Compared with the shadow rate by Krippner (2013, 2015), the Nakajima series is intended to better capture the characteristics of Japanese monetary policy and financial market conditions. Table 2 also shows that the model fit is somewhat better when the Nakajima shadow rate is employed. We therefore use the Nakajima shadow rate as the benchmark specification in the main text, while the corresponding results based on the shadow rate by Krippner (2013, 2015) are reported in Appendix C.

4.3 Posterior Estimates

Table 3 reports the posterior distributions for the models estimated using the shadow rate constructed by Nakajima (2025).

The posterior means of both the future inflation misperception parameter ψ_f and the current inflation misperception parameter ψ_c are large, with $\psi_f = 0.81$ and $\psi_c = 0.52$. The larger estimate of ψ_f implies that future inflation misperception plays a larger role in the intertemporal demand channel. The estimated degree of future inflation misperception is substantially larger than the departures from rational expectations reported in the cognitive discounting studies for Japan by Hirose et al. (2024) and Hirose and Yoo (2026). One possible reason is that our model introduces behavioral distortions only through inflation expectations, whereas cognitive discounting models discount all forward-looking variables.

Most structural parameters are similar across the MI and RE models, including habit persistence, investment adjustment costs, monetary policy coefficients, and shock persistence parameters. The main differences arise in the parameters related to the Euler equation and the Phillips curve.

The posterior mean of the relative risk aversion parameter σ is 0.80 under MI and 0.99 under RE. The estimated standard deviation of preference shocks also differs across the two models, with $\sigma_b = 4.19$ under MI and $\sigma_b = 5.42$ under RE. These differences reflect the role of future inflation misperception in the Euler equation. Because future inflation misperception affects the perceived real interest rate, the MI model changes the intertemporal demand channel. The MI specification therefore requires a lower value of σ and a smaller preference shock volatility.

Nominal rigidities also differ across the two models. The estimated degree of price stickiness is slightly lower in the MI model, with $\xi_p = 0.71$ under MI and $\xi_p = 0.74$ under RE. The estimated degree of wage stickiness also declines from $\xi_w = 0.63$ under RE to $\xi_w = 0.59$ under MI. These differences reflect the role of current inflation misperception in the Phillips curve. Because current inflation misperception affects labor supply and marginal costs, the MI model requires somewhat less nominal rigidity to account for inflation persistence.

Table 3: Posterior Estimates

	Money Illusion		Rational Expectations	
	Mean	90% credible interval	Mean	90% credible interval
ψ_c	0.5181	[0.1956, 0.8523]	–	–
ψ_f	0.8130	[0.6547, 0.9759]	–	–
σ	0.7960	[0.4197, 1.1540]	0.9888	[0.4764, 1.4817]
θ	0.8146	[0.7318, 0.9054]	0.8008	[0.6207, 0.9323]
χ	2.1244	[1.0168, 3.1607]	2.2405	[1.0688, 3.3617]
$1/\zeta$	6.0545	[3.2797, 8.7025]	6.0304	[3.3192, 8.6577]
μ	1.4874	[0.4094, 2.5424]	0.9312	[0.0390, 1.7841]
ϕ	0.0701	[0.0507, 0.0884]	0.0690	[0.0504, 0.0875]
γ_w	0.5881	[0.2569, 0.9588]	0.5565	[0.2003, 0.9406]
ξ_w	0.5852	[0.5027, 0.6663]	0.6333	[0.5436, 0.7250]
γ_p	0.1054	[0.0011, 0.2118]	0.0907	[0.0008, 0.1847]
ξ_p	0.7096	[0.6608, 0.7556]	0.7396	[0.6699, 0.8061]
λ^p	0.2733	[0.1509, 0.3931]	0.2944	[0.1615, 0.4237]
z^*	-0.0637	[-0.1331, 0.0107]	-0.0199	[-0.0996, 0.0591]
$\bar{\ell}$	-0.0006	[-0.0828, 0.0813]	-0.0017	[-0.0835, 0.0826]
π^*	0.0229	[-0.0496, 0.0986]	0.0134	[-0.0635, 0.0912]
r^*	-0.2464	[-0.3148, -0.1776]	-0.2470	[-0.3147, -0.1779]
ϕ_r	0.8549	[0.8137, 0.8980]	0.8446	[0.8028, 0.8867]
ϕ_π	1.7164	[1.5543, 1.8809]	1.7577	[1.5921, 1.9220]
ϕ_y	0.0558	[0.0253, 0.0858]	0.0532	[0.0158, 0.0893]
ρ_z	0.1118	[0.0253, 0.1920]	0.1285	[0.0323, 0.2222]
ρ_b	0.2257	[0.0289, 0.4167]	0.2766	[0.0210, 0.6420]
ρ_i	0.5017	[0.3607, 0.6432]	0.4479	[0.2661, 0.6247]
ρ_g	0.9013	[0.8530, 0.9522]	0.9078	[0.8548, 0.9624]
ρ_w	0.0983	[0.0125, 0.1819]	0.1024	[0.0126, 0.1897]
ρ_p	0.9356	[0.8869, 0.9880]	0.8498	[0.7249, 0.9771]
ρ_r	0.5943	[0.4508, 0.7402]	0.5910	[0.4641, 0.7200]
σ_z	1.8027	[1.5730, 2.0375]	1.7939	[1.5548, 2.0235]
σ_b	4.1870	[1.8500, 6.4820]	5.4189	[1.7670, 8.9768]
σ_i	3.0859	[2.4947, 3.6513]	2.8888	[2.3191, 3.4371]
σ_g	2.7290	[2.3753, 3.0556]	2.7226	[2.3770, 3.0571]
σ_w	1.0319	[0.8805, 1.1836]	1.0169	[0.8645, 1.1683]
σ_p	0.1753	[0.1269, 0.2209]	0.1751	[0.1271, 0.2202]
σ_r	0.0721	[0.0628, 0.0810]	0.0736	[0.0642, 0.0829]

Table 4: Variance Decompositions: Money Illusion vs. Rational Expectations

Money Illusion							
	z^b	z^i	z^g	z^w	z^p	z^r	z^z
$100\Delta \log Y_t$	4.13	25.90	21.74	0.79	7.20	0.46	39.77
$100\Delta \log C_t$	77.32	1.88	0.57	0.41	2.41	0.28	17.13
$100\Delta \log I_t$	0.34	67.74	1.42	1.28	12.78	0.72	15.72
$100\Delta \log W_t$	0.55	0.51	0.09	45.42	23.14	0.12	30.15
$100 \log \ell_t$	1.27	21.17	7.82	7.72	38.97	1.78	21.28
$100\Delta \log P_t$	0.69	3.04	0.72	11.12	49.20	9.18	26.05
$100 \log R^n_t$	1.26	15.43	2.86	6.91	32.05	14.97	26.53
Rational Expectations							
	z^b	z^i	z^g	z^w	z^p	z^r	z^z
$100\Delta \log Y_t$	5.14	24.61	21.45	1.05	5.18	0.85	41.72
$100\Delta \log C_t$	80.98	0.90	0.47	0.36	1.45	0.37	15.48
$100\Delta \log I_t$	0.51	66.29	1.71	1.89	9.65	1.45	18.51
$100\Delta \log W_t$	0.55	0.30	0.06	53.59	17.92	0.14	27.46
$100 \log \ell_t$	2.07	21.31	9.17	10.30	21.70	3.15	32.30
$100\Delta \log P_t$	0.80	2.95	0.99	10.19	50.74	8.06	26.29
$100 \log R^n_t$	1.36	10.31	3.17	6.33	28.86	11.20	38.77

Note: Infinite-horizon forecast error variance decompositions are reported.

4.4 Variance Decomposition

Table 4 reports the infinite-horizon forecast error variance decompositions under the MI and RE models. The dominant sources of fluctuations for real activity variables are similar across the two models. Technology shocks account for the largest share of output fluctuations, preference shocks dominate consumption fluctuations, and investment shocks are the primary source of investment fluctuations in both specifications.

At the same time, the quantitative importance of demand-side disturbances differs across the two models. The contribution of preference shocks to consumption fluctuations declines from 80.98% under RE to 77.32% under MI, indicating that future inflation misperception changes the transmission of demand-side shocks through the Euler equation. The decline in the contribution of preference shocks is not limited to consumption. Their contribution is lower under MI for most variables, suggesting that the estimated MI model assigns a smaller role to preference shocks than the RE model.

More substantial differences emerge for labor and nominal variables. For labor, the contribution of

Table 5: Variance Decompositions: Money Illusion vs. Counterfactual Case

Money Illusion ($\psi_c = 0.5181$, $\psi_f = 0.8130$)							
	z^b	z^i	z^g	z^w	z^p	z^r	z^z
$100\Delta \log Y_t$	4.13	25.90	21.74	0.79	7.20	0.46	39.77
$100\Delta \log C_t$	77.32	1.88	0.57	0.41	2.41	0.28	17.13
$100\Delta \log I_t$	0.34	67.74	1.42	1.28	12.78	0.72	15.72
$100\Delta \log W_t$	0.55	0.51	0.09	45.42	23.14	0.12	30.15
$100 \log \ell_t$	1.27	21.17	7.82	7.72	38.97	1.78	21.28
$100\Delta \log P_t$	0.69	3.04	0.72	11.12	49.20	9.18	26.05
$100 \log R_t^n$	1.26	15.43	2.86	6.91	32.05	14.97	26.53
Counterfactual Case ($\psi_c = \psi_f = 0$)							
	z^b	z^i	z^g	z^w	z^p	z^r	z^z
$100\Delta \log Y_t$	4.10	26.10	21.49	0.82	6.58	0.77	40.13
$100\Delta \log C_t$	77.02	1.80	0.56	0.42	2.10	0.50	17.60
$100\Delta \log I_t$	0.34	67.48	1.39	1.32	11.74	1.18	16.54
$100\Delta \log W_t$	0.57	0.56	0.10	45.79	22.72	0.19	30.08
$100 \log \ell_t$	1.28	21.38	7.80	7.78	36.81	2.76	22.19
$100\Delta \log P_t$	0.57	2.57	0.71	9.35	52.64	10.78	23.38
$100 \log R_t^n$	0.97	11.35	2.66	5.15	37.70	10.41	31.75

Note: The lower panel reports a counterfactual variance decomposition obtained by setting $\psi_c = \psi_f = 0$ while holding all other parameters at their posterior means under the money illusion specification.

price markup shocks rises from 21.70% under RE to 38.97% under MI. At the same time, the contribution of technology shocks falls from 32.30% to 21.28%. Under MI, labor fluctuations are therefore linked more strongly to nominal disturbances than under RE. This pattern follows from the role of current inflation misperception in labor supply and marginal cost determination.

Inflation decompositions remain similar across the two models. Price markup shocks explain roughly one half of inflation fluctuations in both specifications. By contrast, the decomposition of the nominal interest rate changes more noticeably. Under MI, the contribution of investment shocks rises from 10.31% to 15.43%, price markup shocks from 28.86% to 32.05%, and monetary policy shocks from 11.20% to 14.97%. The contribution of technology shocks, in contrast, declines from 38.77% under RE to 26.53% under MI. Future inflation misperception increases the contribution of monetary policy and price markup shocks to nominal interest rate fluctuations.

Table 5 compares the estimated MI model with a counterfactual specification that sets $\psi_c = \psi_f = 0$

while holding all other parameters fixed at their posterior means under the MI model. Unlike the comparison between the estimated MI and RE models, the counterfactual exercise keeps all structural parameters fixed. The comparison isolates the propagation effects of inflation misperception.

This exercise is closely related to the impulse response analysis in Figure 1. The impulse responses in the simple model show that future inflation misperception affects the transmission of technology, price markup, and monetary policy shocks through the perceived real interest rate channel, while current inflation misperception affects inflation dynamics through labor supply and marginal cost movements. The counterfactual variance decomposition examines how these propagation channels affect the relative importance of shocks in the estimated model.

The dominant sources of fluctuations remain similar across the two cases. Technology shocks account for the largest share of output fluctuations in both cases, preference shocks remain the dominant source of consumption fluctuations, and investment shocks remain the primary source of investment fluctuations. The contribution of preference shocks to consumption also changes little in the counterfactual comparison. This contrasts with the MI–RE comparison in Table 4, where preference shocks play a smaller role under the estimated MI model. This difference suggests that the decline in the importance of preference shocks mainly reflects shifts in estimated structural parameters rather than the direct propagation effect of inflation misperception.

Somewhat larger differences appear for inflation and nominal interest rate fluctuations, while changes in labor fluctuations are relatively modest. For inflation, price markup shocks remain the dominant source in both cases, but their contribution falls from 52.64% in the counterfactual case to 49.20% under MI. The contribution of monetary policy shocks also declines from 10.78% to 9.18%, while the contribution of wage markup shocks rises from 9.35% to 11.12%. Labor fluctuations exhibit only modest changes. The contribution of price markup shocks rises from 36.81% in the counterfactual case to 38.97% under MI, whereas the contribution of technology shocks declines slightly from 22.19% to 21.28%. For the nominal interest rate, the contributions of investment shocks and monetary policy shocks rise from 11.35% to 15.43% and from 10.41% to 14.97%, respectively, whereas the contributions of price markup and technology shocks decline from 37.70% to 32.05% and from 31.75% to 26.53%.

The counterfactual exercise supports the propagation mechanisms highlighted by the impulse responses, particularly for inflation and nominal interest rate fluctuations. The effects on labor fluctuations are present but more modest than those in the comparison between the estimated MI and RE models because the counterfactual exercise holds the other structural parameters fixed. This suggests that the estimated differences between the MI and RE models in Table 4 reflect both the propagation effects of inflation misperception and shifts in the estimated structural parameters.

5 Concluding Remarks

This paper investigated the role of money illusion in the Japanese business cycle using a medium-scale New Keynesian DSGE model with inflation misperceptions. The model distinguishes between current inflation misperception and future inflation misperception, which affect labor supply and intertemporal demand decisions, respectively.

The theoretical analysis showed that the two forms of inflation misperception have different implications for equilibrium determinacy and shock propagation. Future inflation misperception affects perceived real interest rates and changes the transmission of technology and monetary policy shocks through the intertemporal demand channel. Current inflation misperception affects labor supply and marginal costs, altering the propagation of nominal disturbances through the Phillips curve.

The empirical analysis using Japanese macroeconomic data for 1995Q1–2019Q4 showed that the money illusion model outperforms the rational expectations model in terms of marginal likelihood. The results indicate that allowing for inflation misperceptions improves the empirical fit of a medium-scale New Keynesian DSGE model. Both current and future inflation misperceptions are estimated to be quantitatively important, although the estimated degree of money illusion is larger for future inflation misperception.

The variance decompositions and counterfactual exercises support the propagation mechanisms highlighted by the impulse response analysis. Current inflation misperception increases the role of nominal disturbances in labor fluctuations, whereas future inflation misperception affects the contribution of monetary policy and other shocks to nominal interest rate dynamics. At the same time, the counterfactual exercise shows that these propagation effects have modest implications for the variance decompositions when the other structural parameters are held fixed.

These results imply that inflation misperceptions mainly affect shock propagation while leaving the dominant sources of fluctuations broadly unchanged. They also suggest that departures from full rational expectations may become quantitatively important in a prolonged low-interest-rate environment.

Several extensions remain for future research. One extension is to introduce inflation misperceptions on the firms' side, where misperceptions in pricing or demand expectations may further affect inflation dynamics. Another is to examine whether similar mechanisms operate in other economies with different inflation experiences and monetary policy regimes.

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Appendix A Proof of Proposition 1

To analyze the determinacy of the equilibrium, we examine the eigenvalues of the matrix $\mathbb{M} = \mathbb{A}^{-1}\mathbb{B}$. Since both $\tilde{y}_t$ and $\tilde{\pi}_t$ are forward-looking variables, the system is unique and determinate if and only if both eigenvalues of $\mathbb{M}$ lie strictly inside the unit circle.

The eigenvalues μ of $\mathbb{M}$ are the roots of the characteristic polynomial $P(\mu) = \det(\mu\mathbb{I} - \mathbb{M}) = 0$. This is equivalent to solving the following equation: $F(\mu) \equiv \det(\mu\mathbb{A} - \mathbb{B}) = 0$. Substituting the definitions of $\mathbb{A}$ and $\mathbb{B}$, we obtain:

$$\mu\mathbb{A} - \mathbb{B} = \begin{pmatrix} \mu(1 + \phi_y/\sigma) - 1 & \frac{\mu\phi_\pi - (1-\psi_f)}{\sigma} \\ -\mu\lambda(\chi + \sigma) & \mu(1 + \lambda\psi_c) - \beta \end{pmatrix}.$$

The determinant $F(\mu)$ is a quadratic polynomial $F(\mu) = c_2\mu^2 - c_1\mu + c_0$, where the coefficients are given by:

$$\begin{aligned} c_2 &= \left(1 + \frac{\phi_y}{\sigma}\right)(1 + \lambda\psi_c) + \frac{\lambda(\chi + \sigma)\phi_\pi}{\sigma} > 1, \\ c_1 &= \beta\left(1 + \frac{\phi_y}{\sigma}\right) + (1 + \lambda\psi_c) + \frac{\lambda(\chi + \sigma)(1 - \psi_f)}{\sigma}, \\ c_0 &= \beta. \end{aligned}$$

According to the Jury stability criterion, the necessary and sufficient conditions for the roots to lie within the unit circle are: (i) $|c_0| < c_2$, (ii) $F(-1) > 0$, and (iii) $F(1) > 0$.

Condition (i) is satisfied because $c_0 = \beta < 1 < c_2$. This ensures that the product of the two roots is less than unity, ruling out the possibility of both roots lying outside the unit circle simultaneously. Condition (ii) is verified as:

$$\begin{aligned} F(-1) &= c_2 + c_1 + c_0 \\ &= \left(2 + \frac{\phi_y}{\sigma}\right)(1 + \beta + \lambda\psi_c) + \frac{\lambda(\chi + \sigma)}{\sigma}(\phi_\pi + 1 - \psi_f) > 0, \end{aligned}$$

which holds since $\psi_f \in [0, 1]$ and all other parameters are positive. Finally, condition (iii) $F(1) > 0$ requires:

$$\begin{aligned} F(1) &= \det(\mathbb{A} - \mathbb{B}) \\ &= \det\begin{pmatrix} \phi_y/\sigma & (\phi_\pi + \psi_f - 1)/\sigma \\ -\lambda(\chi + \sigma) & 1 - \beta + \lambda\psi_c \end{pmatrix} \\ &= \frac{\phi_y}{\sigma}(1 - \beta + \lambda\psi_c) + \frac{\lambda(\chi + \sigma)}{\sigma}(\phi_\pi + \psi_f - 1) > 0. \end{aligned}$$

This inequality is satisfied if and only if:

$$\phi_y(1 - \beta + \lambda\psi_c) + \lambda(\chi + \sigma)(\phi_\pi + \psi_f - 1) > 0.$$

Q.E.D.

Appendix B Linearized Medium-Scale DSGE Model with Money Illusion

This appendix presents the log-linearized equations of the medium-scale DSGE model with money illusion. The variable with tilde $\tilde{x}_t$ is defined as the log-deviation of x_t from its steady-state value x^*:

$$\tilde{x}_t = \log(x_t) - \log(x^*).$$

1. Households

The marginal utility of consumption $\tilde{\lambda}_t$ is determined by

$$\left(1 - \frac{\theta}{z^*}\right)\left(1 - \frac{\theta}{r^*}\right)\tilde{\lambda}_t = -\sigma\left(\tilde{c}_t - \frac{\theta}{z^*}(\tilde{c}_{t-1} - z_t^z)\right) + \left(1 - \frac{\theta}{z^*}\right)z_t^b + \frac{\theta}{r^*}\left[\sigma\left(\tilde{c}_{t+1} + z_{t+1}^z - \frac{\theta}{z^*}\tilde{c}_t\right) - \left(1 - \frac{\theta}{z^*}\right)z_{t+1}^b\right], \tag{A.1}$$

where θ is the degree of habit persistence and σ is the relative risk aversion.

The Euler equation incorporating the misperception of expected future inflation ψ_f is

$$\tilde{\lambda}_t = E_t\tilde{\lambda}_{t+1} - \sigma E_t z_{t+1}^z + \tilde{r}_t^n - (1 - \psi_f)E_t\tilde{\pi}_{t+1}, \tag{A.2}$$

where $\tilde{r}_t^n$ and $\tilde{\pi}_t$ denote the log deviations of the nominal interest rate and inflation rate from their steady-state values, respectively.

The capital utilization $\tilde{u}_t$ and the investment Euler equation are

$$\tilde{u}_t = \mu(\tilde{r}_t^k - \tilde{q}_t), \tag{A.3}$$

$$\zeta^{-1}(\tilde{i}_t - \tilde{i}_{t-1} + z_t^z + z_t^i) = \tilde{q}_t + \zeta^{-1}\frac{z^*}{r^*}E_t(\tilde{i}_{t+1} - \tilde{i}_t + z_{t+1}^z + z_{t+1}^i), \tag{A.4}$$

where $\tilde{r}_t^k$ is the real rental rate of capital and $\tilde{q}_t$ is Tobin's q.

The capital accumulation process is

$$\tilde{k}_t = \frac{1-\delta}{z^*}(\tilde{k}_{t-1} - z_t^z) - \frac{r^{k*}}{z^*}\tilde{u}_t + \left(1 - \frac{1-\delta}{z^*}\right)\tilde{i}_t, \tag{A.5}$$

where $r^{k*} = (z^*)^\sigma/\beta - 1 + \delta$.

The dynamic equation for Tobin's q is

$$\tilde{q}_t = E_t\tilde{\lambda}_{t+1} - \tilde{\lambda}_t - \sigma E_t z_{t+1}^z + \frac{1}{r^*}\left(r^{k*}E_t\tilde{r}_{t+1}^k + (1-\delta)E_t\tilde{q}_{t+1}\right). \tag{A.6}$$

Note that although the inflation misperception parameter ψ_f does not explicitly appear in the linearized equation for $\tilde{q}_t$, it implicitly distorts the dynamics of Tobin's q through the marginal utility $\tilde{\lambda}_t$. From the Euler equation, the term $E_t\tilde{\lambda}_{t+1} - \tilde{\lambda}_t - \sigma E_t z_{t+1}^z$ is equivalent to $(1 - \psi_f)E_t\tilde{\pi}_{t+1} - \tilde{r}_t^n$. This implies that households discount future returns on capital using a subjective real interest rate affected by misperceptions of future inflation.

The log-linearized wage-setting condition can be written in terms of the effective wage gap as

$$\tilde{w}_t - \tilde{w}_{t-1} + \tilde{\pi}_t - \gamma_w \tilde{\pi}_{t-1} + z_t^z = \frac{z^*}{r^*}\left(E_t \tilde{w}_{t+1} - \tilde{w}_t + E_t \tilde{\pi}_{t+1} - \gamma_w \tilde{\pi}_t + E_t z_{t+1}^z\right) + \frac{(1-\xi_w)(1-\xi_w z^*/r^*)\lambda_w}{\xi_w(\lambda_w + \chi(1+\lambda_w))}(\tilde{m}_t - \tilde{w}_t) + z_t^w. \tag{A.7}$$

The effective marginal labor supply cost is

$$\mathcal{M}_t = MRS_t \left(\frac{\Pi_t}{\Pi^*}\right)^{-\psi_c}. \tag{A.8}$$

Since

$$MRS_t = \frac{\exp(z_t^b)\exp(z_t^\ell)\ell_t^\chi}{\lambda_t}, \tag{A.9}$$

where $\lambda_t = \Lambda_t / Z_t^{1-\sigma}$, the log-linearized effective marginal labor supply cost is

$$\tilde{m}_t = \chi \tilde{\ell}_t - \tilde{\lambda}_t + z_t^b + z_t^\ell - \psi_c \tilde{\pi}_t. \tag{A.10}$$

The labor supply shock z_t^ℓ is not separately identified from the wage markup shock and is absorbed into the composite wage shock z_t^w. Then, the wage Phillips curve is

$$\tilde{w}_t - \tilde{w}_{t-1} + \tilde{\pi}_t - \gamma_w \tilde{\pi}_{t-1} + z_t^z = \frac{z^*}{r^*}\left(E_t \tilde{w}_{t+1} - \tilde{w}_t + E_t \tilde{\pi}_{t+1} - \gamma_w \tilde{\pi}_t + E_t z_{t+1}^z\right) + \frac{(1-\xi_w)(1-\xi_w z^*/r^*)\lambda_w}{\xi_w(\lambda_w + \chi(1+\lambda_w))}\left(\chi \tilde{\ell}_t - \tilde{\lambda}_t - \tilde{w}_t + z_t^b - \psi_c \tilde{\pi}_t\right) + z_t^w. \tag{A.11}$$

2. Firms and Market Clearing

Real marginal cost $\widetilde{mc}_t$ and the cost minimization condition are

$$\widetilde{mc}_t = (1-\alpha)\tilde{w}_t + \alpha \tilde{r}_t^k, \tag{A.12}$$

$$\tilde{w}_t - \tilde{r}_t^k = \tilde{u}_t + \tilde{k}_{t-1} - \tilde{\ell}_t - z_t^z. \tag{A.13}$$

The production function is

$$\tilde{y}_t = (1+\phi)\left[(1-\alpha)\tilde{\ell}_t + \alpha(\tilde{u}_t + \tilde{k}_{t-1} - z_t^z)\right]. \tag{A.14}$$

The price Phillips curve is

$$\tilde{\pi}_t - \gamma_p \tilde{\pi}_{t-1} = \frac{z^*}{r^*}(E_t \tilde{\pi}_{t+1} - \gamma_p \tilde{\pi}_t) + \frac{(1-\xi_p)(1-\xi_p z^*/r^*)}{\xi_p}\widetilde{mc}_t + z_t^p. \tag{A.15}$$

The resource constraint and the monetary policy rule are

$$\tilde{y}_t = c_y^* \tilde{c}_t + i_y^* \tilde{i}_t + g_y^* z_t^g, \tag{A.16}$$

$$\tilde{r}_t^n = \phi_r \tilde{r}_{t-1}^n + (1-\phi_r)\left[\phi_\pi \frac{\tilde{\pi}_t + \tilde{\pi}_{t-1} + \tilde{\pi}_{t-2} + \tilde{\pi}_{t-3}}{4} + \phi_y(\tilde{y}_t - \tilde{y}_t^{pot})\right] + z_t^r. \tag{A.17}$$

The potential output is given by $\tilde{y}_t^{pot} = -\alpha(1+\phi)z_t^z$.

Appendix C Robustness Check: Krippner Shadow Rate

The main text uses the shadow rate estimated by Nakajima (2025). This appendix examines whether the posterior estimates and variance decompositions are robust to using the alternative shadow rate measures proposed by Krippner (2013, 2015). The prior distributions are the same as those reported in Table 1, except for the prior mean of r^*, which is set to -0.382 to match the sample mean implied by the Krippner shadow rate series.

Table A.1 reports the posterior estimates using Krippner shadow rate. The estimated money illusion parameters are close to those in the baseline case, with $\psi_c = 0.52$ and $\psi_f = 0.80$. As in the baseline results, both current and future inflation misperceptions are quantitatively important, with future inflation misperception playing the larger role.

Most structural parameters are similar across the MI and RE models. The main differences again appear in the parameters related to the Euler equation and nominal rigidities. The relative risk aversion parameter is lower under MI, with $\sigma = 0.87$ under MI and $\sigma = 1.02$ under RE. The estimated standard deviation of preference shocks also declines from $\sigma_b = 7.19$ under RE to $\sigma_b = 5.19$ under MI. These differences reflect the role of future inflation misperception in the intertemporal demand channel. Nominal rigidities are also lower under MI. The estimated degree of price stickiness declines from $\xi_p = 0.76$ under RE to $\xi_p = 0.71$ under MI, while wage stickiness declines from $\xi_w = 0.64$ to $\xi_w = 0.58$. As in the baseline case, current inflation misperception reduces the degree of nominal rigidity required to account for inflation persistence.

Table A.2 reports the variance decompositions. The qualitative decomposition patterns are similar to those in the baseline case. Technology shocks remain the largest source of output fluctuations, while preference shocks and investment shocks account for most consumption and investment fluctuations, respectively. In the MI–RE comparison, price markup shocks play a larger role in labor fluctuations under MI, while technology shocks play a smaller role. For the nominal interest rate, the contributions of investment, price markup, and monetary policy shocks are larger under MI, whereas the contribution of technology shocks is smaller.

The counterfactual comparison shows that the direct propagation effects of inflation misperception are more limited. Holding all other parameters fixed, the main differences appear in inflation and nominal interest rate fluctuations. For inflation, the contribution of price markup shocks remains around one half in both cases, while the contributions of wage markup and technology shocks change modestly. For the nominal interest rate, the contributions of investment and monetary policy shocks rise under MI, whereas the contributions of price markup and technology shocks decline.

The results using the Krippner shadow rate therefore support the robustness of the baseline findings.

Table A.1: Posterior Estimates: Krippner Shadow Rate

	Money Illusion		Rational Expectations	
	Mean	90% credible interval	Mean	90% credible interval
ψ_c	0.5191	[0.1922, 0.8486]	–	–
ψ_f	0.7965	[0.6298, 0.9722]	–	–
σ	0.8661	[0.4754, 1.2562]	1.0230	[0.5046, 1.5294]
θ	0.8384	[0.7708, 0.9099]	0.8495	[0.7763, 0.9360]
χ	2.1741	[1.0996, 3.2062]	2.2719	[1.0932, 3.3792]
$1/\zeta$	6.3282	[3.5521, 9.0185]	6.5413	[3.7025, 9.1686]
μ	1.1019	[0.3934, 1.7845]	0.8799	[0.1968, 1.5825]
ϕ	0.0693	[0.0506, 0.0876]	0.0684	[0.0498, 0.0866]
γ_w	0.6048	[0.2736, 0.9673]	0.5272	[0.1557, 0.9127]
ξ_w	0.5797	[0.4980, 0.6613]	0.6383	[0.5433, 0.7343]
γ_p	0.1075	[0.0012, 0.2143]	0.0885	[0.0005, 0.1783]
ξ_p	0.7072	[0.6556, 0.7567]	0.7642	[0.6856, 0.8510]
λ^p	0.2854	[0.1606, 0.4072]	0.2957	[0.1667, 0.4262]
z^*	-0.0577	[-0.1263, 0.0136]	-0.0273	[-0.1012, 0.0473]
$e\bar{l}l$	-0.0019	[-0.0842, 0.0803]	-0.0021	[-0.0841, 0.0784]
π^*	0.0206	[-0.0568, 0.0973]	0.0150	[-0.0651, 0.0922]
r^*	-0.3132	[-0.3817, -0.2424]	-0.3139	[-0.3833, -0.2453]
ϕ_r	0.7859	[0.7281, 0.8459]	0.7785	[0.7216, 0.8366]
ϕ_π	1.7090	[1.5453, 1.8703]	1.7414	[1.5748, 1.9110]
ϕ_y	0.0763	[0.0392, 0.1133]	0.0763	[0.0297, 0.1215]
ρ_z	0.1074	[0.0234, 0.1857]	0.1268	[0.0294, 0.2175]
ρ_b	0.1939	[0.0317, 0.3485]	0.1990	[0.0242, 0.3602]
ρ_i	0.4973	[0.3643, 0.6350]	0.4673	[0.3098, 0.6280]
ρ_g	0.9002	[0.8526, 0.9492]	0.9010	[0.8523, 0.9519]
ρ_w	0.0973	[0.0124, 0.1804]	0.1012	[0.0133, 0.1866]
ρ_p	0.9272	[0.8734, 0.9843]	0.7896	[0.5789, 0.9716]
ρ_r	0.5559	[0.4262, 0.6887]	0.5538	[0.4349, 0.6754]
σ_z	1.8114	[1.5776, 2.0410]	1.8187	[1.5802, 2.0592]
σ_b	5.1913	[2.5515, 7.7480]	7.1936	[2.3363, 11.7390]
σ_i	3.2210	[2.5809, 3.8238]	3.0834	[2.4539, 3.7098]
σ_g	2.7903	[2.4338, 3.1396]	2.7844	[2.4271, 3.1324]
σ_w	1.0307	[0.8792, 1.1770]	1.0061	[0.8559, 1.1525]
σ_p	0.1774	[0.1270, 0.2254]	0.1694	[0.1221, 0.2135]
σ_r	0.1135	[0.0991, 0.1274]	0.1148	[0.1003, 0.1289]

Table A.2: Variance Decompositions: Krippner Shadow Rate

Money Illusion ($\psi_c = 0.5191$, $\psi_f = 0.7965$)							
	z^b	z^i	z^g	z^w	z^p	z^r	z^z
$100\Delta \log Y_t$	3.39	28.63	22.05	0.80	5.95	0.42	38.75
$100\Delta \log C_t$	81.22	1.87	0.50	0.29	1.45	0.17	14.49
$100\Delta \log I_t$	0.44	69.55	1.77	1.34	10.62	0.69	15.59
$100\Delta \log W_t$	0.74	0.43	0.07	46.05	22.36	0.11	30.24
$100 \log \ell_t$	1.02	24.66	8.55	7.83	33.07	1.66	23.22
$100\Delta \log P_t$	0.79	3.23	0.80	11.80	49.95	6.88	26.55
$100 \log R_t^n$	1.42	18.38	3.38	6.69	27.62	16.83	25.68
Rational Expectations							
	z^b	z^i	z^g	z^w	z^p	z^r	z^z
$100\Delta \log Y_t$	4.07	29.30	22.31	0.79	3.36	0.74	39.43
$100\Delta \log C_t$	85.63	0.78	0.29	0.16	0.57	0.16	12.42
$100\Delta \log I_t$	0.52	71.14	1.74	1.46	6.33	1.34	17.47
$100\Delta \log W_t$	0.77	0.36	0.06	56.42	15.50	0.11	26.78
$100 \log \ell_t$	1.64	28.67	10.18	9.57	13.64	3.12	33.17
$100\Delta \log P_t$	0.65	2.51	0.92	9.40	49.93	4.72	31.87
$100 \log R_t^n$	1.33	12.87	3.32	5.62	24.75	13.90	38.21
Counterfactual Case ($\psi_c = \psi_f = 0$)							
	z^b	z^i	z^g	z^w	z^p	z^r	z^z
$100\Delta \log Y_t$	3.43	29.17	22.01	0.76	5.27	0.61	38.75
$100\Delta \log C_t$	81.32	1.79	0.49	0.27	1.22	0.25	14.66
$100\Delta \log I_t$	0.43	70.17	1.71	1.28	9.44	0.99	15.98
$100\Delta \log W_t$	0.75	0.49	0.08	46.52	21.98	0.16	30.02
$100 \log \ell_t$	1.09	25.66	8.74	7.62	30.75	2.31	23.83
$100\Delta \log P_t$	0.67	2.84	0.89	9.54	50.50	6.87	28.69
$100 \log R_t^n$	1.14	13.84	3.14	5.16	31.76	11.31	33.64

Note: Infinite-horizon forecast error variance decompositions are reported. The counterfactual case is obtained by setting $\psi_c = \psi_f = 0$ while holding all other parameters at their posterior means under the money illusion specification.